Effects of Plant Identification on the Generalized Predictive Controllers Design

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Key Words: System identification; optimization; regressive models; generalized predictive controller.

Abstract. The effects of the estimated plant models accuracy on the control system signals quality after generalized predictive controllers design are studied in the paper. Two identification approaches are used for different in structures discrete-time models – by optimization procedure in Optimization Toolbox based on the plant step response as a standard deterministic plant characteristic and by functions in System Identification Toolbox after experiments with random signals on the plant. The generalized predictive controllers are design according to the estimated models. The processes in the simulated control systems are analyzed concerning the effects of different kind plant models on the designed controllers.

1. Introduction

The generalized predictive controller (GPC) [1] belongs to a class of structurally-optimizing discrete controllers, whose structure and parameters depend mainly on discrete plant model with description in terms of the input signal u and output y

$$A(q^{-1})y(k) = B(q^{-1})q^{-d}u(k-1) + \frac{C(q^{-1})}{\Delta}\xi(k),$$

$$(1) \quad A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{na}q^{-na},$$

$$B(q^{-1}) = b_1q^{-1} + \dots + b_{nb}q^{-nb},$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \dots + c_{nc}q^{-nc},$$

where the model dead-time is represented by d numbers discrete sampling intervals T_0 , $b_0 = 0$ corresponds to the use of a zero-order hold in sampling the continuous plant, by $\Delta = 1 - q^{-1}$ the controller has integral action to eliminate (compensate) the undesired effects after load disturbances on the plant, and the white Gaussian noise ξ presents the plant description error.

When the plant model (1) is unknown, it must be identified on the basis of collected plant data. The modern methods for processing measured data cover both aspects of the process: determination of the most appropriate model (1) structure (model order $n = na = nb$, dead-time d , sampling interval T_0 etc.) and estimation of the parameters $a_1, a_2, \dots, b_1, b_2, \dots$ and so on. Closest to the intuitive understanding of the plant dynamics are the non-parametric models – step and impulse response functions obtained experimentally at the plant output. This type of primary dynamic descriptions are subject to further processing by

the appropriate graph-analytical methods [2] or functions in Optimization Toolbox [3] to determine the secondary dynamic descriptions via parametric models in the form of continuous-time or discrete transfer functions. A similar final result can be achieved through alternative solution to identification. Together with applying the random test signal at the plant input the output response is measured and the observed data is processing through functions in System Identification Toolbox [4] to estimate mainly the parameters of discrete models with different structures. The choice of plant identification approach, the plant model structures and parameter estimation methods are the **first problem** to solve, standing in front of controller designers. They should know the factors that influence on the identification. The **second problem** facing them is a choice of design process conditions to realize the control system (CS) with desired behaviours. For this purpose specific factors in performance index (2) are used to GPC design

$$J(N_1, N_2, N_u, \lambda) =$$

$$(2) \quad = \sum_{i=N_1}^{N_2} [y(k+i) - r(k+i)]^2 + \lambda \sum_{i=1}^{N_u} [\Delta u(k+i-1)]^2$$

where r is the CS reference, the vector u contains the control (GPC output), $\Delta u(k) = u(k) - u(k-1)$, N_1 is the minimum prediction horizon, most often corresponding to d , N_2 – the maximum prediction horizon and N_u – the control horizon. By weighting factor the λ , ($\lambda_{\max} < \lambda < 0$) controller energy is distributed between two conflicting goals: achieving accuracy in CS error by $\lambda \rightarrow 0$, and control signal limitation by $\lambda \rightarrow \lambda_{\max}$. Designers face a **third major problem** due to the fact that each plant model is untruthful to a certain degree, because it is not fully consistent with the real plant dynamics and the operation noisy environment. Therefore, the control algorithm designed for a specific plant model will not provide the desired system behavior to control the actual plant. These undesirable effects demand the assumed control strategy to be tested and verified by the designer if the CS has satisfactory performance in terms of inaccurate information about the process dynamics. This final step in control practice means a further adjustment (similar to the advanced tuning in PID controllers) to be done, or a correction in the control scheme to be made for minimum signal sensitivity to the plant uncertainty, or a robust control to be realized in order to determine the appropriate factors for quality system behavior.

