MODELING AND FORECASTING THE HOUSEHOLD WATER CONSUMPTION IN SAUDI ARABIA

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Abstract. The demand for water in the kingdom of Saudi Arabia increases due to population growth urbanization, industrialization and the expansion of irrigated agricultural lands. Analyzing the current situation of the water consumption and predicting water demand in the future are essential for the authorities to improve and manage production quantities. The objective of this paper is to find the best model for water consumption in Saudi Arabia and use the fitted model for forecasting. A data for water consumption between January 2010 and July 2017 were analyzed using a multiplicative seasonal autoregressive integrated moving average model (SARIMA). Through our research, we concluded that the best SARIMA model for fitting water consumption in Saudi Arabia is SARIMA (1,0,1) $\times$ (1,1,2)₁₂.

Keywords: *water consumption, time series, SARIMA, ACF, PACF, Box-Jenkins*

Introduction

It is widely recognized that the kingdom of Saudi Arabia is one of many countries of the world are entering an era of severe water shortage. Saudi Arabia, with an area of 2.15 million km², is a desert-like country lying in the Middle East in a zone where temperatures are very high in summer and low in winter. The country receives very little of precipitation throughout the year and also does not contain perennial rivers or stable bodies of water. The desert lands throughout the country frequently lose more moisture through evaporation than they receive from precipitation. In fact, more than half of the area of Saudi Arabia is desert. Recently, the arid and water deficit threaten the stability of the country.

The causes of water scarcity are varied; some are natural and others are as a result of human activities. Population growth and the climatic conditions pose a continual challenge and cause the limitation of underground water resources. Also, urbanization, industrialization and the expansion of irrigated agricultural lands have contributed to a dramatic increase in water consumption over the past few decades. Many researchers have discussed the influence of the depletion of underground water resources. They were interested in studying, analyzing and predicting the future demand for water. The journals “Land Economics” and “Water Resources Research” have dedicated much space to this study. Water consumption and demand estimation in developed countries have been at the core of many empirical papers, starting with the work of Gottlieb (1963) and Howe and Lina weaver (1967). Many studies have been made in a large set of countries including Canada (Kulshreshtha, 1996), Denmark (Hansen, 1996), France (Nauges and Thomas, 2000), Spain (Martínez-Espiñeira, 2002), Sweden (Höglund, 1999), and the US (Foster and Beattie, 1979; Agthe and Billings, 1980; Chicoine et al.,

1986; Nieswiadomy and Molina, 1989). Comprehensive reviews of the literature can be found, for examples, in Arbués-Gracia et al. (2003) and Dalhuisen et al. (2003).

Statistical methodologies have been used by many researchers as a tool for understanding and predicting future water demand. Kambale et al. (2016) used autoregressive integrated moving average (ARIMA) model for predicting the climate change scenarios by Inter-Governmental Panel for Climate Change (IPCC) and Indian Network for Climate Change Assessment (INCCA). Hongyan et al. (2017) combined ARIMA model and neural network to propose a prediction model. They used it to predict the 15 years supply and demand for water resources in Shandong Province. Generally, there are many studies dealt with different types of statistical models. We cite, for instance, Cornillon et al. (2008) for principal component analysis (PCA); Rita et al. (2013) and Samsuri et al. (2017) for multiple linear regression model.

In the Kingdom of Saudi Arabia, a general study for the most concise summary of groundwater resources was presented by Al-Ibrahim (1990). He stated that groundwater is the most important source of water in Saudi Arabia. Later, Abushammala and Bawazir (2017) developed an artificial neural network (ANN) model and used it to predict the annual and monthly domestic water demand in Makkah city, Saudi Arabia. Recently, Muhamed et al. (2018) proposed a hybrid model of ANN and ARIMA for predicting the water quality data. They found that the proposed hybrid model is better than the traditional methods and ANN.

None of the previous studies used and discussed the seasonal autoregressive integrated moving average (SARIMA) for predicting water demand in Saudi Arabia. In this paper, we aim to understand the current household water consumption and build a model that will encourage sustainable consumption and conservation of water resources.

