A systems approach to the modeling of racing wheelchair propulsion

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Abstract—This paper presents a systems model for racing wheelchair propulsion. Recently, an interest has developed in computer and mathematical models of racing wheelchair propulsion with the goal of improving performance and efficiency. The model in this paper is first presented in block diagram form and later described in greater detail. Equations are presented that describe the motion of a racing wheelchair on the road/track and an inertial roller system, as well as kinematic equations describing the stroke biomechanics. Because the model is used to describe a human/machine system, it is nonlinear. Some thoughts on methods of developing a model of the physiology of racing wheelchair propulsion are also presented.

Key words: *human/machine system, mathematical models, physiological and physical factors, racing wheelchair propulsion, stroke mechanics.*

INTRODUCTION

Many mobility-impaired persons rely on manual wheelchairs for propulsion. There have been several investigations into the physiological and physical factors affecting manual wheelchair propulsion in response to a growing interest in racing wheelchair propulsion (15,16). This subject is of particular interest because efficiency and optimal performance are desirable. Wheelchair-racing individuals place high physical and physiological demands on their bodies. Measurement of these demands yields insight into the response of the body to physical disability.

Although wheelchair sports science and medicine have not reached maturity, the need has been expressed for modeling and computer simulation of racing wheelchairs and wheelchair racing (6,7). The interaction between the individual and the wheelchair is an area of critical importance (as is true with most human/machine systems). Developing mathematical models based upon physical laws and actual experiences are ways of gaining further insight into the interaction between the individual and the racing wheelchair. As with many complicated human/machine systems, the model must define the ecosystem with which the system is to interact (17).

The purpose of this paper is to describe a theoretical model of racing wheelchair propulsion. A general block diagram model for racing wheelchair propulsion is presented and possible models for each of the blocks are described.

MULTIVARIABLE BLOCK DIAGRAM MODEL

There are many factors that define the interaction of the individual/racing wheelchair system as related to propulsion. A possible general block diagram model is presented in **Figure 1**.

In this model the physiological engine represents the internal function of the individual's body and its ability to convert energy to the muscle forces required to generate propulsion torques. Several factors affect the efficiency with which the body converts energy to muscle force: body structure and composition, age, gender, level and nature of disability, sensorimotor and autonomic nervous system

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function, medical complications, medications, diet/hydration, fatigue, strength, flexibility, anaerobic and aerobic power, psychological motivation, and feedback (3). Hence, a multi-input model is appropriate.

The biomechanical linkage represents a kinematic model for converting the individual joint torques (i.e., the trunk, shoulder, elbow, and wrist) appropriate for racing wheelchair propulsion to the forces at the push-rings. The biomechanics for racing wheelchair propulsion are affected by the fit of the chair to the individual, as well as the position of the person in the chair. In general, the motions of the arms are symmetrical and the same model can be used for both arms. In cases where there are anthropometric differences simple modifications in a symmetric model can be made.

The racing wheelchair is perhaps the simplest of the subsystems of this model. It can generally be modeled as a simple linear system which converts the input forces at the push-rings to the output velocity of the center of mass. The model is further simplified because the rules for wheelchair racing permit only a single set of push-rings.

The velocity of the center of mass of the individual/wheelchair system is influenced by the environment, which is used to describe the effect of air resistance and the changing slope of the rolling surface.

The actual velocity of the individual/wheelchair system is used as a feedback variable to the physiological engine. The purpose of wheelchair racing is to win the race or to complete the predesignated course in a minimum amount of time. With this in mind, the response of the physiological engine must be influenced by velocity.

The block diagram of **Figure 1** can be rewritten to reflect the *a priori* knowledge about the system. The functional block diagram representation using operator notation is depicted in **Figure 2**. Here the physiological engine, the biomechanical linkage, and the environment have been represented by the nonlinear operators $N_p(\cdot)$, $N_k(\cdot)$, and $N_e(\cdot)$, respectively. The racing wheelchair is represented by the linear operator $L_w(\cdot)$. With a representation for the system model more specific descriptions of the subsystem models may be explored.

All the models developed in this paper are two-dimensional, assuming the roll axis is parallel with the center line of the wheelchair.

DIFFERENTIAL EQUATION MODELS FOR RACING WHEELCHAIRS AND THEIR INTERACTION WITH THE ENVIRONMENT

In this section, simplified models of a racing wheelchair and the environment are presented. These models

include racing wheelchair propulsion on a flat road (i.e., no road crown) or track, and for an inertial roller system. Inertial roller systems are being used more frequently in laboratory simulation of wheelchair racing; therefore it is important to compare the models of wheelchair propulsion on the road/track to an inertial roller system.