This article discusses approaches to solve the first

Figure 1. Scheme for optimal tuning of model parameters

two problems by the designer. Section 2 presents a research implementing optimization procedures for different plant models with proposed structures and parameters to achieve maximum proximity in the time domain between the real plant and its model step response functions. In the same section an alternative solution is applied to identification using regression models. Then the generalized predictive controllers are designed based on the defined models, whose impact on the plant is examined in simulated control systems in Section 3. Here ideas on appropriate choice of coefficients in the performance index (2) are explored. The analysis of the identification accuracy and the design procedure sensitivity has been carried out in Section 4.

2. Plant Identification

Let the continuous-time plant to be controlled has dynamics presented by discrete models in the form of

$$W_i^{(j)}(z) = \frac{B_i^{(j)}(z)}{A_i^{(j)}(z)} z^{-d_i^{(j)}} \quad (3)$$

$$= \frac{b_{i,1}^{(j)} z^{-1} + b_{i,2}^{(j)} z^{-2} + \dots + b_{i,i}^{(j)} z^{-i}}{1 + a_{i,1}^{(j)} z^{-1} + a_{i,2}^{(j)} z^{-2} + \dots + a_{i,i}^{(j)} z^{-i}} z^{-d_i^{(j)}}$$

where i takes the value of the model order – $i = 1, 2, \dots$ and j – the corresponding variant for the model depending on the assumed number of time intervals for the model dead-time.

2.1. Optimization Approach to Identification

The approach to estimate the coefficients of the parametric model (3) with selected structure representing the plant dynamics is based on the requirement that the model step response to be as close to the plant step response under the same conditions for their formation. Thus, in an optimization procedure according to the scheme of *figure 1* [2], the best tuning model parameters are looking for that provide this closeness.

The implementation of the procedure in the scheme is carried out after the following conditions are specifying: first, N step response values $h_e(t) \approx h_e(kT_0) \equiv h_e(k)$, $k = 0, 1, 2, \dots, N$, $NT_0 = T$ in the interval T equivalent to the process settling time or the time of process observation; second, the model structure in the form of discrete transfer function with initial values of adjustable parameters – coefficients in the model numerator and denominator as a vector for a particular choice of the dead-time; third, the programming conditions for the model step response $h_m(k)$

and the error vector $e(k)$ calculation as a measure of closeness in the formulated optimization index $J(e) = \sum_{k=0}^N e^2(k)$; fourth, other elements of the optimization – optimization procedure algorithm, constraints conditions, number of operating steps, attainable accuracy of the decision and so on.

2.2. Regression Approach to Identification

This approach follows the standard methodology of experimental plant input-output data processing. For this purpose it uses a generator of purely random signal with a normal (Gaussian) distribution, known as the White Gaussian Noise (WGN), which forms the input identification signal on the plant and produces a corresponding output response. With this pair of large amount of data a procedure is realized for parameter estimation of regression ARX and ARMAX models [2] by software functions in the System Identification Toolbox. For each model a visual test is displayed verifying the accuracy of the resulting estimates. Time and frequency characteristics between the plant and the models are compared.

2.3. Experimental Results after Identification

A dynamic plant is proposed with a transfer function

$$(4) \quad W(s) = \frac{(13.81s + 1)^2 (18.4s + 1)}{(59s + 1)^5}.$$

Its step response is settled in 700 s and a suitable sampling time interval $T_0 = 15$ s is chosen.

2.3.1. Results from the Optimization Approach

After the realized optimization for each of the estimated models $W_i^{(j)}(z)$ (3) an integral square error (ISE) is calculated as a reasonable criterion of closeness between the plant (4) and model (3) step responses in the range T to specify the best value $d_i^{(j)}$ for a model. Due to the quality results in the time domain, only Nyquist and Bode frequency characteristics transformed for continuous descriptions are shown in *figures 2.1-2.3*.

The results of this identification approach show a serious lack of correspondence in the high frequencies between the plant and all models. Obviously, more recent studies can be done for identification using optimization approach by simultaneously requirements looking for closeness between plant and models not only in time but also in the frequency domain.