A monthly data for the household water consumption in Saudi Arabia will be analyzed and modeled using the SARIMA model. There are two types of this model; additive and multiplicative. For monthly data, an additive model assumes that the difference in the water consumption values between the beginning and middle of each year is approximately the same. For a multiplicative model, the absolute differences in the consumption values are of less interest and importance than the percentage changes. We use the multiplicative SARIMA model since the pattern of the seasonal factor varies with the time series. The monthly water demand model, based on the SARIMA model, is essential for public authorities to improve monthly water production quantities. This study will be utilized for future development programs consideration; especially capital expenditures and water quantities.

The paper is structured as follows: In “Materials and methods” the household water consumption data used in this research is described and the notation of the multiplicative SARIMA model is established. Some models for the data are built and discussed in “Results and discussion”. Also, the best SARIMA model is stated. Finally, “Conclusion” is devoted to the conclusion and recommendation.

Materials and methods

Data collection

Data, for the household water consumption, between January 2010 and July 2017 were obtained from the Ministry of Environment, Water and Agriculture in Saudi Arabia. The data was monthly for the household water consumption, and it is measured in cubic meters (m³) as the water follows through the water meter. The lowest total

water consumption was in February 2010 with $161 \times 10^6 \text{ m}^3$ while the highest was in May 2017 with $285 \times 10^6 \text{ m}^3$. The data values are displayed in the Appendix.

Multiplicative seasonal ARIMA model

The dependence on groundwater supplies often emerges in importance during seasonal lags. Natural phenomena such as temperature also have strong components corresponding to seasons. Because of this, it is appropriate to introduce autoregressive and moving average polynomials that identify with the seasonal lags.

The seasonal autoregressive moving average model, say, ARMA (p, q), takes the form (Eq. 1)

$$\Phi_p(B^S)x_t = \theta_q(B^S)\omega_t, \quad (\text{Eq.1})$$

with the following definition. The operators (Eq. 2)

$$\Phi_p(B^S)x_t = 1 - \phi_1 B^S - \phi_2 B^{2S} - \dots - \phi_p B^{pS} \quad (\text{Eq.2})$$

and (Eq. 3)

$$\theta_q(B^S) = 1 + \theta_1 B^S + \theta_2 B^{2S} + \dots + \theta_q B^{qS}, \quad (\text{Eq.3})$$

are the seasonal autoregressive operator and the seasonal moving average operator of orders P and Q, respectively, with seasonal period S.

The multiplicative seasonal autoregressive integrated moving average (SARIMA) model of Box Jenkins (Box and Jenkins, 1970) is given by Equation 4:

$$\Phi_p(B^S)\varphi(B)\nabla_S^D\nabla^d x_t = \theta_q(B^S)\theta(B)\omega_t, \quad (\text{Eq.4})$$

where ω_t is the usual Gaussian white noise process. The one shorthand notation for the model is

$$\text{ARIMA}(p,d,q) \times (P,D,Q)S$$

With p = non-seasonal AR order, d = non-seasonal differencing, q = non-seasonal MA order, P = seasonal AR order, D = seasonal differencing, Q = seasonal MA order, and S = time span of repeating seasonal pattern. The non-seasonal difference components are $\nabla^d = (1 - B)^d$ and the seasonal is $\nabla_S^D = (1 - B^S)^D$ (Davis, 2005; Dickey et al., 1979). Identification of AR(p)s, MA(Q)s and then an ARMA model is often best done with the ACF and PACF. Table 1 gives a guideline for choosing the best model.

Table 1. Behavior of the ACF and the PACF for the Seasonal ARMA model (Chicoine et al., 1986)

	AR(P)s	MA(Q)s	ARMA (P,Q)s
ACF	Tails off at lags ks, k = 1,2,...	Cuts off after lag Qs	Tails off at lag ks
PACF	Cuts off after lag Ps	Tails off at lags ks k = 1,2,...	Tails off at lag ks

Results and discussion

Data exploration

The plot of the household water consumption series is given in *Figure 1*.

