DUALITY IN NONLINEAR FRACTIONAL PROGRAMMING PROBLEM USING FUZZY PROGRAMMING AND GENETIC ALGORITHM

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ABSTRACT

In this paper we have considered nonlinear fractional programming problem with multiple constraints. A pair of primal and dual for a special type of nonlinear fractional programming has been considered under fuzzy environment. Exponential membership function has been used to deal with the fuzziness. Duality results have been developed for the special type of nonlinear programming using exponential membership function. The method has been illustrated with numerical example. Genetic Algorithm as well as Fuzzy programming approach has been used to solve the problem.

KEYWORDS

Fuzzy Mathematical Programming, Nonlinear Fractional Programming, Exponential membership function, Decision Analysis, Genetic Algorithm.

1. INTRODUCTION

Several factors in the real world imply the increase in use of nonlinear programming models. There are several classes of problems where nonlinear programming had had a great impact for example oil and petrochemical industries, nonlinear network problems and economic planning models. The area where nonlinear programming can be used in these several classes of problems is given in detail in (Ladson et al., 1980). The concept of fuzzy programming in decision making problem was first proposed by (Bellmann and Zadeh, 1970). Many authors have applied fuzzy programming approach in different area of linear programming (Zimmermann, 1978, Stancu-Minasian et al., 1978, Chakraborty and Gupta, 2002). In (Jimenez, 2005), author has considered a non linear programming problem with fuzzy constraints and the solution has been obtained by multi objective evolutionary Algorithms. An interactive cutting plane algorithm for fuzzy multi objective nonlinear programming problems has been presented in (Kanaya, 2010). In (Jameel, 2012), author has obtained accurate results for solving non linear programming using fuzzy environment by using properties of fuzzy set and fuzzy number with linear membership function.

Fractional programs arise in management decision making as well as outside of it. They also occur sometimes indirectly in modeling where initially no ratio is involved. The efficiency of a system is sometimes characterized by a ratio of technical and/or economical terms. Maximizing system efficiency then leads to a fractional program. List of frequently occurring objectives are maximization of productivity, maximization of return on investment, maximization of return/risk,

minimization of cost/time, maximization of output/input, Non-Economic Applications. There are a number of management science problems which indirectly give rise to a fractional program; see e.g. (Schaible, 1981, 1983, 1995). A new method has been given by (Borza et al. 2012) for solving the linear fractional programming problems with interval coefficients in the objective function.

An algorithm has been developed by (Saad et al., 2011) for multi objective integer nonlinear fractional programming problem under fuzziness. In order to defuzzify the problem, he has developed the concept of α -level set of the fuzzy number is given and for obtaining an efficient solution to the problem (FMOINLFP), a linearization technique is presented to develop the solution algorithm. In the paper (Biswas and Bose, 2012) author presented a fuzzy programming procedure to solve nonlinear fractional programming problems in which the parameters involved in objective function are considered as fuzzy numbers.

The duality theory for nonlinear multiobjective optimization problems in the field of the optimization theory has intensively developed during the last decades. In (Rodder and Zimmermann, 1980), a generalization of maxmin and minmax problems in a fuzzy environment is presented and thereby a pair of fuzzy dual linear programming problems is constructed. An economic interpretation of this duality in terms of market and industry is also discussed in that paper. In (Bector and Chandra, 2002), a pair of linear programming primal-dual problem is introduced under fuzzy environment and appropriate results were proved to establish the duality relationship between them. In (Liu, 1995) a constructive approach has been proposed to duality for fuzzy multiple criteria and multiple constraint level linear programming problems. (Biswas and Bose, 2012) gives a parametric approach for the duality in fuzzy multi criteria and multi constraint level linear programming problem. In (Gupta and Mehlawat, 2009), a study of a pair of fuzzy primal–dual linear programming problems has been presented and calculated duality results using an aspiration level approach using exponential membership function, while a discussion of fuzzy primal dual linear programming problem with fuzzy coefficients has been presented in (Bector and Chandra, 2002; Liu, 1995). In (Bector and Chandra, 2002), a pair of linear programming primal-dual problem is introduced under fuzzy environment and appropriate results were proved to establish the duality relationship between them. Also in (Chakraborty et al. 2014), author has presented a pair of linear primal – dual programming using linear and exponential membership function using fuzzy programming approach and genetic algorithm approach.