Figure 2.1. $W_1^{(7)}(z) = \frac{0.088z^{-1}}{1-0.913z^{-1}} z^{-7}$ with ISE = 0.047

Figure 2.2. $W_3^{(3)}(z) = \frac{0.034z^{-1} - 0.042z^{-2} + 0.0338z^{-3}}{1 - 1.1z^{-1} - 0.42z^{-2} + 0.5467z^{-3}} z^{-3}$ with ISE = 0.00137

Figure 2.3. $W_5^{(3)}(z) = \frac{0.03z^{-1} - 0.0058z^{-2} - 0.006z^{-3} - 0.005z^{-4} + 0.028z^{-5}}{1 - 0.81z^{-1} - 0.39z^{-2} - 0.08z^{-3} + 0.27z^{-4} + 0.057z^{-5}} z^{-3}$ with ISE = 0.00066

2.3.2. Results from Regression Approach

Time and frequency characteristics of the estimated model

$$W_5^{(1)}(z) = \frac{0.0008z^{-1} + 0.00046z^{-2} - 0.0012z^{-3} + 0.0005z^{-4} - 0.0007z^{-5}}{1 - 3.87z^{-1} + 6.01z^{-2} - 4.66z^{-3} + 1.8z^{-4} - 0.28z^{-5}} z^{-1}$$

completely coincide with those of the plant. The conclusion is that the low order ARX models cannot present the plant

$$W_3^{(1)}(z) = \frac{0.0008z^{-1} + 0.0014z^{-2} + 0.00009z^{-3}}{1 - 2.745z^{-1} + 2.543z^{-2} - 0.795z^{-3}} z^{-1}$$

Figure 2.5.

dynamics whereas the same type regression models but in order, close to the dynamics of the plant, present successful behavior even in high frequencies, unlike the models produced by an optimization approach in Section 2.1. only in the time domain.

When the next experiments are done, there was no significant difference in quality between the ARX and other types of models, i.e. no improved models of lower order are reported.

3. GPC Design and CS Simulation

3.1. Survey on the Recommended Values of Parameters in the Criterion (2)

The criterion (2) is remarkable with its sufficiently significant level of complexity to make effective control of a wide class of plants as the design procedure depends mainly on three major parameters N_2 , N_u , λ . These features allow GPC to deal effectively with unknown or variable plant dead-time, and to ensure stable control for non-minimum phase discrete models. A lot of studies offer conditions to adjust the structural factors in (2). The main set of rules requires a closed system with GPC to be stable if the following conditions are satisfied [5]:

$$(5) \quad N_1 \geq n_b, \quad N_u \geq n_a, \quad N_2 - N_u \geq n_b - 1, \quad N_2 - N_1 \geq n_a - 1.$$

For plant models with delay another set of rules have been proposed [6]

$$(6) \quad N_1 \geq d + 1, \quad N_2 \geq d + N = d + n_A - 1, \quad N_u \geq N = n_A - 1.$$

The choice of the above structural parameters in the GPC design procedure is a designer task, which he intends to achieve the best decision for signals into CS. Unfortunately, there is no 100 percent guarantees to obtain good results, even according to the authors of the method [1] if the conditions (5) and (6) are satisfied. When the coefficients take values above inequalities, GPC behaves like a-periodic (dead-beat) controller, provided that the plant is controllable and observable, and the model order coincides with the plant order. Here are some recommendations for the values of adjustable parameters, generalized into the literature [7]:

- **Recommendation 1.** Inequality $\lambda > 0$ must always be fulfilled. In principle the system behavior depending on λ can be explained in this way: increasing the value of λ reduces the control activity, it becomes less aggressive and the plant output response becomes slower. Conversely, by decreasing this value the control restriction reduces, as a result the speed of system signals improves, but its stability worsens. Therefore, λ balances system behavior (desirable slight deviations of output signal) and its robustness (desirable slight deviations of control signal).

- **Recommendation 2.** Too small values of λ are not appropriate for non-minimum phase plants.

- **Recommendation 3.** A formula about λ is derived depending on the first polynomial non-zero coefficient (b_i ($i \neq 0$)) in the numerator ($b_i z^{-i} + b_{i+1} z^{-i+1} + \dots$) of the plant model discrete transfer function (1): $\lambda = b_i 10^i$.

- **Recommendation 4.** $N_1 = d$, if the dead-time value is known in advance.

- **Recommendation 5.** In practical applications of GPC N_2 is a value recommended to cover (50-90)% of the number of sampling intervals from the system output initial value to the values closed to the reference (rise-time interval). If N_2 is too smaller, it can cause unstable control. If $N_2 \rightarrow \infty$ the closed-loop system poles match those of the open system, i.e. stable control is possible only for stable plants.

- **Recommendation 6.** $N_u < N_2$, even $N_u \ll N_2$, for example, $N_u \leq N_2/2$, i.e. control signal variation has to complete before the reference been worked off by the system

output. As an effect a smooth signal is observed. The relation between the two coefficients is given by inequality $N_u < N_2 < 3N_u$, provided that $N_u = N_1 = d \neq 0$.

