Studies on the spatial variability of rebound hammer test results recorded at in-situ testing

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Abstract

Variability of the in-situ rebound hammer tests is composed of the inherent variability of the measuring method and the spatial variability of the concrete performance properties. In-situ, Schmidt rebound hammer tests were performed at the bottom surface of a concrete slab, with dimensions of 25.0 m × 7.5 m. The studied region on was 22.0 m × 6.0 m. The analysis of the inherent variability of the rebound hammer tests was limited to the comparison of the statistical location parameters; mean, median and mode values. The spatial variability was analysed by contour plots and semivariograms of the statistical location parameters. It was demonstrated that the geostatistical methods may be applicable for the much smaller scale concrete structures than the scale of geological formations. Semivariograms and semivariogram models seem to be useful tools for the characterisation of spatial variability of concrete structures.

Keywords: concrete, rebound hardness, Schmidt hammer, spatial variability, statistical location parameters, semivariogram

1. Introduction

The magnitude of the variation in the strength of structural concrete is a result of the level of quality control over the concrete production, transportation, compacting and curing procedures. The compressive strength of the concrete is not measured in the structure during in-situ non-destructive testing (NDT), but some other property is measured instead that is correlated to the compressive strength. The strength of concrete is estimated from a previously established relationship between the measured NDT property and the compressive strength. The uncertainty of the estimated compressive strength, therefore, depends both on the variability of the in-situ measurements and the uncertainty of the relationship between the measured NDT property and compressive strength. *Variability of the in-situ measurements* is composed of the *inherent variability* of the measuring method (repeatability and reproducibility; attributed to the operation mode and the calibration conditions of the testing device, to the operator and to environmental influences) and the *spatial variability* of the concrete performance properties due to differences of compaction and curing, possible non-uniform supply of material and structural effects (reinforcement, changes in dimension, corners, connecting structural elements etc.). Inherent variability phenomena can be considered as point properties (local parameter). Spatial variability phenomena can be considered as regional properties (regional or volume parameter). Local phenomena can be analysed by classical statistical methods. Analysis of regional phenomena needs spatial models – that are typically used in *geostatistics*.

Development of geostatistical tools is attributed to Prof. George Matheron who published his treatise on the Theory of Regionalised Variables and its Applications based on earlier

empirical work in the 1960s [1], nevertheless, the basic idea of *kriging* (Gaussian piecewise-polynomial spline interpolation regression) is first used in a technical context by Prof. Danie G. Krige, in his study about average gold grades at the Witwatersrand reef complex in South Africa [2]. Historically, Prof. Andrey N. Kolmogorov was the first who recognized spatial correlation. He defined the so-called *structure function* for its representation and developed how to use the structure function for optimal interpolation (without bias and with minimum variance) [3]. Kolmogorov was unable to use the method due to the lack of computer power in those days [4]. Kolmogorov's structure function is known today as the *variogram* and his interpolation technique as *kriging*.

Geostatistics deals with spatially autocorrelated data (autocorrelation = correlation between elements of a series and others from the same series separated from them by a given interval). Some spatially autocorrelated parameters of interest in geostatistics are facies, reservoir thickness, porosity, permeability [5].

Fig. 1. Schematic representation of lag vectors in 2D [7]
1. ábra Kétdimenziós helyvektorok sematikus megjelenítése [7]

Geostatistics usually assumes that the differences between the values of samples are determined by the relative spatial orientation of the samples and the mean and variance of

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the differences depend only on the relative orientation [6]. Therefore, the semivariogram is introduced in spatial correlation analyses that plots the semivariance of samples as the function of the separation between two spatial locations (referred to as *lag*).

For the development of the semivariogram, let us suppose a 2D field of values of a variable $f(x,y)$ as indicated in Fig. 1. Lags are schematically shown as h_1 , h_2 and h_3 .