Road/track model

The motion of a racing wheelchair on a flat road (i.e., no road crown), as defined in **Figure 3**, is described in the following differential equation. The equation is derived by summing all of the external forces as indicated in **Figure 3** (1,4).

$$F(R/r_{pr}) = M\dot{v} + I\dot{v}/R + I_f \dot{v}/r + F_a + M_B/r_{ax} + M_b/r_{axf} + F_R + F_r + W\sin\Theta(x) \quad [1]$$

In Equation 1, (R/r_{pr}) represents the gear ratio of the pushing to the wheel.

The following list of definitions applies to Equation [1] and all other equations in this section.

- F = The force tangential to the pushrings.
- R = The radius of the rear wheels.
- r_{pr} = The radius of the pushrings.
- M = The mass of the individual and wheelchair.
- v = the velocity of the individual/wheelchair system ($v = x = dx/dt$).
- I = The inertia of the rear wheels.
- I_f = The inertia of the front wheels.
- r = The radius of the front wheels.
- F_a = The air resistance of the individual wheelchair system.
- M_B = The bearing resistance of the rear hubs.
- M_b = The bearing resistance of the front hubs.
- r_{ax} = The radius of the rear axle.
- r_{axf} = The radius of the front axles.
- F_R = The rolling resistance of the rear wheels.
- F_r = The rolling resistance of the front wheels.
- W = The weight of the individual/wheelchair system.
- $\Theta(x)$ = The angle of inclination (i.e., changes in elevation).
- x = The distance traversed.

To be of greater use the relationship of the external forces to the velocity must be revealed. The equation we used for the air resistance is as follows:

$$F_a = C v^2$$

where C is the drag coefficient of the individual and wheelchair.

The equations for rolling resistances are as follows:

Figure 1.
A block diagram description of racing wheelchair propulsion.

Figure 2.
An operator notation description of racing wheelchair propulsion.

$$F_R = (W_R b_R / R) \cos \Theta(x) \quad F_f = (W_f b_f / r) \cos \Theta(x)$$

where W_R is the weight upon the rear wheels, W_f is the weight upon the front wheels, b_R is the coefficient of rolling resistance for the rear wheels, and b_f is the coefficient of rolling resistance for the front wheels.

After substitution of the models for air resistance and rolling resistance, Equation [1] becomes Equation [2].

$$F(R/r_{pr}) = (M + I/R + I_f/r) \dot{v} + C v^2 + M_B/r_{ax} + M_b/r_{axf} + (W_R b_R / R + W_f b_f / r) \cos \Theta(x) + W \sin \Theta(x) \quad [2]$$

It will be assumed that the bearing resistances are

proportional to the velocity (1), and that the bearing resistances can be combined for simplicity. Therefore the bearing resistances can be modeled as follows:

$$K_3 v = M_B/r_{ax} + M_b/r_{axf}$$

Hence, Equation [2] reduces to Equation [3].

$$F(R/r_{pr}) = (M + I/R + I_f/r) \dot{v} + C v^2 + K_3 v + (W_R b_R / R + W_f b_f / r) \cos \Theta(x) + W \sin \Theta(x) \quad [3]$$

By substituting the definition of $v = dx/dt = \dot{x}$ into Equation [3], it can be shown that Equation [3], and thus Equation [1], is of the following form with $\ddot{x} = d^2x/dt^2$.

$$F = K_1\ddot{x} + K_2\dot{x}^2 + K_3\dot{x} + K_4\cos\Theta(x) + K_5\sin\Theta(x) \quad [4]$$

This is a nonlinear differential equation describing the interaction of a racing wheelchair with the environment. Without the interaction of the environment (level ground—no change in slope—without air), Equation [4] can be reduced to Equation [5].

$$F = K_1\ddot{x} + K_3\dot{x} + K_4 \quad [5]$$

Equation [5] represents a simple linear system for the racing wheelchair model.

Inertial roller model

Equation [6] is a differential equation describing the motion of the rear wheels of a racing wheelchair while on an inertial roller system (simplistically, a drum upon which the rear wheels roll with flywheels for inertia and some means of creating resistance) with the external torques, as defined in **Figure 4**. The differential equation is derived by summing the torques acting upon the individual/wheelchair system.

$$Fr_{pr} = I\dot{w} + I_r\dot{w}_r + T_a + M_B + F_R R + T(w,z,t) \quad [6]$$

In Equation [6], Fr_{pr} represents the torques of the rear wheel.

