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MATHEMATICAL MODELING AND EXPERIMENTAL STUDY OF FORMING AND RELAXATION OF RESIDUAL STRESSES IN PLANE SAMPLES MADE OF EP742 ALLOY AFTER ULTRASONIC HARDENING UNDER HIGH-TEMPERATURE CREEP CONDITIONS

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ABSTRACT

The authors carried out a complex (computational and experimental) research of residual stresses in prismatic samples made of EP742 alloy after ultrasonic hardening under temperature of 650 °C for 100 hours without load. The laws of residual stresses distribution over the thickness of the surface-hardened layer were discovered. The experiments proved that the ultrasonic hardening of the sample caused compression residual stresses in the surface layer. The maximum stresses were observed in the subsurface layer and they decreased on reaching the surface. When the temperature was exposed, the induced compression residual stresses became relaxed, the level of residual stresses decreased by 1.4-1.6 times and its maximum was displaced deep into the sample, but the thickness of the compressed layer was kept to about 200 micron.

We developed the mathematical model of residual stresses forming in prismatic samples after surface plastic deformation and relaxation under high-temperature creep conditions. We used the hardened half-space as a model object for the prismatic sample, because the hardened layer was rather thin. We introduced Cartesian coordinate system, where xOy plane coincided with the half-space hardened surface, and Oz axis was directed into the depth of the hardened layer.

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1. Introduction

Increase of the life time of construction members and parts is performed in the engineering practice in different directions: creation of new ultra-strong materials, modification and improvement of properties of existing materials and alloys by means of different alloying additives, design solutions at the engineering stage and other approaches. A special place in this set is held by the process methods of enhancement of operational characteristics of parts of various purpose related to the procedure of surface plastic deformation which does not lead to the increase of material intensity of finished parts. Typical industrial hardening technologies include surface hydraulic and air shot blasting, hardening with microballoons, rolling, thermoplastic hardening and some others. Positive influence of hardening technologies is related to formation of compressive residual stresses in the thin hardened layer which prevent emergence of various vacancies which, in their turn, are micro-sites of fracture of the part surface material. Efficiency of these methods for increase of fatigue resistance, wear resistance and micro-hardness of surface layers at normal temperatures was observed in a variety of papers of both Russian [1-6] and foreign [7-12] researchers.

The present paper focuses on theoretical and experimental research of ultrasonic hardening of plane parts with free microballoons which is performed in a special working chamber where under the influence of the forcing (controlled) ultrasonic field with small amplitude and frequency up to 10^5 Hz, microballoons are moving in a random way and are interacting with the item surface being hardened.

This method has taken a fixed place in process operations of part hardening of various purpose [6, 13-17]. However, all above mentioned works are strictly experimental and are aimed at determination of influence of the hardening type on characteristics of fatigue resistance, quality of the surface layer, change of the metal physical structure of material in the hardened layer of parts, construction of simple approximation dependences for experimental dependences of residual stresses.

In-depth theoretical research of the problem for evaluation of the stress-strain state in hardened cylindrical parts after the hardening procedure was performed in papers [18, 19], and further publications [20, 21] propose a mathematical model for calculation of residual stresses with consideration of effects of strain anisotropy in case of surface plastic hardening. Apparently, the lack of methods of calculation of the initial stress-strain state in hardened structures (after the hardening procedure) was the main constraint of the theoretical solution of the problem of relaxation of induced residual stresses in parts under high-temperature creep conditions, though the needs of, for example, aircraft engine building and power plant engineering encouraged purely experimental researches in this direction [22-28].

The publication [29] was the first one to present the direct numerical method of solution of the boundary problem of kinetics of residual stresses in a hardened cylindrical item under creep conditions, it was applied in paper [30] towards the problem of relaxation of residual stresses for a hardened cylinder under the action of tensile load, and in paper [31] - under the combined action of tensile axial load and torque. However, nowadays there is no theoretical approach for solution of problems of this kind for plain samples. It is this that is the aim of the present paper along with experimental examination of the corresponding method of solution of the boundary problem for plane samples made of EP742 alloy after ultrasonic hardening under high-temperature exposure at temperature 650 °C for 100 hours.

2. Method of Calculation of Residual Stresses and Plastic Deformations after the Hardening Procedure

We consider a prismatic sample (rectangular parallelepiped) one side of which was subjected to the procedure of surface plastic deformation. As a rule, the thickness of the hardened layer is 100-200 μm . Assume that values of three dimensions of the prismatic sample are by 2-3 orders of magnitude greater than thickness of the hardened layer. Then, the problem of the hardened half-space may be taken as a model one. We introduce the Cartesian coordinate system, bring plane xOy into coincidence with the hardened surface and direct axis Oz into the depth of the hardened layer.

