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TRANSFORMATION AND INTERACTION OF MICROCRACKS IN METAL UNDER HIGH-ENERGY PULSED ELECTROMAGNETIC FIELD

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ABSTRACT

Simulation of processes occurring in metals when they are treated with short pulses of high density electric current is of interest primarily due to studying the phenomenon of electroplasticity; the physical mechanism of which is still unknown to researchers. The effect of healing micro-defects in metals is one of existing explanations for this phenomenon.

The present paper considers the processes of transformation and interaction related to flat microcracks with linear sizes of about 10 microns when processing metal samples with short pulses of high-density currents. The investigation is carried out numerically on basis of coupled quasi-stationary model of impact using high-energy electromagnetic field on the pre-damaged thermal elastoplastic material with defects. The model accounts for melting and evaporation of the metal and the dependence of its physical and mechanical properties on the temperature. The system of equations is solved numerically by finite elements method with adaptive mesh using alternative Euler-Lagrange's method.

The calculations show that the crack welding and micro-defects healing occur under the short pulse of current. The healing occurs due to a simultaneous reduction in length, the ejection of the molten metal into the cracks and closing of micro-crack shores which leads to the fact that the shores of the crack come into contact with the jet stream; and in the end of these processes the jet's material is completely jammed by the cracks shores.

This paper studies the influence of distance between the cracks and their relative position with respect to each other on deformation and healing of micro-defects; also the choice of the integration regions and conditions at its boundaries.

Numerical modeling shows that it is enough to study microcracks healing by considering one representative element (or one-quarter of the axisymmetric representative element) as a region of integration by setting the electrical potential which is certain for the element without defects (when it is "unperturbed" by the presence of microcracks) on its borders that are the axes of symmetry. When the distances between the cracks exceed their lengths by 5–6 times, the healing processes will occur in the same manner regardless of the fact that we model them in the region of integration consisting of one or several representative elements.

When the distance between the cracks increases, the influence of mutual arrangement of micro-cracks on the healing process is decreased. Thus, if the distances between the microcracks exceed their lengths by six times, in fact, the healing of microcracks is the same for any position of cracks compared to each other.

Interaction between microcracks begins to significantly affect their healing processes when the distance between them is reduced to 5-6 lengths of microcracks. If the distance between the cracks exceeds their six lengths, the processes of microcracks healing become practically independent of the distance between the defects or the position of defects with regard to each other.

Decreasing the distance between the cracks up to 1-2 of their linear sizes (taking into account their relative position changes) does not qualitatively change the described healing process of microcracks, however it results in a considerable slowing down: the ejection of a molten material in a crack is retained, but the crack reduction is significantly reduced especially in the transverse direction.

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1. Introduction

The study of the processes occurring in conductive materials during samples exposure to short-term pulses of high-density electric current is of considerable interest both in connection with study of the electroplastic deformation phenomenon, the physical mechanism of which remains incomprehensible to researchers, and in connection with study of the defects healing phenomenon (from microdefects to macrodefects of fatigue origin as an example), which also could not been explained until recently.

This work is focused on microdefects with linear dimensions around 10 μm , formed by surfaces of neighboring single crystals (grains). They are most often found in polycrystalline metal (inter-grain defects), are always produced in certain amount after casting, and can also develop in the metal during processing (for example, as a result of electroplastic deformation) or operation (for example, as a result of multi-cycle loading) of any products made of it. The defects are accumulated in the material under load. With a certain amount of damage, this leads to the product fracture. In terms of subsequent macrofracture, the defects like flat microcracks are most dangerous of the microdefects.

The assumptions that defects healing (change) in conductive material can occur under short pulses of high-density current were made in [2-5] in the context of studying the electroplasticity phenomenon.

While, experimental evidence of the defects healing occurring in one form or another in conductive materials has appeared without regard to the study of electroplasticity.

For the first time, the possibility of cracks retardation in the plates exposed to short pulses of high-density current was experimentally established in [6-8]. The experiments showed melting, micro-explosions, crater formation in crack tips of different materials (silicon steel, copper and other non-ferrous metals), which leads to retardation of crack propagation.

In [9-10], by measuring the propagation speed of ultrasonic waves in samples of rail steel subjected to cyclic loading, it was established that fatigue microcracks are getting healed after a sample containing such cracks has been exposed to current pulses. Such healing significantly increased sample lifetime until its fatigue failure under the cyclic loading.

In [11], by determining the density by hydrostatic weighing performed for samples of austenitic steels (with different carbon content) subjected to cyclic loading before and after their exposure to electric current, it was found that the samples exposure to current pulses caused microcracks healing, which was evidenced by the increase of the material density after the exposure.

In [12], when studying the electroplasticity phenomenon in tensile tests of titanium samples using optical microscopy, it was found that the microdefects caused by plastic deformation were healed after the samples

had been exposed to current pulses, which was evidenced by photographs of the material microstructure.

In [13], using Scanning Electron Microscope for examining stainless steel samples in tests for multi-cycle stretching before and after exposure to electric current, it was established that a fatigue mesocrack in a sample got healed after it had been exposed to current pulses. The crack edges welded together.

Thus, it became possible to assert that the theoretical presumptions of defects healing under short pulses of high-density current made in [2-5] have now transformed from a hypothesis to an experimentally observed fact [6-13].

