UDC: 577.3.01; 577.38 SOME EXACT AND NUMERICAL SOLUTION OF THE PROBLEM OF KOLMOGOROV-FISHER TYPE BIOLOGICAL POPULATION TASK WITH DOUBLE NONLINEAR DIFFUSION

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Abstract

In this paper, we study the global solvability of a task of Kolmogorov-Fisher type biological population task with double nonlinear diffusion and qualitative properties of the solution of the task based on the self-similar analysis.

Keywords: double nonlinearity, cross-diffusion, biological population, parabolic system of quasilinear equations, convective transport, numerical solution, iterative process, self-similar solutions.

1 INTRODUCTION

Population model are studied for a long time, first such work was performed by Fisher, Gause, and mathematical justification was performed by Kolmogorov, Petrovsky (KPP) and Piskunov (1937) in famous work (see. [8] and references therein). They were interested in the behavior of the velocity wave solutions and the resulting estimate of the speed of wave propagation. Then there were appear other models of the population. In recent years, were studied intensively nonlinear model taking into account diffusion and revealed new properties of the finite speed of propagation of diffusive waves (see [3] and the literature cited therein). We have proposed a model, which is described by a nonlinear system of competing species. Identified new properties such as the finite speed of propagation, localization outbreaks in a specific area. In particular, in the critical case the rate type KPP generalizes their result.

Consider in the field $Q = \{(t, x) : 0 < t, x \in R\}$ class of parabolic systems of two quasilinear equations of reaction-diffusion with double nonlinear diffusion [1-9]

$$\begin{cases} \frac{\partial u_1}{\partial t} = \operatorname{div}(D_1 u_2^{m_1-1} |\nabla u_1|^{p-2} \nabla u_1) + f_{\beta_1}(u_1) \\ \frac{\partial u_2}{\partial t} = \operatorname{div}(D_2 u_1^{m_2-1} |\nabla u_2|^{p-2} \nabla u_2) + f_{\beta_2}(u_2) \end{cases}$$

$$u_1|_{t=0} = u_{10}(x), \quad u_2|_{t=0} = u_{20}(x),$$

which describes the process of Kolmogorov-Fisher type biological population in a nonlinear two-component environment at

$$\begin{aligned} f_{\beta_1}(u_1) &= k_1 t u_1 (1 - u_1^{\beta_1}), \\ f_{\beta_2}(u_2) &= k_2 u_2 (1 - u_2^{\beta_2}), \quad \text{where} \quad k_1 = 1/\beta_1, \\ k_2 &= 1/\beta_2 \quad [10] \text{ and} \end{aligned}$$

$$\begin{cases} \frac{\partial u_1}{\partial t} = \frac{\partial}{\partial x} (D_1 u_2^{m_1-1} \left| \frac{\partial u_1}{\partial x} \right|^{p-2} \frac{\partial u_1}{\partial x}) + k_1 u_1 (1 - u_1^{\beta_1}) \\ \frac{\partial u_2}{\partial t} = \frac{\partial}{\partial x} (D_2 u_1^{m_2-1} \left| \frac{\partial u_2}{\partial x} \right|^{p-2} \frac{\partial u_2}{\partial x}) + k_2 u_2 (1 - u_2^{\beta_2}) \end{cases}, \quad (1)$$

Diffusion coefficient of which is equal to $D_1 u_2^{m_1-1} \left| \frac{\partial u_1}{\partial x} \right|^{p-2}$,

$$D_2 u_1^{m_2-1} \left| \frac{\partial u_2}{\partial x} \right|^{p-2}, \quad \text{where } m_1, m_2, p, \beta_1, \beta_2 - \text{positive}$$

real numbers, $u_1 = u_1(t, x) \geq 0, \quad u_2 = u_2(t, x) \geq 0$ - desired solution. At $\beta_i \rightarrow 0$

$$\lim_{\beta_i \rightarrow 0} f_{\beta_i}(u_i) = u_i \lim_{\beta_i \rightarrow 0} \left[\frac{1}{\beta_i} (1 - u_i^{\beta_i}) \right] = \bar{f}(u_i),$$

$$\bar{f}(u_i) = -u_i \ln u_i, \quad i = 1, 2 \quad [10].$$

Below we investigate the qualitative properties of the considered problem by constructing self-similar system of equations for (1).