Let us denote through σ_x , σ_y , σ_z and q_x , q_y , q_z diagonal tensor elements of residual stresses and residual plastic deformations after the hardening procedure. We neglect off-diagonal tensor elements of stresses and plastic deformations because their values, as a rule, are by an order of magnitude less than those of diagonal elements, which is really observed for a set of hardening technologies (thermoplastic hardening, hydraulic and air shot blasting, nitriding and other ones) including ultrasonic hardening which will be considered hereinafter. We will also assume that hardening was performed uniformly over the entire plane of the half-space.

With the imposed constraints, residual compression stresses and plastic deformations do not depend on x and y coordinates but depend only on z variable. Then, $\sigma_i = \sigma_i(z)$, $q_i = q_i(z)$ ($i = x, y, z$). Hereinafter, we will omit z variable in writing for the sake of brevity, wherever possible, and still we will always consider that $0 \leq z < +\infty$.

As the thickness of the hardened layer is only 100–200 μm , it can be considered as "glued" to the plane limiting the half-space. Then the plane section hypothesis may be introduced (xOz and yOz) for total deformations, and as $0 \leq z < +\infty$, it will be written mathematically in the following way:

$$\varepsilon_x(z) = 0, \quad \varepsilon_y(z) = 0, \quad (1)$$

where $\varepsilon_x(z)$ and $\varepsilon_y(z)$ are tensor elements of total deformations.

Let us introduce into consideration the hypothesis of anisotropic surface hardening in the form of

$$q_x = \alpha q_y, \quad (2)$$

where α – is a parameter of anisotropic hardening (phenomenological parameter), the method of determination of which in a cylindrical hardened sample for a wide range of hardening technologies is given in [20, 21] and may be used for plane samples with little to no fundamental changes.

Proceeding from the condition of plastic incompressibility $q_x + q_y + q_z = 0$ and (2), we obtain

$$q_z = -(1 + \alpha)q_y. \quad (3)$$

Considering that $\varepsilon_i(z) = e_i(z) + q_i(z)$ ($i = x, y, z$), where e_i are tensor components of elastic deformations, we will write down relations (1) in expanded form:

$$\begin{aligned} \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] + q_x &= 0, \\ \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] + q_y &= 0 \end{aligned} \quad (4)$$

where E is the Young modulus, ν is the Poisson ratio. In these relations $\sigma_z \equiv 0$ which results from the condition of equilibrium

$$\frac{\partial \sigma_z}{\partial z} = 0 \quad (\lim_{z \rightarrow +\infty} \sigma_z(z) = 0).$$

Therefore, from (2) and (4) we have

$$\sigma_y = \frac{1 + \alpha \nu}{\alpha + \nu} \sigma_x. \quad (5)$$

Now, it is not difficult to obtain the following formula from (2)-(5)

$$\begin{aligned} q_x &= -\frac{\alpha(1 - \nu^2)}{E(\alpha + \nu)} \sigma_x, \quad q_y = -\frac{1 - \nu^2}{E(\alpha + \nu)} \sigma_x, \\ q_z &= \frac{(1 + \alpha)(1 - \nu^2)}{E(\alpha + \nu)} \sigma_x \end{aligned} \quad (6)$$

Thus, it is resulted from formulas (5), (6) that if one knows a tensor element of residual stresses $\sigma_x = \sigma_x(z)$, $0 \leq z < +\infty$ and an anisotropy factor α , all tensor elements of residual stresses and plastic deformations are determined through these values. In other words, in order to determine the stress-strain state in the hardened half-space, it is enough to know the experimental $\sigma_x = \sigma_x(z)$ and the value α . In special cases, for example, in case of thermoplastic hardening, shot peening, ultrasonic surface hardening, the value $\alpha = 1$ [20, 21] and formulas (5), (6) are simplified:

$$\sigma_y = \sigma_x, \quad q_x = q_y = -\frac{1 - \nu}{E} \sigma_x, \quad q_z = \frac{2(1 - \nu)}{E} \sigma_x. \quad (7)$$

As the procedure of ultrasonic hardening is considered hereinafter, it is resulted from (7) that the value $\sigma_y(z)$ and values of residual plastic deformations $q_i(z)$ ($i = x, y, z$) after ultrasonic hardening are determined through the dependence $\sigma_x = \sigma_x(z)$. Hence, the following phenomenological method is proposed to determine the stress-strain state after hardening: value $\sigma_x = \sigma_x(z)$ is determined experimentally (on the basis of the strip method [32] in the present paper), and the other elements are found with the use of formulas (7).