For the determination of a semivariogram corresponding to a 2D field of values of a variable $f(x,y)$, the followings can be considered:

u vector of spatial coordinates (with 2D components x and y),

$f(\mathbf{u})$ variable under consideration as a function of spatial location,

h lag vector representing separation between two spatial locations,

$f(\mathbf{u}+\mathbf{h})$ lagged version of variable under consideration,

$N(\mathbf{h})$ the number of data pairs separated by lag **h**.

Semivariance $\gamma(\mathbf{h})$ can be determined for lag **h** as of Eq. (1):

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{i=1}^{N(\mathbf{h})} [f(\mathbf{u}_i + \mathbf{h}) - f(\mathbf{u}_i)]^2 \quad (1)$$

Semivariance is a measure of the dissimilarity of the values. The empirical function of $\gamma(\mathbf{h})$ can be considered as a population parameter. The diagram showing the semivariance versus lag is referred to as the *semivariogram*.

Geostatistical analyses prefer using the semivariogram since the semivariogram tends to filter the influence of a spatially varying mean by averaging the squared differences of the variable. Also, the semivariogram can be applied whenever the covariance function cannot be defined. In such a case the semivariance may keep increasing with increasing lag, rather than leveling off, corresponding to an infinite global variance; and the covariance function is undefined [5].

2. Spatial variability analysis for in-situ rebound hammer testing

Typical relative strength variation for structural concrete in a wall is illustrated in Fig. 2 [8]. The strength gradients are reasonably uniform due to gravity and the variations in compaction and supply is visible in the variability indicated by the relative strength contours. Anisotropy is always realised in the compressive strength of structural members in vertical direction (including columns, walls, beams and thick slabs).

Considering the bottom surface of a slab, the variability of the compressive strength in horizontal direction is expected to be random due to only compaction and supply inconsistencies. The study of the spatial variability of concrete performance properties is, therefore, reasonable to be carried out for a bottom surface of a slab where the vertical bias in the compressive strength is excluded and the clear random influences due to only compaction and supply differences can be analysed.

For such an analysis, the bottom surface of the top concrete slab of a framed, monolithic, subsoil concrete tunnel was

selected, with dimensions of 25.0 m $\times$ 7.5 m with a thickness of 0.48 m. The studied region on the bottom surface was 22.0 m $\times$ 6.0 m.

A total number of 42 testing locations were selected for Schmidt rebound hammer testing. N-type original Schmidt rebound hammer was used. Eleven individual rebound index readings were recorded at each location. The measurements were performed by the same operator.

Fig. 2. Relative strength variation for a structural concrete wall [8]
2. ábra Relatív nyomószilárdság változása beton falban [8]

3. Results

The present paper discusses the spatial variability of the measured rebound indices (R_i) based exclusively on the statistical location parameters, i.e. test location *mean* – $E[R]$ values, test location *median* – $m[R]$ values and test location *mode* – $Mo[R]$ values.

A total number of 462 individual rebound indices were recorded at the 42 testing locations. The frequency histogram of the individual rebound indices without any separation by location is indicated in Fig. 3 together with the best fit probability density function.

Fig. 3. Frequency histogram of the individual rebound indices for the 42 testing locations without any separation by location

3. ábra A 42 mérési helyen rögzített visszapattanási értékek együttes relatív gyakorisági histogramja

Considering the rebound hammer test, it can be assumed that the rebound index reading sets of separate testing locations are independent and identically distributed (i.i.d.) random variables since it can be supposed that the probability distribution of the rebound index does not change by location within the same concrete structure and the separate testing locations can be considered to be mutually independent [9]. Based on these assumptions, the central limit theorem applies for the rebound hammer test; i.e. the probability distribution of the mean of the rebound index reading sets of separate testing locations (each with finite mean and finite variance) approaches the normal distribution if sufficiently large number of the i.i.d. random variables is available. It can be realised that the mean of the 42 independent testing locations approaches rather well to the normal distribution.