The following is a list of the definitions for the newly added variables:

- I_r = The inertia of the roller and all of its components.
- w = The rotational velocity of the rear wheels.
- w_r = The rotational velocity of the roller.
- T_a = The air resistance of the rear wheels.
- $T(w,z,t)$ = The variable external resistance.
- z = The distance travelled on the roller.
- t = The time spent pushing on the roller.

The following is a rotational model of the air resistance with c being the coefficient of air resistance for the rear wheels.

$$T_a = cw^2$$

The moment of rolling resistance with W_R defined as the weight upon the rear wheels, and b_R being the coefficient of rolling resistance for the rear wheels (assumed to be the same as the road in this case), is as follows:

$$F_{RR} = W_R b_R$$

By substituting the models of the air resistance of the rear wheels and the moment of the bearing resistances into Equation [6], the result is Equation [7]. Assuming there is no slip between the rear wheels and the roller (i.e., $w_r = [R/r_r]w$, where r_r = the radius of the roller).

$$Fr_{pr} = (I + I_r R/r_r)\dot{w} + cw^2 + M_B + W_R b_R + T(w,z,t) \quad [7]$$

Equation [7] is of the form:

Figure 3.
Diagram of a racing wheelchair rolling over a flat road/track.

$$F = k_1 w + k_2 \dot{w}^2 + k_3 \dot{w} + k_4 + T(w, z, t) \quad [8]$$

Again assuming no slip between the roller and the rear wheels then $w = v/R = (dx/dt)/R = \dot{x}/R$, $\ddot{x} = \dot{v} = d^2x/dt^2$ and $z = x$.

$$F = k_1 \ddot{x} + k_2 \dot{x}^2 + k_3 \dot{x} + k_4 + T(\dot{x}, x, t) \quad [9]$$

This is also a nonlinear differential equation and the nonlinearities once again arise from the interaction of the individual/wheelchair system with the environment.

Comparison of the road/track model to the inertial roller system model

The validity of inertial roller system testing is dependent upon the similarity of racing wheelchair propulsion on a roller system to that of a road or track. The differential equation models derived for the road/track and an inertial roller system are both nonlinear and have the same order. To initiate the same response the coefficients of the differential models for the road/track and inertial roller system must be set as equal.

$$F = K_1 \ddot{x} + K_2 \dot{x}^2 + K_3 \dot{x} + K_4 \cos\Theta(x) + K_5 \sin\Theta(x) = k_1 \ddot{x} + k_2 \dot{x}^2 + k_3 \dot{x} + k_4 + T(\dot{x}, x, t) \quad [10a]$$

We want to determine the required inertial roller coefficients with the following physical restrictions:

$$k_2 = c \neq C = K_2$$

$$k_3 = M_B/r_{ax} \neq (M_B/r_{ax} + M_b/r_{ax} t) = K_3$$

$$K_4 = (W_R b_R + W_r b_r) \neq (W_R b_R) = k_4 \quad [10b]$$

Therefore, to satisfy the inequalities of Equation [10b] the constants c_i are defined as follows:

$$k_1 = K_1, (k_2 + c_2) = K_2, (k_3 + c_3) = K_3$$

$$k_4 + c_4 \cos\Theta(x) + c_4 = K_4 \cos\Theta(x)$$

hence

$$T(x, x, t) = c_2 \dot{x}^2 + c_3 \dot{x} + c_4 \cos\Theta(x) + c_4 + K_5 \sin\Theta(x) \quad [10c]$$

For Equation [10c] to be implemented it must be transformed back into its original coordinate system. The equations show that without a variable external torque applied to the inertial roller system, equality cannot be established. If given equal conditions of slope and rolling resistance for both of the models described, a variable external torque is still required for air resistance simulation.

Model of a rear wheel

Because many racing wheelchairs have the push-rings mounted to the spokes of the rear wheels, as shown in **Figure 5**, it cannot be assumed that all of the force tangential to the push-rings goes into propulsion.

Figure 4. Diagram of a racing wheelchair rolling on an inertial roller system.

Figure 5.
Distribution of the push-ring forces on a racing wheelchair rear wheel.