In this paper we have extended the work done by (Gupta and Mehlawat, 2009) in which he has solved duality in convex fractional programming using linear membership function. We have taken the same kind of problem and proved the duality theorems considering exponential membership function for fuzzified objective function and constraints and then solved using LINGO and Genetic Algorithm also and the results have been analysed.

The paper has been organized as follows: In section 2, the nonlinear fractional programming problem and its dual has been defined. The fuzzified and the crisp formulation have been defined for the primal and dual. In section 3, the necessary duality results have been developed. The numerical example defined in (Gupta and Mehlawat, 2009) has been illustrated in section 4. Finally, analysis of the solution and concluding remarks has been presented in section 5.

2. NON LINEAR FRACTIONAL PROGRAMMING PROBLEM

The nonlinear fractional programming problem (Gupta and Mehlawat, 2009) can be defined as:

$$\text{Max } f(x) = \frac{(c'x)^2}{d'x}$$

$$\text{Subject to } Ax \leq b \\ x \geq 0$$

(1)

The dual form of the above problem can be written as

$$\text{Min } g(u, v) = b'u$$

$$\text{Subject to } A'u + dv^2 \geq 2cv \\ u, v \geq 0$$

(2)

Where A is $m \times n$ matrix, x , c , b are column vectors with n components and b is a column vector of m components.

2.1 Fuzzified Non Linear Fractional Programming Problem

Fuzzified form of nonlinear fractional programming problem can be defined as

$$\text{Find } x \in R^n$$

$$\text{Subject to } f(x) = \frac{(c'x)^2}{d'x} \tilde{\geq} Z_0$$

$$Ax \tilde{\leq} b \\ x \geq 0$$

(3)

Where Z_0 is the aspiration level of the objective function of the primal problem. “ $\tilde{\leq}$ ” and “ $\tilde{\geq}$ ” denotes the flexibility of the objective function and the multi constraints.

Fuzzified form of the dual of the problem can be defined as

$$\text{Find } u \in R^m$$

$$\text{Subject to } g(u, v) = u'b \tilde{\leq} w_0$$

$$A'u + dv^2 \tilde{\geq} 2cv \\ u, v \geq 0$$

(4)

Where w_0 is the aspiration level of the objective function of the dual problem.

2.2 Crisp Formulations using Exponential Membership Function

The advantage of nonlinear membership function over linear membership function is already described in (Gupta and Mehlawat, 2009; Chakraborty et al. 2014). Linear membership function is most commonly used in fuzzy linear programming problem because it is simple and it is defined by fixing two points. Also, linear membership function has been used in a vast range of nonlinear problems (Bector and Chandra, 2002; Gupta and Mehlawat, 2009; Chakraborty et al. 2014). But in many practical situations linear membership function is not a suitable

representation. Furthermore, if the membership function is interpreted as the fuzzy utility of the decision maker, used for describing levels of indifference, preference towards uncertainty, then a nonlinear membership function provides a better representation than a linear membership function. Moreover, unlike linear membership function, for nonlinear membership functions the marginal rate of increase (or decrease) of membership values as a function of model parameters is not constant- a technique that reflects reality better than the linear case (Gupta and Mehlawat , 2009).

Let us consider the following form of exponential membership function (Gupta and Mehlawat , 2009; Chakraborty et al. 2014) for the fuzzy nonlinear fractional programming problem respectively.

$$\mu(x) = \begin{cases} 1 & \text{if } \frac{(c_t x)^2}{d^t x} \geq z_0 \\ \frac{e^{-\alpha \left\{ \left(\frac{(c_t x)^2}{d^t x} - z_0 \right) / -p \right\}} - e^{-\alpha}}{1 - e^{-\alpha}} & \text{if } z_0 - p < \frac{(c_t x)^2}{d^t x} < z_0 \\ 0 & \text{if } \frac{(c_t x)^2}{d^t x} \leq z_0 - p \end{cases}$$

$$\mu_j(x) = \begin{cases} 1 & \text{if } A_j x \leq b_j \\ \frac{e^{-\alpha_j \{(b_j - A_j x) / -q_j\}} - e^{-\alpha_j}}{1 - e^{-\alpha_j}} & \text{if } b_j < A_j x < b_j + q_j \\ 0 & \text{if } A_j x \geq b_j + q_j \end{cases} \quad j=1, 2, \dots, m$$