- **Recommendation 7.** $N_u = 1$ is selected for low-order models or $N_u = (2 - 3)n_A$ in case of load disturbances.

3.2. Novel Ideas to Select the Appropriated Values in (2)

In author publications [7,8] three approaches to suitable choice of parameters N_1 , N_2 , N_u , λ in criterion (2) have been proposed and successfully used based on the quality of the signals into the control system – by rules for their changes, by scanning into a set of intervals, and by multicriteria optimization procedures. Below in this section some rules for choosing appropriated values are worked out, thanks to which better quality signals can be achieved in the CS. Consecutively GPC are designed based on models of optimization and regression approaches from Section 2. Everywhere in the figures 3.1-3.6 the improved CS and the factors that lead to this effect are shown on the right side.

3.2.1. GPC Design on Models of Optimization Approach

Figure 3.1. CS signals for GPC based on the model $W_1^{(7)}(z)$
(The stable system below is provided by increasing the restriction on the control)

Figure 3.2. CS signals for GPC based on the model $W_3^{(3)}(z)$ (The stable system on the right is provided by increasing the restriction on the control)

Figure 3.3. CS signals for GPC based on the model $W_5^{(3)}(z)$ (The system on the right accelerates the reference working off)

3.2.2. GPC Design on Models of Regression Approach

Using the model $W_1^{(7)}(z)$ an unstable CS is simulated or it works off the reference for a long time. *Figure 3.4.* and *figure 3.5.* show stable systems with GPC designed respectively on models $W_3^{(1)}(z)$ and $W_5^{(1)}(z)$. Based on the

results in section 3.2 conclusions can be expressed that the structural parameters in (2) possess a leading role in the GPC design process and they can compensate in a certain degree the inaccurate plant models after identification.

Figure 3.4. CS signals for GPC based on the model $W_3^{(1)}(z)$ (The system on the right accelerates the reference work off by the constraint weakening on the control)

Figure 3.5. CS signals for GPC based on the model $W_s^{(1)}(z)$ (Option 2 smoothes the control by increasing the constraint on it)

3.3. GPC and Classic PID Controller

With the optimization procedure on the step response of the plant (4) an approximated continuous time first order plus time delay (FOPTD) model is obtained

$$(7) \quad W_1(s) = \frac{k}{Ts+1} e^{-\tau s} = \frac{1.03}{160s+1} e^{-105s},$$

which parameters are used to calculate the coefficients K_p , T_i , T_d of the classic continuous PID controller according to the method of Cohen-Coon. After sampling by $T_0 = 15$ s a corresponding digital PID controller is formed

$$(8) \quad (u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)),$$

$$q_0 = K_p \left(1 + \frac{T_0}{T_i} + \frac{T_d}{T_0} \right), q_1 = K_p \left(-1 - 2 \frac{T_d}{T_0} \right), q_2 = K_p \frac{T_d}{T_0}.$$

Signals into the control system with PID controller (6) are shown in *figure 3.6*.

For the test plant dynamics substantial reasons cannot be pointed out for some advantage of GPC to discrete PID, moreover, the design procedure and PID coefficients tuning are relatively simple and routine engineering operations.

Figure 3.6. CS signals for discrete PID based on the model (6) (The system on the right provides better processes in the CS by additional continuous PID parameter tuning)

4. Summary Notes

4.1. Scanning Procedure for N_1 , N_2 , N_u , λ

The GPC design can be automated by the scanning procedure [8] to search for the most appropriate values of coefficients N_1 , N_2 , N_u , λ in (2) for a CS with desired behavior as time of a reference working off, degree of overshoot, maximum control amplitude or speed etc. Below an

algorithm to select the best coefficients is presented:

A. Preliminary works

A1. The planning intervals for searching the coefficients N_1 , N_2 , N_u , λ are specified in the form $N_{1min} \leq N_1 \leq N_{1max}$, $N_{2min} \leq N_2 \leq N_{2max}$, etc.

A2. The scanning step S into the planning intervals

is defined for each of the coefficients in the research range – $N_{1min} : S_{N_1} : N_{1max}$, etc.

A3. Conditions for coefficients N_1, N_2, N_u, λ are chosen depending on the structural model parameters in accordance with the recommended values given in section 3.1.

A4. Criteria for quality control system behavior are formulated required to be calculated during the scanning process after control system simulation using the designed GPC with the scan parameters:

K1. Maximum absolute system output error $e_y^{max} = \max |e_y|$ behind the prediction horizon N_2 (scalar), subject to a maximum absolute error of the output value $e_y^{max} : e_y^{ultimate}$.

