Lecture

Some Highlights of 20th Century Mathematics in India - A Personal Perspective[#]

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Speaking at a meeting of the Moscow Mathematical Society in 1935 Andre Weil, one of 20th century's greatest mathematicians, had this to say about mathematics in India: "The intellectual potentialities of the Indian nation are unlimited and not many years would perhaps be needed before India can take a worthy place in world mathematics". Weil was well acquainted with the Indian situation in mathematics, having spent two years as Head of the Department of Mathematics at Aligarh Muslim University.

The arrival of the legendary Srinivasa Ramanujan on the scene towards the end of the second decade of the last century was of course a major factor in A Weil arriving at that assessment. But there were other mathematicians who had done "world class work" well before Ramanujan. There was for example Shyamadas Mukhopadhyaya whose "Four Vertex Theorem" had attracted international attention. In fact about 10 years ago there was a celebratory conference in the US commemorating the centenary of the publication of the theorem and that was some years before the famous letter of Ramanujan to Hardy. Asutosh Mukherjee, the visionary vice-chancellor of Kolkata University was a mathematician of some standing, even if he was not the equal of Shyamadas Mukhopadhyaya. Another mathematician with an international standing was Ganesh Prasad who too had chosen the pursuit of mathematics long before the Ramanujan story broke out. All three were products of the much maligned colonial educational system devised by Thomas Babington Macaulay. In fact the system produced some 20 odd mathematicians who became well-known internationally.

Among them was Ananda Rao who was somewhat younger than Ramanujan and whose decision to pursue mathematics antedates Ramanujan's name getting widely known. Ananda Rao was a mentor to some half a dozen outstanding mathematicians; the first two were S S Pillai and T Vijayaraghavan. Ramanujan was of course a source of inspiration for a whole lot of people to embark on a career in mathematics and as I said a good number made their mark on the international scene. All these mathematicians worked in a handful of institutions: Universities of Calcutta, Madras, Benares, Allahabad and Delhi or rather colleges affiliated to these institutions. In fact in most cases they worked at colleges affiliated to them. And it is their work, not just Ramanujan's that led Andre Weil to his optimistic prognosis made more than 80 years ago.

Mathematics in India has indeed registered some impressive progress since then and much of it has come after independence. Yet we cannot quite say that we have got to our "rightful place" in world mathematics. Countries like the US, UK, France, Germany and Russia are still way ahead of us. In the developing world we had primacy even before independence and that state of affairs continues; not many developing countries have made much progress in mathematics in the last century or this; exceptions are China, which challenges our primacy, Brazil, Korea and Vietnam. Two institutions set us on the path of achieving our present good standing in mathematics in the comity of nations: the Indian Statistical Institute (ISI) in Kolkata and the Tata Institute of Fundamental Research in Mumbai (TIFR) even while the leading

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pre-independence institutions declined rapidly. ISI was founded in 1931 and TIFR in 1945. The former's main preoccupation was with Statistics and it was somewhat later that mathematics proper came to be pursued there. And the academic leadership passed on to one of our greatest statisticians C R Rao.

TIFR too was founded with the principal aim of promoting Physics, but early on, the founder Homi Bhabha wanted mathematics also to be pursued there. And one of his early recruits (in 1949) was K Chandrasekharan, an outstanding mathematician whose talent and vision for organization were superior even to his considerable mathematical talents. Under his leadership the school of mathematics at TIFR evolved into one of the leading international centres for mathematics in the short space of a decade and a half. There were in that period – the sixties-already a number of research papers which were turning points in several mathematical areas – Number Theory, Algebraic Geometry, Differential Geometry, Lie Theory; and they were all by the people trained at TIFR. Parallely, at ISI, work of the highest calibre in mathematics (as also in) Probability and Statistics was being produced by young people trained there. Both these institutions received a lot of academic support by way of visitors from abroad. The visitors to ISI were mostly from the erstwhile Soviet Union while TIFR drew most of its visitors from the West, especially France and Germany. Practically all world-class mathematics in the country during the post-independence period right up to the eighties came from these two institutions with TIFR having a decided edge. The situation began to change in the last three decades with some other institutions in the country recruiting good young faculty and quality work emerging from these places as well, while TIFR and ISI continue to this day to be very much in the lead. Mathematical research of high quality is now coming out of some nine or ten institutions, among them IISC, IMSC, HRI, some IITs and some IISERs. Most of it though is in Pure Mathematics and some in Applied Mathematics related to Computer Science. TIFR and ISI have had a role in this development. The new institutions that are doing well have on their faculty, products of these two institutions and their graduate schools are modelled on those of ISI and TIFR.

The TIFR Centre for Applicable Mathematics is one institution that is producing consistent world-

class work in Applied Maths (mostly to do with non-linear partial differential equations). One of its products – Siddhartha Mishra - has been awarded the Infosys Prize for Mathematics for 2019.

The number of good mathematicians working in the country is perhaps more than 100, a far cry from the colonial days; and by “good”, I mean that they deserve to be elected to this academy. Our international standing has more than maintained itself at the level to which it was raised in the sixties of the last century.

