OPTIMAL INVENTORY CONTROL POLICY OF A HYBRID MANUFACTURING – REMANUFACTURING SYSTEM USING A FUZZY LINEAR PROGRAMMING APPROACH

Kittiphan Nuamchit^{1*} and Navee Chiadamrong¹

¹*Sirindhorn International Institute of Technology, Thammasat University, Thailand*

Abstract: Remanufacturing is the process of bringing used products back to like-new products. In this study, production planning of inventory control policies in remanufacturing of used products with different prioritizations (remanufacturing vs manufacturing) are investigated. For this hybrid manufacturing/remanufacturing production planning problem, imprecise on customer demand, related operating costs, and amount of returned used components are considered to be the uncertainty parameters. To defuzzilize imprecise data, the Fuzzy Linear Programming (FLP) with the weighted average method is applied to the problem. The proposed approach maximizes the most likely value of the profit, minimizes the risk of obtaining a lower profit, and maximizes the possibility of obtaining a higher profit for each production planning policy at the same time. The result shows that the Priority-To-Remanufacturing (PTR) policy shows a higher profit and a higher satisfaction value of the linear membership function of each objective function than the policy of giving more Priority-To-Manufacturing (PTM), while FLP can help decision makers to be well aware of the risk and effect of uncertainty towards its plans.

Keywords: Hybrid manufacturing/remanufacturing, Fuzzy theory, Optimization

Introduction

In the remanufacturing processes, the uncertainty of quantity, quality and timing of returning used products dramatically increase the difficulty in the production planning and inventory management. Planners need to make decisions on how many used products or how many new components are required and especially when such orders should be placed. Moreover, customer demand and related operating costs are also subject to uncertainty. Customer demand may normally be varied with time due to errors in forecasting, or lack of information while certain operating costs are difficult to be fixed in advance. To handle such issues, a certain approach can be used to optimize such problems under the uncertainty. Instead of using a typical linear programming where the uncertainty is disregarded, Fuzzy Linear Programming (FLP) is one of the approaches that can be used to incorporate fuzzy data and optimize this problem. Our main contribution is to introduce the Fuzzy Linear Programming model (FLP) to solve the production planning and inventory control in a hybrid manufacturing/remanufacturing system and recommend the best policy between the Priority-To-Remanufacturing (PTR) and Priority-To-Manufacturing (PTM) under the uncertainty environment.

Literature Review

Actual production planning and control problems, input data or related parameters are imprecise/fuzzy, such as various costs of operations, customer demand, amount of returned products. Zadeh (1977) presented the theory of possibility, which is related to the theory of fuzzy sets by applying the concept of a possibility distribution as a fuzzy restriction, acting as an elastic constraint. To incorporate the uncertainty, Zimmermann (1978) then

introduced the fuzzy set theory into traditional linear programming problems. His study considered linear programming problems with fuzzy goal and constraints. Zadeh (1977) presented the theory of possibility, which is related to the theory of fuzzy sets by applying the concept of a possibility distribution as a fuzzy restriction, acting as an elastic constraint. Buckley (1989) presented a procedure of Possibilistic Linear Problem (PLP) in standard form with no equality constraints.

Our study applies Fuzzy Linear Programming approach for solving the production planning and inventory control policy for a hybrid manufacturing/remanufacturing with imprecise forecast demand, related operating costs, and amount of returned component. The proposed approach attempts to maximize the most likely value of the imprecise total profits, minimize the risk of obtaining a lower profit, and maximize the possibility of obtaining higher profits of each policy simultaneously. This approach can provide realistic results, which are better than a typical deterministic approach as it can handle fuzzy data and recommend a range of possible optimal objective values. This helps decision makers to be aware of possible outcomes in all scenarios from most likely, optimistic, and pessimistic cases. So, they can well prepare themselves in advance from such scenarios that might occur.

Methodology

Two optimization solving methods are selected in order to evaluate their performances on a hybrid manufacturing/remanufacturing system under an uncertainty environment. The system is modeled with both deterministic and stochastic conditions, using Mixed-Integer Linear Programming (MILP) and Fuzzy Linear Programming (FLP) respectively.

