Cross Phase Incoherent Modulational Stability Due to Interacting Laser Pulses in a Magnetized Plasma

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Abstract: A new form of modulational instability is analyzed in case of the propagation of two laser pulses in an electron-ion plasma, under the influence of a magnetic field. It is actually the influence of one nonlinear wave on the nonlinear phase change in the other one. Our next important observation is the effect of incoherence of the initial state on such cross phase modulation. This is actually done with the formulation of Wigner and is also important due to quantum studies on plasma. Lastly it is seen that the external magnetic field has an important influence on such stability.

Keywords: Magnetized plasmas, Nonlinear phenomena, Laser-plasma interactions, Waves, oscillations, Instabilities in plasmas and intense beams.

1. INTRODUCTION

Stability studies in nonlinear wave propagation in plasma is very important both in classical and quantum context [1]. In fact modulational stability holds very important position in such analysis [2]. For the case of modulational stability people have derived the necessary Nonlinear Schrödinger (NLS) equation in various cases [3]. But up till now the study of modulational stability in interacting waves in plasma has drawn little attention [4].

The modulational stability induced by the cross phase modulation was considered in case of nonlinear optics by Agarwal *et al.* [5-7] long back. They observed that there was a significant change in gain and decay. It was quite natural that the phenomenon was governed by a set of coupled NLS equation. One should note that such coupled equations also occur in the description of upper hybrid waves in plasma. This has been discussed in the works of Kourakis *et al.* [8]. Even in case of Alfvénic wave propagation Mjølhus *et al.* [9] have shown the existence of such a pair of NLS equation. But such coupled systems have never been studied when the initial density is incoherent, *i.e.* it is not a definite steady state but has a distribution in the sense of probability. Now-a-days in present day quantum plasma a reformation of modulational stability is very important when there is incoherence and cross phase modulation [10].

In this communication we have studied the instability growth rate for two oppositely propagating

laser pulses resulting in a cross phase modulation situation, with the help of Wigner formalism which gives us result not only when the interaction is coherent but also when it is incoherent.

2. FORMULATION

We consider a electromagnetic wave in a cold plasma with relativistic electrons in a constant magnetic field $\vec{B} = B_0 \hat{z}$. We consider heavier ions to be stationary. If we assume the electromagnetic wave to be propagating along z-axis then the governing equations in one-dimensional approximation can be written as [11];

$$\frac{\partial^2 \vec{A}}{\partial t^2} - \frac{\partial^2 \vec{A}}{\partial z^2} = \frac{\partial^2 \phi}{\partial t \partial z} \hat{z} + \frac{N}{\gamma} \vec{P} \quad (1)$$

$$\frac{\partial^2 \phi}{\partial z^2} = N - 1 \quad (2)$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial z} (NV_z) = 0 \quad (3)$$

$$\frac{\partial}{\partial t} (\vec{P} - \vec{A}) = \frac{\partial}{\partial z} (\phi - \gamma) \hat{z} + \vec{V} \times \left(\frac{\partial}{\partial z} \hat{z} \right) \times (\vec{P} - \vec{A}) - \vec{V} \times \vec{B}_0 \quad (4)$$

Where $\vec{A}$ is the vector potential, ϕ is the electrostatic potential, $\vec{V} = \vec{P}/\gamma$ is the fluid velocity and $\gamma = \sqrt{1 + P^2}$ the relativistic factor. N represents the electron density. Here the electron density is normalized by N_0 , the scalar and vector potentials by mc^2/e , $\vec{B}$ by $m\omega_{pe}/e$, the electron velocity by velocity of light c and momentum by mc , the length is normalized c/ω_{p0} and time scaled by ω_{p0}^{-1} .

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We do not go into the detail of the derivation of the following. If we write $\vec{P} = \vec{p} + \gamma u \hat{z}$ where $\vec{p}$ represents the transverse part of the electron momentum and $u = V_z \hat{z}$. Also in Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$, we have $A_z = 0$. By separating the longitudinal and transverse motion we get a simplified normalized set of equation as follows,

$$\frac{\partial^2 \vec{A}}{\partial t^2} - \frac{\partial^2 \vec{A}}{\partial z^2} = \frac{\partial^2 \phi}{\partial t \partial z} \hat{z} + \frac{N}{\gamma} \vec{P} \quad (5)$$

$$\frac{\partial^2 \phi}{\partial z^2} = N - 1 \quad (6)$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial z} (NV_z) = 0 \quad (7)$$