Figure 1. Monthly water consumption in Saudi Arabia between January 2010 and July 2017

From *Figure 1* we can see that there are clear patterns of seasonality. The seasonal variation increases as we move across time. Also, *there is a clearly increasing trend in water consumption over time.*

Building SARIMA model

To build a multiplicative seasonal autoregressive integrated moving average (SARIMA) model based on the Box Jenkins method (Box and Jenkins, 1970), the stages of this method for analyzing time series data are followed: identification, estimation, diagnostic checking and forecasting. For identification, the autocorrelation function (ACF) and partial autocorrelation function (PACF) are used for checking stationarity of the data series and determining if possible the values of p , P , q , Q . *Figures 2 and 3* display the results of ACF and PACF.

Figure 2. The ACF for the monthly water consumption data in Saudi Arabia between January 2010 and October 2016

The autocorrelations of the water consumption are significant for a large number of seasonal lags, but perhaps the autocorrelations at lags 12 and above are merely due to the propagation of the autocorrelation at lag 1.

Figure 3. The PACF for the monthly water consumption data in Saudi Arabia between January 2010 and October 2016

Figure 3 shows that there is a spike in the PACF at lags 12, but nothing at seasonal lags in the ACF in Figure 2. This may be suggestive of a seasonal AR(1) term. In the non-seasonal lags, there are three significant spikes in the PACF, suggesting a possible AR(1) term. Note that from Figure 1 it can be seen that the water consumption data needs differencing seasonally, i.e. $D = 1, d = 0$. The pattern in the ACF is not indicative of any simple model. Consequently, this initial analysis suggests that a possible model for this data is an ARIMA(1,0,q)(1,1,Q)[12].

Estimated autocorrelations

The estimated autocorrelations for the monthly water consumption data in Saudi Arabia are shown in Table 2. Table 2 depicts the estimated autocorrelations between values of water consumption at various lags. The lag $k = 1, 2, \dots, 24$ autocorrelation coefficient measures the correlation between values of data at time t and time $t - k$, where $t = 1, 2, \dots, 82$. Also shown are 95.0% probability limits around 0. The lower and upper limits of the 95% probabilities are shown in Figure 2; these limits are computed by $0 \pm z_{\alpha/2} se(\hat{\eta}_i)$, where $z_{\alpha/2} = 1.96$ (at $\alpha = 5\%$) and $se(\hat{\eta}_i)$ is the standard error of the corresponding autocorrelation $\hat{\eta}_i$. If the probability limits at a particular lag do not contain the corresponding autocorrelation, there is a statistically significant correlation at that lag at the 95% confidence level.

Fitting SARIMA model

Choosing the best model will involve obtaining some important values in fitting like the root mean squared error (RMSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE), the mean percentage error (MPE) and Akaike Information Criterion (AIC), which will be compared to their corresponding values when applying the following seasonal ARIMA models:

$$(A) \text{ ARIMA}(1,0,1) \times (1,1,2)_{12},$$

(B) ARIMA(1,1,0) $\times$ (1,1,2)₁₂,

(C) ARIMA(2,1,0) $\times$ (1,1,2)₁₂,

(D) ARIMA(0,1,2) $\times$ (1,1,2)₁₂,

(E) ARIMA(1,1,1) $\times$ (1,1,2)₁₂.