A number of mathematical models have been proposed by researchers [14-25] to study physical processes occurring in the vicinity of cracks under intense current pulses and to explain the healing effect. These models demonstrated a possibility of compressive normal stresses developed at crack tips in the direction perpendicular to the crack surface, and a possibility of closing-in movement of cracks edges accompanied by craters (pores) melt out at the tips. According to the author, this prevented further propagation of the cracks in material and meant the defects healing.

However, the mentioned experiments [6-13] showed a change in the defect form up to the complete removal of the defect rather than just development of compressive normal stresses accompanied by some craters (pores) melt out at the tips. Thus, the mathematical models [3-4, 14-25] could not be used to accurately explain the observed facts. This was described in details in [1].

To solve this problem, the authors of [1, 26-27] proposed models of the electromagnetic field impact on pre-damaged defected material. These models successfully mathematically described the experimentally observed transformation of microdefects in conductive material. In particular, it was shown in [1, 27] that a microcrack can be completely healed under certain conditions. The process is a combination of simultaneous shortening of a microcrack, ejection of molten metal from the tip inward the crack and the crack closing. As a result, the crack edges become welded together by molten metal. According to [1], the model correctly reflects main experimentally observed parameters specific to electrothermomechanical processes in the vicinity of the cracks.

Meanwhile, simulation (solution of the initial-boundary problem) was carried out for a certain material representative element (periodic cell) under selected boundary conditions on its surface [1, 27]. In this regard, it is important to find out to what extent the identified process of microcracks healing (and the stress-strain state in the defects vicinity) is affected by the distance between the adjacent microcracks and their position relative to each other.

To address this issue, one should, first, assess the mistake that we make solving the problem for a single representative element rather than for the entire sample with a set of defects; and second, understand how the considered processes depend on the boundary conditions that can be used in the model.

2. Objectives, Main Equations and Boundary Conditions

The considered material is conductive and contains similar defects—flat microcracks with rounded tips. The defects are assumed to be located either orderly or randomly in the material, as shown in Fig. 1. In the first case (Fig. 1, *a, b*), the selection of a representative element of the volume (a periodicity cell) is not difficult. In the second case, we assume that, in general, the defects are distributed uniformly over the volume, so that the range of the distances between the centers of microdefects is small. In this case, a representative element will be a ball (a disk if in a plane)

with a radius r_{aver} equal to the average distance between defects in the material, as shown in Fig. 1, *c*.

The material is exposed to short pulses of high-energy electromagnetic field of certain intensity by applying potential difference to the outer borders of the sample. This creates current at the borders with a density vector perpendicular to the plane of microcracks, as shown in Fig. 1. The focus is on the electromagnetic fields causing current to flow in the sample with the density from 10^8 to 10^{10} A/m² for the period of 10^{-5} - 10^{-4} sec.

The solution is sought in integration regions containing from one to six microcracks or from one to six parts of representative elements, as shown in Fig. 2.

Fig. 1. The defects location in the material (at the top) and respective representative elements (at the bottom) of the microdefects: *a*) in the nodes of a rectangular lattice, *b*) in the nodes of a hexagonal lattice, *c*) when located randomly

Fig. 2. Considered integration regions for rectangular (*a-d*), hexagonal (*e-h*) and disk (*i-j*) representative elements

In this setting the objective is to investigate the processes of change in the electric and temperature fields in the material and their influence on phase transformations and stress-strain state, as well as to study the microdefects healing process. A model of electromagnetic field impact on pre-damaged defective material is used to get this objective [1]. The solution of the resulting system of equations is sought by the finite element method.

The following physical processes take place in the material under the considered impact: electromagnetic, mechanical and thermal. The characteristic time of each of these processes is approximately inversely proportional to the propagation speed of the corresponding perturbations. Consequently, the time required for establishing the electromagnetic and mechanical processes that are considered in this work is 10^{-13} - 10^{-12} sec. and 10^{-8} - 10^{-7} s, respectively, which is significantly less than the time of external influence of the current source (10^{-5} - 10^{-4} s). In this regard, to obtain electric potential in a conductive material, the charge conservation law is used (under the assumption that the current in the sample is steady) in a variational formulation [1]: in addition, Ohm's law is considered correct for the conductor. The displacement field is determined from the principle of virtual work [28]: the displacements are assumed to be finite. The additivity of the speeds of elastic, plastic and temperature deformations is accepted in the study. The Hooke's law for an isotropic body and the associated law of flow with the Mises plasticity condition are assumed correct for the speeds of elastic and plastic deformations.

The temperature field is determined by energy conservation law. Since the time of electromagnetic impact on the material is short (10^{-5} - 10^{-4} s), while temperature gradients in the vicinity of the crack tips is great (10^7 °C/m) [6-7], thermal conductivity should be neglected and the process should be considered adiabatic. The resulting temperature evolutionary equation takes into account the heat released in a volume unit of the current body configuration per a time unit due to the flow of electric current in accordance with Joule-Lenz law, and the heat released during plastic deformation, as well as the latent heat absorbed in the process of melting and evaporation.