Deterministic condition using MILP

The system is modelled and optimized profit using MILP. There is no uncertainty in this situation. As a result, the ideal solution from this method will be used as a benchmark for the comparison.

Stochastic condition using FLP

FLP method can handle the problem with uncertainty. Uncertainty environment in this case contains uncertainty of customer demand in each week, related operating costs, and amount of returned components arriving in each week. FLP would aim to convert these uncertainty/fuzzy data to crisp ones. There are four main steps to solve FLP:

Step 1. Model the uncertainty data with triangular possibility distribution

The triangular distribution of an uncertainty coefficient $\tilde{x}=(x^P, x^M, x^O)$ is employed to represent uncertain data. In general, we can construct the triangular distribution based on the three important data as follows.

- The most pessimistic value (x^P) that has the lowest possibilistic degree to the set of available values (possibilistic degree = 0)
- The most likely value (x^M) that has the highest possibilistic degree to the set of available values (possibilistic degree = 1)
- The most optimistic value (x^O) that has the lowest possibilistic degree to the set of available values (possibilistic degree = 0)

Step 2. Develop three new crisp objective functions of multi objective linear programming (MOLP)

The imprecise objective function in this model has a triangular distribution z_3 . This imprecise objective function is defined by three important points $(z^P, 0)$, $(z^M, 1)$, and $(z^O, 0)$. The imprecise objective can be maximized by pushing three important points to the right. Because of the vertical coordinate of important points are fixed at either 1 or 0, so three horizontal coordinates are only considered. Solving the imprecise objective requires maximizing z^P , z^M , and z^O , simultaneously. Rather simultaneously maximizing z^P , z^M , and z^O the proposed approach maximizes the profit z^M , minimizes the range of the profit of $(z^M - z^P)$ and, maximizes the range of the profit of $(z^O - z^M)$. This proposed approach involves maximizing the most likely value of the imprecise total profit (z^M), minimizing the risk of obtaining a lower profit ($z^M - z^P$), and maximizing the possibility of obtaining a higher profit ($z^O - z^M$). Figure 1 presents the strategy for maximizing the imprecise objective functions.

Figure 1, The strategy for maximizing the imprecise objective function.

As presented in Figure 1, the possibility distribution $\tilde{A}$ is preferred to the possibility distribution $\tilde{B}$. The results for three new crisp objective functions are presented as follows:

$$\text{Max } z_1 = z^M \quad (1)$$

$$\text{Min } z_2 = (z^M - z^P) \quad (2)$$

$$\text{Max } z_3 = (z^O - z^M) \quad (3)$$

Constraint (1) to Constraint (3) are equivalent to simultaneously maximizing the most likely value of total profit, minimizing the risk of obtaining a lower profit (area A of the possibility distribution in Figure. 1), and maximizing the possibility of obtaining a higher profit (area B of the possibility distribution in Figure. 1).

Step 3. Convert uncertainty constraints into crisp constraints using the weighted average method

We consider the situation when a number of returned products, related operating costs, and customer demand are uncertain and have the triangular distribution with the most likely and least possible values. The problem is to obtain crisp numbers for the uncertainty of a number of returned products, related operating costs, and customer demand by applying the weighted average method to convert them into crisp values, where w_1 , w_2 , and w_3 denote the weight of the pessimistic, most likely, and optimistic cases respectively. The weight w_1 , w_2 , and w_3 can be determined by the experience of decision makers and $w_1 + w_2 + w_3 = 1$.

Step 4. Specify the linear membership functions for three new objective functions, and then convert the auxiliary MOLP problem into an equivalent linear programming model using FLP

- Find the lower bound and the upper bound of each objective.