(5)

Where α and α_j are user defined parameters which determine the shape of the membership function. p and q_j ($j = 1, 2, \dots, m$) are subjectively chosen constant of admissible violations such that p is associated with nonlinear fractional objective function and q_j 's are associated with linear constraints of the problem.

Therefore using exponential membership function the crisp formulation of the problem can be given by

Max λ

$$\text{Subject to } \lambda \leq \frac{e^{-\alpha \left\{ \left(\frac{(c_t x)^2}{d^t x} - z_0 \right) / -p \right\}} - e^{-\alpha}}{1 - e^{-\alpha}}$$

$$\lambda \leq \frac{e^{-\alpha_j \{(b_j - A_j x) / -q_j\}} - e^{-\alpha_j}}{1 - e^{-\alpha_j}} \quad j=1, 2, \dots, m$$

$$\lambda \leq 1, \lambda \geq 0, x \geq 0. \quad (6)$$

or Max λ

$$\begin{aligned} \text{Subject to } & p \log \{ \lambda(1 - e^{-\alpha}) + e^{-\alpha} \} \leq \alpha \left(\frac{(c_t x)^2}{d' x} - z_0 \right) \\ & q_j \log \{ \lambda(1 - e^{-\alpha_j}) + e^{-\alpha_j} \} \leq \alpha_j (b_j - A_j x) \quad , j=1, 2, \dots, m \\ & \lambda \leq 1, \lambda \geq 0, x \geq 0. \end{aligned} \quad (7)$$

The exponential membership function for the dual problem objective function and nonlinear constraints are given by

$$\begin{aligned} \mu(w) &= \begin{cases} 1 & \text{if } g(u, v) \leq w^0 \\ \frac{e^{-\beta \{(w^0 - g(u, v)) / -r\}} - e^{-\beta}}{1 - e^{-\beta}} & \text{if } w^0 < g(u, v) < w^0 + r \\ 0 & \text{if } g(u, v) \geq w^0 + r \end{cases} \\ \mu_j(w) &= \begin{cases} 1 & \text{if } A^t u + d_j v^2 \geq 2c_j v \\ \frac{e^{-\beta_j \{(A^t u + d_j v^2 - 2c_j v) / -s_j\}} - e^{-\beta_j}}{1 - e^{-\beta_j}} & \text{if } 2c_j v - s_j < A^t u + d_j v^2 < 2c_j v \\ 0 & \text{if } A^t u + d_j v^2 \leq 2c_j v - s_j \end{cases} \quad j = 1, 2, \dots, n \end{aligned} \quad (8)$$

Where β and β_j 's are user defined parameters which determine the shape of the membership function. r and s_j ($j = 1, 2, \dots, n$) are subjectively chosen constant of admissible violations such that r is associated with objective function and s_j 's are associated with n linear constraints of the dual problem.

Min $(-\eta)$

$$\begin{aligned} \text{Subject to } & \eta \leq \frac{e^{-\beta \{(w^0 - g(u, v)) / -r\}} - e^{-\beta}}{1 - e^{-\beta}} \\ & \eta \leq \frac{e^{-\beta_j \{(A^t u + d_j v^2 - 2c_j v) / -s_j\}} - e^{-\beta_j}}{1 - e^{-\beta_j}} \quad j=1, 2, \dots, n \\ & \eta \leq 1, \eta \geq 0, u, v \geq 0. \end{aligned} \quad (9)$$

or Min $(-\eta)$

$$\text{Subject to } r \log \{ \eta(1 - e^{-\beta}) + e^{-\beta} \} \leq \beta (w^0 - g(u, v))$$

$$s_j \log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\} \leq \beta_j (A_j^t u + d_j v^2 - 2c_j v) \quad , j=1, 2, \dots, n$$

$$\eta \leq 1, \eta \geq 0, u, v \geq 0.$$

(10)