The temperature in these processes varies from the room temperature to the temperature of metal evaporation [6-8]. For this reason, in the proposed model all physical and mechanical properties of material (density, specific heat capacity, electrical conductivity, thermal expansion coefficient, elastic moduli, yield strength, etc.) included in the above equations depended on temperature. All physical properties of the material sharply changed in the points where it had melted, while in the points where the material had completely evaporated, the current density vector and the stress tensor were assumed to be zero, and the temperature was assumed to be equal to the evaporation temperature.

Thus, the complete system of equations for the displacement vector $\mathbf{u}$, the electric potential φ and the

temperature T of the electrothermomechanical model under consideration has the following form [1]:

$$\int_V \nabla \delta \varphi \sigma^E(T) \nabla \varphi dV = \int_S \delta \varphi j dS, \quad \mathbf{j} = \sigma^E(T) \mathbf{E} = -\sigma^E(T) \frac{\partial \varphi}{\partial \mathbf{x}} \quad (1)$$

$$\int_V \boldsymbol{\sigma} : \delta \boldsymbol{\varepsilon} dV = \int_S \mathbf{t} \cdot \delta \mathbf{u} dS + \int_V \mathbf{f} \cdot \delta \mathbf{u} dV, \quad \dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{el} + \dot{\boldsymbol{\varepsilon}}^{pl} + \dot{\boldsymbol{\varepsilon}}^{th}, \quad (2)$$

$$\dot{\boldsymbol{\sigma}} = \lambda(T) \dot{\boldsymbol{\varepsilon}}^{el} : \mathbf{I} + 2\mu(T) \dot{\boldsymbol{\varepsilon}}^{el}, \quad \dot{\boldsymbol{\varepsilon}}^{pl} = \dot{\Lambda} \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} = \dot{\Lambda} \mathbf{s}, \quad \bar{\boldsymbol{\sigma}} = \sigma_Y(T),$$

$$\bar{\boldsymbol{\sigma}} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}}, \quad d\boldsymbol{\varepsilon}^{th} = \alpha(T) \mathbf{I} dT \quad (3)$$

$$\rho(T) c(T) \dot{T} = r^E + r^{pl} + r^{melt} + r^{evap},$$

$$r^E = \eta^E \mathbf{j} \cdot \mathbf{E} = \eta^E \nabla \varphi \cdot \sigma^E \cdot \nabla \varphi, \quad r^{pl} = \eta^{pl} \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{pl}, \quad (4)$$

$$T = T_{melt}, \quad t_s \leq t \leq t_1, \quad \int_{t_s}^{t_1} (r^E + r^{pl}) dt = \rho \Lambda_{melt}, \quad (5)$$

$$T = T_{evap}, \quad t_{el} \leq t \leq t_{ev}, \quad \int_{t_{el}}^{t_{ev}} (r^E + r^{pl}) dt = \rho \Lambda_{evap}, \quad (6)$$

where V is arbitrary volume, bounded by a piecewise smooth surface S ; $\mathbf{n}$ is the outward normal to S ; T is temperature, $\mathbf{E}$ is intensity vector of the electric field, defined as negative gradient of electric potential $\mathbf{E} = -\partial \varphi / \partial \mathbf{x}$, $\sigma^E(T)$ is electrical conductivity, $j = -\mathbf{j} \cdot \mathbf{n}$ – current density normal to surface S , $\delta \varphi$ is variations of the electric potential, satisfying the boundary conditions of the problem, $\mathbf{u}$ is displacement vector, $\boldsymbol{\sigma}$ is stress tensor, $\boldsymbol{\varepsilon}$ is total deformation tensor, $\mathbf{f}$ is vector of volume forces, $\mathbf{t}$ is vector of surface forces, $\delta \mathbf{u}$ and $\delta \boldsymbol{\varepsilon}$ is variation of displacements and total deformations correspondent to them $\dot{\boldsymbol{\varepsilon}}^{el}$, $\dot{\boldsymbol{\varepsilon}}^{pl}$, $\dot{\boldsymbol{\varepsilon}}^{th}$ are speed tensors of elastic, plastic and temperature deformations, $\lambda(T)$, $\mu(T)$ are Lamé elastic moduli, $\sigma_Y(T)$ is yield strength, $\mathbf{s}$ is deviator of the stress tensor, $\mathbf{I}$ is unit tensor, $\alpha(T)$ is coefficient of thermal expansion, $\rho(T)$ is density, $c(T)$ is specific heat capacity, $\dot{T}$ – material derivative of temperature, r^k is heat released in a volume unit in the current body configuration per a unit of time ($k = E, pl, melt, evap$), where r^E is heat released by electric current flow, r^{pl} is heat released during plastic deformation, r^{melt} is heat, absorbed in the melting process, r^{evap} is heat, absorbed in the evaporation process, η^E is an empirical coefficient defined as the fraction power of the electric current per a volume unit that has dissipated into heat, η^{pl} is an empirical coefficient that is defined as the plastic fraction power of the current per a volume unit that has dissipated into heat, T_{melt} is the material melting point, t_s is the time when the material begins to melt, t_1 is the time when the material gets completely melted, Λ_{melt} is latent melting heat, T_{evap} is the material evaporation temperature,

t_{el} is the time when the material begins to evaporate, t_{ev} is the time when the material gets completely evaporated, Λ_{evap} is latent evaporation heat.