However, it must be acknowledged that the quality of the mathematics research done in institutions other than these handful of institutions leaves much to be desired. And some, even in institutions with an outstanding past record, is dismally poor. One reason for this state of affairs is the faulty policies and methods in the recruitment of faculty in our universities and other institutions of higher learning. Once a person of poor calibre is recruited, he or she will eventually rise to be a decision maker and will inevitably make the wrong decisions in recruitments. Bureaucratic devices are of no help in this matter; in fact, far from ensuring quality recruitment they end up impeding it.

When one creates a large number of institutions in a short space of time - as we have done in the last seven decades – it is inevitable that there will be some fall in standards, but with some care the dismal fall in many of our institutions need not have happened. Good future recruitment of faculty in our universities and institutions of higher learning is the only way to go forward from here. And for that greater autonomy in this matter rather than stricter bureaucratic rules is likely to produce good results. Also we have over the years given little importance to scholarship. This was the result of the perception that our traditions placed a lot of value on arid scholarship and thereby discouraged original thinking. There is no doubt there is some truth in that, but we seem to have swung to the other extreme when young people with poor scholarship embark on ‘research’ which often is of indifferent quality: it is only in a milieu of good scholarship that sustained good research is possible. I should think that the success of TIFR has much to do with the emphasis on the acquisition of true scholarship in that institution. In many of our

institutions of higher learning, students who join the graduate school are almost immediately given a problem to work on and start specialising in the related area. Inevitably he/she has a very narrow mathematical scholarship which is all that is needed to solve the problem; and the problem itself is more often than not, not interesting or challenging – the only purpose it serves is to equip the student with a degree and the guide to enhance the number of PhDs he has guided in his CV.

There have been efforts by the National Board for Higher Mathematics, (an agency set up in 1983 by the Department of Atomic Energy to promote mathematics), towards raising the general standards of mathematics across the country. It has been organizing many instructional schools which offer opportunities to graduate students to learn subjects that they are not taught at their universities. It has also run programmes where standard courses, as are taught at TIFR or ISI, are offered to graduate students. It provides travel support for attending international conferences to students as well as to faculty. It has also provided extensive support to libraries all over the country making up for the progressively declining support from the universities themselves. It also offers scholarships to graduate students (selected through a rigorous test) to pursue a PhD at an institution of their choice. It appears that some of these initiatives have paid off and the mathematical community at large seems to be better informed about mathematics at the cutting edge of research.

There are some themes in Indian mathematics that seem to have a remarkable continuity of quality research over time. I mention two here but there must be others that I have not noticed, especially in areas I am unfamiliar with. The most striking for me is the Indian contribution to the Waring problem. David Hilbert showed that given any positive integer k , there is a minimal positive integer $g(k)$ such that every whole number can be expressed as a sum of $g(k)$ k th powers of integers. This led people to the problem of finding a formula for $g(k)$. Lagrange - long before Hilbert - had determined that $g(2)=4$. In 1950 the Indian mathematician S S Pillai achieved the exact determination $g(k)$ for $k>5$, the case $k=6$ being particularly difficult. $g(3)$ and $g(5)$ were determined

a little before that. But the case $k=4$, proved to be tough. It eventually yielded (in 1984) to the efforts of another Indian, R Balasubramanian working in collaboration with two Frenchmen. Meanwhile in 1968 or thereabouts, C P Ramanujam at TIFR solved a problem closely connected to Waring's problem. Hilbert's theorem generalises to number fields: if F is a number-field there is an integer $g(k,F)$ such that every integer in F which is the sum of some number of k th powers, is the sum of $g(k,F)$ k th powers of integers in F . Ramanujam showed that $g(k,F)$ can be chosen to be independent of F . A second theme relates to the heat equation to which some interesting contributions were made by Ganesh Prasad in the thirties. Minakshi Sundaram worked on the asymptotic expansion of the so called heat kernel on general Riemannian manifolds, a study from a purely mathematical perspective; and some 25 years later, V K Patodi came up with a proof of the Atiyah-Singer index theorem, (one of the great theorems of the 20th century) based on Minakshisundaram's work (that Minakshi Sundaram's work could be used to prove the index theorem was an observation due to McKean and Singer).

Summing up, I think that it is a fair assessment to say that we are in a much better state in regard to mathematics than we were in the wake of our independence. One indicator of this is that since 1962, barring two occasions at least one Indian mathematician working in India has given a prestigious invited talk at the International Congress of Mathematicians, an event that takes place once in four years. However, considering our size as a country, we are still a good distance away from a truly 'worthy place' in world mathematics that Andre Weil envisaged for us.

There is however a continuing perception among the general public as well as the scientific community at large that mathematics research is in bad shape in the country. I do not think that mathematics is in any worse shape than other sciences like physics or chemistry. In fact there is a case to be made that the peaks in mathematics are more than comparable to those in other sciences. Unfortunately, this is not reflected in indicators such as the Fellowship of the academies: the elections have not kept up with the growth in numbers of good mathematicians.