3. THEOREMS ON DUALITY

We shall prove some duality theorems. In the theorems, $Q = (q_1, q_2, \dots, q_m)$, $S = (s_1, s_2, \dots, s_n)$ are column vectors

Theorem 3.1 (Modified fuzzy weak duality theorem): Let (x, λ) be the feasible solution of crisp primal problem (7) and (u, v, η) be feasible solution of crisp dual problem (10). Then

$$\sum_{i=1}^m \frac{\log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\}}{\beta_j} S^t x \leq u^t b - \frac{(c^t x)^2}{d^t x}$$

Proof: Since (x, λ) be the feasible solution of crisp primal problem (7) and (w, η) be feasible solution of crisp dual problem (10), then

$$q_i \log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\} \leq \alpha_i (b_i - A_i x), i=1, 2, \dots, m$$

$$s_j \log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\} \leq \beta_j (A_j^t u + d_j v^2 - 2c_j v), j=1, 2, \dots, n$$

$$\text{Or } \sum_{i=1}^m \frac{\log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\}}{\alpha_i} Q \leq b - Ax \quad (11)$$

$$\sum_{j=1}^n \frac{\log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\}}{\beta_j} S \leq A^t u + dv^2 - 2cv \quad (12)$$

Multiplying the equation (11) by transpose of u and taking transpose of equation (12) and multiplying equation by x, we have

$$\sum_{i=1}^m \frac{\log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\}}{\alpha_i} u^t Q \leq u^t b - u^t Ax$$

$$\sum_{j=1}^n \frac{\log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\}}{\beta_j} S^t x \leq u^t Ax + v^2 d^t x - 2v^t c^t x$$

Adding the above two inequalities, we have

$$\sum_{i=1}^m \frac{\log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\}}{\beta_j} S^t x \leq u^t b + v^2 d^t x - 2v^t c^t x$$

$$\text{Or } \sum_{i=1}^m \frac{\log\{\lambda(1 - e^{-\alpha_i}) + e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1 - e^{-\beta_j}) + e^{-\beta_j}\}}{\beta_j} S^t x \leq u^t b + \left\{ (d^t x)^{1/2} v^t - \frac{(c^t x)}{(d^t x)^{1/2}} \right\}^2 - \frac{(c^t x)^2}{d^t x}$$

$$\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x \leq \left\{ u^t b - \frac{(c^t x)^2}{d^t x} \right\} + \left\{ (d^t x)^{\frac{1}{2}} v - \frac{(c^t x)}{(d^t x)^{\frac{1}{2}}} \right\}^2$$

$$\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x \leq \left\{ u^t b - \frac{(c^t x)^2}{d^t x} \right\}$$

Remark: When $\lambda=1$, $\eta=1$ in the above inequality we get the objective function of primal problem is imprecisely less than the objective function of the dual $\left\{ u^t b - \frac{(c^t x)^2}{d^t x} \right\}$ which can be defined as the standard fuzzified weak duality theorem in a multi objective linear programming problem.

Theorem 3.2: Let $(\hat{x}, \hat{\lambda})$ be feasible solution of crisp primal problem (7) and $(\hat{w}, \hat{\eta})$ be feasible solution of crisp dual problem (10) such that

- (i)
$$\sum_{i=1}^m \frac{\log\{\hat{\lambda}(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} \hat{w}^t Q + \sum_{j=1}^n \frac{\log\{\hat{\eta}(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t \hat{x} = \hat{u}^t b - \frac{(c^t \hat{x})^2}{d^t \hat{x}}$$
- (ii)
$$\frac{\log\{\hat{\lambda}(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha} p + \frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta} r = \left(\frac{(c^t \hat{x})^2}{d^t \hat{x}} - \hat{u}^t b \right) + (w_0 - z_0)$$
- (iii) The aspiration levels z_0 and w_0 satisfy $(z_0 - w_0) \leq 0$

Then $(\hat{x}, \hat{\lambda})$ is optimal solution of primal problem and $(\hat{w}, \hat{\eta})$ be optimal solution of dual problem.

