BOUND STATE SOLUTION TO SCHRODINGER EQUATION WITH MODIFIED HYLLERAAS PLUS INVERSELY QUADRATIC POTENTIAL USING SUPERSYMMETRIC QUANTUM MECHANICS APPROACH

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ABSTRACT

In this work, we obtained an approximate bound state solution to Schrodinger equation with modified Hylleraass plus inversely quadratic potential using Supersymmetric quantum mechanics approach. Applying perkeris approximation to the centrifugal term, we obtained the eigen-energy and the normalized wave function using Gauss and confluent hypergeometric functions. We implement Fortran algorithm to obtained the numerical result of the energy for the screening parameter $\alpha = 0.1, 0.2, 0.3, 0.4$ and 0.5 . The result shows that the energy increases with an increase in the quantum state. The energy spectrum shows increase in angular quantum state spacing as the screening parameter increases.

KEYWORDS

Schrodinger, Supersymmetric Quantum Mechanics Approach, Modified Hylleraass plus Inversely Quadratic potential.

1. INTRODUCTION

Schrodinger wave equation belongs to non-relativistic wave equation. The total wave function of any quantum mechanical system basically provides implicitly the relevant information about the physical behavior of the system. Bound state solutions most time provides negative energies because oftenly, the energy of the particle is less than the maximum potential energy therefore, causing the particle to be trapped within the potential well. However, in a well that is infinitely long, the particles can have positive energies and are still trapped within the potential well, hence we can conclude that for infinitely long potential well, bound state energy of a particle is either less than the potential at negative infinity ($E < -\infty$) or less than the potential at positive infinity ($E < \infty$) which provides the reason for obtaining both negative and positive bound state energies predominantly in Klein-Gordon equation. A lot of authors developed interest in studying bound state solutions majorly due to its scientific applications in both physical and chemical sciences in particle, high energy Physics and molecular dynamics . [1-3].

Different analytical techniques have been adopted by different authors in providing solutions to relativistic and non-relativistic wave equations. These are: Nikiforov-Uvarov method, exact quantisation, asymptotic iteration method, supersymmetric quantum mechanics approach, factorization method, tridiagonalisation method etc.[4-10]. Some of the potentials under consideration are: woods-Saxon plus modified exponential coulomb potential, Hulthen plus generalized exponential coulomb potential, Rosen-Morse, Hulthen, pseudo harmonic, Poschl-Teller, kratzer fues and Mie-Type potential, Eckart potential and P-T symmetric Hulthen potential etc [11-20]. This paper is organized as follows: section 1 is the introduction. In section 2, we introduced the concept of supersymmetric quantum mechanics approach. In section 3, we apply the concept of supersymmetry to provide the solution to Schrodinger equation using the proposed potential and obtained the energy eigen value and the wave function.. In section 4, we implement an algorithm to obtained numerical computation for the resulting energy.

The modified Hylleraas plus inversely quadratic potential is given by

$$V(r) = \frac{-v_0}{\chi_2} \left(\frac{\chi_1 e^{-2\alpha r}}{1 - e^{-2\alpha r}} \right) + \frac{\chi_3}{r^2} \quad (1)$$

Where χ_3 is a constant, χ_1 and χ_2 are Hylleraas parameter. The graph of this potential against the inter-nuclear distance with various values of the screening parameter $\alpha = 1.0, 2.0, 3.0$ and 4.0 is shown below. The chosen α is to enable one sees the nature of the graph which is quite different from the one for numerical computation.

Figure a.

