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Анализ обратной кинематики робота
на примере KUKA KR Agilus

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Inverse Kinematics Analysis Using KUKA KR Agilus Robot

Данная работа посвящена анализу обратной кинематики роботов на примере промышленного робота KUKA из серии KR Agilus. Обратная кинематика рассчитывается геометрическим методом. Также в работе обсуждается выбор возможного подходящего положения робота после расчета обратной кинематики

Ключевые слова: обратная кинематика, анализ, моделирование, KUKA KR Agilus

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This paper is devoted to the inverse kinematics analysis of industrial KUKA robot of KR Agilus series. The inverse kinematics for robot manipulator is analyzed systemically using geometrical method. Selection of possible closest solution is discussed after the inverse kinematics is provided. There is no areas dealing with robots where the inverse kinematics would not be required

Key words: inverse kinematics, KUKA KR Agilus, robotic manipulator, analysis, modeling

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Inverse kinematics is the opposite process to forward kinematics. There the robot orientation in terms of joint angles are computed by the given Cartesian position of the robot end-effector (EE). If the robot have tool then the inverse kinematics solving is consist of 2 steps: first find transformation of robot flange relative to base, and then the inverse kinematics are used to solve for joint angles. By the first step, it means to multiply the given transformation matrix to inverted matrix of tool position relative to flange [1].

To solve inverse kinematics is a big challenge. There is no easy way like in forward kinematics to find the solution to for robots like KUKA KR Agilus. The main problem of solving the kinematic equations of a manipulator is a non-linear one. So for robots of series KUKA KR Agilus there is no analytical solution to find joint angles from the given transformation matrix [2].

Method that was used to solve the inverse kinematics for KUKA KR Agilus is geometrical one. In this method, the angles must be calculated in the following order: $\theta_1, \theta_3, \theta_2, \theta_5, \theta_4, \theta_6$.

One thing that should be said that inverse kinematics solution might have multiple solutions for the given Cartesian position of EE as in opposite to forward kinematics that have always only one solution.

Figure 1. KUKA Agilus schematic view

Analyzing θ_1

Angle θ_1 can be calculated from projection of frame K4 on the x_0 - y_0 plane of frame K0 (see Fig. 1). At first, we have to compute the position of K4. Twist joints don't change the position but the orientation of its subsequent frame. In KUKA KR Agilus we have three twist joints: A1, A4, A6. As shown in Fig. 4, the angular state of joint 6 (frame K6) influences just the orientation of the TCP, not its position. So

$$\vec{p}_{46} = d_6 \cdot \vec{n}$$

The unit vector $\vec{n}$ points to the z-axis of K6 is retrieved as a third column from rotation submatrix of transformation matrix 0_6T . So

$$\vec{p}_{04} = \vec{p} - \vec{p}_{46} = \vec{p} - d_6 \cdot \vec{n} = \begin{bmatrix} p_{04x} \\ p_{04y} \\ p_{04z} \end{bmatrix} = \begin{bmatrix} p_x - d_6 n_x \\ p_y - d_6 n_y \\ p_z - d_6 n_z \end{bmatrix}$$

Finally, θ_1 will be calculated as following

$$\theta_1 = \text{Atan2}(p_{04y}, p_{04x}) + \pi n$$

where $n = -1, 0, 1$.

Analyzing θ_3

Via the knowledge of $\vec{p}_{14}$ we get angle γ which in turns leads to joint angle θ_3 . As was mentioned above, the angular state of joint A4 influences the orientation of frame K4, not its position. Hence vector $\vec{p}_{14}$ can be calculated. We get the position of $\vec{p}_{01}$ from the translation part of 0_1T :

$$\vec{p}_{14} = \vec{p}_{04} - \vec{p}_{01} = \begin{bmatrix} p_x - d_6 n_x \\ p_y - d_6 n_y \\ p_z - d_6 n_z \end{bmatrix} - \begin{bmatrix} a_1 \cos \theta_1 \\ a_1 \sin \theta_1 \\ d_1 \end{bmatrix}$$

Consider the triangle K2K3K4, sum of all angles in a triangle is equal to 180° . Adjacent angle to φ is equal to $180^\circ - \varphi$. Therefore

$$a) \gamma = 180^\circ - \omega - (180^\circ - \varphi) = \varphi - \omega$$

$$b) \gamma = \varphi + \omega$$

We compute angle φ though the law of cosine:

$$\varphi = \text{Acos}\left(\frac{a_2^2 + l_4^2 - |\vec{p}_{14}|^2}{2a_2l_4}\right)$$

Angle ω we get though the right triangle:

$$\omega = \text{Atan2}(a_3, d_4)$$

Finally knowledge of γ offers 2 possible solution to θ_3 :

$$1) \theta_3 = n(180^\circ - (\varphi - \omega))$$

$$2) \theta_3 = n(180^\circ - (\varphi + \omega))$$

where $n = -1, 1$.

Analyzing θ_2

As shown in Fig. 2, angle θ_2 can be computed via β_1 and β_2 which in turn we get though the vector $\vec{p}_{14}$.