Proof: Let (x, λ) be feasible solution of crisp primal problem (7) and (w, η) be feasible solution of crisp dual problem (10). Then from Theorem 3 we have,

$$\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x - \left(u^t b - \frac{(c^t x)^2}{d^t x} \right) \leq 0$$

(13)

From condition (i) and Equation (13) we have

$$\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x - \left(u^t b - \frac{(c^t x)^2}{d^t x} \right)$$

$$\leq \sum_{i=1}^m \frac{\log\{\hat{\lambda}(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} \hat{u}^t Q + \sum_{j=1}^n \frac{\log\{\hat{\eta}(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t \hat{x} - \left(\hat{u}^t b - \frac{(c^t \hat{x})^2}{d^t \hat{x}} \right)$$

Thus, $(x, \hat{\lambda}, w, \hat{\eta})$ is optimal to the problem whose maximum objective value is 0.

$$\text{Max} \sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x - \left(u^t b - \frac{(c^t x)^2}{d^t x} \right)$$

Subject to

$$\begin{aligned} p \log\{\lambda(1-e^{-\alpha})+e^{-\alpha}\} &\leq \alpha \left(\frac{(c^t x)^2}{d^t x} - Z_0 \right) \\ q_i \log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\} &\leq \alpha_i (b_i - A_i x) \quad (i = 1, 2, \dots, m) \\ r \log\{\eta(1-e^{-\beta})+e^{-\beta}\} &\leq \beta (w_0 - u^t b) \\ s_j \log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\} &\leq \beta_j (A_j^t u + d_j v^2 - 2c_j v) \quad (j = 1, 2, \dots, n) \\ \lambda \leq 1, \lambda \geq 0, x \geq 0, \eta \leq 1, u, v \geq 0, \eta \geq 0. \end{aligned}$$

Adding condition (i) and (ii), we have

$$\begin{aligned} &\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x - \left(u^t b - \frac{(c^t x)^2}{d^t x} \right) + \\ &\frac{\log\{\hat{\lambda}(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha} p + \frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta} r - \left(\frac{(c^t x)^2}{d^t x} - u^t b \right) - (w_0 - z_0) = 0 \end{aligned}$$

Or

$$\begin{aligned} &\sum_{i=1}^m \frac{\log\{\lambda(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q + \sum_{j=1}^n \frac{\log\{\eta(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x + \\ &\frac{\log\{\hat{\lambda}(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha} p + \frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta} r + (z_0 - w_0) = 0 \end{aligned}$$

Each term in the above sum is non positive as $\hat{\lambda}, \hat{\eta} \leq 1$.

$$\begin{aligned} \text{Therefore, } \sum_{i=1}^m \frac{\log\{\hat{\lambda}(1-e^{-\alpha_i})+e^{-\alpha_i}\}}{\alpha_i} u^t Q &= 0 \\ \sum_{j=1}^n \frac{\log\{\hat{\eta}(1-e^{-\beta_j})+e^{-\beta_j}\}}{\beta_j} S^t x &= 0 \\ \frac{\log\{\hat{\lambda}(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha} p &= 0 \end{aligned}$$

$$\frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta}r=0$$

$$z_0 - w_0 = 0$$

Since $\frac{\log\{\lambda(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha}p \leq 0$ and $\frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta}r \leq 0$, because $\hat{\lambda}, \hat{\eta} \leq 1$, we get

$$\frac{\log\{\lambda(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha}p \leq \frac{\log\{\hat{\lambda}(1-e^{-\alpha})+e^{-\alpha}\}}{\alpha}p$$

$$\frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta}r \leq \frac{\log\{\hat{\eta}(1-e^{-\beta})+e^{-\beta}\}}{\beta}r$$

or $\lambda \leq \hat{\lambda}$ and $\eta \leq \hat{\eta}$

Thus, $(\hat{x}, \hat{\lambda})$ is optimal solution of (7) and $(\hat{w}, \hat{\eta})$ be optimal solution of (10).