2.THE CONCEPT OF SUPERYSYMMETRIC QUANTUM MECHANICS(SUSYQM)

The supersymmetric approach deals with the partner Hamiltonian of the form

$$H_{\pm} = \frac{p^2}{2m} + V(x) \quad (2)$$

Where P is the momentum and V(x) is the effective potential which can be expressed in terms of super-potential as

$$V_{eff\pm}(x) = \phi^2(x) \pm \phi'(x) \quad (3)$$

The ground state energy is obtained as

$$\phi_0^{-1}(x) = Ce^{-N} \quad (4)$$

where N is the normalization constant which for very simple cases can be determined using the expression

$$N(x) = \int_{x_0}^x \phi(r) dr \quad (5)$$

However, the superpotential satisfies shape invariant condition of the form

$$V_+(a_0, x) = V_-(a_1, x) + R(a_1) \quad (6)$$

Where a_1 is a new set of parameter uniquely determined from the old set a_0 through the mapping $f : a_0 \rightarrow a_1 = f(a_0)$ (7)

The supersymmetric energy is determined as

$$E_n = \sum_{s=1}^n R(a_s) \quad (8)$$

While higher order state solutions are obtained through the expression

$$\phi_n^-(a_0, x) = \prod_{s=0}^{n-1} \left(\frac{A^\dagger(a_s)}{(E_n - E_s)^{\frac{1}{2}}} \right) \phi_0^-(a_n, x) \quad (9)$$

Where $A^\dagger(a_s)$ is a raising ladder operator expressed as

$$A_s^\dagger = -\frac{\partial}{\partial x} + \phi(a_s, x) \quad (10)$$

3. RADIAL SOLUTION OF SCHRODINGER EQUATION

Schrodinger equation is given by

$$\frac{d^2 R}{dr^2} + \left[\frac{2\mu}{\hbar^2} (E_{nl} - V(r)) - \frac{l(l+1)}{r^2} \right] R(r) = 0 \quad (11)$$

Substituting equation (1) into (11) gives

$$\frac{d^2 R}{dr^2} + \left[\frac{2\mu}{\hbar^2} \left(E_{nl} + \frac{v_0}{\chi_2} \left(\frac{\chi_1 e^{-2\alpha r}}{1 - e^{-2\alpha r}} \right) - \frac{\chi_3}{r^2} \right) - \frac{l(l+1)}{r^2} \right] R(r) = 0 \quad (12)$$

Let's define suitable approximation to the centrifugal term as

$$\frac{1}{r^2} = \alpha^2 \left[D_0 + \frac{D_1}{(1 - e^{-2\alpha r})} \right] \quad (13)$$

Substituting equation (13) into (12) and re-arranging gives

$$\begin{aligned} \frac{d^2 R}{dr^2} + \frac{1}{(1-e^{-2\alpha r})} \left[\frac{2\mu v_0 \chi_1 e^{-2\alpha r}}{\hbar^2 \chi_2} - \frac{2\mu v_0 \chi_3 \alpha^2 D_1}{\hbar^2} - l(l+1)\alpha^2 D_1 \right] R(r) \\ = - \left[\frac{2\mu E}{\hbar^2} - l(l+1)\alpha^2 D_0 - \frac{2\mu v_0 \chi_3 \alpha^2 D_0}{\hbar^2} \right] R(r) \end{aligned} \quad (14)$$

This can also be represented as

$$\begin{aligned} \frac{d^2 R}{dr^2} - \frac{1}{(1-e^{-2\alpha r})} \left[\frac{2\mu v_0 \chi_3 \alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu v_0 \chi_1 e^{-2\alpha r}}{\hbar^2 \chi_2} \right] R(r) \\ = - \left[\frac{2\mu E}{\hbar^2} - l(l+1)\alpha^2 D_0 - \frac{2\mu v_0 \chi_3 \alpha^2 D_0}{\hbar^2} \right] R(r) \end{aligned} \quad (15)$$

Let's define second order differential equation containing effective potential as

$$-\frac{d^2 R_{nl}(r)}{dr^2} + V_{eff}(r) R_{nl}(r) = \tilde{E}_{nl} R_{nl}(r) \quad (16)$$

In order to represent equation (15) in the form of equation (16), then equation (15) is multiply by -1.