Below are the initial conditions and detailed electrical (for potential) and mechanical (for $\mathbf{u}$ and) boundary conditions for the case of plane deformation that are to be logically applied to this problem.

By virtue of symmetry of the representative elements (Fig. 1), the integration regions consisted of halves or quarters of the representative elements, as shown in Fig. 2.

The boundary conditions for φ were set based on the assumption that the electric field at the boundary of the integration region can be considered unperturbed by the presence of defects (that is, as if there were no defects in the material). In a defectless sample to the external boundary of which the potential difference was applied as shown in Fig. 1 (center), the unperturbed potential φ_* is known and is a linear function of one y -coordinate: $\varphi_* = \varphi_*(y') = U_*(y'/L)$ where U_* is the value of the potential difference applied to the outer boundaries of the sample, L is the sample length, y' is the coordinate in the coordinate system associated with the sample (see Fig. 1).

Based on this, at the boundaries of the integration regions (see Fig. 2) that do not cover the surfaces of cracks and the axes of symmetry of the representative elements, one can set either “unperturbed” potential or corresponding “unperturbed” current density, the relationship between which is determined by Ohm's law (second equation (1)). However, the model material contains the defects, so it is clear that such boundary conditions of the integration regions should be satisfied approximately. In addition, the accuracy of such satisfaction increases with the increase of the distance from the defect to the region boundary (Fig. 2). Moreover, it is necessary to figure out which of these conditions better reflects the state at the boundary, and, consequently, which of them should be preferred in relation to the problem.

If the boundary of the integration region passed along the vertical axis of symmetry of the representative element, then the condition for zero derivative of the normal to the axis was set, which corresponded to the absence of a current density vector normal to the boundary. If the boundary of the integration region passed along the horizontal axis of symmetry of the representative element, then the condition for constant potential at the given boundary was set on it, which corresponded to the absence of a current density vector tangent to the boundary.

Since the crack did not conduct current, in cases where the boundary of the integration region passed over crack surface, the condition of zero potential derivative φ normal to the boundary was set, which corresponded to the absence of a current density vector normal to the boundary.

In addition, it was assumed that the shape of the electromagnetic pulse is such that the electrical boundary conditions do not depend on time, i.e. the potential

difference or current density set at the boundary was considered constant throughout the duration of the electromagnetic pulse τ_0 .

As a result of the considered electric field impact, the stress-strain state (SSS) in the representative element with the defect under consideration, surrounded by other similar defects was found to be either the same (in the cases shown in Fig. 1, *a, b*), or approximately the same (in the case shown in Fig. 1, *c*) as the SSS of surrounding defects. Therefore, at the boundary of the integration region that did not pass along crack surface and the symmetry axes of the representative element, the component of the displacement vector normal to this boundary and the tangential component of the stress tensor can be assumed zero. If the boundary of the integration region passed along the vertical and horizontal axis of symmetry of the representative element, then there was set the condition of zero displacement vector component normal to this boundary and zero tangent component of the stress tensor.

Crack surface assumed to be free of stress.

Specifying the above, we should write out the electrical and mechanical boundary conditions for the integration regions shown in Fig. 2, *a-b, c-f, i-j* and containing one crack. Boundary conditions for the integration regions shown in Fig. 2 and containing more than one crack were set similarly.

There were two different conditions on horizontal sections of the region boundary (with the exception of the crack surface) for the integration regions consisted of rectangular elements (Fig. 1, *a*, Fig. 2, *a, b*): either a spatially constant potential difference or a spatially constant vector of electric current density normal to the boundary (the values of the potential difference and current density are related to each other, as indicated below). The vertical boundaries of the integration region were conditioned by the absence of the derivative potential normal to the boundary.

Electrical boundary conditions for the integration region equal to a half of the representative element shown in Fig. 2, *b* were, respectively, as follows:

$$\varphi(x, b, t) = \varphi_*(b), \quad (7a)$$

$$\varphi(x, -b, t) = -\varphi_*(b), \quad (7b)$$

$$j_y(x, b, t) = j_*, \quad j_x(x, b, t) = 0, \quad (8a)$$

$$j_y(x, -b, t) = j_*, \quad j_x(x, -b, t) = 0, \quad (8b)$$

$$\partial\varphi/\partial x|_{x=0} = 0, \quad (9a)$$

$$\partial\varphi/\partial x|_{x=a} = 0. \quad (9b)$$

The values φ_0 and j_0 in (7)-(8) are assumed to be related by

$$j_0 = j_* = \sigma^E(T_0)\varphi_*(b)/b \quad (10)$$

meaning that the electric field at the boundary is considered as unperturbed. The boundary conditions (9b) are the result

of one more problem's symmetry axis present in the case under consideration, and do not contradict to the above condition of the field stability on the boundary.