3. Method of Identification of Approximation Parameters for Tensor Elements of Residual Stresses $\sigma_x = \sigma_x(z)$

In accordance with model approximation of the hardened prismatic sample with the hardened half-space, the diagram $\sigma_x = \sigma_x(z)$, considering the natural condition $\lim_{z \rightarrow +\infty} \sigma_x(z) = 0$ after the procedure of surface plastic hardening, shall be of the schematic form presented in Fig. 1. Still, one considers the case when maximum residual compression stresses are both in the subsurface layer ($z^* \neq 0$) and on the hardened surface ($z^* = 0$). However, the experimental method [32] allows to determine values $\sigma_x = \sigma_x(z)$ only in the thin hardened layer $0 \leq z \leq z_0$, i.e. in the area of material compression (refer to fig. 1). But in order to solve the problem of relaxation of residual stresses under creep conditions, it is necessary to have continuous fields of residual stresses and plastic deformations all over the whole integration domain (all over the whole half-space). For this reason, the first problem is approximation of the experimental curve $\sigma_x = \sigma_x(z)$ for all $0 \leq z < +\infty$. In other words, it is necessary to perform extrapolation from the compression area $0 \leq z \leq z_0$, where experimental values are determined, to the area $z_0 < z < +\infty$ but in such a way that the condition of self-equilibrium of residual stresses is fulfilled

$$\int_0^{+\infty} \sigma_x(z) dz = 0, \quad (8)$$

because no external forces are applied to the half-space.

Fig. 1. Schematic experimental curve $\sigma_x = \sigma_x(z)$

Let us choose $\sigma_x = \sigma_x(z)$ of the form of

$$\sigma_x(z) = \sigma_0 \exp\left(-\frac{(z-z^*)^2}{l^2}\right) - \sigma_1 \exp\left(-\frac{(z-z^*)^2}{b^2}\right), \quad (9)$$

where σ_0 , σ_1 and b are parameters to determine which (at a first approximation) one may use curve values $\sigma_x = \sigma_x(z)$ in representative points: $\sigma_x(0) = \sigma^*$, $\sigma_x(z^*) = \sigma_{\min}$, $\sigma_x(z_0) = 0$ and condition (8); parameter $l = 1$ and has the same dimensions of the length as z . As a result, considering (9) we obtain the following system of equations:

$$\begin{aligned} \sigma_0 \exp(-z^{*2}) - \sigma_1 \exp(-z^{*2}/b^2) &= \sigma^*, \\ \sigma_0 - \sigma_1 &= \sigma_{\min}, \\ \sigma_0 \exp(-(z_0 - z^*)^2) - \sigma_1 \exp(-(z_0 - z^*)^2/b^2) &= 0, \quad (10) \\ \int_0^{+\infty} \sigma_x(z) dz &= 0. \end{aligned}$$

The system of equations (10) is superdefinite (3 parameters and 4 equations). In order to eliminate superdefiniteness, one may withdraw from strict fulfillment of one of conditions for curve $\sigma_x = \sigma_x(z)$, for example, its passing through point $(z_0, 0)$. Then, dropping the third relation in (10) and substituting (9) into (8), we will get the system of three equations in three unknowns σ_0 , σ_1 and b :

$$\begin{aligned} \sigma_0 \exp(-z^{*2}) - \sigma_1 \exp(-z^{*2}/b^2) &= \sigma^*, \\ \sigma_0 - \sigma_1 &= \sigma_{\min}, \\ \sigma_0 \int_0^{+\infty} \exp(-(z - z^*)^2) dz - & \quad (11) \\ - \sigma_1 \int_0^{+\infty} \exp(-(z - z^*)^2/b^2) dz &= 0, \end{aligned}$$

which is solved numerically with the use of function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt.$$

Adjustment of σ_0 , σ_1 and b parameter values (the second approximation) is performed as a result of minimization of deviations of estimated data $\sigma_x(z_i)$ from experimental data $\sigma_x^3(z_i)$ in the following norm:

$$\Delta = \sqrt{\frac{\sum_{i=1}^n [\sigma_x(z_i) - \sigma_x^3(z_i)]^2}{\sum_{i=1}^n [\sigma_x^3(z_i)]^2}}, \quad (12)$$

where z_i are discretization points, n is the number of discretization points of dependence $\sigma_x = \sigma_x(z)$. For this purpose, starting from the first approximation, σ_0 , σ_1 values vary, and value b is determined from the third

equation (11) (in doing so, self-equilibrium of curve $\sigma_x = \sigma_x(z)$ is preserved) until the functional (12) reaches its minimum. According to the same formula, the value of deviation of estimated data from experimental ones (approximation error) is calculated on a percentage basis.