$$\begin{aligned} -\frac{d^2 R}{dr^2} + \frac{1}{(1-e^{-2\alpha r})} \left[\frac{2\mu v_0 \chi_3 \alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu v_0 \chi_1 e^{-2\alpha r}}{\hbar^2 \chi_2} \right] R(r) \\ = \left[\frac{2\mu E}{\hbar^2} - l(l+1)\alpha^2 D_0 - \frac{2\mu v_0 \chi_3 \alpha^2 D_0}{\hbar^2} \right] R(r) \end{aligned} \quad (17)$$

Comparing equation (17) to (16), we obtained the following:

$$\begin{aligned} V_{eff}(r) &= \frac{1}{(1-e^{-2\alpha r})} \left[\frac{2\mu v_0 \chi_3 \alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu v_0 \chi_1 e^{-2\alpha r}}{\hbar^2 \chi_2} \right] \\ \tilde{E}_{nl} &= \left[\frac{2\mu E}{\hbar^2} - l(l+1)\alpha^2 D_0 - \frac{2\mu v_0 \chi_3 \alpha^2 D_0}{\hbar^2} \right] \end{aligned} \quad (18b)$$

$$\text{Let } A = \frac{2\mu v_0 \chi_3 \alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 \quad \text{and} \quad B = -\frac{2\mu v_0 \chi_1}{\hbar^2 \chi_2} \quad (19)$$

Then, the effective potential reduced to

$$V_{eff}(r) = \frac{1}{(1-e^{-2\alpha r})} [A + B e^{-2\alpha r}] \quad (20)$$

The super-potential suitable for the effective potential is given as

$$\phi(r) = \frac{-q_1}{(1-e^{-2\alpha r})} + q_2 \quad (21)$$

In order to construct partner potential, we apply equation (6) by first taking the square and first derivative of equation (21)

$$\phi'(r) = \frac{2\alpha q_1 e^{-2\alpha r}}{(1 - e^{-2\alpha r})^2} \quad (22)$$

$$\phi^2(r) = q_2^2 - \frac{2q_1 q_2}{(1 - e^{-2\alpha r})} + \frac{q_1^2}{(1 - e^{-2\alpha r})^2} \quad (23)$$

The partner potentials are

$$V_{eff}^+(r) = \phi^2(r) + \phi'(r) = q_2^2 - \frac{2q_1 q_2}{(1 - e^{-2\alpha r})} + \frac{q_1^2}{(1 - e^{-2\alpha r})^2} + \frac{2\alpha q_1 e^{-2\alpha r}}{(1 - e^{-2\alpha r})^2} \quad (24)$$

$$V_{eff}^-(r) = \phi^2(r) - \phi'(r) = q_2^2 - \frac{2q_1 q_2}{(1 - e^{-2\alpha r})} + \frac{q_1^2}{(1 - e^{-2\alpha r})^2} - \frac{2\alpha q_1 e^{-2\alpha r}}{(1 - e^{-2\alpha r})^2} \quad (25)$$

Equation (24) and (25) satisfies shape invariant condition.

3.1 CALCULATION OF GROUND STATE ENERGY

The ground state energy can be calculated by solving associated Riccati equation. This equation is given as

$$\phi^2(r) - \phi'(r) = V_{eff}^-(r) - \tilde{E}_{0l} \quad (26)$$

Where $\tilde{E}_{0l}$ is the ground state energy.

Substituting equation (20) and (25) into (26) and simplifying in decreasing order of exponent gives rise to three pairs of simultaneous equations

$$A + \tilde{E}_{0l} = q_1^2 + q_2^2 - 2q_1 q_2 \quad (27)$$

$$B - A - 2\tilde{E}_{0l} = -2q_2^2 + 2q_1 q_2 - 2\alpha q_1 \quad (28)$$

$$\tilde{E}_{0l} - B = q_2^2 \quad (29)$$

Solving the following simultaneous equations then

$$q_1 = q_2 \pm \sqrt{q_2^2 + (A + B)} \quad (30)$$

$$q_2 = -\left[\frac{A + B + 2\alpha q_1}{2q_1} \right] \Rightarrow q_2^2 = \left[\frac{A + B + 2\alpha q_1}{2q_1} \right]^2 \quad (31)$$