K2. Maximum absolute system control error $e_u^{max} = \max |e_u|$ behind the control horizon N_u (scalar), subject to a maximum admissible absolute control error $e_u^{max} : e_u^{ultimate}$.

K3. Control system overshoot $o_u = |\max(y) - y_{\infty}|$ (scalar), subject to a maximum overshoot $o_u^{max} : o_u^{ultimate}$.

K4. Current control deviation from the settling value: $e_u(k) = |u_f - u_k|$, (vector).

A5. The parameters for quality measure of the control system processes are defined. For this purpose some quantities have to be calculated during the simulation:

- Current system error (a measure for a current deviation of the system output from the reference): $e_{ry} = |r - y|$ (vector);
- Mean-square error of the system error into the time interval T : $S = \text{sqrt}(e_{ry}^T e_{ry})/T$, (scalar), subject to a maximum admissible absolute average square system error $S : S^{ultimate}$.

A6. A summarized measure is expressed for eligibility of factor combinations in the GPC design:

$$(9) \quad (S < S^{ultimate}) \ \& \ (o_u < o_u^{ultimate}) \ \& \ (e_u^{max} < e_u^{ultimate}) \ \& \ (e_u^{max} < e_u^{ultimate})$$

Satisfying the formulated measure (7) leads to a substantial reduction in the number of tuning parameters combinations. The best combination of N_1, N_2, N_u, λ among this multitude is chosen.

B. Activities during the scanning procedure

B1. The GPC design is implemented for coefficients in (2) of the permissible combinations (7) if the A3 conditions are satisfied.

B2. For each designed GPC a control system is simulated and a quality measure A5 is calculated.

C. Selection of the best combination(s)

C1. The best combination(s) is (are) determined to find the design solution in accordance with the selected criteria.

C2. The results of the scan are demonstrated.

The described algorithm involves repeatedly realization of the GPC design for each of the planned combinations of structural parameters, (optional) relevant indicators display for the control system functioning, and automatic (optional) selection of the best decision variant.

4.2. Implementation of an Optimization Search Procedure

The main task of the GPC designer is to assign (make a choice) or to calculate the appropriate values. When such values N_1, N_2, N_u, λ are connected to satisfy certain conditions or additional criteria, there is a multi-criteria optimization problem whose objective function has more than one extreme within the limits of the feasible multitude. Most often, this problem is solved by multi optimization procedures [7,8].

5. Conclusion

Studies conducted have confirmed expectations that:

First, regardless of the approach, the accuracy achieved in the plant identification by different order models is good enough and does not affect significantly on the behavior of the control system with the appropriate GPC. This result explains the researcher interest to GPC design with lower order with time delay models, similar to the methods for tuning the widely used PID controllers.

Second, although a quality control can be achieved for seemingly less accurate models, it is still desirable to use identification procedures that carry a maximum closeness between the models and the plant both in the time and frequency area.

Third, by scanning the tolerances of amending coefficients N_1, N_2, N_u, λ an automation of the most appropriate values selection can be achieved, similar to the application of optimization approaches.

Fourth, the identification stage takes a significant time in the design of control system with GPC. This study is an attempt to understand even the inaccurate models in the GPC design, highlighting the leading role of the structural parameters N_1, N_2, N_u, λ in this task.

References

1. Clarke, D. W., C. Mohtadi, and P. S. Tuffs. Generalized Predictive Control. – *Automatica*, 23,1987, 137-160.
2. Garipov, E. Identification Systems – Part 1 and 2 (Third Edition). TU Sofia, 2007 (in Bulgarian).
3. MathWorks. MATLAB Optimization Toolbox, 2011.
4. MathWorks. MATLAB System Identification T., 2011.
5. Bao-Cang, Ding. Modern Predictive Control. Taylor and Francis Group, 2010.
6. Normey-Rico, J. E., E. F. Camacho. Control of Dead-time Process, Springer, 2006.

7. Garipov, E., V. Ruykova, T. Peltekov. Generalized Predictive Control with Varying Degrees of Uncertainty in the Plant Model. FA'2014, Sozopol, 2014 (in Bulgarian).

8. Garipov, E., V. Ruykova, T. Peltekov. Appropriate Values Selection for Adjustable Factors in the Generalized Predictive Controller Design. SAI'2014, Sofia, 2014 (in Bulgarian).

Manuscript received on 21.02.2017

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