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right) (\vec{p} - \vec{A}) = \frac{\vec{B}_c \times \vec{p}}{\gamma} \quad (8)$$

$$\frac{\partial}{\partial t} (\gamma u) = \frac{\partial}{\partial z} (\phi - \gamma) + \frac{1}{\gamma} \left[\vec{p} \cdot \frac{\partial}{\partial z} (\vec{p} - \vec{A}) \right] \quad (9)$$

where, ' $\vec{B}_c = B_c \hat{z}$ ' is the electron cyclotron frequency normalized by plasma frequency and the relativistic factor has got a slight change in expression i.e.

$$\gamma = \sqrt{\frac{1+p^2}{1-u^2}}$$

Now proceeding with weakly relativistic situation, $\gamma \approx 1 + \frac{p^2}{2}$ and assuming that the polarized laser pulse of the form;

$$\vec{A} = \frac{1}{2} (\hat{x} + j\alpha\hat{y}) a_1(z, t) \exp[j\theta_1] + \frac{1}{2} (\hat{x} - j\alpha\hat{y}) a_2(z, t) \exp[i\theta_2] + c.c \quad (10)$$

where ' a_1, a_2 ' are slowly varying amplitude whereas $\theta_i = k_i z - \omega_i t$, where $i = 1, 2$ and $j = \sqrt{-1}$.

Whenever an electromagnetic wave is circularly polarized, it can rotate in one of two possible senses: 'right circular polarization' in which the electric field vector rotates in a right-hand sense with respect to the direction of propagation, and 'left circular polarization' in which the vector rotates in a left-hand sense. ' $\alpha=1$ ' stands for the right hand circular polarization (RCP) state and ' $\alpha=-1$ ' for the left hand polarization (LCP) state.

The corresponding electron momentum can be written as

$$\vec{p} = \frac{1}{2} (\hat{x} + j\alpha\hat{y}) p_1 \exp[i\theta_1] + \frac{1}{2} (\hat{x} - j\alpha\hat{y}) p_2 \exp[i\theta_2] + c.c \quad (11)$$

Following [11] we find

$$p_1 = c_1 a_1 - \frac{\alpha B_c c_1^2}{2\omega_1} (c_1^2 |a_1|^2 + 2c_2^2 |a_2|^2) a_1 \quad (12)$$

$$p_2 = c_2 a_2 - \frac{\alpha B_c c_2^2}{2\omega_2} (2c_1^2 |a_1|^2 + c_2^2 |a_2|^2) a_2 \quad (13)$$

where $c_1 = \omega_1 / (\omega_1 - \alpha B_c)$ and $c_2 = \omega_2 / (\omega_2 - \alpha B_c)$ and the corresponding dispersion relations for left hand circular polarization (LCP) and right hand circular polarization (RCP) are respectively;

$$\omega_1^2 - k_1^2 = \frac{\omega_1}{\omega_1 - \alpha B_c} \quad (14)$$

$$\omega_2^2 - k_2^2 = \frac{\omega_2}{\omega_2 - \alpha B_c} \quad (15)$$

Following the standard procedure the two coupled Nonlinear Schrödinger equations are

$$j \left(\frac{\partial}{\partial \tau} + \delta \frac{\partial}{\partial \xi} \right) a_1 + S_1 \frac{\partial^2 a_1}{\partial \xi^2} + (q_{11} |a_1|^2 + q_{12} |a_2|^2) a_1 = 0 \quad (16)$$

$$j \left(\frac{\partial}{\partial \tau} + \delta \frac{\partial}{\partial \xi} \right) a_2 + S_2 \frac{\partial^2 a_2}{\partial \xi^2} + (q_{21} |a_1|^2 + q_{22} |a_2|^2) a_2 = 0 \quad (17)$$

$q_{il} (i, l = 1, 2)$ are the nonlinear co-efficient written as;

$$q_{11} = \frac{c_1^3 v_{g1}}{4k_1} \quad (18)$$

$$q_{12} = \frac{c_2 v_{g1}}{4k_1} \left\{ 2c_1 c_2 - \frac{(c_1 k_2 + c_2 k_1)(k_1 + k_2)}{(\omega_1 + \omega_2)^2 - 1} \right\} \quad (19)$$

$$q_{21} = \frac{c_2^3 v_{g2}}{4k_2} \quad (20)$$

$$q_{22} = \frac{c_1 v_{g2}}{4k_2} \left\{ 2c_1 c_2 - \frac{(c_1 k_2 + c_2 k_1)(k_1 + k_2)}{(\omega_1 + \omega_2)^2 - 1} \right\} \quad (21)$$

