MATHEMATICAL APPARATUS USED IN “METHOD OF MODELS” FOR THE STUDY OF FOUR-BAR LINKAGE MECHANISM (RRRR)

Roumen Roussev

Faculty of Technics and Technologies of Yambol, Trakia University of Stara Zagora, Bulgaria

Faculty of Engineering, South-West University "Neofit Rilski", Blagoevgrad, Bulgaria

e-mail: roussev_r@abv.bg

Abstract: “The method of models” is a graphical method for determining kinematic characteristics of links and points of polycontoured mechanism. For this purpose must find their images in the “secondary” and “tertiary” mechanism’s model. In present paper mathematical approach, replacing graphical work with analytical calculations is proposed, to analyze four- bar linkage mechanism.

Key words: method of models, kinematic characteristics, four-bar linkage, mathematical apparatus.

1. INTRODUCTION

Method of models is proposed by Piperkov [1] and developed by Tenchev [2]. Trough it can be determined kinematic characteristics of random links and points of polycontoured mechanism without calculate it for previous ones. Only need to draw a mechanism in the desired position, which is assumed to be “primary model”. This is displacement function for selected value of the input parameter. Graphic representation of “secondary model” / for determining velocities/ and “tertiary model” / for determining accelerations/ of mechanism is carried out according to certain rules. Advantage of this method is that it can obtain correct results for trigonometric functions for angles $\varphi = k \cdot \pi/2$ ($k=1,2, \dots$). The indefiniteness of arcs functions is being avoided. The disadvantage of this method is great graphics work. Usually this method is used to analyze 12 positions of the mechanism or to clarify “conflict” positions obtained by analytical methods.

Typically polycontoured mechanisms are constructed from elementary four-linkage mechanisms.

In this work mathematical apparatus is available for determining the coordinates of the characteristic points of the four-bar linkage mechanism in its models without graphic representation. This enables research to be conducted for any number of positions of the mechanism.

2. NATURE OF “METHOD OF MODELS” FOR FOUR-BAR LINKAGE MECHANISM.

The mechanism is built for the desired position. The centre of the model has been selected, for RRRR it is the axis of rotation of input link – point O (picture below).

2.1. Secondary model

1'. The main link of the mechanism 1 and its secondary image 1' coincide, i.e. $1 \equiv 1'$ and $A \equiv A'$.

2'. If the secondary image A' of point A of the rocker is known, then secondary image B' of point B is located on the line which passes through A' and is parallel to line AB. Points $A'B'$ determine the position of the secondary image of the rocker in secondary model of the

mechanism. The velocity $V_B = \omega_1 \cdot OB'$ of point B is perpendicular to the segment OB' and is with direction corresponding of ω_1 relative to the center O . Angular velocity of the rocker AB

is: $\omega_{AB} = \frac{A'B'}{AB} \cdot \omega_1$. If directed segments AB and $A'B'$ are unidirectional, then angular velocity

ω_{AB} has the same direction as ω_1 . If they are not, angular velocity ω_{AB} has opposite direction of ω_1 .

Secondary image M' of third point M on the rocker AB is obtained by triangle $A'B'M'$, which is similar and equally oriented with triangle ABM . Velocity of point M : $V_M = \omega_1 \cdot OM'$ is

perpendicular to the segment OM' with direction corresponding of ω_1 relative to the center O .

3'. Secondary image C' of point C of the rocker coincide with center O , and secondary image B' of point B lies on line that passes through the center O , parallel to the line BC . Points C' and B' determine the position of secondary image of the rocker BC in the secondary model of

the mechanism. The magnitude of the angular velocity of the rocker BC is: $\omega_{BC} = \frac{C'B'}{CB} \cdot \omega_1$. If

directed segments CB and $C'B'$ are unidirectional, then angular velocity ω_{BC} has direction of ω_1 . If they are not, angular velocity ω_{BC} has opposite direction of ω_1 .

Figure 1.