2. SELF-SIMILAR SYSTEM OF EQUATIONS

Self-similar system of equations we will construct by the method of nonlinear splitting [1].

Substitution in (1)

$$u_1(t, x) = e^{k_1 t} v_1(t, x), \quad u_2(t, x) = e^{k_2 t} v_2(t, x),$$

Lead (1) to the form:

$$\begin{cases} \frac{\partial v_1}{\partial \tau_1} = \frac{\partial}{\partial x} (D_1 v_2^{m_1-1} \left| \frac{\partial v_1}{\partial x} \right|^{p-2} \frac{\partial v_1}{\partial x}) + k_1 e^{[(\beta_1 - p + 2)k_1 - (m_1 - 1)k_2]t} v_1^{\beta_1+1} \\ \frac{\partial v_2}{\partial \tau_2} = \frac{\partial}{\partial x} (D_2 v_1^{m_2-1} \left| \frac{\partial v_2}{\partial x} \right|^{p-2} \frac{\partial v_2}{\partial x}) + k_2 e^{[(\beta_2 - p + 2)k_2 - (m_2 - 1)k_1]t} v_2^{\beta_2+1} \end{cases} \quad (2)$$

$$v_1|_{t=0} = v_{10}(\eta), \quad v_2|_{t=0} = v_{20}(\eta).$$

Choosing

$$\tau = \frac{e^{[(m_1-1)k_2 + (p-2)k_1]t}}{(p-2)k_1 + (m_1-1)k_2} = \frac{e^{[(m_1-1)k_1 + (p-2)k_2]t}}{(p-2)k_2 + (m_2-1)k_1};,$$

obtain the following system of equations:

$$\begin{cases} \frac{\partial v_1}{\partial \tau} = \frac{\partial}{\partial x} \left(D_1 v_2^{m_1-1} \left| \frac{\partial v_1}{\partial x} \right|^{p-2} \frac{\partial v_1}{\partial x} \right) - a_1 \tau^{b_1} v_1^{\beta_1+1} \\ \frac{\partial v_2}{\partial \tau} = \frac{\partial}{\partial x} \left(D_2 v_1^{m_2-1} \left| \frac{\partial v_2}{\partial x} \right|^{p-2} \frac{\partial v_2}{\partial x} \right) - a_2 \tau^{b_2} v_2^{\beta_2+1} \end{cases} \quad (3)$$

Where

$$a_1(t) = k_1(t) \cdot [(p-2)k_1 + (m_1-1)k_2]^{b_1};$$

And $\tau = \tau(t)$ is chosen as

$$\begin{aligned} \tau(\tau) &= \int_0^\tau v_1^{-(p-2)}(t) v_2^{-(m_1-1)}(t) dt = \int_0^\tau v_2^{-(p-2)}(t) v_1^{-(m_2-1)}(t) dt = \\ &= \begin{cases} \frac{1}{1 - [\alpha_1(p-2) + \alpha_2(m_1-1)]} (T + \tau)^{-(\alpha_1(p-2) + \alpha_2(m_1-1)) + 1}, & \text{if } 1 - [\alpha_1(p-2) + \alpha_2(m_1-1)] \neq 0, \\ \ln(T + \tau), & \text{if } 1 - [\alpha_1(p-2) + \alpha_2(m_1-1)] = 0, \end{cases} \end{aligned}$$

if $\alpha_1(p-2) + \alpha_2(m_1-1) = \alpha_2(p-2) + \alpha_1(m_2-1)$.

Then for $w_i(\tau, x)$, $i = 1, 2$ we get the system of equations

$$b_1 = \frac{(\beta_1 - p + 2)k_1 - (m_1 - 1)k_2}{(p-2)k_1 + (m_1 - 1)k_2};$$

$$a_2 = k_2 \cdot [(p-2)k_2 + (m_2-1)k_1]^{b_2};$$

$$b_2 = \frac{(\beta_2 - p + 2)k_2 - (m_2 - 1)k_1}{(p-2)k_2 + (m_2 - 1)k_1};$$

With the aim of obtaining self-similar system for the system of equations (3) we find first the solution of a system of ordinary differential equations

$$\begin{cases} \frac{d\bar{v}_1}{d\tau} = -a_1 \tau^{b_1} \bar{v}_1^{\beta_1+1}, \\ \frac{d\bar{v}_2}{d\tau} = -a_2 \tau^{b_2} \bar{v}_2^{\beta_2+1}, \end{cases}$$