4. NUMERICAL EXAMPLE

Let us consider the same numerical example which is defined in [20]. The primal dual nonlinear fractional programming problem can be defined as

$$\text{Min } f(x) = \frac{(2x_1 + x_2)^2}{x_1 + 2x_2}$$

Subject to $2x_1 + x_2 \geq 6$

$$x_1 + 3x_2 \geq 8$$

$$x_1, x_2 \geq 0$$

$$\text{Max } g(u, v) = 6u_1 + 8u_2$$

Subject to $2u_1 + u_2 + v^2 - 4v \leq 0$

$$u_1 + 3u_2 + 2v^2 - 2v \leq 0$$

$$u_1, u_2, v \geq 0$$

The fuzzified nonlinear fractional programming problem can be written as:

Find $x \in R^n$

Min

$$\text{Subject to } f(x) = \frac{(2x_1 + x_2)^2}{x_1 + 2x_2} \leq 1$$

$$2x_1 + x_2 \geq 6$$

$$\begin{aligned} x_1 + 3x_2 &\geq 8 \\ x_1, x_2 &\geq 0 \end{aligned} \quad (14)$$

$$\begin{aligned} \text{Max } g(u, v) &= 6u_1 + 8u_2 \geq 1 \\ \text{Subject to } 2u_1 + u_2 + v^2 - 4v &\leq 0 \\ u_1 + 3u_2 + 2v^2 - 2v &\leq 0 \\ u_1, u_2, v &\geq 0 \end{aligned} \quad (15)$$

Let us take $p = 2, q_1 = 1, q_2 = 1, \alpha = 2, \alpha_1 = 2, \alpha_2 = 1$ for primal problem. Using exponential membership function, the crisp model of (14) can be written as:

$$\begin{aligned} \text{Min } & -\lambda \\ \text{Subject to } & 4x_1^2 + x_2^2 + 4x_1x_2 \geq (x_1 + 2x_2) \log(0.865\lambda + 0.1353) \\ & 4x_1 + 2x_2 - 12 \geq \log(0.865\lambda + 0.1353) \\ & x_1 + 3x_2 - 8 \geq \log(0.865\lambda + 0.1353) \\ & \lambda \leq 1, \lambda \geq 0 \\ & x_1, x_2 \geq 0. \end{aligned} \quad (16)$$

Let us take $r = 1, s_1 = 1, s_2 = 1, \beta = 2, \beta_1 = 2, \beta_2 = 1$ for dual problem (15). Using exponential membership function, the crisp model can be written as:

$$\begin{aligned} \text{Max } & \eta \\ \text{Subject to } & \log(0.865\eta + 0.1353) \leq 12u_1 + 16u_2 - 2 \\ & \log(0.865\eta + 0.1353) \leq -4u_1 - 2u_2 - 2v^2 + 8v \\ & \log(0.865\eta + 0.1353) \leq -u_1 - 3u_2 - 2v^2 + 2v \\ & 0 \leq \eta \leq 1, u_1, u_2, v \geq 0. \end{aligned} \quad (17)$$

5. SOLUTION AND ANALYSIS

Using LINGO software, the solution of the primal problem (16) is given by

Local optimal solution found.

Objective value:	-1.000000
Infeasibilities:	0.000000
Extended solver steps:	5
Total solver iterations:	25
Model Class:	NLP
Total variables:	3
Nonlinear variables:	3
Integer variables:	0

Total constraints:	5
Nonlinear constraints:	3
Total nonzeros:	11
Nonlinear nonzeros:	5

Variable	Value
LAMBDA	1.000000
X1	1.234568
X2	0.1000000E+08

Minimum value of objective function is -1.
The solution of the dual problem (17) is given by

Local optimal solution found.

Objective value:	1.000000
Infeasibilities:	0.000000
Extended solver steps:	5
Total solver iterations:	37
Model Class:	NLP
Total variables:	4
Nonlinear variables:	2
Integer variables:	0
Total constraints:	5
Nonlinear constraints:	3
Total nonzeros:	13
Nonlinear nonzeros:	5

Variable	Value
ETA	1.000000
U1	0.1037300
U2	0.4665636E-01
V	0.1359680

Maximum value of the objective function is 1.

