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Воропай Р.А., Доманов Е.В., Кирсанов М.Н. Анализ прогиба плоской фермы с двойной треугольной решеткой

Voropai R.A., Domanov E.V., Kirsanov M.N.
Analysis of the deflection of planar truss with
double triangular lattice

Получено точное выражение для прогиба статически определимой симметричной балочной фермы с двумя опорами под действием сосредоточенной силы для произвольного числа панелей. Сдвоенные нисходящие раскосы образуют треугольники, восходящие – параллельны между собой. Решение, полученное в системе Maple, имеет вид кубического полинома по числу панелей. Исследованы асимптотические свойства решения
Ключевые слова: ферма, прогиб, формула Максвелла-Мора, Maple

Obtained the exact expression for the deflection of statically determinate symmetric girder beam with two supports under the action of concentrated forces for an arbitrary number of panels. Double down braces form triangles, ascending – parallel to each other. The solution obtained in the system Maple, has the form of a cubic polynomial in the number of panels. Asymptotic properties of the solution are investigated
Key words: truss, deflection, Maxwell-Mohr' formula, Maple

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Unlike standard schemes of lattices of triangular or rectangular type, the truss (Fig. 1) has double braces. This design can be used in cases where the elements of the reduced area of the cross-sections are used or to give the construction additional architectural expression. Here the calculation has some difficulties. Induction method, developed in [1-9] in the calculation of the planar and in [10-11] spatially regular

(periodic) trusses allows to obtain a General formula for the deflection of the structure and efforts in the critical (the most compressed and stretched) rods for an arbitrary number of panels. Applying the algorithm [12] and the corresponding program in the system Maple the formula for deflection in this case are derived. Restrict ourselves to the case of concentrated load at Midspan.

Fig. 1 $n=3$

The algorithm uses the method of cutting all nodes and completing the global matrix of the system of equilibrium equations. The guides of the cosines of the efforts are the coefficients in the equilibrium equations. For their definition, you must specify the coordinates of the nodes. The special feature of this design is uneven, but periodic location of the hinges. Here's the code fragment in the language of the computer algebra system Maple (origin located at the left support):

```
> for i to n do
> x[2*i-1]:=(i-1)*5*a:
> x[2*i]:=(i-1)*5*a+a:
> x[2*i+2*n]:=((i+n)*5-1)*a:
> x[2*i+2*n+1]:=(i+n)*5*a: od:
> for i to 2*n do x[2*i+4*n]:=(i-1)*5*a+2*a:
> x[2*i+4*n+1]:=(i-1)*5*a+3*a: od:
> for i to 4*n+1 do y[i]:=0: od:
> for i to 4*n do y[i+4*n+1]:=h: od:
```

The structure of the connection rods and units sets the system of conditional vectors $\mathbf{N}$. Here the example of the corresponding vectors of upper and bottom zones:

```
> for i to 4*n do N[i]:=[i,i+1]; od:
> for i to 4*n-1 do N[i+4*n]:=[i+4*n+1,i+4*n+2]; od:
```

The matrix elements of the equilibrium equations G are introduced in the cycle of number of rods $m=16n+2$

```
> for i to m do
> Lxy[1]:=x[N[i][2]]-x[N[i][1]]:
> Lxy[2]:=y[N[i][2]]-y[N[i][1]]:
> L[i]:=sqrt(Lxy[1]^2+Lxy[2]^2):
> for j to 2 do
> jj:=2*N[i][2]-2+j:
> if jj<=m then G[jj,i]:=-Lxy[j]/L[i]:fi:
> jj:=2*N[i][1]-2+j:
```

```
> if jj<=m then G[jj,i]:= Lxy[j]/L[i]:fi;
> od;
> od;
```

The solution of efforts are obtained in analytical (symbolic) form. The deflection determined by the formula of Maxwell-Mora, which in this case has a simple form $\Delta = P \sum_{i=1}^{m-3} S_i^2 l_i / (EF)$, where S_i – the forces in the rods of the truss isolated from the action of external loads, l_i – the lengths of the rods EF – their stiffness. Sequential settlement of trusses by number of panels 1, 2, ..., 10 gives a series of solutions. The method of induction with the use of Maple operators working out and solution of recurrent equations obtained for the $\Delta = P\Delta / (EF)$ General solution $\Delta = (a^3 A_n + (c^3 + f^3) C_n) / (2h^2)$, where $A_n = (5/3)n(7 + 50n^2)$, $C_n = n$, $c = \sqrt{4a^2 + h^2}$, $f = \sqrt{9a^2 + h^2}$. In order to find the General term of A_n the sequence 95, 690, 2285, 5380, 10475, 18070, 28665, 42760, 60855, 83450 is investigated. The recurrence equation was derived and its solution by use of the operator **rsolve** gives required formula. In figure 2 the curves are obtained depending on n when $a = L / (5n)$, $L=50$ m and different heights h .

Fig. 2

To assess the strength and stability it is necessary to know expressions for efforts in critical elements. Obviously one bar (compressed) is in the middle of the upper belt, the other (stretched) – in the middle of the bottom belt. The stress in the compressed rod obtained by induction $S^- = -5Pna / (2h)$. Similarly, an expression for the

efforts of the most extended rod is $S^+ = (5n - 3)Pa / (2h)$. Note that the standard methods adopted to determine the stresses in the bars of the truss based on the method of sections (Ritter) are failed, these values can not be found.

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