In the form

$$\bar{v}_1(\tau) = \tau^{-\alpha_1}, \quad \bar{v}_2(\tau) = \tau^{-\alpha_2},$$

Where

$$\alpha_1 = \frac{b_1 + 1}{\beta_1}, \quad \alpha_2 = \frac{b_2 + 1}{\beta_2}.$$

Then the solution of system (3) is sought in the form

$$v_1(t, x) = \bar{v}_1(t) w_1(\tau, x), \quad v_2(t, x) = \bar{v}_2(t) w_2(\tau, x) \quad (4)$$

$$\begin{cases} \frac{\partial w_1}{\partial \tau} = \frac{\partial}{\partial x} \left(D_1 w_2^{m_1-1} \left| \frac{\partial w_1}{\partial x} \right|^{p-2} \frac{\partial w_1}{\partial x} \right) + \psi_1 (w_1^{\beta_1+1} - w_1) \\ \frac{\partial w_2}{\partial \tau} = \frac{\partial}{\partial x} \left(D_2 w_1^{m_2-1} \left| \frac{\partial w_2}{\partial x} \right|^{p-2} \frac{\partial w_2}{\partial x} \right) + \psi_2 (w_2^{\beta_2+1} - w_2) \end{cases}, \quad (5)$$

Where

$$\psi_i = \frac{1}{(1 - [\alpha_i(p-2) + \alpha_{3-i}(m_i-1)])\tau}, \quad i = 1, 2 \quad (6)$$

Consider the self-similar solution of the system (5) of the form

$$w_1(t, x) = f_1(\xi), \quad w_2(t, x) = f_2(\xi), \quad \xi = |x| / (T + \tau) \quad (7)$$

Then substituting (7) into (5) with respect to $f_1(\xi), f_2(\xi)$ obtain the following system of nonlinear degenerate self-similar equations:

$$\begin{cases} \frac{d}{d\xi} (f_2^{m_1-1} \left| \frac{df_1}{d\xi} \right|^{p-2} \frac{df_1}{d\xi}) + \frac{\xi}{p} \frac{df_1}{d\xi} + \theta_1 (f_1 - f_1^{\beta_1+1}) = 0, \\ \frac{d}{d\xi} (f_1^{m_2-1} \left| \frac{df_2}{d\xi} \right|^{p-2} \frac{df_2}{d\xi}) + \frac{\xi}{p} \frac{df_2}{d\xi} + \theta_2 (f_2 - f_2^{\beta_2+1}) = 0, \end{cases} \quad (8)$$

Where

$$\theta_i = \frac{1}{(1 - [\alpha_i(p-2) + \alpha_{3-i}(m_i-1)])\tau}$$

Let us build an upper solution for system (8).

3. CONSTRUCTION OF UPPER SOLUTIONS

If

$$\beta_i = [(p-2)^2 - (m_1-1)(m_2-1)] / [(p-1)(p-(m_i+1))]$$

$$p > 2 + \sqrt{(m_1-1)(m_2-1)}, \quad i = 1, 2, \quad \text{equation (8)}$$

has a local solution

$$\bar{f}_1(\xi) = A(a - \xi^\gamma)_+^{n_1}, \quad \bar{f}_2(\xi) = B(a - \xi^\gamma)_+^{n_2},$$

Where

$$(b)_+ = \max(0, b), \quad \gamma = p / (p-1),$$

$$n_1 = \frac{(p-1)(p-(m_1+1))}{(p-2)^2 - (m_1-1)(m_2-1)};$$

$$n_2 = \frac{(p-1)(p-(m_2+1))}{(p-2)^2 - (m_1-1)(m_2-1)}.$$

Then in the field Q according to the comparison principle of solutions [1,8] we have

Theorem 1: Let $u_i(0, x) \leq u_{i\pm}(0, x), x \in R$.

Then for the solution of the problem (1) in the domain Q we have the estimate

$$u_1(t, x) \leq u_{1+}(t, x) = e^{k_1 t} \tau^{-\alpha_1} \bar{f}_1(\xi),$$

$$u_2(t, x) \leq u_{2+}(t, x) = e^{k_2 t} \tau^{-\alpha_2} \bar{f}_2(\xi),$$

$$\xi = |x| / \tau^{1/p}$$

where $\bar{f}_1(\xi), \bar{f}_2(\xi)$ and $\tau(t)$ - certain of the above functions.