The ground state energy is calculated using equation (29)

$$\tilde{E}_{0l} = \left[\frac{A + B + 2\alpha q_1}{2q_1} \right]^2 + B \Rightarrow -\tilde{E}_{0l} = -\left[\frac{A + B + 2\alpha q_1}{2q_1} \right]^2 - B \quad (32)$$

3.2 CALULATION OF HIGHER ORDER SUPERSYMMETRIC ENERGY

Using the condition of shape invariant, higher order supersymmetric energy can be calculated as using

$$\bar{E}_{nl} = \sum_{k=1}^{\infty} R(a_k^-) \quad (33)$$

Equation (33) satisfying shape invariant condition can be evaluated as follows

$$\begin{aligned} R(a_1) &= R(a_0) - R(a_1) \\ R(a_2) &= R(a_0) - R(a_2) \\ R(a_3) &= R(a_0) - R(a_3) \\ R(a_n) &= R(a_{n-1}) - R(a_n) \end{aligned} \quad (34)$$

Thus, there is one to one corresponding mapping as shown below.

$$\begin{aligned} q_1 &\mapsto q_1 + \alpha \\ a_0 &\mapsto a_0 + \alpha \\ a_n &\mapsto a_n + \alpha n \\ \Rightarrow q_1 &\mapsto a_0 \\ \Rightarrow q_1 &\mapsto a_n \end{aligned} \quad (35)$$

Therefore, using (35), equation (31) can be written as

$$q_2^2 = \left[\frac{A+B+2\alpha q_1}{2q_1} \right]^2 \Rightarrow q_2^2 = \left[\frac{A+B+2\alpha q_0}{2q_0} \right]^2 \quad (36)$$

Using equation (34), the following evaluations is carried out

$$\begin{aligned} R(a_1) &= \left[\frac{A+B+2\alpha q_0}{2q_0} \right]^2 - \left[\frac{A+B+2\alpha q_1}{2q_1} \right]^2 \\ R(a_2) &= \left[\frac{A+B+2\alpha q_1}{2q_1} \right]^2 - \left[\frac{A+B+2\alpha q_2}{2q_2} \right]^2 \\ R(a_3) &= \left[\frac{A+B+2\alpha q_2}{2q_2} \right]^2 - \left[\frac{A+B+2\alpha q_3}{2q_3} \right]^2 \\ \bar{E}_{nl} &= \sum_{k=1}^{\infty} R(a_k^-) = \left[\frac{A+B+2\alpha a_0}{2a_0} \right]^2 - \left[\frac{A+B+2\alpha a_n}{2a_n} \right]^2 \end{aligned} \quad (37)$$

3.3 CALCULATION OF TOTAL ENERGY

The total energy is the sum of the ground state energy and higher order supersymmetric energy. This is given as

$$\tilde{E}_{nl} = \bar{E}_{nl} + \tilde{E}_{0l} \quad (38)$$

Sometimes, the ground state energy can be negative (bound State condition) like in hydrogen atom which is about -13.6eV.

Hence, equation (38) can in this manner be expressed as

$$\tilde{E}_{nl} = \bar{E}_{nl} + (-\tilde{E}_{0l}) \quad (39)$$

Substituting equation (32) and (37) into (39) gives

$$\tilde{E}_{nl} = - \left\{ \left[\frac{A + B + 2\alpha a_n}{2a_n} \right] + B \right\} \quad (40)$$

Let's recall equation (30)

$$q_1 = q_2 \pm \sqrt{q_2^2 + (A + B)} \Rightarrow q_1 = \alpha \pm \sqrt{\alpha^2 + (A + B)} \quad , \quad q_2 \mapsto \alpha$$