To convert the auxiliary MOLP problem into an equivalent single-goal linear programming problem, the fuzzy decision method from Bellman & Zadeh (1970), and Zimmermann (1978)'s fuzzy programming method are employed. The Negative Ideal Solutions (NIS) and Positive Ideal Solution (PIS) of three objective functions from step two are required by developing three new crisp objective functions of the multi objective linear programming can be stated as follows:

$$z_1^{PIS} = \max z^M, z_1^{NIS} = \min z_3^{NIS} \quad (4)$$

$$z_2^{PIS} = \min (z^M - z^P), z_2^{NIS} = \max (z^M - z^P) \quad (5)$$

$$z_3^{PIS} = \max (z^O - z^M), z_3^{NIS} = \min (z^O - z^M) \quad (6)$$

- Find the corresponding linear membership function of each objective function

The corresponding linear membership function for each objective function is defined by

$$f1(z1) = \begin{cases} 1, z_1 < z_1^{PIS}, \\ \frac{z_1 - z_1^{NIS}}{z_1^{PIS} - z_1^{NIS}}, z_1^{NIS} \leq z_1 < z_1^{PIS}, \\ 0, z_1 > z_1^{NIS}, \end{cases} \quad (7)$$

$$f2(z2) = \begin{cases} 1, z_2 < z_2^{PIS}, \\ \frac{z_2^{NIS} - z_2}{z_2^{NIS} - z_2^{PIS}}, z_2^{PIS} \leq z_2 < z_2^{NIS}, \\ 0, z_2 > z_2^{NIS}, \end{cases} \quad (8)$$

$$f3(z3) = \begin{cases} 1, z_3 < z_3^{PIS}, \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}}, z_3^{NIS} \leq z_3 < z_3^{PIS}, \\ 0, z_3 > z_3^{NIS}, \end{cases} \quad (9)$$

Each linear membership function can be obtained by using a case study to specify imprecise objective value in an interval (0-1).

- Find the maximum overall satisfaction

The fuzzy decision method from Bellman & Zadeh (1970), and Zimmermann (1978) is used to formulate with a single goal linear programming model, which is minimax of the satisfaction values from the three above mentioned objective functions.

Max λ

Subject to

$$\lambda \leq f_i(z_i), i = 1, 2, 3$$

$$0 \leq \lambda \leq 1$$

i = number of linear membership function of each objective.

Case Study

For this case study of production planning and inventory control optimization in a hybrid manufacturing/remanufacturing under an uncertainty environment, Figure 2 shows a general flow diagram of the system. This system requires two types of components for production: New components and Returned components. Returned components use the reorder cycle policy to control its inventory as returned components arrive in batch at the beginning of each week. A certain percentage of returned components need to be disposed to prevent surplus inventory. Accepted returned components are then stored in a Returned Component Inventory (RCI), waiting for the remanufacturing processes. It is assumed that the remanufacturing time of the returned components is less than the manufacturing time of new components according to the quality of returned components. There is also a New Component Inventory (NCI), which is also reviewed every week. Ordering lead time for the new components is constant at one week. After arrival, these new components are kept in the NCI before they are used in the manufacturing processes. One week is also required for a manufacturing week. Finished products are then stored in a Finished Product Inventory (FPI), awaiting for the customer demand.

Figure.2: General flowchart of a hybrid manufacturing/remanufacturing system

Customer demand arrives every week. Lost sales occur when there are not enough finished products. Otherwise, customers pick up the finished products from the FPI. Finished products, quality from remanufacturing and manufacturing is considered to be the same. When the inventory level of the FPI is reduced, the system triggers the production. The purpose is to restore back the inventory level of the FPI. An initial inventory quantity is equal to the FPI target inventory level, which is one of the decision variables. Two inventory control policies (Priority-To-Remanufacturing vs Priority-To-Manufacturing) are imposed here to decide which process (remanufacturing vs manufacturing) has a higher priority. Operational details of each policy can be presented as follows:

Priority-To-Remanufacturing (PTR), with the priority-to-remanufacturing, returned components are given a higher priority than new components to be selected for production unless they are not available. Manufacturing production is provoked only when there are not enough returned components in RCI. Controlled decision variables in this policy include Disposal Rate (disR), Target Inventory Level of NCI (TinvN), and Target Inventory Level of FPI (TinvF). Certain returned components are disposed according to disR. New components are ordered up to TinvN in every review cycle (a week). The initial inventory level of NCI is equal to TinvN. When finished products are sold to customers, upstream components are pulled to replenish the taken products by filling FPI up to TinvF where the initial inventory level of FPI is also equal to TinvF. However, Priority-To-Manufacturing (PTM), is opposite to PTR while new components have a higher priority over returned components. Returned components are used for production only when there is a shortage of new components. Inventories are controlled by the decision variables, similar to PTR. The system is investigated for one year (or 50 weeks). With 5 days a week and 8 hours a day, there are 120,000 minutes a year. Maximum levels of the target inventory for new component inventory and finished product inventory depend on the maximum of

production capacity. Overall customer service level in this case study is set at least at 85% while the returned component ratio is set at 0.6: This is the ratio of returned component arrival per total customer demand. Both returned component arrival and total customer demand fluctuate under the normal distribution in an uncertain environment. Mean and standard deviation for total customer demand is 100 and 20 units per week, respectively. The symbol “~” refers to ambiguous data that are determined to be fuzzy in this study, and the supplier has ability to provide new component parts not more than 60 units per week.

Model

The mathematical model presenting in this case study is formulated as the linear programming model. Notations and the analytical model formulation are presented below where t refers to time in weeks, ranging from 1 to 50.

Parameters: Production planning parameters (units)

- ReA_t = Quantity of returned component arrival in week t
- $dispose_t$ = Quantity of returned component disposal in week t
- RO_t = Quantity of returned components accepted to inventory in week t
- RCI_t = Quantity of ending inventory in the Returned Component Inventory (RCI) in week t
- RI_t = Quantity of returned components sent to remanufacturing in week t
- $R2_t$ = Quantity of finished products from remanufacturing in week t
- $FPIr_t$ = Quantity of ending inventory in Finished Product Inventory (FPI) from remanufacturing in week t
- $R3_t$ = Quantity of finished products from remanufacturing sent to customer in week t
- $order_t$ = Quantity of new components ordered in week t
- NCI_t = Quantity of ending inventory in the New Component Inventory (NCI) in week t
- $M1_t$ = Quantity of new components sent to manufacturing in week t
- $M2_t$ = Quantity of finished products from manufacturing in week t
- $FPI m_t$ = Quantity of ending inventory in Finished Product Inventory (FPI) from manufacturing in week t
- $M3_t$ = Quantity of finished products from manufacturing sent to customer in week t
- FPI_t = Quantity of ending inventory in Finished Product Inventory (FPI) in week t
- D_t = Quantity of customer demand in week t
- LS_t = Quantity of lost sales in week t
- PC_t = Quantity of machine capacity to produces a product in week t

Cost parameters (\$)

- RC = Total returned component disposal cost
- LS = Total lost sales cost
- RMM = Total remanufacturing and manufacturing cost
- NC = Total new component cost
- RP = Total returned component cost
- CH = Total component holding cost
- FH = Total finished product holding cost
- TC = Total costs
- $Revenue$ = Total income from selling finished products
- $Profit$ = Total income after deducting total costs

Decision Variables

$TinvN$ = Quantity of target inventory level in NCI (units)

$TinvF$ = Quantity of target inventory level in FPI (units)

$disR_t$ = Disposal rate in week t (%)

Objective Function:

$$\text{Maximize} \quad \text{Profit} = \text{Revenue} - \text{TC} \quad (10)$$

subject to

inventory balance constraints:

$$ReA_t = \text{dispose}_t + R0_t \quad \text{for all } t \quad (11)$$

$$R1_t = R2_t \quad \text{for all } t \quad (12)$$

$$M2_t = \begin{cases} 0 & \text{where } t=1 \\ M1_{t-1} & \text{where } t=2 \text{ to where } t=50 \end{cases} \quad (13)$$

$$R3_t + M3_t + LS_t = D_t \quad \text{for all } t \quad (14)$$

$$FPI_t = FPIr_t + FPIIm_t \quad \text{for all } t \quad (15)$$

$$PC_t \geq R1_t + M1_t \quad \text{for all } t \quad (16)$$

$$RCI_t = \begin{cases} R0_t - R1_t & \text{where } t=1 \\ RCI_{t-1} + R0_t - R1_t & \text{where } t=2 \text{ to } t=50 \end{cases} \quad (17)$$