Similarly, [9] permanent α can be found from the condition

$$\int_{-\infty}^{\infty} E_{p_1}(x, \tau) dx = P_1, \quad \int_{-\infty}^{\infty} E_{p_2}(x, \tau) dx = P_2,$$

Where

$$E_{p_1}(x, \tau) = \tau^{-\mu_1} (a - \xi_1^\gamma)_+^{n_1}, \quad E_{p_2}(x, \tau) = \tau^{-\mu_2} (a - \xi_2^\gamma)_+^{n_2}$$

$$\xi_i = |x| / \tau^{1/\mu_i},$$

$$\mu_i = m_i + 2p - 3, \quad i = 1, 2.$$

For this we prove the following theorem:

Theorem 2: If $E_{P_i}(x, \tau)$, $i=1,2$ is solution to the following system

$$\begin{cases} \frac{\partial u_1}{\partial \tau} = \frac{\partial}{\partial x} \left(D_1 u_2^{m_1-1} \left| \frac{\partial u_1}{\partial x} \right|^{p-2} \frac{\partial u_1}{\partial x} \right), \\ \frac{\partial u_2}{\partial \tau} = \frac{\partial}{\partial x} \left(D_2 u_1^{m_2-1} \left| \frac{\partial u_2}{\partial x} \right|^{p-2} \frac{\partial u_2}{\partial x} \right), \end{cases}$$

Then for solution of the system (1) is true the following expression

$$\lim_{\tau \rightarrow \infty} |u_1(x, \tau) - E_{P_1}(x, \tau)| = 0,$$

$$\lim_{\tau \rightarrow \infty} |u_2(x, \tau) - E_{P_2}(x, \tau)| = 0 \quad (9)$$

In the set of

$$\{x \in R : |x| < c\tau^{1/\mu}, c = \max\{c_1, c_2\} > 0\}.$$

Proof.

Let

$$u_i(x, \tau), i=1,2$$

Are solution of the following system

$$\begin{cases} \frac{\partial u_1}{\partial \tau} = \frac{\partial}{\partial x} \left(D_1 u_2^{m_1-1} \left| \frac{\partial u_1}{\partial x} \right|^{p-2} \frac{\partial u_1}{\partial x} \right) - a_1 \tau^{b_1} k^{\beta_1+3-m_1-2p} u_1^{\beta_1+1}, \\ \frac{\partial u_2}{\partial \tau} = \frac{\partial}{\partial x} \left(D_2 u_1^{m_2-1} \left| \frac{\partial u_2}{\partial x} \right|^{p-2} \frac{\partial u_2}{\partial x} \right) - a_2 \tau^{b_2} k^{\beta_2+3-m_2-2p} u_2^{\beta_2+1}. \end{cases}$$

From theorem 1 the family of solutions

$$u_{ik}(x, \tau) = k u_i(kx, k^{\mu_i} \tau), i=1,2$$

is a compact set, and

$$u_{ik}(x, 1) = k u_i(kx, k^{\mu_i}) = \left[a - |x|^{\frac{p}{p-1}} \right]^{\frac{(p-1)(p-(m_i+1))}{(p-2)^2-(m_1-1)(m_2-1)}} = E_{P_i}(x, 1). i=1,2 \quad (10)$$

Be the solution of system (1). We define the family of functions

$$u_{ik}(x, \tau) = k u_i(kx, k^{\mu_i} \tau), i=1,2$$

$$k > 0.$$

$$\begin{cases} k^{\mu_1+1} \frac{\partial u_1}{\partial \tau} = k^{m_1+2p-2} \frac{\partial}{\partial x} \left(D_1 u_2^{m_1-1} \left| \frac{\partial u_1}{\partial x} \right|^{p-2} \frac{\partial u_1}{\partial x} \right) + a_1 \tau^{b_1} k^{\beta_1+1} u_1^{\beta_1+1} \\ k^{\mu_2+1} \frac{\partial u_2}{\partial \tau} = k^{m_2+2p-2} \frac{\partial}{\partial x} \left(D_2 u_1^{m_2-1} \left| \frac{\partial u_2}{\partial x} \right|^{p-2} \frac{\partial u_2}{\partial x} \right) + a_2 \tau^{b_2} k^{\beta_2+1} u_2^{\beta_2+1} \end{cases}$$