A simple model of a propulsive force applied to a rear wheel is:

$$F = F_{pr} - \sum_{i=1}^N f_i = F_{pr} - \sum_{i=1}^N k_i d_i \quad [11]$$

where

f := deflection force for a spoke.

k := spring constant of a spoke in the wheel.

d := displacement of the spoke tangential to the push-ring.

N := number of spokes used to attach the push-ring.

F_{pr} := tangential force on the push-ring.

This completes the modeling of the racing wheelchair and the interaction with the environment. A method for modeling the biomechanical linkage follows.

PLANAR KINEMATIC MODEL FOR RACING WHEELCHAIR STROKE DYNAMICS

Racing wheelchair stroke dynamics are usually symmetrical and the objective of the individual is to generate the greatest possible average force tangential to the push-ring. Therefore, most of the force will be tangential to the push-ring. However, the elbow/hand trajectories are in essence three-dimensional and would be more accurately modeled as such, but this will be left for future investigation. A graphical representation of the planar kinematic model is shown in **Figure 6**.

There is considerable interest in the possible relationships between an individual's physical dimensions and his/her position a racing wheelchair to performance. A general model of racing wheelchair stroke dynamics would be useful for studying possible relationships.

Joint trajectory equations

The model developed and presented here uses four links to represent the trunk, upper arm, forearm, and hand in the pushing position. The origin is taken to be the hip joint. To be of use in modeling the stroke kinematics, the equations for the path of the hand from the joint positions and the inverse problem of determining the joint positions for a specific hand path must be derived. Kinematic models of this type are regularly used in robotics (8,9,13).

The equations describing the position of each joint relative to the next most proximal joint are presented in Equations [12-16], using Cartesian coordinates with the hip joint as the origin.

Equation [12] describes the position of the shoulder at each instant.

$$(A) (x_A, y_A) = (l_t \cos \Theta_t, l_t \sin \Theta_t) \quad [12]$$

Equation [13] describes the position of the elbow with respect to the shoulder at each instant.

$$(B) (x_B, y_B) = (x_A - l_u \cos(\Theta_s - \Theta_t), y_A + l_u \sin(\Theta_s - \Theta_t)) \quad [13]$$

Equation [14] describes the position of the wrist with respect to the elbow at each instant.

$$(C) (x_c, y_c) = (x_B - l_f \cos(\Theta_t - \Theta_s - \Theta_e), y_B + l_f \sin(\Theta_t - \Theta_s - \Theta_e)) \quad [14]$$

Equation [15] describes the position of the end of the hand with respect to the elbow at each instant.

$$(D) (x_D, y_D) = (x_c - l_h \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w), y_c + l_h \sin(\Theta_t - \Theta_s - \Theta_e - \Theta_w)) \quad [15]$$

During propulsion the hand is forced to follow the push-ring. Hence, the Equation [15] can be described by Equation [16] during the propulsion phase.

$$(D_p) (x_{Dp}, y_{Dp}) = (x + R \cos(\Theta_0 - \Theta), -y - R \sin(\Theta_0 - \Theta)) \quad [16]$$

In Equation [16] Θ_0 (which is a constant) is the $\tan^{-1}(y_0/x_0)$ (**Figure 6**) and Θ is the angle made with respect to vertical of the point of contact of the hand (D) with the push-ring.

Figure 6.

A four-link kinematic model for racing wheelchair propulsion.

During recovery the hand is free to follow numerous paths. An example path for point (C) of an elite wheelchair racer is depicted in **Figure 7**. **Figure 8** shows a simulation of the joint paths which were generated using Equations [12-16], the equations describing the change in the joint angles ($\Theta_t, \Theta_s, \Theta_e, \Theta_w, \Theta_0, \Theta$) were derived by fitting quintic polynomials to motion analysis data (13). Notice that these are closed kinematic paths (8). The simulation results of **Figure 8** are similar in appearance to the actual results report by Ridgway *et al.*, and Sanderson and Sommer (11,12).

The inverse problem is derived by setting the origin as (x_D, y_D) and working backwards to solve for the positions of the joints.

Joint torque differential equations

The external forces acting on the links are depicted in **Figure 9**. The primary external forces are the gravitational forces acting on each of the links, in addition to the forces on the push-ring. Each of the links influences the other and the interaction is nonlinear. Nevertheless some insight can be gained from the derivation of these relationships.

The differential equations for each of the joint angles are presented because they are directly dependent upon the joint torques, which can be considered the output variables from the physiological engine.