Thus, after identification of approximation parameters (9), value $\sigma_x = \sigma_x(z)$ will have analytic representation for all $0 \leq z < +\infty$, meaning that the other tensor elements of residual stresses and plastic deformations will also have analytic representations in accordance with formulas (5), (6) at $\alpha \neq 1$ or (7) at $\alpha = 1$.

4. Method of Calculation of Kinetics of Stresses in the Surface Hardened Half-Space under Creep Conditions

Let us consider the problem of calculation of kinetics of residual stresses induced in the process of surface plastic deformation of the half-space under material high-temperature creep conditions in the absence of external forces. Hereinafter, we will refer to such a mode as thermal exposure (holding at high temperature at no load). As initial conditions of the boundary problem, we use the stress-strain state which emerges after the hardening procedure and was formed at the initial moment of time $t = 0 - 0$ at temperature T_1 (as a rule, ambient temperature). Let us assume that at $t = 0$ instantaneous warming-up of the half-space happens from temperature T_1 to temperature T_2 ($T_2 > T_1$) at which the process of material creep emerges. Let us denote through E_1 the Young modulus at temperature T_1 , and through E_2 at temperature T_2 . We also assume that new plastic deformations do not emerge during warming-up and they are determined by equations (6) at $E = E_1$. Hereinafter, thermal deformations are not taken into account as they give only uniform increase in volume and do not lead to changing residual stresses by virtue of uniform warming-up of a body. Hereinafter, we will consider the general case of anisotropic hardening $\alpha \neq 1$.

At the moment of time $t = 0 - 0$ at temperature T_2 by virtue of preservation of the plane section hypothesis (1) and permanence of tensor elements of residual plastic deformations, we have the following distribution for stresses:

$$\sigma_x(z) = -\frac{E_2(\alpha + \nu)}{1 - \nu^2} q_y(z), \quad \sigma_y(z) = \frac{1 + \nu\alpha}{\alpha + \nu} \sigma_x(z).$$

During the time period $t \in [0, t^*]$, let the sample be held at temperature T_2 at which creep strains emerge. Under the action of self-equilibrated stresses, the creep strain the components of which are denoted through $p_j = p_j(z, t)$ ($j = x, y, z$) will accumulate in the half-space. Then we have

$$\varepsilon_x(z, t) = \frac{1}{E_2} [\sigma_x(z, t) - \nu \sigma_y(z, t)] + q_x(z) + p_x(z, t) = 0,$$

$$\varepsilon_y(z, t) = \frac{1}{E_2} [\sigma_y(z, t) - \nu \sigma_x(z, t)] + q_y(z) + p_y(z, t) = 0. \quad (13)$$

The strain component $\varepsilon_z(z, t)$ is "passive" and, with regard to incompressibility of creep strain $p_x + p_y + p_z = 0$, is determined through the relation

$$\varepsilon_z(z, t) = -\frac{\nu}{E_2} [\sigma_x(z, t) + \sigma_y(z, t)] + q_z(z) - (p_x(z, t) + p_y(z, t)).$$

By solving the system of equations (13) in $\sigma_x(z, t)$ and $\sigma_y(z, t)$, we obtain relations describing kinetics of these stresses in time due to the creep:

$$\begin{aligned} \sigma_x(z, t) &= \frac{E_2}{\nu^2 - 1} [q_x(z) + p_x(z, t) + \nu(q_y(z) + p_y(z, t))], \\ \sigma_y(z, t) &= \frac{E_2}{\nu^2 - 1} [q_y(z) + p_y(z, t) + \nu(q_x(z) + p_x(z, t))]. \end{aligned} \quad (14)$$

Thus, if values of tensor elements of creep strains $p_x(z, t)$ and $p_y(z, t)$ are known, values $\sigma_x(z, t)$ and $\sigma_y(z, t)$ are determined through (14).