Mechanical boundary conditions were as follows:

$$x = 0: u_x = 0, \sigma_{xy} = 0, \quad (11a)$$

$$x = a: u_x = 0, \sigma_{xy} = 0, \quad (11b)$$

$$y = b: u_y = 0, \sigma_{xy} = 0, \quad (11c)$$

$$y = -b: u_y = 0, \sigma_{xy} = 0. \quad (11d)$$

Electrical boundary conditions for the integration region equal to a quarter of the representative element shown in Fig. 2, a were assumed as (7a) or (8a), and the following condition was used instead of condition (7b):

$$\varphi(x, 0, t) = \varphi_*(0) = 0 \quad (7c)$$

either the following one was used instead of condition (8b):

$$j_x(x, 0, t) = 0. \quad (8c)$$

Mechanical boundary conditions were as in (11a)-(11c), the following one was used instead of condition (11d):

$$y = 0: u_y = 0, \sigma_{xy} = 0. \quad (11e)$$

For the integration regions consisting of hexagonal and disk elements (Fig. 1, b , c), the following boundary conditions were chosen:

For G_1 boundary area: $y = \Phi(x)$, when such area does not pass over crack surface and symmetry axes of the representative element (Fig. 2, e , f , i , j); the following was set: unperturbed potential, absence of a displacement vector component normal to the boundary, and absence of a tangential component of stress tensor:

$$\varphi(x, y, t)|_{\Gamma_1} = \varphi_*(y), \quad (7d)$$

$$\mathbf{u}_n|_{\Gamma_1} = 0, \quad \sigma_\tau|_{\Gamma_1} = 0, \quad (11f)$$

where $\mathbf{u}_n$ is normal component of displacement vector on G_1 boundary, σ_τ is tangent components of stress tensor on G_1 boundary.

For G_2 boundary area of the integration region, when such area passed along the horizontal axis of symmetry of the representative element (Fig. 2, e , f , i , j), the following conditions were set: constant potential on the given boundary (7c) and absence of a displacement vector component normal to the boundary as well as of a tangent component of the stress tensor in the form of condition (11c).

For G_3 boundary area of the integration region, when such area passed along the vertical axis of symmetry of the representative element (Fig. 2, e , f , i , j), the following conditions were set: absence of a vector component of current density normal to the boundary (9a), and absence of displacement vector component normal to the boundary, as

well as absence of a tangent component of the stress tensor in the form of condition (11a).

For G_4 boundary area of the integration region, when such area passed crack surface, the condition of zero potential derivative normal to the surface was set, and the surface was considered free from stresses.

The initial fields of temperature, displacements, and electric potential were assumed to be homogeneous ($T_0 = 20^\circ\text{C}$, $\mathbf{u}_0 = 0$, $\varphi_0 = 0$).

3. Numerical Simulation Results

All equations of system (1)-(6) are related. They should be solved numerically together with boundary conditions (7)-(11) and the initial conditions. The calculations were performed for plane deformation using linear four-node isoparametric and three-node finite elements. The grid was restructured on the basis of mixed Euler-Lagrangian method. Temperature evolutionary equation was solved directly at each integration point using the approximation of temperature derivative by inversed time difference. The calculations were performed with the use of ASTRA computational package developed under the guidance of prof. N.G. Burago in Solids Mechanics Simulation Laboratory of Ishlinsky Institute for Problems in Mechanics RAS.

The simulation was performed for zinc samples [1, 27]. The initial length of all microcracks l_0 was set at $l_0 = 10 \mu\text{m}$. The initial distance between the edges was set at $h_0 = 1 \mu\text{m}$, and the curvature radius at a microcrack tip – $r_0 = 0.5 \mu\text{m}$.

Representative element dimensions varied between 20 and 240 μm , which corresponded to the distances between crack tips of 10-230 μm (Fig. 1). Let us dwell in more detail on the reasons for choosing such distances between the defects. Indeed, the average size of grains of the vast majority of metals and alloys, including zinc, are several dozens of microns; therefore, such grain sizes do not make it feasible to consider the distances of less than 10 microns between the intergranular crack tips. In addition, we believe that the material with smaller distance between microcracks can not be considered intact in terms of macrofracture (mesofracture). On the other hand, it does not make sense to consider distances greater than 230 μm , since, as it will be seen from the simulation results below, further distance increase does not affect the studied processes in any way. Thus, in fact, the interval in question covers the entire range of possible distances between microdefects that precede the macrofracture and could occur in the material due to its electroplastic deformation or operation.

The potential difference (per a length unit) used in the calculations was 534.3 mV/mm, which corresponded to the current density of 8.95 kA/mm² in the defectless material.

The calculations show that when electric current flows through the defected sample, there are large electric field gradients in the vicinity of microcracks. These gradients

lead to a significant increase of the current density in the vicinity of the crack tips compared to the current density applied to the sample. This causes rapid non-homogeneous local heating in the vicinity of a microcrack tip and material melting accompanied by its thermal expansion (at the same time, central part of the crack edges is not getting hot). This leads to large compressive stresses (the pressure can be over 100 MPa) in the vicinity of the microcrack and, as a result, to simultaneous closure of its edges (crack narrowing), shortening of its length and release of molten material into the crack (Fig. 3).

Fig. 3, *a* and *b* show the boundaries of a microcrack being healed and the temperature field in its vicinity when potential difference is applied to the boundary of the integration region for different distances between the defects (but with the same length of the healed crack). The dashed line in the figures shows the initial boundary of the crack (at $t = 0$). The distance between the centers of adjacent microcracks is 60 μm and 20 μm , respectively. The provided simulation results are of a microcrack with a center at $x = 0$, $y = 0$ in the integration region shown in Fig. 2 *g* (3 microcracks with hexagonal arrangement (Fig. 1, *b*)).