Genetic Algorithm gives global optimal solution. The NSGA used here is a real parameter GA that works directly with the parameter values. NSGA (Nondominated Sorting Genetic Algorithm) uses niching, as well as nondominated sorting of the solutions in every generation to ensure that the “good solutions get preference in selection for procreation”. The non-dominated sorting GA uses a ranking selection method to emphasize good solutions and then a niche building procedure to maintain a stable sub population of good solutions. Since multi objective GAs can find multiple pareto optimal solutions in one single run, the proposed technique is capable of finding multiple solutions to the problems.

The parameters used to solve the problem (16) in Genetic Algorithm are as follows:

Number of objective functions : 1

Objective function # 1 : Minimize

CROSSOVER TYPE : Binary GA (Single-pt)
 STRATEGY : 1 cross - site with swapping
 Population size : 100
 Total no. of generations : 100
 Cross over probability : 0.9000
 Mutation probability : 0.1000
 String length : 60
 Number of variables
     Binary : 3
     Integer : 0
     Enumerated : 0
     Continuous : 0
     TOTAL : 3

Epsilon for closeness : 0
 Sigma-share value : 0.2320
 Sharing Strategy : sharing on Parameter Space

Lower and Upper bounds :
     0.0000 $\leq \lambda \leq$ 1.0000
     0.0000 $\leq x_1 \leq$ 2.0000
     0.0000 $\leq x_2 \leq$ 2.0000

Table 1 gives a set of solution of problem (16).

Table 1: Primal Solution

λ	x_1	x_2
0.942	1.952	1.946
0.992	1.673	1.976
0.962	1.794	1.974
0.942	1.884	1.916
0.977	1.514	1.922

The parameters used to solve the problem (17) in Genetic Algorithm are as follows:
 Number of objective functions : 1

Objective function # 1 : Maximize

CROSSOVER TYPE : Binary GA (Single-pt)
 STRATEGY : 1 cross - site with swapping
 Population size : 100
 Total no. of generations : 100
 Cross over probability : 0.9000
 Mutation probability : 0.1000
 String length : 40
 Number of variables
     Binary : 4

Integer : 0
 Enumerated : 0
 Continuous : 0
 TOTAL : 4

Epsilon for closeness : 0
 Sigma-share value : 0.2810
 Sharing Strategy : sharing on Parameter Space
 Lower and Upper bounds :
 0.0000 $\leq u_1 \leq$ 2.0000
 0.0000 $\leq u_2 \leq$ 2.0000
 0.0000 $\leq v \leq$ 2.0000
 0.0000 $\leq \eta \leq$ 1.0000

Using Genetic algorithm, a set of solutions of dual problem (17) is given in the form of table in Table 2.

Table 2: Dual Solution

H	u_1	u_2	V
0.956	1.863	1.961	1.844
0.956	1.867	1.607	0.956
1.000	1.836	0.741	1.273
0.979	0.716	1.118	1.994
0.953	1.935	1.075	1.894

From the above table, we can clearly see that the solution obtained satisfies the fuzzy constraints as well as the fuzzy objective in both primal and dual problem. Also, it has been observed that difference between the tolerance limit and aspiration level is less. In other words, the decision makers have been provided with enough flexibility to choose satisfying solutions that maximize or minimize their utility functions. Genetic Algorithm gives multiple pareto optimal solutions in one single run. Therefore, it provides set of solutions to the decision maker. The following graph clearly shows comparison between the primal and dual objective function of different sets of solutions.

Figure 1: Primal and Dual objective function of different solution using GA

6. CONCLUSIONS

In this paper, an approach has been presented for a specific kind of fuzzy nonlinear fractional programming problem by constructing a pair of fuzzy non linear fractional primal and dual problems. Crisp form of the above primal and dual problems has been obtained by using exponential membership function. Duality results have been established to prove the duality relationship between above primal and dual problem and are illustrated by an example. The duality results fully satisfy the aspiration levels or the tolerance levels of the objective functions and the system constraints made by the decision maker. The difference between the achieved level and the allowable limit of the satisfying solutions of the decision maker is very less. The numerical example has also been solved by Genetic Algorithm Approach. Genetic Algorithm gives multiple pareto optimal solution. The decision maker can choose any optimal solution according to the convenience. The results of the present paper encourage us to apply duality results in variety number of fields of optimization problem.

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