Values $p_x(z, t)$ and $p_y(z, t)$ are calculated numerically with time increments on the basis of the selected creep theory which will be described further. The principle of the method is as follows. Let us assume that time discretization $0 = t_0 < t_1 < \dots < t_n = t^*$ in increments of $\Delta t_i = t_{i+1} - t_i$ ($i = 0, 1, \dots, n-1$) is performed, and we know values $p_j(z, t)$ ($j = x, y, z$). Then, on the basis of the selected creep theory, incremental creep strains $\Delta p_j(z, t_i)$ are calculated for the time increment Δt_i for all z , and values $p_j(z, t_{i+1}) = p_j(z, t_i) + \Delta p_j(z, t_i)$ are found. Then, with regard to $\sigma_z(z) \equiv 0$, values $\sigma_j(z, t_{i+1})$ ($j = x, y$) are determined according to formulas (14).

Let us assume that at the moment of time $t = t^*$ temperature removal from $T = T_2$ to $T = T_1$ happens, and at the same time the Young modulus changes instantaneously from E_2 to E_1 . Then, at $T = T_1$ formulas (14) take the form of

$$\begin{aligned} \sigma_x(z, t^* + 0) &= \\ &= \frac{E_1}{\nu^2 - 1} [q_x(z) + p_x(z, t^* + 0) + \nu(q_y(z) + p_y(z, t^* + 0))], \\ \sigma_y(z, t^* + 0) &= \frac{E_1}{\nu^2 - 1} \times \\ &\times [q_y(z) + p_y(z, t^* + 0) + \nu(q_x(z) + p_x(z, t^* + 0))]. \end{aligned} \quad (15)$$

It is relations (15) that set final expressions for stresses after creep (thermal exposure) at the moment of time $t = t^* + 0$.

5. Experimental Investigation of the Influence of Thermal Exposure on Residual Stresses of Prismatic Samples Made of EP742 alloy at Temperature 650 °C after Ultrasonic Hardening

With the aim of experimental confirmation of the foregoing methods of calculation of residual stresses after the hardening procedure and their relaxation due to the creep, a set of experimental investigations was performed to evaluate the influence of thermal exposure on residual stresses of plane samples (rectangular parallelepipeds, i.e. squared beams) with square section 10×10 mm and length 100 mm made of EP742 alloy after double-sided ultrasonic hardening. The processing of opposite faces of the sample is performed in a special container filled with microballoons to which oscillations with small magnitude and high frequency (16-30 kHz) are imparted through an acoustic speed transformer (a concentrator of elastic oscillations). As a result of disordered collisions of balloons with the surface being processed, its hardening takes place. Ultrasonic hardening modes: distance between the sample and the concentrator (the unit bit) – 7 mm; diameter of balloons – 1 mm; material of balloons – steel SHKH15; weight of balloons – 4.6 g; amplitude of oscillation – 50 μm; frequency – 20 kHz; diameter of the planar concentrator – 40 mm; duration of processing – 20 s (mode No. 1), 40 s (mode No. 2), 60 s (mode No. 3), 80 s (mode No. 4). After ultrasonic hardening, a part of hardened layers (for all modes) was used to determine residual stresses, and the other part was subject to thermal exposure (holding at high temperature) at temperature 650 °C within 100 hours, and residual stresses were determined after removal of temperature.

The method of determination of residual stresses in hardened layers is based on the computational-experimental strip method [1, 32]. All samples were cut on the electrical discharge machine to get two strips with thickness h along the middle plane which is parallel to the processed faces of the sample. According to the strip method, additional residual stresses due to the squared beam cutting are taken into account by measuring deflection $f(0)$ of each strip in accordance with the scheme given in Fig. 2 within a reference length l . After deflection $f(0)$ measurement within a reference length l , layers of the material with thickness a are removed on the side of the investigated surface of the strip by means of electrochemical etching, and emerging deflections $f(a)$ are measured (Fig. 3). The measured deflections $f(0)$ and $f(a)$ are used in calculation of residual stresses [1, 32]:

$$\sigma_y(a) - \nu \sigma_x(a) = -\frac{8E}{l^2} f(0) \left(\frac{h}{2} - a \right) +$$

$$+ \frac{4E}{3l^2} \left[(h-a)^2 \frac{df(a)}{da} - 4(h-a)f(a) + 2 \int_0^a f(\xi) d\xi \right], \quad (16)$$

where E is the Young modulus, ν is the Poisson ratio, h is the strip thickness prior to etching, $0 \leq \xi \leq a$. The composition of the etching solution and the etching technology are given in paper [20].