If

$$\mu_i = m_i + 2p - 3, i=1,2,$$

Then

$$u_{ik}(x, \tau) = k u_i(kx, k^{\mu_i} \tau), i=1,2$$

If we take $kx = x'$, $k^\mu = \tau'$,

Then

$$x = \frac{x'}{k}, \quad k = \tau'^{\frac{1}{\mu}}$$

and taking into account (10)

$$\tau'^{\frac{1}{\mu_i}} u_i(x', \tau') = E_{P_i} \left(\frac{x'}{k}, 1 \right) = \tau'^{\frac{1}{\mu_i}} \tau'^{-\frac{1}{\mu_i}} \left[a - \left(\frac{k|x|}{k} \right)^{\frac{p}{p-1}} \right]^{\frac{(p-1)(p-(m_i+1))}{(p-2)^2 - (m_1-1)(m_2-1)}} = \tau'^{\frac{1}{\mu_i}} E_{P_i}(x', \tau') \quad i = 1, 2 \quad (11)$$

Because $\tau'^{\frac{1}{\mu_i}} \neq 0$ from (11)

$$u_i(x', \tau') = E_{P_i}(x', \tau') \quad i = 1, 2$$

Thus, in a compact set holds following expression

$$\lim_{\tau \rightarrow \infty} |u_1(x, \tau) - E_{P_1}(x, \tau)| = 0, \\ \lim_{\tau \rightarrow \infty} |u_2(x, \tau) - E_{P_2}(x, \tau)| = 0.$$

Theorem 2 is proved.

In view of theorem 2, we define a in the following way

If the condition (9), and is defined as follows:

$$a = (P_1 \gamma / B(1/\gamma, 1 + n_1))^{\frac{\gamma}{n_1}} = (P_2 \gamma / B(1/\gamma, 1 + n_2))^{\frac{\gamma}{n_2}}.$$

Because

$$\begin{cases} \tau^{-\frac{1}{\mu_1}} \int_{-\infty}^{\infty} (a - \xi_1^\gamma)_+^{n_1} dx = P_1 \\ \tau^{-\frac{1}{\mu_2}} \int_{-\infty}^{\infty} (a - \xi_2^\gamma)_+^{n_2} dx = P_2 \end{cases}$$

$$\xi_i = \frac{x}{\tau^{1/\mu_i}}, \quad \left[\frac{x}{\tau^{1/\mu_i}} \right]^{\frac{p}{p-1}} = a\eta,$$

$$x = a^{\frac{p-1}{p}} \tau^{\mu_i} \eta^{\frac{p-1}{p}}, \quad dx = a^{\frac{p-1}{p}} \tau^{\mu_i} \frac{p-1}{p} \eta^{\frac{p-1}{p}-1} d\eta.$$

$$\begin{cases} \tau^{-\frac{1}{\mu_1}} \int_{-\infty}^{\infty} (a - \xi_1^\gamma)_+^{n_1} dx = a^{\frac{n_1}{\gamma}} \frac{1}{\gamma} \int_0^1 \eta^{\frac{1}{\gamma}-1} (1-\eta)^{n_1} d\eta = a^{\frac{n_1}{\gamma}} \frac{1}{\gamma} B\left(\frac{1}{\gamma}, 1+n_1\right) = P_1 \\ \tau^{-\frac{1}{\mu_2}} \int_{-\infty}^{\infty} (a - \xi_2^\gamma)_+^{n_2} dx = a^{\frac{n_2}{\gamma}} \frac{1}{\gamma} \int_0^1 \eta^{\frac{1}{\gamma}-1} (1-\eta)^{n_2} d\eta = a^{\frac{n_2}{\gamma}} \frac{1}{\gamma} B\left(\frac{1}{\gamma}, 1+n_2\right) = P_2 \end{cases}$$

Here

$$a = [P_1 \gamma / B(\frac{1}{\gamma}, 1+n_1)]^{\frac{\gamma}{n_1}} = [P_2 \gamma / B(\frac{1}{\gamma}, 1+n_2)]^{\frac{\gamma}{n_2}}.$$

Obtained computational experiments and numerical results.

4. CONCLUSION

Thus, the adequate study of nonlinear equations, biological populations with double-linearity based on the method of nonlinear splitting and numerical study of nonlinear processes described by equations with a double nonlinearity and analysis of the results, based on the obtained estimates of the solutions gives a comprehensive picture of the process of multicomponent systems competing biological population.