Equation [17] presents a differential equation describing the motion of the trunk.

$$I_t \ddot{\Theta}_t = T_t + (M_t/2 + M_{hd}) g l_t \cos \Theta_t + M_u g (X_A - (l_u/2) \cos(\Theta_s - \Theta_t)) + M_f g (X_B + (l_f/2) \cos(\Theta_t - \Theta_s - \Theta_e)) + M_h g (X_c - (l_h/2) \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w)) + (M_t l_t \ddot{x}_A/2) \sin \Theta_t - (M_t l_t \ddot{y}_A/2) \cos \Theta_t + F_A l_t \sin \Theta_s \quad [17]$$

Equation [18] presents a differential equation describing the motion of the shoulder.

$$I_u \ddot{\Theta}_s = T_s + (M_u g l_u/2) \cos(\Theta_s - \Theta_t) + (M_f g l_f/2) \cos(\Theta_t - \Theta_s - \Theta_e) + M_h g (l_f \cos(\Theta_t - \Theta_s - \Theta_e) - l_h/2 \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w)) - (M_u l_u/2) (\ddot{x}_B - \ddot{x}_A) \sin(\Theta_s - \Theta_t) - (M_u l_u/2) (\ddot{y}_B - \ddot{y}_A) \cos(\Theta_s - \Theta_t) - F_B l_u \sin \Theta_e \quad [18]$$

Equation [19] presents a differential equation describing the motion of the elbow.

$$\begin{aligned}
 I_f \ddot{\Theta}_e = & T_e + (M_u g l_f / 2) \cos(\Theta_t - \Theta_s - \Theta_e) \\
 & - (M_f l_f / 2) (\ddot{x}_C - \ddot{x}_B) \sin(\Theta_s + \Theta_e - \Theta_t) \\
 & + (M_f l_f / 2) (\ddot{y}_C - \ddot{y}_B) \cos(\Theta_t - \Theta_s - \Theta_e) \\
 & - [F_A + M_u g \sin(\Theta_s - \Theta_t)] l_f \sin \Theta_e \\
 & - M_h g [l_f \cos(\Theta_t - \Theta_s - \Theta_e) - (l_h / 2) \cos(\Theta_t - \Theta_s \\
 & - \Theta_e - \Theta_w)] \\
 & - (M_u l_u / 2) [(\ddot{x}_A - \ddot{x}_B) \sin(\Theta_s - \Theta_t) + (\ddot{y}_A - \\
 & \ddot{y}_B) \cos(\Theta_s - \Theta_t)] \\
 & + F_C L_f \sin \Theta_w
 \end{aligned} \quad [19]$$

Equation [20] presents a differential equation describing the motion of the wrist.

$$\begin{aligned}
 I_h \ddot{\Theta}_w = & T_w + M_u g l_f \sin(\Theta_s - \Theta_t) \cos \Theta_e \\
 & - (M_f g l_f / 2) \cos(\Theta_t - \Theta_s - \Theta_e) \\
 & - (M_h g l_h / 2) \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \\
 & + F_B l_h \sin(\Theta_t - \Theta_s - \Theta_e) \sin(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \\
 & - F_B l_f \cos(\Theta_t - \Theta_s - \Theta_e) \sin(\Theta_t - \Theta_s - \Theta_e) \\
 & + (M_f l_f / 2) (\ddot{x}_B - \ddot{x}_C) \cos(\Theta_t - \Theta_s - \Theta_e) \\
 & - (M_f l_f / 2) (\ddot{y}_B - \ddot{y}_C) \sin(\Theta_t - \Theta_s - \Theta_e) \\
 & - (M_h l_h / 2) (\ddot{x}_D - \ddot{x}_C) \sin(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \\
 & - (M_h l_h / 2) (\ddot{y}_D - \ddot{y}_C) \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \\
 & + F_D l_f \cos(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \sin(\Theta_t - \Theta_s - \Theta_e) \\
 & + F_D l_h \sin(\Theta_t - \Theta_s - \Theta_e - \Theta_w) \cos(\Theta_t - \Theta_e - \Theta_s - \Theta_w)
 \end{aligned} \quad [20]$$

Based on Equations [17-20], the forces at the joints of each of the links can be calculated for a specific path, provided the torque at each of the joints is known. In general the dynamic torques at each of the joints are not available for measurement. Some investigators have attempted to determine joint torque from the electromyograms of the prime movers. Due to the highly nonlinear nature of the relationships presented in this section, the equations can only be solved numerically.