Fig. 2. Scheme of deflection measurement $f(0)$

Fig. 3. Scheme of deflection measurement $f(a)$

In case of ultrasonic hardening, $\sigma_y(a) = \sigma_x(a)$ and formula (16) takes the form of

$$\sigma_y(a) = \sigma_x(a) = \frac{1}{1-\nu} \left\{ -\frac{8E}{l^2} f(0) \left(\frac{h}{2} - a \right) + \right. \\ \left. + \frac{4E}{3l^2} \left[(h-a)^2 \frac{df(a)}{da} - 4(h-a)f(a) + 2 \int_0^a f(\xi) d\xi \right] \right\}. \quad (17)$$

In order to increase the resolution capability of the described method, during removal of strip layers by etching, a scheme of displacement measurement was used which is based on the single-sided support of the sample (Fig. 4). Relation among measured displacements of the strip with the extension $f(a)^*$ and deflections $f(a)$ used in formulas (16) and (17) is written with the use of the Mohr integral [2] in the following form:

$$f(a) = \frac{f(a)^*}{4(1+2c/l)},$$

where c is length of the extension, l is reference length of the strip.

Fig. 4. Scheme of displacement measurement $f(a)^*$

Continuous lines in Fig. 5 show results of experimental determination of residual stresses in samples after the ultrasonic hardening procedure for all four modes of surface

processing. Analysis of these data proves that compression stresses may reach the value of minus 1100 MPa (mode No. 1). The change of a stress sign emerges at the distance of $a \approx 0.2$ mm from the surface. In all modes of ultrasonic hardening, the drop of residual stresses is observed in the direction to the sample surface. Increase of processing time from 20 to 80 s leads to decrease of the maximum of compression residual stresses with the rise of its depth of occurrence and insignificant increase of the hardened layer thickness.

Fig. 7 (the continuous line with No. 4) and Fig. 8 (continuous lines) show experimental dependences of residual stresses $\sigma_x = \sigma_x(z)$ after ultrasonic hardening at temperature of 20 °C, further thermal exposure at temperature of 650 °C within 100 hours and temperature removal to 20 °C. Analysis of these data proves that the influence of thermal exposure has a great impact on the values of residual stresses, in particular, it leads to decrease of maximum compression stresses 1.4-1.6 times. The character of stress distribution does not undergo any significant changes. The drop of residual stresses in the direction to the surface is preserved, and the less the hardening time is the higher maximum stresses are. The depth of occurrence of compression residual stresses increases by approximately 0.02 mm, and, moreover, it is higher for the samples processed within a longer time.

6. Experimental Examination of the Mathematical Model of Formation of Residual Stresses after Ultrasonic Hardening and their Relaxation under High-Temperature Creep Conditions

In order to perform experimental examination of the method of calculation of the stress-strain state in a hardened plane sample (model approximation is the hardened half-space) after ultrasonic hardening, we will use experimental data for four hardening modes given in Fig. 5. As already mentioned, processing with ultrasonic hardening refers to the isotropic process of surface plastic hardening, thus in this case the anisotropy factor $\alpha = 1$ in the relation (2), and hereinafter formulas (7) are used to evaluate residual stresses and plastic deformations. As ultrasonic hardening is performed at normal temperature 20 °C, reference values $E = 2.21 \cdot 10^5$ MPa and $\nu = 0.3$ were used for EP742 alloy in (7). All approximation parameters (9) were determined according to the method given in section 2, with the use of experimental data for $\sigma_x = \sigma_x(z)$ in Fig. 5. For evaluation of the first approximation of values of σ_0 , σ_1 and b in (9), values of representative points for graphs z_0 , z^* , σ^* , $\sigma_{\min}$ were used which are given in the Table. Estimated values of dependence $\sigma_x = \sigma_x(z)$ for all four hardening modes are given in Fig. 5 with dotted lines. The last line of the table contains values of deviation of estimated data from experimental ones which are calculated on the basis of (12)

in percentage terms. In general, reconciliation of estimated and experimental data is satisfactory. As an example, figure 6 shows estimated dependences for distribution of tensor elements of residual plastic deformations for the first hardening mode.

Now, let us consider results of numerical implementation of the method of calculation of residual stresses relaxation in samples after holding at temperature 650 °C within 100 hours. One of the main stages of solution of this problem is the choice of the creep theory. Paper [33] provides experimental data on EP742 alloy creep at temperature 650 °C, and their analysis proves that at the given temperature the creep strain is significant for those levels of residual stresses that emerge after the hardening procedure as a result of which relaxation of residual stresses occurs.