If $a_n \mapsto a_n + \alpha n$, for $q_1 \mapsto a_n$

$$\text{Then } a_n = \alpha n + \alpha \pm \sqrt{\alpha^2 + (A + B)} \Rightarrow a_n = \alpha \left[(n+1) \pm \sqrt{\alpha^2 + (A + B)} \right] \quad (41)$$

Substituting equation (18b) and equation (41) into (40) and simplifying gives the total energy as

$$E_{nl} = -\frac{\hbar^2}{2\mu} \left\{ \frac{\left(\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\chi_2} \right) + 2\alpha^2 \left(n+1 \pm \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)}{2\alpha \left(n+1 \pm \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)} - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2} \right\} \\ + \chi_3\alpha^2 D_0 + \frac{\hbar^2 l(l+1)\alpha^2 D_0}{2\mu} \quad (42)$$

Equation (42) is the energy equation for Hylleraas plus inversely quadratic potential. However, because of the plus and minus accompanied by the square root sign, equation (42) can be expressed as follows:

$$E_{nl} = -\frac{\hbar^2}{2\mu} \left\{ \frac{\left(\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\chi_2} \right) + 2\alpha^2 \left(n+1 + \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)}{2\alpha \left(n+1 + \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)} - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2} \right\} \\ + \chi_3\alpha^2 D_0 + \frac{\hbar^2 l(l+1)\alpha^2 D_0}{2\mu} \quad (43)$$

$$E_{nl} = -\frac{\hbar^2}{2\mu} \left\{ \frac{\left(\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\chi_2} \right) + 2\alpha^2 \left(n+1 - \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)}{2\alpha \left(n+1 - \sqrt{\frac{2\mu\chi_3\alpha^2 D_1}{\hbar^2} + l(l+1)\alpha^2 D_1 - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2}} \right)} \right\} - \frac{2\mu\nu_0\chi_1}{\hbar^2\chi_2} \quad (44)$$

$$+ \chi_3\alpha^2 D_0 + \frac{\hbar^2 l(l+1)\alpha^2 D_0}{2\mu}$$

3.4 CALCULATION OF THE WAVE FUNCTION

Furthermore, in order to calculate the radial wave function, we used the coordinate transformation $s = e^{-2\alpha r}$ into equation (16) and obtained the following

$$\left\{ \frac{d^2 R}{dr^2} + \frac{(1-s)}{s(1-s)} \frac{dR}{ds} + \frac{1}{s^2(1-s)^2} \right\} + \left\{ -\left(\frac{\nu_0\chi_1}{4\alpha^2\chi_2} + \frac{l(l+1)D_0}{4} + \frac{\chi_3 D_0}{4} - \frac{2\mu E}{4\hbar^2\alpha^2} \right) s^2 + \left(\frac{\nu_0\chi_1}{4\alpha^2\chi_2} + \frac{2\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{2l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{\mu E}{\hbar^2\alpha^2} \right) s - \left(\frac{\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{2\mu E}{4\hbar^2\alpha^2} \right) \right\} \quad (45)$$

The corresponding radial wave function is then given by

$$R_{nl}(r) = N_n \left(e^{-2\alpha r} \right)^{\sqrt{\frac{\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{2\mu E}{4\hbar^2\alpha^2}}} (1 - e^{-2\alpha r})^{-1 - \sqrt{\frac{\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{2\mu E}{4\hbar^2\alpha^2}}} \times P_n \left[\left(1 + 2\sqrt{\frac{\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{2\mu E}{4\hbar^2\alpha^2}} \right) \left(3 + 2\sqrt{\frac{\chi_3 D_0}{4} + \frac{\chi_3 D_1}{4} + \frac{l(l+1)D_0}{4} + \frac{l(l+1)D_1}{4} - \frac{2\mu E}{4\hbar^2\alpha^2}} \right) \right] (1 - 2e^{-2\alpha r}) \quad (46)$$

4. NUMERICAL COMPUTATION OF THE ENERGY EQUATION

Using equation (43), we implement Fortran algorithm to compute for the energy of the equation in electron volt with various values of the screening parameter α . We use the following values

for our computation. $V_0 = \hbar = \mu = 1.0$, $D_0 = \frac{1}{12}$, $D_1 = 1.0$, $\chi_1 = -1$, $\chi_2 = 2$, $\chi_3 = 1$.