The joint torques described by Equations [17-20] may be estimated using an inverse dynamic approach, where push-ring torque (force), anthropometric, and motion analysis data enable the calculation of net joint torques. This may lead to a means of describing the relationship between the physiological engine and the individual joint torques. The push-ring propulsion force (F), for one of the rear wheels is equal to F_C . This force can be measured if strain gages are mounted appropriately to the push-rings. It is reasonable to assume that all of the forces (F_A , F_B , F_C) are zero during the recovery phase of the propulsion recovery cycle. This may provide a means to calibrate an estimation scheme for the joint torques. For simplicity, the output forces at the push-rings could be summed and substituted for F in the equations in the previous section. The outputs of this subsystem would be the propulsion force at each of the push-rings while the most appropriate inputs would be the joint torques. Logically, it would be desirable to generate a model of the physiological engine which has the joint torques as output variables.

Figure 7.

A typical path for the wrist of an elite wheelchair racer.

Figure 8.
Simulated paths of the joint trajectories for an elite wheelchair racer.

THE PHYSIOLOGICAL ENGINE

This block of the model is the most difficult to describe; a complete model describing the relationship of all of the possible input variables to the appropriate joint torques may not be attainable. However, it is helpful to investigate possible relationships that may be useful in predicting the appropriate joint torques from a reasonable number of input variables, which can be selected based upon empirical rules and insight. Some factors have greater influence on an individual's ability to produce the joint torques required to propel a wheelchair.

At present, there is little appropriate experimental data available to develop a model of a physiological engine for racing wheelchair propulsion. Insight can be gained from models developed for similar sports such as running. Because of the complex nonlinear behavior of this subsystem, these models are of little use without experimental data for racing wheelchair propulsion.

In the modeling of complex physiological systems such as this one, a general approach usually works best (2). In cases where simplifying assumptions cannot be made, such as linearity or a simple nonlinear model (usually one that is quadratic in a single variable, bilinear, or is a linear

Because of the desire of practitioners in the area of wheelchair racing medicine and sports science to predict the performance of an individual given the appropriate input variables, it would be most useful to develop a parametric model of the physiological engine.

This paper presented a general block diagram model of an individual/racing wheelchair system and described the major components of each of the blocks. Modeling of racing wheelchair propulsion and the interaction between the individual and his/her racing wheelchair is in the preliminary stages. Previously, investigators were interested in attempting to understand the effects wheelchair racing had on an individual's wellness and rehabilitation. The trend has shifted toward attempting to understand the factors which affect athletic performance and enhance athletic performance. Thus, some interest has developed for modeling racing wheelchair propulsion.

The model of the racing wheelchair and its interaction with the environment presented in this paper is useful for relating the output of the biomechanical system to variables that are tangible to the practitioner. The constraints presented to establish the equivalence of racing wheelchair propulsion on the road/track to an inertial roller system can be used to measure the validity of experiments performed in the laboratory when applied to actual use.

With the establishment of a preliminary biomechanical (kinematic) model that relates the joint torques to the propulsion force, work may progress toward determining optimal trajectories for the hand so that push-ring force is maximized. Tools need to be developed to measure dynamic forces and torques at the push-rings; so far only two attempts in this direction have been made (5,14). After the development of a reliable dynamic force/torque sensing device, the device could be combined with standard cinematographic techniques to study the joint torques and trajectories for manual wheelchair propulsion.

The individual/racing wheelchair system is complex and further complicated by the desire of the individual to achieve optimal performance. Thus, the physiological system does not operate at some nominal value but often at levels of super maximal work. Further experimentation measuring performance-related parameters, in order to

facilitate the modeling of an individual's physiological response to racing wheelchair propulsion, is needed. Individuals with varying degrees and types of disabilities have different physiological responses to stress; thus, it is important for the purposes of modeling to determine the underlying similarities of their physiological response to racing wheelchair propulsion. Furthermore, electromyographic analysis of dynamic force near maximal force is not well understood, and the kinematic model may be helpful in exploring this problem.

The models presented here need to be verified with experimental data. Instruments need to be developed to carry out model verification and refinement, as necessary. There has been little work in the area of measuring the parameters and specifications of racing wheelchairs. After these models have been verified, a three-dimensional kinematic model can be addressed.

The majority of investigation remains in the areas of (1) modeling the physiological system (physiological engine) with respect to racing wheelchair propulsion and (2) describing the effects of the body's position within the racing wheelchair to race performance. The physiological response to manual wheelchair propulsion may be further understood through the development of mathematical models and the use of system identification tools.

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