Fig. 4. Correlations between the mean, median and mode values for the 42 testing locations

4. ábra A visszapatannási értékek 42 mérési helyre vonatkozó átlagának, mediánjának és móduszának korrelációi

Fig. 5. Frequency histograms for the mean, median and mode values for the 42 testing locations

5. ábra A visszapatannási értékek 42 mérési helyre vonatkozó átlagának, mediánjának és móduszának gyakorisági histogramjai

For the 462 individual rebound indices, the global mean value is $E[R] = 52.6$, the global median value is $m[R] = 53$ and the global mode value is $Mo[R] = 53$. Fig. 4 illustrates the rather close correlations between the statistical location parameters corresponding to the 42 testing locations.

Fig. 5 indicates the frequency histograms of the statistical location parameters; mean – $E[R]$ values, median – $m[R]$ values and mode – $Mo[R]$ values, respectively. Findings confirm that the three location parameters are not equally sensitive to the extreme individual values of which possible influences seem to rule out for the mean values (frequency histogram and best fit probability density function have almost zero skewness or excess kurtosis), on the contrary to the median values (where frequency histogram and best fit probability density function show positive skewness) and the mode values (where frequency histogram and best fit probability density function show positive excess kurtosis).

In a practical situation, when noticing or localising of the presence of eventually weaker regions of the structural element can be one purpose, the analysis of mean responses may be misleading; in the case of sufficiently large number of testing locations the central limit theorem applies and global observations can hide the influence of local extremities.

Contour plots may indicate local information in a visible way. Maps of the statistical location parameters for mean – $E[R]$ values, median – $m[R]$ values and mode – $Mo[R]$ values are shown in Fig. 6, respectively. It can be seen that the three parameters broadly show similar differences in the measured values, however, the extremities are observable the best at the contour plot of the mode values. Contour plot of the mean values seems to be the least sensitive in this sense. It can be also noted that different performance areas can be observed

on particular maps that are not visible on the other two maps. Results evidently show the influences due to compaction and supply differences. The observed behaviour can be considered, however, result of random influences. No clear tendencies in the differences of the performance properties are realised.

In a practical situation, when eventually weaker regions of the structural element are present, it can be interesting to know that at which extent and in which orientation the discrepancy is present and has influence on performance properties. Geostatistical approaches can be used for the analysis of spatial variation. The method of kriging is applicable for modelling Gaussian random processes, based on models of semivariograms for a variable [1,4,6]. Semivariograms may indicate the lag distance (=range) over which the values of the variable are not correlated (independent). This distance can be referred to as *correlation distance*. The *sill* is the semivariance value at which the semivariogram levels off. The *nugget* is the semivariance value at distances smaller than the minimum lag of measurements [4,6]. Typical models for semivariograms are the spherical, exponential, Gaussian, wave, nugget [4-7]. Combined models are also used.

Fig. 7 represents the empirical semivariograms for the statistical location parameters; mean – $\gamma(E[R])$, median – $\gamma(m[R])$ and mode – $\gamma(Mo[R])$, respectively. The best fit spherical models are also indicated. It can be realised that the semivariogram models level off at a range about ten meters. In geostatistics, a common rule of thumb is accepted for the maximum lag in a semivariogram restricted to half of the diagonal of data extent [10]. In the present example, the concrete slab has dimensions of 25.0 m $\times$ 7.5 m of which the studied region is 22.0 m $\times$ 6.0 m. Semivariograms in Fig. 7 are, however, not limited to about 12 meters since one dimension

Fig. 6. Contour plots for the mean, median and mode values over the tested area
6. ábra A visszapatantási értékek átlagának, mediánjának és móduszának szintvonalas ábrázolása

Fig. 7. Semivariograms for the mean, median and mode values for the 42 testing locations together with the best fit spherical model
7. ábra A visszapatantási értékek 42 mérési helyre vonatkozó átlagának, mediánjának és móduszának félvariogramjai, szférikus modellel közelítve

of the slab is three times larger than the other. Restricting the semivariograms at maximum lag about half of the diagonal of data extent would not allow the diagrams to visibly level off and the sill could not be clearly determined. It can be realised that the spatial variability of the mean and median values is of the same magnitude, and the spatial variability of the mode values is a little higher. It can be concluded that the testing locations can provide uncorrelated (independent) results for the rebound index readings over a lag distance of about ten meters.