Bound State energy with $\alpha = 0.1$ for different quantum state

n	l	$E_n(\alpha = 0.1)eV$	n	l	$E_n(\alpha = 0.1)eV$	n	l	$E_n(\alpha = 0.1)eV$	n	l	$E_n(\alpha = 0.1)eV$
0	0	-1.81557810	0	1	-1.83414400	0	2	-1.87091950	0	3	-1.92523000
1	0	-1.39580210	1	1	-1.40959080	1	2	-1.43698190	1	3	-1.47761940
2	0	-1.18513580	2	1	-1.19601170	2	2	-1.21765080	2	3	-1.24983630
3	0	-1.05848570	3	1	-1.06744430	3	2	-1.08528610	3	3	-1.11186640
4	0	-0.97394794	4	1	-0.98155713	4	2	-0.99672174	4	3	-1.01933810
5	0	-0.91351247	5	1	-0.92012280	5	2	-0.93330324	5	3	-0.95297605
6	0	-0.86815786	6	1	-0.87399983	6	2	-0.88565266	6	3	-0.90305570
7	0	-0.83286536	7	1	-0.83809840	7	2	-0.84853950	7	3	-0.86414030
8	0	-0.80462086	8	1	-0.80935950	8	2	-0.81881640	8	3	-0.83295200

Table 1

Bound State energy with $\alpha = 0.2$ for different quantum state

n	l	$E_n(\alpha = 0.2)eV$	n	l	$E_n(\alpha = 0.2)eV$	n	l	$E_n(\alpha = 0.2)eV$	n	l	$E_n(\alpha = 0.2)eV$
0	0	-1.25588080	0	1	-1.29197690	0	2	-1.36170600	0	3	-1.46096090
1	0	-1.04142170	1	1	-1.06843020	1	2	-1.12113100	1	3	-1.19725980
2	0	-0.93265150	2	1	-0.95405185	2	2	-0.99604530	2	3	-1.05721940
3	0	-0.86688805	3	1	-0.88456786	3	2	-0.91938436	3	3	-0.97037660
4	0	-0.82283480	4	1	-0.83788250	4	2	-0.86758830	4	3	-0.91125740
5	0	-0.79126420	5	1	-0.80435646	5	2	-0.83024790	5	3	-0.86841345
6	0	-0.76752913	6	1	-0.77911305	6	2	-0.80205260	6	3	-0.83593720
7	0	-0.74903460	7	1	-0.75942045	7	2	-0.78000940	7	3	-0.81047153
8	0	-0.73421750	8	1	-0.74362910	8	2	-0.76230270	8	3	-0.78996754

Table 2

Bound State energy with $\alpha = 0.3$ for different quantum state

n	l	$E_n(\alpha = 0.3)eV$	n	l	$E_n(\alpha = 0.3)eV$	n	l	$E_n(\alpha = 0.3)eV$	n	l	$E_n(\alpha = 0.3)eV$
0	0	-1.11232240	0	1	-1.16417930	0	2	-1.26113510	0	3	-1.39352740
1	0	-0.96447120	1	1	-1.00370690	1	2	-1.07845380	1	3	-1.18307350
2	0	-0.88827163	2	1	-0.91957426	2	2	-0.97985350	2	3	-1.06547440
3	0	-0.84179723	3	1	-0.86777450	3	2	-0.91814520	3	3	-0.99038374
4	0	-0.81049335	4	1	-0.83267320	4	2	-0.87588680	4	3	-0.93828130
5	0	-0.78797410	5	1	-0.80731654	5	2	-0.84513450	5	3	-0.90001200
6	0	-0.77099670	6	1	-0.78814130	6	2	-0.82175210	6	3	-0.87071250
7	0	-0.75773966	7	1	-0.77313274	7	2	-0.80337375	7	3	-0.84755990
8	0	-0.74710070	8	1	-0.76106584	8	2	-0.78854835	8	3	-0.8288036

