ANALYSIS OF CIRCUITS WITH OPERATIONAL AMPLIFIERS

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Abstract: The method and the dependencies for transformation of parallelly connected active bipolars into one equivalent (theorem of Millman) during the analysis of single-line analogy circuits with operational amplifiers is presented. Three examples of the application of the theorem of Millman for determining certain parameters and dependencies for circuits of measuring amplifiers and converters are considered.

Keywords: operational amplifiers, theorem of Millman, measuring converter.

1. INTRODUCTION

Two approaches are basically used for the analysis of different circuits with operational amplifiers (OA). The first one is based on the theory of feedback (FB). The second approach makes use of the laws of Kirchoff, considering OA to be ideal.

The present work examines the application of the method for transformation of parallelly connected active bipolars (voltage generators) in one equivalent [1,7]. According to it, an electric circuit, consisting of n parallelly connected branches with conductivities $Y_1, Y_2, \dots, Y_n$ and voltage sources respectively $e_1, e_2, \dots, e_n$ ... (Figure 1) can be replaced with an equivalent source of voltage U_{OA} and its own resistance Z :

$$Z = \frac{1}{\sum_{k=1}^n Y_k}, \quad U_{OA} = \frac{\sum_{k=1}^n e_k Y_k}{\sum_{k=1}^n Y_k} = \frac{\sum_{k=1}^n Y_k U_{Ok}}{\sum_{k=1}^n Y_k} \quad (1)$$

Where $e_k = U_{Ok}$ ($k = 1, 2, \dots, n$).

The sum (1) is algebraic and the voltage signs are determined against a common reper, e.g. in respect to junctions O and A. The expression is valid even when part of the parallel branches are passive. Then the value of the respective voltage is taken to be equal to zero [1]. This dependency (theorem of Millman) can be used for determining some parameters of single-line analogy circuits with OA. Such circuits find wide application in the systems of automation as measuring amplifiers, measuring converters, various regulators, etc. [4, 5, 6, 8, 9, 10]. Different correctional circuits as well as circuits for FB, generally with RC elements, participate in them.

Figure 1. Parallely connected active bipolars

In all cases OA are considered to be ideal – with amplification coefficient of voltage without FB $K_{U \rightarrow \infty}$, with input impedance $Z_i \rightarrow \infty$ and input current $I_i \rightarrow 0$, with output impedance $Z_o \rightarrow 0$ (with open circuit for FB), voltage and current of asymmetry $u_{10} \rightarrow 0$ and $i_{10} \rightarrow 0$ [2, 3, 4, 6, 9, 10].

It is characteristic for AO that it “estimates” the status of its output and input and with the assistance of the external circuit for FB, it passes voltage from the output to the input in such a way that as a result the difference between the voltage of the output and input becomes zero [2, 3, 4, 6].

2. RESULTS AND DISCUSSION

Figure 2 shows the scheme of an inverting amplifier with a T-shaped divider in the circuit for FB [9]. All potentials in the marked junctions are considered against the common “mass” – junction O. According to the above mentioned characteristics of OA and the principle of “conventional ground” [2, 3, 4, 6], the potentials at the entries of OA are equal, i.e. $V_a = V_b = 0$. According to the theorem of Millman (1) we can write down:

$$V_b = \frac{\frac{V_i}{R_1} + \frac{V_a}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = 0 \quad (3)$$

$$V_a = \frac{\frac{V_b}{R_2} + \frac{V_0}{R_3}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \quad (4)$$

Figure 2. Inverting amplifier with a T-shaped divider in the circuit for FB

When (3) is solved against V_a and is equalized to (4), it is easy to obtain the expression for the amplification coefficient of voltage for the circuit:

$$K = \frac{V_0}{V_i} = - \frac{R_2 R_3 + R_3 R_4 + R_2 R_4}{R_1 R_4} = - \frac{1}{R_1} \left(R_2 + R_3 + \frac{R_2 R_3}{R_4} \right) \quad (5)$$

Figure 3 shows the scheme of a measuring differential amplifier with high input resistance, which can work in intensive disturbing signals (broom, impulse interference, etc.), appearing single-phase at both entries [10]. The amplification coefficient of this circuit can be regulated by modifying R_3 . Similarly to the previous scheme there is an equality of the potentials at the entries of the OA, i.e. $V_a = V_c$ and $V_b = V_e$ whereas

$$U_i = V_a - V_b \quad (6)$$

According to dependency (1) for the case is found:

$$V_a = V_c = \frac{\frac{V_d}{R_2} + \frac{V_e}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}, \quad V_b = V_e = \frac{\frac{V_d}{R_2} + \frac{V_c}{R_3} + \frac{U_0}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad (7)$$

Replacing the expressions (7) in (6) and provided that $R_4 = R_2$ and $R_5 = R_1$, after ordinary algebraic transformations the following is obtained for the amplification coefficient:

$$K = \frac{U_0}{U_i} = 1 + R_1 \left(\frac{2}{R_3} + \frac{1}{R_2} \right) \quad (8)$$

Figure3. Measuring differential amplifier

The following scheme (Figure 4) finds application in automation as a proportional-integral-differential (PID) regulator as well as in the analog measuring technique for differentiating and separating rapidly-changing signals, finding the average of alternating quantities, etc. [6]. On the conductivities of the input circuit Y_1 and the FB circuit Y_2 in operational form [7], it can be written down:

$$Y_1 = \frac{1}{R_1} + pC_1, \quad Y_2 = \frac{1}{R_2 + \frac{1}{pC_2}} \quad (9)$$

According to the considered features of OA, it follows that $V_a = V_b = 0$. Then for junction a according to (1) it is obtained:

$$\frac{U_i(p) \left(\frac{1}{R_1} + pC_1 \right) + \frac{U_0(p)}{R_2 + \frac{1}{pC_2}}}{\frac{1}{R_1} + pC_1 + \frac{1}{R_2 + \frac{1}{pC_2}}} = 0 \quad (10)$$

When (10) is solved against $U_0(p)$ we obtain the link between the output and input voltage in operational form:

$$U_0(p) = - \left[\left(\frac{R_2}{R_1} + \frac{C_1}{C_2} \right) + pR_2C_1 + \frac{1}{pR_1C_2} \right] U_i(p) \quad (11)$$

Figure 4 Proportional-integral-differential regulator

After the reverse transformation of Laplas it is easy to obtain the link between the originals of the voltages:

$$U_0(t) = - \left[\left(\frac{R_2}{R_1} + \frac{C_1}{C_2} \right) U_i(t) + R_2C_1 \frac{dU_i(t)}{dt} + \frac{1}{R_1C_2} \int_0^t U_i(t) dt \right] \quad (12)$$

3. CONCLUSION

The use of the method and the dependencies for transformation of parallelly connected active bipolars in one equivalent (theorem of Millman) can considerably simplify the analysis of singleline analogy schemes with OA. The proposed approach can be applied for obtaining certain parameters and dependencies for measuring converters, regulators, etc. It is particularly helpful in more complex schemes with more than one OA.

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