Fig. 3. Healing of a microcrack (the dashed line shows the initial boundary of the crack) and temperature isolines $^{\circ}\text{C}$ (1 – 25, 2 – 50, 3 – 100, 4 – 200, 5 – 300, 6 – 400; dark gray – melting area ($T \geq 419^{\circ}\text{C}$), black – evaporation area $T = 906^{\circ}\text{C}$) (a) at time $t = 19.2 \mu\text{s}$ and the distance between microcracks of 60 μm , (b) at time $t = 71.0 \mu\text{s}$ and a distance between microcracks of 20 μm

This process continues until the jet of molten metal is completely clamped by the crack edges (Fig. 4). As a result, the crack edges become welded together by molten metal. Further, the entire process above is repeated again for the microcrack of new smaller sizes. This leads to complete healing of the microcrack.

Fig. 4. Microcrack boundaries and isolines of the intensity of plastic deformations (1 – 0.010, 2 – 0.0375, 3 – 0.0650, 4 – 0.0925, 5 – 0.120, 6 – 1.00) when exposed to a current pulse of 8.95 kA/mm²; the distance between the centers of adjacent microcracks is 180 microns (the dashed line shows the initial boundary of the crack)

Below you will find description of how the healing processes depend on the boundary conditions (7)-(8), and estimation of the fulfillment of boundary conditions (7), (8) and (11) on a border of one representative element evidenced from integration regions consisting of square ($a = b$) representative elements shown in Fig. 1, *a*. This requires to solve a problem of electromagnetic field impact on 4 and 6 microcracks, respectively, in the integration regions shown in Fig. 2, *c*, *g*. Either potential difference (7) or corresponding current density (8) was set at the boundaries of these regions. The considered representative element had the center at $x = 0$, $y = 0$ and was located inside the integration regions shown in Fig. 2, *c*, *g* (hereinafter – the internal representative element). The values of electric potential, current density, normal displacements and tangential stresses on the vertical and horizontal straight lines forming the boundaries of the internal representative element, as well as on the vertical and horizontal axes of element symmetry and deviations of these calculated values from the values determined from conditions (7)-(11) were studied. The deviation magnitude determined the fulfillment accuracy of conditions (7)-(11) at the boundaries and symmetry axes of the internal representative element. At the same time, the investigation was focused on the influence of boundary conditions (7)-(8) on the microcracks healing process and SSS in the internal representative element. These calculations were carried out over the entire range of distances between microcracks (from 10 to 230 μm) and compared with calculations for the integration regions containing one representative element, when setting conditions (7)-(11) on its boundaries. These integration regions are shown in Fig. 2, *a* and *b* respectively.

For the considered integration regions at the boundaries of the internal representative element and its horizontal axis, the simulation showed that all boundary conditions, (7)-(8), (9) and (11), are fulfilled to high accuracy under the distances between cracks from 60 to 230 μm . Moreover, with an increase of this distance from 60 to 160 μm , the differences between the solutions gradually decreased, and when the distances between the cracks were longer than

160 μm at the boundaries and inside the internal representative element, the solutions coincided with each other when setting any of the conditions, (7) or (8), on the boundaries of the integration region.

The calculations for the distances between cracks in the range from 60 to 20 μm showed good fulfillment of only conditions (7), (9) and mechanical boundary conditions (11). Obviously, the maximum deviation from the fulfillment of these conditions at the boundary of the internal representative element will be observed at the smallest of the considered distances between the cracks. The fulfillment accuracy of conditions (7), (8), (11) is determined as the maximum modulus of deviation of the value, calculated at the boundaries of the internal representative element, from the value determined by this condition, referred to the maximum measured value, in per cent. The simulation for the distance between the cracks of 20 μm showed that the fulfillment accuracy of the conditions was: for condition (7a) and (7b) – 0.017 %, (7d) – 0.017 %, for the first condition of (11b) – 0.010 %, for the first condition of (11c) – 0.011 %, for the second condition of (11b) – 0.019 %, for the second condition of (11c) – 0.057 %, and for conditions (7c) – 0.004 %, for the first and second conditions of (11e) – 0.005 % and 0.008 %, respectively. With an increase of the distance between the crack tips up to 60 μm , this accuracy further improved.

At the same time, conditions (8a)-(8b) on the horizontal border of the internal representative element were fulfilled with insufficient accuracy. For the distances between cracks of 20 μm , the accuracy of fulfilling conditions (8a)-(8b) was determined as 15.6 %. That is significantly lower than the accuracy for conditions (7a)-(7b). Therefore, it is unreasonable to set unperturbed density of electric current in this model on the border of the representative element for the integration regions shown in Fig. 2.

Fig. 5-6 illustrate the above. Figure 5a shows the dependences of crack length (the distances l between crack tips is as in Fig. 1, a) on time for different integration regions (Fig. 2, a-d) with different boundary conditions on their horizontal surfaces: when the potential difference was set (condition (7)) or the corresponding current density (condition (8)). Fig. 5, b, in turn, shows similar dependences of the microcrack width (the maximum distance h between the crack edges, which is observed at its center at $x = 0$, $y = 0$ (Fig. 1)) on time.