Table 2. The estimated autocorrelations between values of data at various lags

Lag	Autocorrelation (r_i)	Std. error (se)	Lower 95.0% Prob. limit	Upper 95.0% Prob. limit
1	0.874044	0.110432	-0.216442	0.216442
2	0.830243	0.175579	-0.34413	0.34413
3	0.73833	0.218267	-0.427796	0.427796
4	0.653835	0.246853	-0.483823	0.483823
5	0.618248	0.267139	-0.523583	0.523583
6	0.539642	0.284052	-0.556734	0.556734
7	0.572324	0.296291	-0.580722	0.580722
8	0.551189	0.30948	-0.60657	0.60657
9	0.584533	0.321228	-0.629597	0.629597
10	0.61312	0.333948	-0.654527	0.654527
11	0.584835	0.347405	-0.680902	0.680902
12	0.63271	0.359211	-0.704041	0.704041
13	0.515042	0.372554	-0.730193	0.730193
14	0.468674	0.381138	-0.747018	0.747018
15	0.377864	0.388103	-0.760669	0.760669
16	0.301906	0.392563	-0.769412	0.769412
17	0.270082	0.395385	-0.774942	0.774942
18	0.198321	0.397628	-0.779339	0.779339
19	0.228487	0.398833	-0.781699	0.781699
20	0.218865	0.400426	-0.784822	0.784822
21	0.251958	0.401882	-0.787676	0.787676
22	0.277167	0.403804	-0.791443	0.791443
23	0.250263	0.406117	-0.795977	0.795977
24	0.29124	0.407994	-0.799655	0.799655

Table 3 represents the comparison results of fitting different models. The model with the lowest value of the Akaike Information Criterion (AIC) is model (A). In addition, the model ARIMA(1,0,1) $\times$ (1,1,2)₁₂ has the lowest values in RMSE, MPE and AIC. Therefore, the model ARIMA(1,0,1) $\times$ (1,1,2)₁₂ is the best selected model. Table 4 also summarizes the performance of the currently selected model in fitting the historical data. The parameter estimates of the best model are also shown in Table 4.

Table 3. A comparison fitting models

Model	RMSE	MAE	MAPE	MPE	AIC
(A)	2.87793E6	2.20079E6	0.954075	-0.0270854	29.8671
(B)	2.91554E6	2.26702E6	0.978276	0.0428601	29.8687
(C)	2.88255E6	2.1878E6	0.9434	0.0419334	29.8703
(D)	2.89092E6	2.22338E6	0.959992	0.0608748	29.8761
(E)	2.8966E6	2.20382E6	0.952618	0.0371123	29.8800

Table 4. The parameter estimates of $ARIMA(1,0,1) \times (1,1,2)_{12}$

Parameter	Estimate	Std. error	T	P-value
ϕ_1	1.00174	0.001590	629.884	0.000000
θ_1	0.470192	0.108953	4.31553	0.000056
Φ_1	-1.24844	0.106268	-11.748	0.000000
Θ_1	-0.459525	0.128718	-3.57001	0.000678
Θ_2	0.897565	0.115493	7.77157	0.000000

The equation of the $ARIMA(1,0,1) \times (1,1,2)_{12}$ is given as below(Eqs. 5-7). If

$$(1 - \phi_1\beta)(1 - \Phi_1\beta^{12})(1 - \beta^{12})WC_t = (1 - \theta_1\beta)(1 - \Theta_1\beta^{12} - \Theta_2\beta^{24})e_t, \quad (\text{Eq.5})$$

then

$$WC_t = \phi_1 WC_{t-1} + (\Phi_1 + 1)WC_{t-12} - \phi_1(\Phi_1 + 1)WC_{t-13} - \Phi_1 WC_{t-24} + \phi_1 \Phi_1 WC_{t-25} + (\theta_1 + \Theta_1)e_{t-12} - (\Theta_2 - \theta_1 \Theta_1)e_{t-24} + \theta_1 \Theta_2 e_{t-36} + e_t, \quad (\text{Eq.6})$$

$$WC_t = 1.00174 WC_{t-1} - 0.24844 WC_{t-12} + 0.31062 WC_{t-13} + 1.24844 WC_{t-24} - 1.25061 WC_{t-25} + 0.01067 e_{t-12} - 1.11363 e_{t-24} + 0.21606 e_{t-36} + e_t. \quad (\text{Eq.7})$$

Forecasting

We used Equation 7 to forecast data from November 2016 to July 2017; the forecasting results are given in Table 5.

