AVAILABILITY OF THE NETWORKS COMPOSED OF THE MULTIPLE INTERCONNECTED RINGS IN TWO NODES

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Abstract: This paper presents a new approach for modeling the availability of the networks composed of the multiple interconnected rings in two nodes. Method of algebraic formulation is used for the availability modeling. Using this method, the general expression for the availability between two terminals of multiple ring networks is derived through the availability of multiple ring networks consisting of two and three-rings connected to two nodes. To perform the expression some real assumptions have been taken. Analytical calculation showed that the use of these expressions under these assumptions provides real values for availability between the two terminals of multiple ring networks. Information on the availability of links and nodes are taken from previously published works.

Key words: availability, ring, WDM, path.

1. INTRODUCTION

Telecommunications services have an extremely important role in the development of modern society. Rapid growth in demand for data transmission and Internet traffic are requiring high-capacity transmission systems. The only transmission medium that can meet the needs of so large capacity is an optical fiber. Unlike other transmission media, only the optical fiber has a large unused capacity that can effectively utilize the application of technologies based on Wavelength Division Multiplexing - WDM. In such networks, disconnection for any reason, an equipment failure or human error can cause isolation in terms of telecommunications, and heavy losses to users and network operators. Therefore, the availability of telecommunication networks is becoming a very important factor for network operators. The basic problem is how to ensure the survival of links between two nodes within large networks in terms of failure and how to ensure high availability. The development of models of reliability and availability of single self-healing WDM rings, the authors in [1] used the optic add/drop multiplexers in nodes. There are a lot of papers that have been done by calculating availability using shared-path protection, and it has come to the conclusion that the use of this method allows to achieve guaranteed availability and better use of spare capacities, which is particularly important for operators as service providers [9]. Mello and authors [5] have investigated how to design a network that is able to restore the connection for all cases of single link failure between any two nodes in the ring (span protection) with less investment. They also concluded that some linked networks are very robust to the simultaneous double link failures. Some authors have investigated how the availability of services depends on the total capacity and the allocated amount of protection resources necessary for the protection in the event of failure to the working path [2] [10]. In this paper, the model of availability for multiple ring networks starting from the example of a network made up of two rings interconnected in two nodes is developed. This model uses a new method of algebraic formulation as a mathematical basis.

2. THE MATHEMATICAL BASIS FOR THE FORMULATION OF NETWORK AVAILABILITY MODELS

2.1. General on availability

Availability is the probability that a system is correct at some point in time t , provided that it properly worked at time $t = 0$ and that have appeared malfunctions (maintenance or failures) are always being repaired and return the system to its operating state. Also the availability of a system at point in time is defined as the ratio of time for which the system is correct in relation to the total time. If are known mean time to failure (MTTF, Mean Time To Failure) and the mean time to repair (MTTR-Mean Time To Repair) availability can be calculated by the formula:

$$A = \frac{MTTF}{MTTF + MTTR} \quad (1)$$

Unavailability U is probability complementary to the availability, ie

$$U = 1 - A \quad (2)$$

and it is likely that the system is not working properly at some point in time. In reporting about system/network performances, the unavailability is often expressed as MDT (Mean Down Time) in minutes per year, ie

$$MDT = 365 \cdot 24 \cdot 60 \cdot U \quad (3)$$

2.2. The Methods of algebraic formulation

Although there are several methods suitable for modeling the availability of the network (enumeration of elementary paths ...), in this paper the algebraic formulation method is used [8].

Instead of working with probabilities, the emphasis is on algebraic objects (polynomials) and their appropriate transformation. Let $G = (N, E)$ be directed network with source node s and the destination node t . Suppose that the nodes in the network G are ideal and that links $m = |E|$ are subject to failures that occur randomly and are repairable. x_k is the corresponding variable to each link $k = 1, 2, \dots, m$. A is the x_i event for $1 \leq i \leq m$, which indicates that the link e_i is in working order then its availability $A_i = P_r[x_i]$ is. Events $x_1, x_2, \dots, x_m$ are independent. If x_1 i x_2 are events, then $x_1 + x_2$ is union of mutually exclusive events and $x_1 \cdot x_2$ is the case of two independent events. The goal is to calculate the availability of the two terminals s and t as represented by a polynomial function of x elements

$$A_{st}(x) = A_{st}(x_1, x_2, \dots, x_m) \quad (4)$$

which gives the exact availability of the two terminals when the appropriate variables x_k are replaced by the appropriate links availability A_k . To calculate the availability two new operators are introduced:

- operator $\otimes$ is used to represent the series combination of two or more elements;
- operator $\oplus$ is used to represent the parallel combination of two or more elements.

The union of arbitrary events $x_1 \oplus x_2$ and "product" of arbitrary events $x_1 \otimes x_2$ can not develop from the numerical values because it depends on the individual statistical correlation or dependence of two events as opposed to union of two mutually exclusive events, or the product of independent events whose probability is easily calculated [4]. Most of the algebraic approaches for exact calculation of availability between the two terminals using the fact that $\otimes$ is commutative and the fact that $x_i \otimes x_i = x_i$ and dampens any repetition of events in $\otimes$ "product" and replacing them by multiplying the ordinary.