Graphic constructions for Secondary and Tertiary model

2.2. Tertiary model

1". If input link 1 has angular velocity $\omega_1 = \text{const}$ ($\varepsilon_1 = 0$), then link 1 and its tertiary image 1" coincide i.e. $1 \equiv 1''$ and $A \equiv A''$. Points O and A" determine tertiary image of primary link in the tertiary model of the mechanism. If the acceleration of the primary link 1 is $\varepsilon_1 \neq 0$, tertiary image A" of point A is shifted at a distance $AA'' = \frac{\varepsilon_1}{\omega_1^2} \cdot OA$. Directed segment AA" is

perpendicular to the segment OA with direction opposite to acceleration ε_1 .

2". If tertiary image A" of point A on the rocker is known, the tertiary image B" of point B lies on line, perpendicular to segment AB at a distance A"N₁ of image A". Segment A"N₁ is

defined as $A''N_1 = \frac{(A'B')^2}{AB}$. The direction from A" to N₁ corresponds to the direction from A

to B. Points A" and B" determine position of tertiary image of the rocker in the tertiary model of the mechanism. Acceleration $\overrightarrow{a_B}$ in tertiary model is directed from B" to O with magnitude

$a_B = \omega_1^2 \cdot OB''$. If the segment B"N₁ is considered as a vector $\overrightarrow{B''N_1}$ and observes rotation of segment A"B" on it about point A", then it is the direction of angular acceleration ε_{AB} of the

rocker. The magnitude of this acceleration is $\varepsilon_{AB} = \frac{B''N_1}{AB} \cdot \omega_1^2$.

The tertiary image M" of third point M on the rocker is obtained by triangle A"B"M", which is similar and equally oriented with triangle ABM. Acceleration $\overrightarrow{a_M}$ in tertiary model has direction from M" to O with magnitude $a_M = \omega_1^2 \cdot OM''$.

3". The tertiary image C' of point C of the rocker coincide with center O, and tertiary image B' of point B lies on line which is perpendicular to the segment BC at a distance C"N₂ from

the center C" $\equiv O$. The magnitude of C"N₂ is determined by the formula $C''N_2 = \frac{(C'B')^2}{CB}$. The

direction from C" to N₂ corresponds to the direction from C to B. Points C" and B" determine the position of tertiary image of the crank in the tertiary model of the mechanism. If the segment C"N₂ is considered as a vector $\overrightarrow{C''N_2}$ and track rotation of segment C"B" on it about

point C", then it is the direction of angular velocity ε_{BC} of the crank with magnitude

$$\varepsilon_{CB} = \frac{C''N_2}{CB} \cdot \omega_1^2.$$

2.3. Graphic constructions

a. Primary model: Mechanism is built in desired position in scale 1: 1 (picture above). Coordinate axis Ox is assumed to coincide with the segment OC.

b. Secondary model: through the point O line $m \parallel BC$ is plotted, segment AB is extended to its intersection with m; intersection point is p. B'; Line $p \parallel BM$ is plotted through point B', through point A line n (extended segment AM); intersection point between m and p is point M'.

c. Tertiary model: position of point A" is located; position of point N₁ and N₂ is located and through them lines $k \perp AB$ and $q \perp BC$; is plotted; its intersection point is B"; lines s and t are built so that $\triangle A''B''M''$ is similar to $\triangle ABM$.

3. MATHEMATICAL APPARATUS FOR DETERMINING CHARACTERISTIC POINTS OF MECHANISM

From the “primary model” the coordinates of characteristic points is determined:

Point A: $x_A = OA \cdot \cos \varphi$ $y_A = OA \cdot \sin \varphi$

Point B: $x_B = OC + CB \cdot \cos \psi$ $y_B = CB \cdot \sin \psi$

Point C: $x_C = OC$ $y_C = 0$

The angle ξ between rocker AB and coordinate axis Ox is determine from primary model of mechanism or $\cos \xi = \frac{x_B - x_A}{AB}$.