Figure 5 illustrates noticeable influence of the choice of boundary conditions, (7) or (8), on the healing process when the distances between the tips of the microcracks become less than 5-6 lengths of these microcracks. In case of large distances, the choice of the condition makes no difference – both solutions (conditioned by (7) or (8)) tend to each other as the distance between the defects is being increased. However, the differences between these solutions increase as the distance between the cracks decreases, which is also seen from the presented curves (Fig. 5): the healing process is faster if conditioned by a set current density than by a set

potential difference corresponding to such density. This is explained by a current density drop at the boundary (if compared with the unperturbed value (10)) when setting the boundary condition (7).

Fig. 5(a) Dependences of crack length l (μm) on time t (μs) for integration regions of various sizes containing 4 microcracks (Fig. 2, c) when setting a potential difference on their horizontal surfaces ($\phi = \text{const}$) or corresponding current density ($j = \text{const}$), (b) similar dependences of the crack width h (μm) on time t (μs)

Thus, condition (7) is used hereinafter, as more accurate and having a more clear physical meaning. In this case, the value of potential difference at the boundary of the representative element is determined based on the condition of electric potential stability in the material. However, one should keep in mind that both solutions tend to each other when the distance between the cracks is being increased, and the healing process under condition (8) is somewhat faster.

To assess the possibility of solving the problem and simulate the healing processes in a region consisting of one representative element with selected boundary conditions instead of solving it for the entire sample, below we will consider differences in the resulting solutions for the

integration regions consisting of several rectangular ($a = b$) and hexagonal representative elements (Fig. 2, $a-d$ and Fig. 2, $e-h$).

Calculations for the integration regions containing 1, 4 and 6 microcracks (square representative elements) shown in Fig. 2, $a-d$ showed that there is a very good agreement between the results obtained in simulating the healing process of the microcrack (having the center at $x = 0, y = 0$): the dependences of the microcrack length and width on time practically coincided with each other in most of the investigated range of sizes of representative elements (Fig. 6). Even simulation for an element of $20\text{ }\mu\text{m}$ in size (when the strongest interaction of microdefects takes place) showed good agreement between these curves (see Fig. 6).

Fig. 6 (a) Dependences of crack length l (μm) on time t (μs) for integration regions of various sizes containing 1, 4 and 6 microcracks (Fig. 2, $a-d$) in square representative elements, when setting potential difference on their horizontal surfaces ($\varphi = \text{const}$), (b) similar dependences of the crack width h (μm) on time t (μs)

Similar calculations performed for the integration regions containing 1, 3, and 5 microcracks (hexagonal representative elements), shown in Fig. 2, $e-h$, also illustrated a good agreement between the results obtained in simulating the healing process of the microcrack (having the

center at $x = 0, y = 0$). In this case, the dependences of the microcrack length and width on time practically coincided with each other in sufficiently large part of the investigated range of sizes of the representative elements. Quite good agreement between these curves was also obtained for the element of $20\text{ }\mu\text{m}$ (Fig. 7). In this case, the simulation showed that the fulfillment accuracy of conditions (7), (11) is lower than for square representative elements. At the distance between the cracks centers of $20\text{ }\mu\text{m}$, the accuracy was 1.37 % for condition (7d), 3.77 % for the first condition of (11f), 10.75 % for the second condition of (11f), and 0.008 % for conditions (7c), 0.008 % and 0.009 % for the first and the second conditions of (11e), respectively. With an increase in the distance between microcracks up to $60\text{ }\mu\text{m}$, this accuracy significantly increased (for condition (7d) – 0.12 %, for the first condition of (11f) – 0.76 %, for the second condition of (11f) – 2.69 %), which was reflected in almost complete overlap of the curves in Fig. 7 for the integration regions containing 1 and 3 microcracks.

Fig. 7 (a) Dependences of the crack length l (μm) on time t (μs) for integration regions of various sizes containing 1, 3 and 5 microcracks (Fig. 2, $e-h$) in hexagonal representative elements, when setting potential difference at their boundary ($\varphi = \text{const}$), (b) similar dependences of the crack width h (μm) on time t (μs)

As it has been mentioned, the accuracy of this condition fulfillment on the horizontal symmetry axis of the representative element (7c) and (11e) is very high. This is evidenced by the complete coincidence of the calculation results shown in Fig. 6, 7 for the integration regions containing 4 and 6 microcracks (shown in Fig. 2, c, d), and 3 and 5 microcracks (shown in Fig. 2, g, h), as well as 1 and $\frac{1}{2}$ of a microcrack (a half and a quarter of the representative element) shown in Fig. 2, a and b.

Based on the above, when simulating the process of microcracks healing in material with an ordered defect structure (Fig. 1, a and b), it is possible to be confined to a quarter of the representative element (Fig. 2, a and e) taken as the integration region. This will not lead to any significant loss of accuracy provided that the corresponding boundary is conditioned by the potential difference determined based on the condition of electric field stability (7d) and by mechanical boundary conditions (11f). The fulfillment accuracy of these conditions improves as the distance between the adjacent cracks increases, so that when the distances between cracks are as long as 5–6 of the crack lengths, the healing process will proceed equally regardless of whether the simulation is performed for a region consisting of one or several representative elements.