Let S represent the set of all polynomials that can arise by a combination of monoms using these two operators, then let $(S, \oplus, \otimes)$ form distribution petal with smallest element 0 (zero polynomial) and the greatest element 1 (unit polynomial). For $f, g, h \in S$ apply the following axioms [3] from the Table 1:

Table 1. Axioms

$f \otimes f = f$	$f \oplus f = f$
$f \otimes 1 = f$	$f \oplus 0 = f$
$f \otimes g = g \otimes f$	$f \oplus g = g \oplus f$
$f \otimes (g \otimes h) = (f \otimes g) \otimes h$	$f \oplus (g \oplus h) = (f \oplus g) \oplus h$
$f \oplus (g \otimes h) = (f \oplus g) \otimes (f \oplus h)$	$f \otimes (g \oplus h) = (f \otimes g) \oplus (f \otimes h)$
$f \otimes (f \oplus g) = f$	$f \oplus (f \otimes g) = f$

In order to found a connection between the algebraic structures and network availability, P_{st} is defined as the set of all elementary s - t paths P in the network G . It also defines the value of path $v(P)$ as a variables "product" that represent the links along the path P

$$v(P) = \otimes \prod \{x_k : k \in P\} \quad (5)$$

In this case, the availability is values "sum" of $v(P)$ over all elementary paths from s to t

$$A_{st}(x) = \oplus \sum_{P \in P_{st}} v(P) \quad (6)$$

In the $\otimes$ operation each link was included as many times as the number of occurrences in the set P_{st} . Since this operator is $\otimes$, before applying the simple product, repetitions are eliminated.

3. AVAILABILITY OF MULTIPLE RING NETWORKS

Generally in the multiple rings network, the transmitted signal between two nodes S and T passes more rings and interconnecting nodes. The path which signal passes during the normal operation is called *working* path. When analyzing the *restoration* paths, only those paths which include rings (through which working path passes) are taken. Although there are more interconnected rings that could provide connectivity between nodes S and T in case of

failure to working path, the same are not analyzed because the recovery of path would take considerable time. In this way are examined only the subnets consisting of interconnected rings over which goes the working path, while all other combinations are not analyzed. In this paper, models of availability were developed and expressions for availability between the two terminals of network consisting of two and three rings that are connected to two nodes, were derived. The expression valid for the network of two and three rings is also true for multiple rings network consisting of more interconnection rings. The expression of the availability of individual rings was taken from previously published papers [6].

3.1. Availability of two rings networks

Another focus is the network consisting of two rings. Ring 1, ring 2 and the interconnecting node are taken for the possible sites of failure so that there is a total of 7 combinations or 7 renewal paths.

$$\binom{3}{1} + \binom{2}{2} + \binom{3}{3} = 3 + 3 + 1 = 7$$

Restoration paths between nodes S and T need not to be mutually exclusive because a parallel "sum" eliminates identical (duplicate) paths.

For a network consisting of two rings there is working path when no failures and 7 restoration paths that are result of failures in working paths of rings 1 and 2 and primary interconnecting node. Suppose that each ring consists of N links of which m are working ones and that the relationship between $t_{1,1} - t_{1,2}$, and between $s_{2,1} - s_{2,2}$ is achieved with one link.

Figure 1 shows the working path and restoration paths, but always with the correct primary interconnecting node.

Figure 1. Working path and restoration paths with a) no failure b) the failure within the rings 1 c) the failure inside the ring 2 d) the failure within two rings

Figure 2 shows the restoration paths, but always with a faulty primary interconnecting node.

Figure 2. Restoration paths with a) no failure b) the failure within the rings 1 c) the failure inside the ring 2 d) the failure within two rings

The value of path in case without failures is

$$v_{st}(P_0) = (x_{l_{r1}})^m \otimes x_{P1} \otimes (x_{l_{r2}})^m$$

where $x_{l_{r1}}$ i $x_{l_{r2}}$ denote the links that belong to the first (index r1) and second ring (index r2).

For each of these 7 restoration paths are also defined as the values of paths that we mark with $v_{st}(P_1)$ (Figure 1.b) to $v_{st}(P_7)$ (Figure 2.d).

Availability is a "parallel sum" of the path values when no faults and the values of all restoration paths that are a consequence of failures as follows

$$A_{s1;t2}(x) = \oplus \sum_{P_i \in P_{st}} v_{st}(P_i)$$

If it is assumed that the rings do not have a common element which is common practice in the telecom operator, the sign of ordinary multiplication can be used instead of the sign $\otimes$. Then the overall availability between S and T ($s_1; t_2$) is

$$A_{ST}(x) = [(X_{s1;t1,1}(x) \cdot X_{s2,1;t2}(x)) \otimes x_{P1}] + [(X_{s1;t1,2}(x) \cdot X_{s2,2;t2}(x)) \otimes x_{S1}] - [(X_{s1;t1,2}(x) \cdot X_{s2,2;t2}(x)) \otimes (x_{P1} \otimes x_{S1})]$$