Figure 2. Calculation of θ_2

For an easy calculation we will look to $\vec{p}_{14}^{(1)}$ – vector $\vec{p}_{14}$ in terms of K1. We transform $\vec{p}_{14}$ from K0 to K1 using inverse of the submatrix ${}^0_1\mathbf{R}$ of ${}^0_1\mathbf{T}$.

$$\vec{p}_{14}^{(1)} = ({}^0_1\mathbf{R})^{-1} \cdot \vec{p}_{14}$$

Because ${}^0_1\mathbf{R}$ is orthogonal we get its inverse though its transpose:

$$({}^0_1\mathbf{R})^T \cdot {}^0_1\mathbf{R} = \mathbf{I} \rightarrow ({}^0_1\mathbf{R})^{-1} = ({}^0_1\mathbf{R})^T$$

We get β_1 and β_2 though Fig. 2:

$$\beta_1 = \text{Atan2}(\vec{p}_{14}^{(1)}_y, \vec{p}_{14}^{(1)}_x)$$

$$\beta_2 = \text{Acos}\left(\frac{a_2^2 + |\vec{p}_{14}|^2 - l_4^2}{2a_2|\vec{p}_{14}|}\right)$$

Finally, knowledge of β_1 and β_2 leads to following solutions:

$$1) \theta_2 = n(\beta_1 + \beta_2)$$

$$2) \theta_2 = n(\beta_1 - \beta_2)$$

where $n = -1, 1$.

Analyzing θ_5

Joint angle θ_5 can be calculated though the dot product between the unit vectors $\vec{n}$ and $\vec{z}_3$.

$$\vec{z}_3 \cdot \vec{n} = |\vec{z}_3| \cdot |\vec{n}| \cdot \cos \theta_5$$

$$\theta_5 = A \cos(\vec{z}_3 \cdot \vec{n})$$

The orientation of vector $\vec{z}_3$ can be retrieved from the rotation sub matrix of the transformation 0_4T . Matrix 0_4T can be calculated via the results we already have for angles θ_1 , θ_2 and θ_3 . And the vector $\vec{z}_3$ will be the third column from 0_4R .

Finally, we get the following possible solutions for θ_5 :

$$\begin{aligned} 1) \theta_5 &= z_{3x}n_x + z_{3y}n_y + z_{3z}n_z \\ 2) \theta_5 &= -(z_{3x}n_x + z_{3y}n_y + z_{3z}n_z) \end{aligned}$$

Analyzing θ_4

The total rotation can be composed as follows:

$${}^0_6R = {}^0_3R \cdot {}^3_6R \rightarrow {}^3_6R = ({}^0_3R)^{-1} \cdot {}^0_6R$$

Because matrix R is orthogonal its inverse can also be calculated through its transpose:

$$({}^0_3R)^T \cdot {}^0_3R = I \rightarrow ({}^0_3R)^{-1} = ({}^0_3R)^T$$

Hence we get:

$${}^3_6R = ({}^0_3R)^T \cdot {}^0_6R = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -s_4c_6 - c_4c_5s_6 & -c_4s_5 \\ s_4c_5c_6 + c_4s_6 & c_4c_6 - s_4c_5s_6 & -s_4s_5 \\ s_5c_6 & -s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

where $s_i = \sin \theta_i$, $c_i = \cos \theta_i$.

From 3_6R we get following equations:

$$\begin{cases} -s_4s_5 = r_{23} \\ -c_4s_5 = r_{13} \end{cases}$$

Dividing one equation to another we comes to:

$$\tan \theta_4 = \frac{r_{23}}{r_{13}}$$

Finally, we get following solutions for θ_4 :

$$\theta_4 = A \tan 2(r_{23}, r_{13}) + \pi n$$

where $n = -2, -1, 0, 1, 2$.

Calculating θ_6

The same way as for θ_4 we get solution for θ_6 .

$$\begin{cases} -s_5s_6 = r_{32} \\ s_5c_6 = r_{31} \end{cases}$$

Dividing one equation to another we comes to:

$$\tan \theta_6 = \frac{-r_{32}}{r_{31}}$$

Finally, we get following solutions for θ_6 :

$$\theta_6 = A \tan 2(-r_{32}, r_{31}) + \pi n$$

where $n = -2, -1, 0, 1, 2$.

Selection of solution

One big issue for inverse kinematics is existence of multiply solutions. Cartesian position of EE can be reached with different set of joint angles. So the robot should be able to choose one.

The one very reasonable choice would be the closest one. We need to select from the list of available solutions for inverse kinematics the one that requires the smallest twist of joint angles from the current robot position $\theta_i^{(0)}$. For this, we calcu-

late deviation δ_i for every possible solution and choose the solution with minimal deviation.

$$\delta_i = \sum_{n=1}^6 \omega_n |\theta_{n,i}^{(0)} - \theta_{n,i}|$$

where $\omega_n = 6 - n$ – weight for joint. It means that twist of first joint cost more than the next one.

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