Where ' v_{g1} ' and ' v_{g2} ' are group velocities

$$v_{g1} = \frac{2k_1 (\omega_1 - \alpha B_c)^2}{2\omega_1 (\omega_1 - \alpha B_c)^2 + \alpha B_c} \quad (22)$$

$$v_{g2} = \frac{2k_2 (\omega_2 + \alpha B_c)^2}{2\omega_2 (\omega_2 + \alpha B_c)^2 - \alpha B_c} \quad (23)$$

And the dispersion coefficients ' S_1 ', ' S_2 ' are;

$$S_1 = \frac{v_{g1}}{2k_1} \left[1 - \left(\frac{v_{g1} + v_{g2}}{2} \right)^2 \left\{ 1 - \frac{\alpha B_c}{(\omega_1 - \alpha B_c)^2} \right\} \right] \quad (24)$$

$$S_2 = \frac{v_{g2}}{2k_2} \left[1 - \left(\frac{v_{g1} + v_{g2}}{2} \right)^2 \left\{ 1 + \frac{\alpha B_c}{(\omega_2 - \alpha B_c)^2} \right\} \right] \quad (25)$$

If we now make a change of variable

$$a_1 = \psi_1 \exp \left\{ j \left(\frac{\delta^2}{4S_1} \tau - \frac{\delta}{2S_1} \xi \right) \right\} \quad (26)$$

$$a_2 = \psi_2 \exp \left\{ i \left(\frac{\delta^2}{4S_2} \tau + \frac{\delta}{2S_2} \xi \right) \right\} \quad (27)$$

$$\text{Where, } \xi = z - \left(\frac{vg_1}{2} + \frac{vg_2}{2} \right),$$

Then the equations (16) and (17) are transformed into;

$$i \frac{\partial \psi_1}{\partial \tau} + S_1 \frac{\partial^2 \psi_1}{\partial \xi^2} + (q_{11} |\psi_1|^2 + q_{12} |\psi_2|^2) \psi_1 = 0 \quad (28)$$

$$i \frac{\partial \psi_2}{\partial \tau} + S_2 \frac{\partial^2 \psi_2}{\partial \xi^2} + (q_{21} |\psi_1|^2 + q_{22} |\psi_2|^2) \psi_2 = 0 \quad (29)$$

3. THE STABILITY ANALYSIS

The standard treatment of modulational stability proceeds by the linearization of the equations (28) and (29). But due to the presence of incoherence in the initial state we take recourse to the method of Wigner[12]. For this we define the Wigner functions as follows,

$$W_{11}(x, p) = \frac{1}{2\pi} \int dy \exp[ipy] \psi_1 \left(\tau, \xi + \frac{y}{2} \right) \psi_1^* \left(\tau, \xi - \frac{y}{2} \right) \quad (30)$$

$$W_{22}(x, p) = \frac{1}{2\pi} \int dy \exp[ipy] \psi_2 \left(\tau, \xi + \frac{y}{2} \right) \psi_2^* \left(\tau, \xi - \frac{y}{2} \right) \quad (31)$$

Using (28), (30), (31) we get

$$\frac{\partial W_{11}}{\partial \tau} = X + \Sigma \quad (32)$$

where,

$$X = \frac{i}{2\pi} \int_{-\infty}^{\infty} dy \exp[ipy] P \left(\frac{\partial^2 \psi_1}{\partial \xi^2} \psi_1^* - \frac{\partial^2 \psi_1^*}{\partial \xi^2} \psi_1 \right) \quad (33)$$

$$\Sigma = \frac{i}{2\pi} \int_{-\infty}^{\infty} \exp[ipy] [Q_1 |\psi_1|^2 - Q_2 |\psi_2|^2] \times \{ \psi_1(\xi + y/2) \psi_1^*(\xi - y/2) - \psi_1^*(\xi + y/2) \psi_1(\xi - y/2) \} \quad (34)$$

After usual simplification we get;

$$\frac{\partial W_{11}}{\partial \tau} + 2S_1 p \frac{\partial W_{11}}{\partial x} + 2q_{11} n_1(\xi, \tau) \sin \left(\frac{1}{2} \frac{\partial}{\partial x} \frac{\partial}{\partial p} \right) W_{11} + 4q_{12} n_2(\xi, \tau) \sin \left(\frac{1}{2} \frac{\partial}{\partial x} \frac{\partial}{\partial p} \right) W_{11} = 0 \quad (35)$$