$$NCI_t = \begin{cases} TinvN - M1_t & \text{where } t=1 \\ NCI_{t-1} + \text{order}_{t-1} - M1_t & \text{where } t=2 \text{ to } t=50 \end{cases} \quad (18)$$

$$FPIr_t = \begin{cases} R2_t - R3_t & \text{where } t=1 \\ FPIr_{t-1} + R2_t - R3_t & \text{where } t=2 \text{ to } t=50 \end{cases} \quad (19)$$

$$FPIIm_t = \begin{cases} TinvF - M3_t & \text{where } t=1 \\ FPIIm_{t-1} + M2_t - M3_t & \text{where } t=2 \text{ to } t=50 \end{cases} \quad (20)$$

Where ReA_t , and D_t are uncertainty coefficients with the triangular distribution. The objective function (constraint (10)) is to maximize the profit. Constraints (11) - (12) are inventory balance constraints. In Constraint (11), some of returned components are disposed and the rest is sent to Returned Component Inventory (RCI). Constraint (12) ensures that the quantity of returned components sent to remanufacturing processes are the same quality as finished products coming out of the remanufacturing processes within the same week and the remanufacturing time is negligible. Constraint (13) states that the manufacturing lead time is one week. In Constraint (14), customer demand is satisfied by the finished products, otherwise the lost sales occur. In Constraint (15), total units of finished products in remanufacturing and manufacturing are combined in FPI. Constraint (16) describes the constraint of production capacity for remanufacturing and manufacturing in which the amount of components sent to the manufacturing process must less than or equal to the production capacity. Constraints (17) to (20) describe the constraints for returned components, new components, and finished products for remanufacturing and manufacturing. Amount of ending inventory is equal to the amount of previous ending inventory week plus incoming inventory and minus outgoing inventory. In first week, the

amount of ending inventory is equal to the amount of incoming inventory minus the amount of outgoing inventory.

PTR constraints:

$$RCI_t = \begin{cases} 0 & \text{if } D_t \geq R0_t & \text{where } t = 1 \\ 0 & \text{if } D_t \geq R0_t + RCI_{t-1} & \text{where } t = 2 \text{ to } t = 50 \\ \text{any integers} & \text{otherwise} & \text{for all } t \end{cases} \quad (21)$$

$$R0_t = \begin{cases} D_t & \text{if } D_t < ReA_t & \text{for all } t \\ ReA_t & \text{if } ReA_t < D_t & \text{for all } t \end{cases} \quad (22)$$

$$RI_t = \begin{cases} D_t & \text{if } D_t < R0_t + RCI_{t-1} & \text{for all } t \\ \text{any integers} & \text{otherwise} & \text{for all } t \end{cases} \quad (23)$$

Where ReA_t , and D_t are uncertainty coefficients with the triangular distribution. Constraints (21) - (23) describe the policies of the priority-to-remanufacturing, all returned components in RCI are sent to remanufacturing when the customer demand is greater than the amount of incoming returned components plus the amount of the previous week's ending inventory. Otherwise, the number of returned components sent to remanufacturing are equal to the customer demand.

PTM constraints:

$$NCI_t = \begin{cases} 0 & \text{if } D_t \geq TinvN & \text{where } t = 1 \\ order_{t-1} & \text{if } D_t \geq NCI_{t-1} & \text{where } t = 2 \text{ to } t = 50 \\ \text{any integers} & \text{otherwise} & \text{for all } t \end{cases} \quad (24)$$

$$MI_t = \begin{cases} D_t & \text{if } D_t < NCI_{t-1} & \text{for all } t \\ \text{any integers} & \text{otherwise} & \text{for all } t \end{cases} \quad (25)$$

Where D_t is uncertainty coefficients with the triangular distribution. Constraint (24) and Constraint (25) describe the policies of the priority-to-manufacturing, Where the number of new components in NCI are sent to manufacturing equal to the customer demand when it is less than the number of new components. Otherwise, all of returned components are sent to the manufacturing. The amount of the ending inventory in NCI is equal to the amount of incoming orders from the previous week when the customer demand is greater than the amount of previous period's ending inventory.