Table 5. Forecasting data from November 2016 to July 2017

Period	Original (m ³)	Forecast (m ³)
11/16	261071635.00	2.63039E8
12/16	262529383.00	2.67532E8
1/17	264786946.00	2.62222E8
2/17	231301561.00	2.39871E8
3/17	268909016.00	2.64531E8
4/17	269628773.00	2.65293E8
5/17	285437797.00	2.79714E8
6/17	269723694.00	2.78574E8
7/17	273197966.00	2.90016E8

The root mean square error between original data and the forecasted data is calculated, that is $RMSE = 7751794.934$.

From *Figure 4* it can be seen that there is a good agreement between the original and forecasted data. Therefore, we conclude that the fitted model, $ARIMA(1,0,1) \times (1,1,2)_{12}$, is appropriate for forecasting.

Figure 4. Forecast values based on the SARIMA model (green) and the original values (blue)

Conclusions

This research aimed is to understand current household water consumption by analyzing the current behaviour and water consumption patterns in Saudi Arabia, and to build a model that will encourage sustainable consumption and conservation of water resources. In this paper, we present a different time series model for fitting the monthly household water consumption data in Saudi Arabia between January 2010 and October 2016. Based on the data analyses and comparisons of five SARIMA models, the result indicated that the suitable and effective model for representing the household water consumption data of the time series is the multiplied seasonal model $ARIMA(1,0,1) \times (1,1,2)_{12}$. This model is used then to forecast the monthly water consumption in Saudi Arabia from the period November 2016 to July 2017. The forecasted values show the harmony with its counterparts in the original series values. This result provides a future image of the reality of monthly water demand in Saudi Arabia. Therefore, the officials and decision-makers can adopt the results of this study to face monthly water demand in Saudi Arabia (water scarcity). In particular, the identified multiplicative SARIMA model can be employed by the Ministry of Environment, Water and Agriculture in Saudi Arabia to ensure sustainable water resources management in the basin.

For future study, new modern statistics such as functional time series or fractional time series may be used in analyzing the water consumption data in Saudi Arabia, and the results can be then compared to the SARIMA model and its findings.

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APPENDIX

*Monthly water consumption data in Saudi Arabia between January 2010 and July 2017
(Ministry of Environment, Water and Agriculture, Saudi Arabia)*

Year	Month	Water consumption (m ³)	Year	Month	Water consumption (m ³)
2010	1	181939098	2014	1	224952954
	2	160540706		2	208397220
	3	184361371		3	234749774
	4	182276377		4	225968489
	5	190071038		5	249799599
	6	191941944		6	245316101
	7	201353276		7	253530687
	8	201911967		8	250959716
	9	197210257		9	245723978
	10	201739765		10	250966000
	11	195916912		11	241421651
	12	197488790		12	246410894
2011	1	191445402	2015	1	239995376
	2	173252570		2	218625646
	3	197589877		3	242120060
	4	193327291		4	239764261
	5	208026292		5	256585620
	6	204244441		6	256445988
	7	213421578		7	263626853
	8	222289987		8	265025393
	9	205258673		9	260071576
	10	214452752		10	269842591
	11	202692522		11	256272921

	12	207495739		12	257284729
2012	1	204710434	2016	1	255794090
	2	185681806		2	240752352
	3	204947388		3	259296870
	4	200254874		4	251720038
	5	217412155		5	266836225
	6	212579186		6	263387877
	7	220756924		7	272165315
	8	220619235		8	268817999
	9	212082770		9	261302224
	10	218817360		10	270277919
	11	209670441		11	261071635
	12	213826019		12	262529383
2013	1	212915242	2017	1	264786946
	2	188238373		2	231301561
	3	213767580		3	268909016
	4	217117937		4	269628773
	5	230191686		5	285437797
	6	228494648		6	269723694
	7	244452775		7	273197966
	8	244873159			
	9	237947752			
	10	242969366			
	11	228361039			
	12	228428423			