In the same manner;

$$\frac{\partial W_{22}}{\partial \tau} + 2S_2 p \frac{\partial W_{22}}{\partial x} + 4q_{21} n_1(\xi, \tau) \sin \left(\frac{1}{2} \frac{\partial}{\partial x} \frac{\partial}{\partial p} \right) W_{22} + 2q_{22} n_2(\xi, \tau) \sin \left(\frac{1}{2} \frac{\partial}{\partial x} \frac{\partial}{\partial p} \right) W_{22} = 0 \quad (36)$$

In the above derivations we have used;

$$n(\xi + y/2) = \sum_{l=0}^{\infty} \frac{\xi^l}{2^l l!} \frac{\partial^l n}{\partial \xi^l} \Big|_{\xi=0} \quad (37)$$

$$\left. \begin{aligned} W_{11} &= W_{11}^{(0)}(\tilde{K}) + \varepsilon W_{11}^{(1)}(\tilde{K}, \xi, \tau) \\ W_{22} &= W_{22}^{(0)}(\tilde{K}) + \varepsilon W_{22}^{(1)}(\tilde{K}, \xi, \tau) \\ n_A &= n_A^{(0)} + \varepsilon n_A^{(1)}(\xi, \tau) \\ n_B &= n_B^{(0)} + \varepsilon n_B^{(1)}(\xi, \tau) \end{aligned} \right\} \quad (38)$$

Next considering $W_{11}^{(1)}$, $W_{22}^{(1)}$, $n_A^{(1)}$, $n_B^{(1)}$ of the following form;

$$\left. \begin{aligned} W_{11}^{(1)}(\tilde{K}, \xi, \tau) &= g_{11}(\tilde{K}) \exp[j(q\tilde{K} - \Omega\tau)] \\ W_{22}^{(1)}(\tilde{K}, \xi, \tau) &= g_{22}(\tilde{K}) \exp[j(q\tilde{K} - \Omega\tau)] \\ n_A^{(1)}(\xi, \tau) &= G_A \exp[j(q\tilde{K} - \Omega\tau)] \\ n_B^{(1)}(\xi, \tau) &= G_B \exp[j(q\tilde{K} - \Omega\tau)] \end{aligned} \right\} \quad (39)$$

Where, 'Ω' is the modulation frequency and 'q' being the modulation wave number.

Substituting equation(38) and (39) in (35) and (36) we get;

$$\left(1 + q_{11} \int \frac{W_{11}^{(0)}(\tilde{K}+q/2) - W_{11}(\tilde{K}-q/2)}{(\Omega - 2S_1 \tilde{K} q)} d\tilde{K} \right) \times \left(1 + q_{22} \int \frac{W_{22}^{(0)}(\tilde{K}+q/2) - W_{11}(\tilde{K}-q/2)}{(\Omega - 2S_2 \tilde{K} q)} d\tilde{K} \right) - \left(4q_{12} q_{21} \int \frac{W_{11}^{(0)}(\tilde{K}+q/2) - W_{11}(\tilde{K}-q/2)}{(\Omega - 2S_1 \tilde{K} q)} d\tilde{K} \times \int \frac{W_{22}^{(0)}(\tilde{K}+q/2) - W_{22}(\tilde{K}-q/2)}{(\Omega - 2S_2 \tilde{K} q)} d\tilde{K} \right) = 0 \quad (40)$$

The evaluation of the equation(40) requires that we specify the form of the initial distribution. For the usual standard steady state;

$$\left. \begin{aligned} W_{11}^{(0)} &= A_0^2 \delta(\tilde{K}) \\ W_{22}^{(0)} &= B_0^2 \delta(\tilde{K}) \end{aligned} \right\} \quad (41)$$

(Where 'A₀' and 'B₀' are the constant amplitudes) whence the equation(40) leads to

$$\Omega^4 + A\Omega^2 + C = 0 \quad (42)$$

whose solution is

$$\Omega^2 = -\frac{A}{2} \pm \frac{1}{2}\sqrt{A^2 - 4C} \quad (43)$$

with

$$A = -S_2^2 q^4 - S_1^2 q^4 + 2q_{22}S_2 q^2 B_0^2 + 2q_{11}S_1 q^2 A_0^2 \quad (44)$$

$$C = S_1^2 S_2^2 q^4 - 2q_{22}S_2 q^2 B_0^2 S_1^2 q^4 - 2q_{11}S_1 q^2 A_0^2 S_2^2 q^4 + 4S_1 S_2 q_{11} q_{22} q^4 A_0^2 B_0^2 \quad (45)$$