Economic constraints:

$$RD = \sum_t RD_u * dispose_t \quad \text{for all } t \quad (26)$$

$$LS = \sum_t LS_u * LS_t \quad \text{for all } t \quad (27)$$

$$RMM = \sum_t RM_u * RI_t + \sum_t MM_u * MI_t + MM_u * TinvF) \quad \text{for all } t \quad (28)$$

$$NC = \sum_t NC_u * (order_t + initial\ NCI + TinvF) \quad \text{for all } t \quad (29)$$

$$RP = \sum_t RP_u * R0_t \quad \text{for all } t \quad (30)$$

$$CH = \left(\frac{RH_{-y}}{50} * (R0_1 + RCI_1) \right) / 2 + \sum_{t=2}^{50} \left(\frac{RH_{-y}}{50} * (R0_t + RCI_{t-1} + RCI_t) \right) / 2 \quad (31)$$

$$+ \left(\frac{NH_{-y}}{50} * (initial\ NCI + NCI_1) \right) / 2 + \sum_{t=2}^{50} \left(\frac{NH_{-y}}{50} * (order_{t-1} + NCI_{t-1} + NCI_t) \right) / 2$$

$$FH = \left(\frac{FHR_{-y}}{50} * (R2_1 + FPIr_1) \right) / 2 + \sum_{t=2}^{50} \left(\frac{FHR_{-y}}{50} * (R2_t + FPIr_{t-1} + FPIr_t) \right) / 2 \quad (32)$$

$$+ \left(\frac{FHM_{-y}}{50} * (M2_1 + TinvF + FPIIm_1) \right) / 2 + \sum_{t=2}^{50} \left(\frac{FHM_{-y}}{50} * (M2_t + FPIIm_{t-1} + FPIIm_t) \right) / 2$$

$$TC = RD + LS + RMM + NC + RP + CH + FH \quad (33)$$

$$Revenue = \sum_t Price_{-u} * (R3_t + M3_t) \text{ for all } t \quad (34)$$

$$Profit = Revenue - Total\ costs \quad (35)$$

Where LS , LS_t , RM_{-u} , MM_{-u} , NC_{-u} , RP_{-u} , NH_{-y} , RH_{-y} , FHR_{-y} , FHM_{-y} , and $Price_{-u}$ are uncertainty coefficients with the triangular distribution. Constraints (26) – (32) describe cost parameters to calculation total costs (TC). TC consists of Returned Component Disposal Cost (RD), Lost Sales Cost (LS), Remanufacturing and Manufacturing Cost (RMM), New Component Cost (NC), Returned Component Cost (RP), Component Holding Cost (CH), and Finished Product Holding Cost (FH). All holding costs are calculated based on average level of inventory. Constraint (35) states that the profit is calculated by the revenue (Constraint (34)) minusing total costs (Constraint (33)).

Fuzzy Linear Programming (FLP)

We develop the fuzzy linear programming method to solve this problem under the uncertainty by developing three new crisp objective functions of the multi-objective linear programming (MOLP) to replace Constraint (1) to Constraint (3)

$$Max\ z_1 = Profit^M = Revenue^M - TC^M \quad (36)$$

$$Max\ z_2 = Profit^{P-M} = Revenue^{P-M} - TC^{P-M} \quad (37)$$

$$Max\ z_3 = Profit^{M-O} = Revenue^{M-O} - TC^{M-O} \quad (38)$$

Result and Discussion

Tables 1-2 show the results of the sensitivity analysis for both PTR and PTM policies.