4. Conclusions and future work

The present paper has summarised the application possibilities of particular geostatistical tools for modelling the spatial variability of rebound hammer non-destructive testing. Results of in-situ measurements were provided and analysed. The analysis of the inherent variability of the rebound hammer tests was limited to the comparison of the statistical location parameters; mean – $E[R]$ values, median – $m[R]$ values and mode – $Mo[R]$ values. It was demonstrated that the statistical location parameters signify the inherent variability by different sensitivities. The spatial variability was analysed by contour plots and semivariograms of the statistical location parameters. It was demonstrated that the geostatistical methods may be applicable for the much smaller scale concrete structures than the scale of geological formations. Semivariograms and semivariogram models seem to be useful tools for the characterisation of spatial variability of concrete structures.

The spatial variability of geological and hydrological variables is well-described in the geostatistical literature for a long time. The use of the geostatistical methods for concrete structures is, however, still very much limited today. Further research is needed about the probability distribution of NDT measures and their influence on the boundary conditions for geostatistical methods. Size effect studies are also needed since the correlation distances are expected to be considerably different in particular cases – changing in the range of ten centimetres to ten meters –, depending on the actual workmanship and material supply. It is also of interest, which statistical parameter of the recorded NDT measure is the most suitable for a useful spatial variability analysis.

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Mérőhelyek közötti változékonyság értékelése Schmidt-kalapáccsal végzett helyszíni vizsgálat során

A helyszíni Schmidt kalapáccs vizsgálati eredmények változékonysága két határból tevődik össze: a mérőhelyi változékonyságból (ismételhetőség és reprodukálhatóság) és a mérőhelyek közötti változékonyságból, amely a térben eltérő bedolgozási és utókezelési hatások és az esetlegesen eltérő összetételű beton szállítmány eredménye. A mérőhelyi változékonyság statisztikai paraméterei a klasszikus matematikai statisztika eszközeivel kezelhetők. A mérőhelyek közötti változékonyság statisztikai paramétereinek vizsgálata térbeli statisztikai modelleket igényel, amelyeket a geostatistika használ széles körben. A térbeli változékonyság leírására használt modelleket a geostatistika szakirodalmában részletesen, régóta tárgyalja. Ennek ellenére, vasbeton szerkezetek vizsgálatára e módszerek napjainkban még nem terjedtek el. A cikk bemutatja egy 25×7,5 m méretű vasbeton födém 22×6 m méretű szakaszának helyszíni schmidt kalapáccs vizsgálati eredményeit. A mérőhelyi változékonyság vizsgálata során bemutatjuk a mérési helyekre meghatározott helyzeti statisztikai jellemzők (számtani közép, medián, módusz) jellegzetességeik és egymáshoz viszonyított kapcsolatukat. Bemutatjuk, hogy a helyzeti statisztikai jellemzők más-más érzékenységgel jelzik a mérőhelyi változékonyságot. A mérőhelyek közötti változékonyság vizsgálata során bemutatjuk a helyzeti statisztikai jellemzők térbeli eloszlását és azok félvariogramjait. Bemutatjuk, hogy a geostatistikában, nagy léptékben alkalmazott térbeli statisztikai modellek alkalmasak vasbeton szerkezetekre történő adaptálásra, kisebb léptékben is. A félvariogramok és azok közelítő modelljei alkalmasak a vasbeton szerkezetek helyszíni, Schmidt kalapáccs vizsgálati eredményeire vonatkozóan a mérőhelyek közötti változékonyság vizsgálatára.

Kulcsszavak: beton, felületi keménység, Schmidt kalapács, térbeli változékonyság, statisztikai helyzeti jellemzők, félvariogram

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