For the initial incoherence situation we assume that the waves to be Lorentzian. So that;

$$\left. \begin{aligned} W_{11} &= \frac{A_0^2}{\pi} \frac{p}{q^2 + p^2} \\ W_{22} &= \frac{B_0^2}{\pi} \frac{p}{q^2 + p^2} \end{aligned} \right\} \quad (46)$$

A detailed algebraic manipulation leads to;

$$\Omega = -2j(S_1 + S_2)qp \pm \{S_1 q^2 [S_2 q^2 + q_{11}(A_0 + B_0)] \pm S_1 q^2 \sqrt{16q_{12}q_{21}A_0 B_0 + q_{22}^2(A_0^2 - B_0^2)}\}^{1/2} \quad (47)$$

4. RESULTS AND DISCUSSION

To start with we proceed with the standard situation, where the initial density is deterministic. This is shown in Figure (1) for the three values of ω_1 , ω_2 , ω_3 and zero magnetic field ' B_c '.

Figure 1: The instability growth rate for monochromatic case in absence of magnetic field.

Here Ω_i (imaginary part of Ω for monochromatic case) is plotted as a function of ' q ' (modulational wave number). The stability curve in this monochromatic situation is exactly same as in the case of standard modulational stability. Though one should note that the maximum growth rate of instability changes with the

change of wave frequency (ω_1 or ω_2). In the next phase we switch on the magnetic field and the variation of Ω_i is shown if Figure (2).

Figure 2: The instability growth rate for monochromatic case in presence of magnetic field.

Here we observe a very slight change as the magnetic field is non-zero. But due to monochromatic nature a standard variation is obtained. In this case also the maximum growth rate of instability has a variation with the wave frequency ' ω_1 ' while ω_2 is kept fixed. In the third Figure (i.e. Figure (3)) we have tried to show the variation of the stability curve when the magnetic field is non-zero and ω_1 is fixed while ω_2 is varied. Still we do not observe a very different form of the stability curve and just as in Figure (1) or Figure (2) the wave frequency has an effect on the maximum growth rate.

Figure 3: The variation of instability growth rate with varying external magnetic field.

In the fourth diagram (i.e. Figure (4)) we consider the non-monochromatic situation. The variation of Ω_{II} with respect to ' q ' changes completely. Here we have used two different values of the magnetic field ' B_c ' while a fixed value of the wave frequencies ' ω_1 ' and ' ω_2 '. Here we observe that the external magnetic field ' B_c ' has a clear effect on the maximum growth rate of instability.

Figure 4: The instability growth rate for non-monochromatic situation.

Here for the first time we observe a drastic change in the nature of the stability. Here also we have used $\omega_1 = 2$ $\omega_1 = 2$, $\omega_2 = 4$ $\omega_1 = 4$. So in the non-

monochromatic situation the variation is totally different. On the other hand to estimate where there is no magnetic field, we have plotted ' Ω_{II} ' in Figure (5) for three different values of ω_1 but $B_c = 0$.

Figure 5: The instability growth rate for non-monochromatic situation.

We observe again a considerable change in all the three cases. Finally we give some three dimensional variation with respect to q and B_c where the monochromatic and non-monochromatic cases are given in Figure (6) and Figure (7) which shows a completely different behavior.

Figure 6: Instability growth rate 3D plot.

Figure 7: Instability growth rate 3D plot.

5. CONCLUSION

In our above analysis we have discussed the dynamics of two propagating LASER pulses in a nonlinear electron-ion plasma. As shown in the analysis of Borhanian *et al.* [11] the whole process is described by a pair of coupled nonlinear Schrödinger equation. At this point we have considered the new phenomena of nonlinear phase change of one wave due to propagation of the other one. This is actually a new event called cross phase modulation. Though the phenomenon of cross phase modulation is observed as a nonlinear fibre optical effects but it also occurs in plasma when there is a laser plasma interaction [13, 14]. In the next stage we consider the existence of incoherence in the initial state of the plasma particles. This is a new kind stability analysis, known as the crossed phase stability in the sense of Wigner. It was in the initial formulation of Wigner who showed how to take care of incoherence in a stability analysis. On the other-hand since incoherence leads to a distribution of initial state, an essence of quantum nature is introduced in the whole analysis. Also since our initial plasma is placed in a magnetic field, here we have showed how the cross phase modulation can be influenced due to

1. incoherence in initial state
2. the Wigner type treatment
3. the effect of external magnetic field

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