In accordance with [33], the following case is used as a creep model for the uniaxial stress state:

$$\begin{aligned} p(t) &= \sum_{k=1}^s v_k(t) + w(t); \\ \dot{v}_k(t) &= \begin{cases} \lambda_k V_k(t), & V_k(t)\sigma(t) > 0, \\ 0, & V_k(t)\sigma(t) \leq 0; \end{cases} \\ V_k(t) &= a_k (\sigma(t) / \sigma^{**})^n - v_k(t); \\ \dot{w}(t) &= (\sigma(t) / \sigma^{**})^m, \end{aligned} \quad (18)$$

where $v(t) = \sum_{k=1}^s v_k(s)$ is a visco-plastic creep strain component $p(t)$ (describes the first stage of creep); $w(t)$ is viscous flow deformation (describes the second stage of creep); $s, \lambda_k, a_k, n, c, m$ are parameters of the model the method of identification of which is given in [33]. Here, σ^{**} is a nondimensionalizing factor which may be selected randomly for reasons of convenience.

Fig. 5. Experimental (continuous lines) and estimated dotted lines of the curve of residual stresses $\sigma_x = \sigma_x(z)$ in the surface layer (EP742 alloy) at different ultrasonic hardening modes (refer to the table): *a* – mode 1; *b* – mode 2; *c* – mode 3; *d* – mode 4

Values of reference points of the curve $\sigma_x = \sigma_x(z)$, approximation parameters (9) and approximation errors for samples made of EP742 alloy depending on ultrasonic hardening modes

	Processing time, s (ultrasonic hardening mode)			
	20 (1)	40 (2)	60 (3)	80 (4)
Values of reference points of the curve $\sigma_x = \sigma_x(z)$				
z_0 , mm	0.189	0.193	0.204	0.210
z^* , mm	0.037	0.040	0.058	0.073
σ^* , MPa	-872.0	-848.5	-840.4	-840.0
$\sigma_{\min}$, MPa	-1111.5	-1058.6	-1024.3	-1032.3
Values of approximation parameters (9)				
σ_0 , MPa	119.2	103.1	140.9	155.2
σ_1 , MPa	1230.7	1159.1	1167.3	1187.5
b	0.097	0.101	0.121	0.131
Values of approximation errors calculated on the basis of (12)				
Δ , %	4.93	4.02	6.47	7.73

Model (18) describes creep strain within the limits of the first and second stages, moreover, the whole deformation is irreversible. The monograph [33] provides the following parameters for EP742 at temperature 650 °C for model (18): $s = 1$, $\sigma^{**} = 500$ MPa, $\lambda_1 = \lambda = 0.022$, $a_1 = a = 6.1 \cdot 10^{-3}$, $n = 3.29$, $c = 0.722 \cdot 10^{-6}$, $m = 14.3$.

For the combined stress state with regard to $s = 1$, model (18) is generalized as follows [33]:

$$\begin{aligned}
 p_{ij}(t) &= u_{ij}(t) + w_{ij}(t); \\
 v_{\omega\omega}(t) &= (1 + \mu'')\beta_{\omega\omega}(t) - \mu''(\beta_{11}(t) + \beta_{22}(t) + \beta_{33}(t)); \\
 \dot{\beta}_{\omega\omega}(t) &= \begin{cases} \lambda B_{\omega\omega}(t), & B_{\omega\omega}(t)\sigma_{\omega\omega}(t) > 0, \\ 0, & B_{\omega\omega}(t)\sigma_{\omega\omega}(t) \leq 0; \end{cases} \\
 B_{\omega\omega}(t) &= a(S(t)/\sigma^{**})^{n-1}\sigma_{\omega\omega}(t)/\sigma^{**} - \beta_{\omega\omega}(t); \\
 \dot{w}_{ij}(t) &= \frac{3c}{2\sigma^{**}}\left(\frac{S(t)}{\sigma^{**}}\right)^{m-1}\left(\sigma_{ij}(t) - \frac{1}{3}\delta_{ij}\sigma_0(t)\right); \\
 \sigma_0(t) &= \sigma_{11}(t) + \sigma_{22}(t) + \sigma_{33}(t),
 \end{aligned} \quad (19)$$