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$$\begin{cases} \frac{\partial u}{\partial t} = \operatorname{div}(D_1 v^{m_1-1} |\nabla u|^{p-2} \nabla u) + k_1 u(1-u^{\beta_1}), \\ \frac{\partial v}{\partial t} = \operatorname{div}(D_2 u^{m_2-1} |\nabla v|^{p-2} \nabla v) + k_2 v(1-v^{\beta_2}), \end{cases}$$

1. Fast Diffusion: As initial approximation we must take:

$$u_0(x, t) = (T+t)^{-\alpha_1} (a + \xi^\gamma)^{n_1},$$

$$v_0(x, t) = (T+t)^{-\alpha_2} (a + \xi^\gamma)^{n_2},$$

$$\xi = \frac{|x|}{\tau^{\frac{1}{p}}}, \quad \gamma = \frac{p}{p-1},$$

$$n_i = \frac{(p-1)[p-(m_i+1)]}{q},$$

$$i = 1, 2, \quad q = (p-2)^2 - (m_1-1)(m_2-1),$$

Parameter values must be $n_1 > 0, n_2 > 0, q < 0$ The constant a is determined from the condition

$$\int_{-\infty}^{\infty} u_1(x, 0) dx = P_1,$$

$$\int_{-\infty}^{\infty} u_2(x, 0) dx = P_2:$$

$$a = \left(P_1 \gamma / B\left(\frac{1}{\gamma}, 1+n_1\right) \right)^{\frac{\gamma}{n_1}} = \left(P_2 \gamma / B\left(\frac{1}{\gamma}, 1+n_2\right) \right)^{\frac{\gamma}{n_2}},$$

$$k_1 = 1/\beta_1, \quad k_2 = 1/\beta_2$$

Parameter values	$x_1 = 1; x_2 = 1;$ $ x = \sqrt{2}$	$x_1 = 2; x_2 = 2;$ $ x = 2\sqrt{2}$	$x_1 = 3; x_2 = 3;$ $ x = 3\sqrt{2}$	$x_1 = 4; x_2 = 4;$ $ x = 4\sqrt{2}$	$x_1 = 5; x_2 = 5;$ $ x = 5\sqrt{2}$
$m_1 = 2.5, m_2 = 2.4,$ $eps = 10^{-3}$ $n_1 = 0.944 > 0$ $n_2 = 0.708 > 0$ $q = -0.89 < 0$ $\beta_1 = 1, \beta_2 = 1$
$m_1 = 3.7, m_2 = 3.8,$ $eps = 10^{-3}$ $n_1 = 0.59 > 0$ $n_2 = 0.674 > 0$ $q = -3.56 < 0$ $\beta_1 = 2, \beta_2 = 2$
$m_1 = 4.1, m_2 = 4.0,$ $eps = 10^{-3}$ $n_1 = 0.672 > 0$ $n_2 = 0.576 > 0$ $q = -3.54 < 0$ $\beta_1 = 1, \beta_2 = 1$
$m_1 = 3.7, m_2 = 3.3, p = 4$ $eps = 10^{-3}$ $n_1 = 0.95 > 0$ $n_2 = 0.407 > 0$ $q = -2.21 < 0$ $\beta_1 = 2, \beta_2 = 2$

2. Slow Diffusion: As initial approximation should be taken

$$q = (p - 2)^2 - (m_1 - 1)(m_2 - 1)$$

$$u_0(x, t) = (T + t)^{-\alpha_1} (a - \xi^\gamma)_+^{n_1},$$

$$v_0(x, t) = (T + t)^{-\alpha_2} (a - \xi^\gamma)_+^{n_2},$$

$$\xi = \frac{|x|}{\tau^p}, \quad \gamma = \frac{p}{p-1}, \quad \tau, i = 1, 2,$$

Parameter values must be $n_1 > 0, n_2 > 0, q_i > 0$, The constant a is determined from the condition :

$$a = \left(P_1 \gamma / B\left(\frac{1}{\gamma}, 1 + n_1\right) \right)^{\frac{\gamma}{n_1}} = \left(P_2 \gamma / B\left(\frac{1}{\gamma}, 1 + n_2\right) \right)^{\frac{\gamma}{n_2}},$$

$$k_1 = 1/\beta_1, \quad k_2 = 1/\beta_2$$