By analogy, it seems that in case of material with a random structure of defects (Fig. 1, c) it is also possible to be confined to a quarter of the representative element (Fig. 2, c) taken as the integration region if to set conditions (7d), (11f) at its boundaries, which will approximately reflect the state in such material.

Conditions (7c), (11e) and (9a), (11a) are accepted on symmetry axes of any of the considered representative elements.

Thus, the selected boundary conditions make it possible to avoid significant loss of accuracy, when simulating healing processes and microcrack interactions by considering one quarter of a symmetric representative element as an integration region (instead of considering integration regions containing several representative elements).

In the subsequent paragraphs you will find the description of how the identified process of microcracks healing (and the stress-strain state in the defects vicinity) is affected by the distance between the adjacent microcracks and their position relative to each other.

Fig. 8 shows dependences of microcrack length $l(t)$ and width $h(t)$ on time for various integration regions, representing one quarter of square, hexagonal and disk representative elements (Fig. 2, a, e, i) with boundary conditions (7d), (11f). The sizes of representative elements were chosen so that the distances between the centers of neighboring microcracks were the same. In this way, the investigated state is characterized by the same distance between the cracks, but different relative positions of the cracks.

The comparison of these curves demonstrates that all solutions are close enough even at the distance between cracks of 20 μm (that is, at a distance between the tips of the cracks equal to their length). Increase of the distance

between microcracks causes all three solutions to tend to each other. Thus, the increasing distance between the cracks leads to a decrease in the influence of the mutual arrangement of cracks on the processes of their healing. As soon as the distances reach 60 μm and greater, the healing processes are almost equal for any arrangement of the cracks relative to each other.

Fig. 8 (a) Dependences of the crack length l (μm) on time t (μs) for integration regions of various sizes representing a quarter of square, hexagonal or disk representative elements, when setting potential difference at their boundary ($\varphi = \text{const}$), (b) similar dependences of the crack width h (μm) on time t (μs)

The analysis of Fig. 5-8 reveals the beginning of noticeable influence of the microcracks interaction on their healing process, when the distance between them decreases to about 5-6 lengths of the microcracks. If the distance between the cracks exceeds 6 or more of their lengths, the time required to heal the microcracks becomes weakly dependent on both the distance between the defects and the position of these defects relative to each other.

The simulation showed that the interaction of microcracks caused by a decrease of the distance between

them down to 10-20 μm does not qualitatively change the above healing process, however, leads to its significant retardation (which is quite understandable from energy considerations). The release of molten material into the crack still happens. However, the shortening of microcrack length and the closing of microcrack edges slow down (Fig. 3, Fig. 5-8). Moreover, the comparison of the curves in Fig. 5-8 demonstrates that with decrease of the distance between the defects, the closing of crack edges slows down more than the its shortening.

Fig. 9 and Fig. 10 respectively present the comparison of temperature fields T and normal stresses σ_{xx} in the vicinity of a microcrack with a distance between the microcrack tips of 10 and 50 μm , respectively, at a time of 13.9 μs .

Fig. 9. Temperature isolines $^{\circ}\text{C}$ (1 – 25, 2 – 50, 3 – 100, 4 – 200, 5 – 300, 6 – 400, 7 – 419) at time $t = 13.9 \mu\text{s}$ at a distance between the microcracks' centers of (a) 60 μm , (b) 20 μm

Fig. 10. Stress isolines σ_{xx} MPa (1 – 10, 2 – 30, 3 – 50, 4 – 70) at time $t = 13.9 \mu\text{s}$ at a distance between the microcracks' centers of (a) 60 μm , (b) 20 μm

The comparison of these fields reveals that microcracks interaction leads to a decrease of compressive stresses in the vicinity of the defect, while the situations are qualitatively similar. Similar situation is observed for σ_{yy} . Retardation of the healing process leads to evaporation of some molten material in the tip of the jet (Fig. 3, b).

4. Conclusion

When simulating the process of microcracks healing, it is possible to be confined to a quarter of a representative element (Fig. 2, a) taken as the integration region. This will not lead to any significant loss of accuracy provided that the boundaries of such quarter are conditioned by the potential difference (7) determined based on the condition of electric field stability (10) and by mechanical boundary conditions (11). In this case, the fulfillment accuracy of these conditions on the boundary of the internal representative element improves as the distances between the adjacent cracks increase, so that when the distances between cracks exceed 5-6 crack lengths, the healing processes will proceed equally regardless of whether the simulation is performed for an integration region consisting of one or several representative elements.

The increasing distance between the microcracks leads to a decrease in the influence of the defects mutual arrangement on the processes of their healing. As soon as the distances exceed 6 crack lengths, the microcrack healing time is almost equal at any arrangement of the cracks relative to each other.

The simulation revealed the beginning of noticeable influence of the microcracks interaction on their healing, when the distance between them decreased to 5-6 lengths of the microcracks. If the distance between the cracks exceeds six their lengths or more, the microcracks healing process becomes almost independent on both the distance between the defects and the position of these defects relative to each other.

The reduction of the distance between cracks down to 1-2 of their linear dimensions (taking into account the change in their relative position) does not qualitatively change the described healing process, however, leads to its significant retardation: molten material remains to eject into the crack, but the crack reduction, especially in its width, is significantly decreased.

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