Table 3

Bound State energy with $\alpha = 0.4$ for different quantum state

n	l	$E_n(\alpha = 0.4)eV$	n	l	$E_n(\alpha = 0.4)eV$	n	l	$E_n(\alpha = 0.4)eV$	n	l	$E_n(\alpha = 0.4)eV$
0	0	-1.07219960	0	1	-1.13772730	0	2	-1.25619530	0	3	-1.41204810
1	0	-0.95648575	1	1	-1.00673560	1	2	-1.10005060	1	3	-1.22680750
2	0	-0.89565750	2	1	-0.93608590	2	2	-1.01235960	2	3	-1.11805080
3	0	-0.85815040	3	1	-0.89188860	3	2	-0.95620050	3	3	-1.04651460
4	0	-0.83271027	4	1	-0.86163010	4	2	-0.91715777	4	3	-0.99588360
5	0	-0.81432050	5	1	-0.83961457	5	2	-0.88844097	5	3	-0.95816210
6	0	-0.80040705	6	1	-0.82287750	6	2	-0.86643210	6	3	-0.92897120
7	0	-0.78951290	7	1	-0.80972373	7	2	-0.84902660	7	3	-0.90571094
8	0	-0.78075130	8	1	-0.79911387	8	2	-0.83491695	8	3	-0.88674080

Table 4

Bound State energy with $\alpha = 0.5$ for different quantum state

n	l	$E_n(\alpha = 0.5)eV$	n	l	$E_n(\alpha = 0.5)eV$	n	l	$E_n(\alpha = 0.5)eV$	n	l	$E_n(\alpha = 0.5)eV$
0	0	-1.07287570	0	1	-1.15000000	0	2	-1.28518370	0	3	-1.45766960
1	0	-0.97570810	1	1	-1.03571430	1	2	-1.14444880	1	3	-1.28834840
2	0	-0.92349565	2	1	-0.97222220	2	2	-1.06231940	2	3	-1.18440910
3	0	-0.89090130	3	1	-0.93181820	3	2	-1.00849700	3	3	-1.11411000
4	0	-0.86861694	4	1	-0.90384614	4	2	-0.97049820	4	3	-1.06339450
5	0	-0.85241880	5	1	-0.88333330	5	2	-0.94223930	5	3	-1.02507970
6	0	-0.84011304	6	1	-0.86764705	6	2	-0.92040080	6	3	-0.99511280
7	0	-0.83044730	7	1	-0.85526310	7	2	-0.90301790	7	3	-0.97103360
8	0	-0.82265410	8	1	-0.84523810	8	2	-0.88885320	8	3	-0.95126210

Table 5

Figure 1: The graph of energy against the distance for different quantum state with $\alpha = 0.1$.

Figure 2: The graph of energy against the distance for different quantum state with $\alpha = 0.2$.

Figure 3: The graph of energy against the distance for different quantum state with $\alpha = 0.3$.

Figure 4: The graph of energy against the distance for different quantum state with $\alpha = 0.4$.

Figure 5. : The graph of energy against the distance for different quantum state with $\alpha = 0.5$.

CONCLUSION

We used supersymmetric quantum mechanics approach to obtained bound state solution to Hylleraas plus inversely quadratic potential using Schrodinger equation. We also obtained eigen energy and the wave function. The result shows that the energy is negative to ascertain bound state condition and increases with an increase in quantum as shown in tables (1-5). The energy graphical spectrum shows that the interspacing within the quantum state is more pronounce the value of α increases. This can be shown from figures (1-5).

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