If it is assumed that the interconnecting nodes have no common elements, which again is the practice of telecom operators, it gets

$$A_{ST}(x) = (\prod_{i=1}^2 X_{min_i}) \otimes [x_{P1} + x_{S1} - x_{P1} \cdot x_{S1}]$$

If we continue to assume that the rings and interconnecting nodes also have no common elements, $\otimes$ sign may be omitted and the common sign of multiplication may be used

$$A_{ST}(x) = (\prod_{i=1}^2 X_{min_i}) \cdot [x_{P1} + x_{S1} - x_{P1} \cdot x_{S1}]$$

3.2. Availability of three rings networks

A similar analysis can be implemented for a network consisting of three rings, taking into account all the restoration paths that can occur as a result of part failures of each ring, the interconnections of nodes and all their possible combinations. If the number of possible failures is 5, total number of possible combinations is

$$\binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 5 + 10 + 10 + 5 + 1 = 31$$

To avoid examining all possible restoration paths (31), we will take into account the fact that all the failures of working path of individual rings renew within the same so the number of combinations is considerably reduced.

$$\binom{2}{1} + \binom{2}{2} = 2 + 1 = 3$$

Four combinations are possible.

Figure 3. Three rings with a) no failures b) failure of the first interconnect node c) failure of another interconnect node d) failure of both interconnect nodes

If the minimum availability $X_{min_i}, i = 1 \dots 3$ is taken (can be taken because it is a negligible small difference) for each ring that is independent of the link numbers of the working path, it follows

$$A_{ST}(x) = (X_{min_1} \cdot X_{min_2} \cdot X_{min_3}) \otimes [x_{p1} \otimes x_{p2} + x_{s1} \otimes x_{p2} + x_{p1} \otimes x_{s2} + x_{s1} \otimes x_{s2} - x_{p1} \otimes x_{s1} \otimes x_{s2} - x_{s1} \otimes x_{p2} \otimes x_{s2} - x_{p1} \otimes x_{s1} \otimes x_{p2} - x_{p1} \otimes x_{p2} \otimes x_{s2} + x_{p1} \otimes x_{s1} \otimes x_{p2} \otimes x_{s2}]$$

Above expression shows that the overall availability of a network consisting of three rings is actually equal to the product of minimum availability of each ring and the availability of the nodes interconnection.

If you take that a primary and secondary interconnecting nodes are functionally identical or if $x_S = x_P$, it is also realistic because in the event of failure of the primary interconnecting node, restoration is done through secondary interconnect node, so the availability is

$$A_{ST}(x) = \left[\prod_{i=1}^z X_{\min_i} \right] \cdot [x_P \cdot (2 - x_P)]^1$$

For a network of three rings

$$A_{ST}(x) = \left[\prod_{i=1}^3 X_{\min_i} \right] \cdot \{(x_P)^2 \cdot [2 - x_P]^2\}$$

A similar analysis that is made for a network composed of multiple rings shows that the availability between two nodes of multiple ring network equals the product of minimal availability of individual rings and the availability of the nodes interconnection.

4. THE ANALYTICAL RESULTS

The results of the analysis will be presented on a network consisting of two rings to show that the difference is neglected when calculating actual availability and the one that is obtained if minimum availability is taken for an individual ring. The availability of nodes and links is taken from [7].

Table 2. Comparison of availability between S and T when using the actual and the minimum availability of the ring

Number of ring nodes	N=6	N=10	The actual availability	Minimum availability	Difference MDT (min/year)
Links of working path	m=1	m=7	0,9999752896	0,9999752482	0,0217901963
Links of working path	m=3	m=8	0,9999752948	0,9999752482	0,0245122162
Links of working path	m=5	m=3	0,9999752741	0,9999752482	0,0136182825
	N=14	N=16			
Links of working path	m=2	m=13	0,9999750980	0,9999748389	0,1361543751

Table 2. shows that for the typical size of the rings (6-10 nodes) differences between the actual availability and those obtained by taking the minimum availability for each individual ring are in the worst case of 0.0245 min/year, which is negligible so that the previously derived expression can be freely used for the calculation of availability. Even for larger rings of 14 and 16 nodes, those differences were of 0.1 min/year, which is also negligible. This is especially important for SLA (Service Level Agreement) contracts which set a minimum availability threshold and charge penalties for exceeding them.

5. CONCLUSION

This paper presents a new approach for modeling the availability of multiple ring network using an algebraic formulation by working with numeric values. It is shown that the paths need not to be mutually exclusive because the use of operator $\otimes$ eliminates duplicate routes. Analysis showed that the availability computing of multiple ring network in which the rings are connected to two nodes is actually reduced to a product of the minimum availability of individual rings and the availability of the nodes interconnection. Of course, this is done with some realistic assumptions and those are: the rings do not have common elements, the interconnect nodes also have no elements in common and neither rings nor interconnecting nodes have also no common elements. Analytical results showed negligible differences between actual availability and the one obtained when the minimum availability of individual rings is used so that the derivative expression can be used for analysis and calculation of the availability of multi ring networks. The analytical results have confirmed the theoretical analysis so the derived general expression for the availability of the two terminals can be used to analyze and evaluate the availability of multiple ring networks.

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