where $p_{ij}(t)$ is a tensor of creep strain; $v_{ij}(t)$ and $w_{ij}(t)$ are tensors of a visco-plastic (irreversible) component of strains and viscous flow deformation; $S(t)$ is equivalent stress; μ'' is a Poisson ratio for component $v_{\omega\omega}$ (according to a recommendation of [33], one may use $\mu'' = 0.42$); σ^{**} , s , λ , a , n , c , m are parameters having the same meaning as in relations (18) at $s = 1$. Let us denote in (19): $p_{11} = p_x$, $p_{22} = p_y$, $p_{33} = p_z$, $\sigma_{11} = \sigma_x$, $\sigma_{22} = \sigma_y$, $\sigma_{33} = \sigma_z = 0$, $v_{11} = v_x$, $v_{22} = v_y$, $v_{33} = v_z$, $\beta_{11} = \beta_x$, $\beta_{22} = \beta_y$, $\beta_{33} = \beta_z = 0$; summation over the repeated index ω is not performed in (19). For numerical implementation, increments of all creep strain components in relations (19) were calculated according to the Euler method. The following values were used in the calculations: at

temperature $T_1 = 20$ °C $E_1 = 2.21 \cdot 10^5$ MPa, at $T_2 = 650$ °C $E_2 = 1.79 \cdot 10^5$ MPa, $\nu = 0.3$.

Let us analyze the results of calculations for the first mode of hardening in detail. Figure 7 shows that experimental (a continuous line) and estimated (a dotted line) dependences marked with digit 1 correspond to $\sigma_x = \sigma_x(z)$ after hardening at temperature 20 °C ($t = 0 - 0$); digit 2 indicates the same estimated dependence in case of the stepwise change of temperature from 20 °C to 650 °C ($t = 0 + 0$); digit 3 indicates the estimated dependence $\sigma_x = \sigma_x(z, t^* - 0)$ after creep within $t^* = 100$ hours at temperature 650 °C; digit 4 indicates distribution of residual stresses after removal of temperature from $T_2 = 650$ °C to $T_1 = 20$ °C at the moment of time $t = t^* + 0$ (continuous lines are for experimental values, and dotted lines are for estimated ones). Normally (12), deviation of estimated data from experimental ones for final dependences ($t^* = 100$ hours) is $\Delta = 27$ %.

Fig. 8 shows final estimated (dotted lines) and experimental (continuous lines) dependences $\sigma_x = \sigma_x(z, t^* + 0)$ after creep within 100 hours at temperature 650 °C and further removal of temperature for ultrasonic hardening modes 2-4. Deviations of estimated data from experimental ones calculated according to (12) are 27.4 % for mode No. 2, 20.9 % for mode No. 3 and 37 % for mode No. 4.

It should be noted that "the difference of 20 % among deformations in creep curves taken at the same stress for the given temperature on samples made of the single-heat material is not considered to be excessive", [34, section 44, pp. 172-173], and our work did not involve special experiments on EP742 alloy creep for investigated samples, but reference data were used for the creep model (18), (19) from monograph [33]. For this reason, despite the obtained deviations of estimated data from experimental ones, results of calculation of residual stress relaxation should be considered satisfactory.

Fig. 6. Graphs of dependences $q_x = q_x(z)$ (line 1) and $q_z = q_z(z)$ (line 2) for the 1-st mode of ultrasonic hardening (EP742 alloy)

Fig. 7. Kinetics of dependence $\sigma_x = \sigma_x(z, t)$ in the process of creep (mode 1, EP742 alloy)

a

b

c

Fig. 8. Experimental (continuous lines) and estimated (dotted lines) dependences $\sigma_x = \sigma_x(z, t)$ at the moment of time $t = 100$ h after creep and removal of temperature: a – mode 2; b – mode 3; c – mode 4

7. Conclusions

Experimental researches were performed on formation of residual stresses after the procedure of ultrasonic hardening of prismatic samples made of EP742 alloy and on evaluation of the degree of their relaxation under conditions of thermal exposure at temperature 650 °C within 100 hours. Regularities were revealed in distribution of residual stresses over the thickness of the surface hardened layer. It was shown that thermal exposure decreases the level of compression residual stresses 1.4-1.6 times and leads to displacement of their maximum into the depth of the sample. The obtained experimental data play an important and independent part in applied problems as EP742 alloy is used for manufacturing of aircraft engine turbine discs, particularly, operating at temperature of 650 °C, and surface hardening is an in-house technology in aircraft engine building.

A mathematical model was developed concerning formation of residual stresses in prismatic samples after surface plastic deformation and their relaxation under conditions of holding at high temperature. It was shown that relaxation of residual stresses may be related to material creep under conditions of the non-homogeneous stress-strain state.

Comprehensive experimental examination of the mathematical model was performed at normal temperature and under high-temperature creep conditions, with satisfactory correspondence of estimated and experimental data being observed.

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