Table 1. Result of the sensitivity analysis of PTR

	Scenario I			Scenario II			Scenario III			Scenario IV		
	P	M	O	P	M	O	P	M	O	P	M	O
	33%	33%	33%	80%	10%	10%	10%	80%	10%	10%	10%	80%
Overall satisfaction	53.46%			38.34%			53.95%			40.08%		
Z ₁ (\$)	109,350			66,774			114,280			99,616		
Z ₂ (\$)	68,630			66,774			68,479			72,778		
Z ₃ (\$)	68,241			63,701			68,388			71,421		
Maximum the most likely value of profit (\$)	109,350			66,774			114,280			99,616		

Minimize the risk of obtaining lower profit (\$)	40,720	0	45,801	26,838
Maximize of the possibility of obtaining a higher profit (\$)	177,591	130,475	182,668	171,037

It is found that the overall satisfaction is 53.46%, and the outcomes of maximum most likely value of profit, minimum possibility of obtaining a lower profit, and maximum of the risk of obtaining a higher profit are \$109,350, \$40,720, and \$177,591, respectively. The result of this sensitivity analysis shows that Scenario III, which is set the weight for the most likely value of 80% and the weight of optimistic and pessimistic values of 10%, gives the highest overall satisfaction of 53.95%, and gives the highest possible value of the most likely profit, obtains the highest possibility of a higher profit and the highest profit from the risk of obtaining a lower profit. However, the profit generated from the scenario IV, which is supposed to be the highest as it is the most optimistic case, cannot be the highest since the amount of optimistic demand is too high to be able to produce by the manufacturing process. As a result, a large amount of shortage needs to be paid.

Table 2. Result of the sensitivity analysis of PTM

	Scenario I			Scenario II			Scenario III			Scenario IV		
	P	M	O	P	M	O	P	M	O	P	M	O
	33%	33%	33%	80%	10%	10%	10%	80%	10%	10%	10%	80%
Overall satisfaction	53.00%			39.13%			53.40%			42.40%		
Z ₁ (\$)	84,866			66,610			88,882			73,055		
Z ₂ (\$)	68,586			66,610			68,450			72,117		
Z ₃ (\$)	67,968			63,562			68,098			69,531		
Maximum the most likely value of profit (\$)	84,866			66,610			88,882			73,055		
Minimize the risk of obtaining a lower profit (\$)	16,280			0			20,432			938		
Maximize of the possibility of obtaining higher profit (\$)	152,834			130,172			156,980			142,586		

The best result of PTM policy is from setting the weight at the most likely value at 80% and 10% at both pessimistic and optimistic values (Scenario III). Similar to PTR policy, the profit generated from the scenario IV (favor the optimistic case) is lower than the one from scenario III. It is due to the fact that the manufacturing process cannot fulfill the optimistic demand. As a result, a lower revenue and a higher shortage cost incur. As the comparison between two policies, it was found that the PTR policy can outperform the PTM policy in terms of profit in all objective functions. However, it is noticeable that the profit of PTR policy in scenario II when it is likely to be pessimistic condition is not much higher than the profit of PTM policy. In fact, they are equal (at 0) when we try to minimize the risk of obtaining a lower profit. In addition, when the returned ratio is further reduced to be lower than 60% (0.6), the profit of PTM could be even higher than the profit of PTR policy. This is due to the fact that the PTR policy cannot gain an advantage from using the returned components (which are cheaper) and needs to use more new components. This somehow shows the vulnerability of the PTR policy in the case that future pessimistic and uncertain conditions might happen as it needs to uncertainty rely on instability of returned components.

Conclusions

This proposed method in this case study can be used to solve most real world planning problems involving imprecise parameters through an interactive decision-making process. It constitutes a systematic framework that facilitates the decision-making process, enabling the decision maker to interactively modify the imprecise data and related model parameters until a satisfactory solution is found. Notably, the optimal goal values using the LP approach are imprecise since the forecasted demand, related operating costs, machine capacity, and the amount of returned component are always imprecise in nature. This brings the results closer to real world problem comparing with the linear programming under no uncertainty while the range of possible total profits can be presented. As a result, decision makers should have knowledge to prepare themselves and take necessary action for future uncertainty. This study focuses on multi-periods with a single product from a hybrid manufacturing/remanufacturing problem. It also provides information on different policies in response to any designed variations. Additionally, the approach also considers the actual tradeoff between the PTM and PTR policies in each circumstance. This lets us know that each policy would benefit or be vulnerable depending on a certain condition and show profit value in each case in real world scenario (for example worst case scenario—lack of component, lack of information of demand, and etc.).

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