3.1. Secondary model

- Coordinates of p. A' : $x_{A'} = x_A$ $y_{A'} = y_A$; coordinates of p. C' : $x_{C'} = 0$; $y_{C'} = 0$

- Slopes of the lines, determined by segments: $OA \rightarrow Ka = \frac{y_A}{x_A}$; $CB \rightarrow Kb = \frac{y_B}{x_B - OC}$

$AB \rightarrow K_{AB} = \frac{y_B - y_A}{x_B - x_A}$; $AM \rightarrow K_{AM} = \frac{y_M - y_A}{x_M - x_A}$; $BM \rightarrow K_{BM} = \frac{y_M - y_B}{x_M - x_B}$

- Coordinates of p. P (intersection point between line defined by segment AB and coordinate axis Ox)

$$x_P = x_A - \frac{y_A}{K_{AB}}$$

- Length of the segment OB' : $OB' = OC \cdot \frac{x_P}{x_P - x_C}$

- Coordinate of the p. B' : $x_{B'} = OB' \cdot \cos \psi$; $y_{B'} = OB' \cdot \sin \psi$

- Length of the segment $A'B'$: $A'B' = \sqrt{(x_{A'} - x_{B'})^2 + (y_{A'} - y_{B'})^2}$

- Slope of the line, determined by segment $A'M'$ is the same as AM .

- Slope of the line, determined by segment $B'M'$ is the same as BM .

- Coordinates of p. M' : $x_{M'} = \frac{K_{AM} \cdot x_{A'} - K_{BM} \cdot x_{B'} + y_{B'} - y_{A'}}{K_{AM} - K_{BM}}$; $y_{M'} = y_{B'} + K_{BM} \cdot x_{M'} - x_{B'}$

- Length of the segment OM' : $OM' = \sqrt{x_{M'}^2 + y_{M'}^2}$

3.2. Tertiary model

- Length of the segment AA'' : $AA'' = \frac{\varepsilon_1}{\omega_1^2} \cdot OA$

- Coordinates of p. C'' : $x_{C''} = 0$ $y_{C''} = 0$

- Coordinates of p. A'' : $x_{A''} = x_A + AA'' \cdot \cos\left(\frac{\pi}{2} - \varphi\right)$ $y_{A''} = y_A - AA'' \cdot \sin\left(\frac{\pi}{2} - \varphi\right)$

- Length of the segment $A''N_1$: $A''N_1 = \frac{(A'B')^2}{AB}$

- Length of the segment ON_2 : $ON_2 = \frac{(OB')^2}{CB}$

- Coordinates of points N_1 и N_2

$$x_{N1} = x_{A''} + A''N_1 \cdot \cos \xi \quad y_{N1} = y_{A''} + A''N_1 \cdot \sin \xi$$

$$x_{N2} = ON_2 \cdot \cos \psi \quad y_{N2} = ON_2 \cdot \sin \psi$$

- Coordinates of p. B'' :

$$x_{B''} = \frac{\left(\frac{1}{k_{AB}} \cdot x_{N1} - \frac{1}{k_{BC}} \cdot x_{N2} \right) - y_{N2} + y_{N1}}{\frac{1}{k_{AB}} - \frac{1}{k_{BC}}} \quad y_{B''} = y_{N1} + \frac{1}{k_{AB}} \cdot (x_{N1} - x_{B''})$$

- Slope of the lines, determined by segments $A''B''$, $A''M''$ и $B''M''$:

$$K_{A''B''} = \frac{y_{A''} - y_{B''}}{x_{A''} - x_{B''}} \quad K_{A''M''} = \frac{k_{A''B''} + \tan \alpha}{1 - k_{A''B''} \cdot \tan \alpha} \quad K_{B''M''} = \frac{k_{A''B''} - \tan \beta}{1 + k_{A''B''} \cdot \tan \beta}$$

- Coordinates of p. M'' :

$$x_{M''} = \frac{K_{A''M''} \cdot x_{A''} - K_{B''M''} \cdot x_{B''} + y_{B''} - y_{A''}}{K_{A''M''} - K_{B''M''}} \quad y_{M''} = y_{B''} + K_{B''M''} \cdot (x_{M''} - x_{B''})$$

- Length of the segment OD'' : $OD'' = \sqrt{x_{D''}^2 + y_{D''}^2}$

4. CONCLUSIONS

This article provides mathematical apparatus that maintains the advantages of "method of models" and avoids voluminous graphics work.

Presented equations are simple and can be implemented in a computer program. Mathematical apparatus allows studying mechanism for a desired number of positions.

5. REFERENCES

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