Simple basic model for concrete and its application

2. Factors that influence compressive strength and drying shrinkage

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Abstract

By introducing dimensionless concrete composition content indicators, the structural composition of concrete mixes can be graphically described, opening the way for analysing the effects which influence the performance properties of fresh and hardened concrete mixes using an approach that differs slightly from what has gone before. The present paper deals with the continuation of the observations and the analysis of the results. By an analysis of factors that have influence on the compressive strength, it is possible to make further progress towards an explanation for the two problems mentioned in the literature (summarised in the first part of the present series of papers), which have never so far been fully understood. The analysis of drying shrinkage deformation tests of small-scale prisms prepared during the laboratory experiments is also presented, and the relationships between the concrete composition content indicators and the drying shrinkage deformations in hardened concretes are highlighted. An interesting parallel is revealed in the effects that influence compressive strength and those that influence drying shrinkage deformation, and this provides an opportunity to estimate drying shrinkage deformation in advance from the concrete composition content indicators.

Keywords: concrete technology, concrete mix design, concrete composition content indicators

1. Comparison of manufacturing plant observations and laboratory experiments

The series of plant observations are reported in the first part of the paper [1]. The cooperation received support from ÉMI Nonprofit Ltd., which also began an internal *series of laboratory experiments*, with the aim of carrying out further investigations of the effects of concrete composition content indicators. The number of observations *suitable for analysis* has currently reached 220 for field observations and 53 for laboratory mixes. The observed concrete composition content indicators are summarised in Table 5.

A particular observation or experiment can be considered for inclusion in the analysis if both the *composition* of the concrete mix and the *concrete composition content indicators* are known (p – volumetric ratio of paste in the concrete, x – volumetric ratio of free fluid and paste powder in the paste, χ_c – the volumetric ratio of cement in the paste powder, λ_{AD} –

volumetric ratio of admixtures compared to the paste powder, l – volumetric ratio of air in the concrete), and *at least one performance property* of the fresh or hardened concrete has been *measured*.

Measured performance properties during the *industrial* observations can be:

- *consistency* of the fresh concrete (flow table test),
- *compressive strength* of hardened concrete (after 28 days standard curing).

During the *laboratory* experiments (at ÉMI Nonprofit Ltd.), *drying shrinkage* was also tested up to the age one year.

2. Evaluation of compressive strength results

It is assumed that the reader is familiar with the standard procedures for estimating compressive strength in the literature. The most widely known procedures are based on placing the effect of the water-cement ratio (as the *only*

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		Content indicators					Other technical data			
		p	x	χ_c	λ_{AD}	l	Types of cement taken into account	Traditional w/c	a^*	D_{max} mm
ÉMI Nonprofit Ltd. 2010. 53 experiments	min.	0.098	0.565	0.084	0.000	0.000	CEM I 42.5 N,	0.202	0.495	4
	max.	1.000	2.633	0.99	0.055	0.202	CEM III/B 32.5 N-S	3.758	0.769	8
	AVG.	0.303	1.432	0.715	0.015	0.040		0.912	0.666	8
	STD.	0.110	0.566	0.275	0.019	0.028		0.757	0.064	1
Augusztin Betongyártó Ltd. 2008–2010. 220 observations	min.	0.153	1.041	0.552	0.000	0.000	CEM I 42.5 N,	0.384	0.652	4
	max.	0.346	3.378	0.985	0.03	0.081	CEM II A-M (V-LL) 42.5 N,	1.218	0.79	32
	AVG.	0.254	1.683	0.834	0.009	0.018	CEM III/A 32.5	0.685	0.728	24
	STD.	0.023	0.305	0.082	0.010	0.011		0.145	0.019	9

Table 5. Concrete composition content indicators and concrete technology parameters, of analysed mixes from plant observations and from experimental mixes. (*: a – the volumetric ratio of aggregates in the concrete mixes)

5. táblázat Az üzemi megfigyelések és kísérleti keverések során értékelésbe vont keverékek betonösszetéti állapotjelzői és néhány egyéb adatai (a^* : a betonkeverékben lévő adalékanyag térfogataránya)

factor) into mathematical formulations. Regarding methods of estimating strength which are "w/c based", Ujhelyi and Popovics have demonstrated that *besides* the w/c ratio there are *other variables* (e.g. *cement content* or *water content*) which can be incorporated to improve the statistical properties of the estimates [2]. Nevertheless, even these improved estimation methods do not account for the *increase in compressive strength experienced when there is an increase in the content of additions* to concrete mixes with *identical water-cement ratios and identical cement content* [3], which suggests that there may be *additional factors* that influence the compressive strength and have not so far been taken into consideration.

Before turning to the analysis of our own observations, the Feret method is introduced, already over 100 years old, but not frequently used in Hungary. The mathematical formula for the Feret method as published by Ujhelyi [4] is shown in Eq. (29).

$$f_{c,28} = A \cdot \left(\frac{c}{1-a} \right)^2, \text{ where } a \leq a_{\max} \quad (29)$$

where $f_{c,28}$ is the expected compressive strength of a standard cube cured for 28 days [N/mm²],

A is the experimental constant,
 c is the *volume* of cement in the concrete,
 a is the *volume* of aggregate in the concrete, and
 $a_{\max}$ is the *volume* of the maximum amount of aggregate that can be compacted in the concrete

Eq. (29) of the Feret equation is *clearly based on volumes*, and therefore compatible with the simple model for concrete mixes, and so it follows that, with some changes to the equation to give us the new Eq. (30), we end up with the concrete composition content indicators which are discussed in the present series of papers:

$$f_{c,28} = A \cdot \left(\frac{\chi_c}{(1+x) \cdot \left(\frac{l}{p} + 1 \right)} \right)^2 \quad (30)$$

Form (30) of the Feret equation makes possible to *analyse* the effects of particular concrete composition content indicators on strength, and can even be used for estimating the strength of *unsaturated concretes*, although when the air content is extremely high it predicts compressive strengths that are too high to be realistic. Note: concrete is unsaturated if

$$p + a_{\max} < 1. \text{ Then: } l = 1 - a - p, \text{ where } a \leq a_{\max} \quad (31)$$

The Feret equation may be a good source of ideas for seeking relationships that can be interpreted within even broader limits.

2.1 Dependence of compressive strength on concrete composition content indicators

Taking, as an example, the 98 mixes of the *industrial observations* which were made using CEM I 42.5 N cement, and which also contained limestone powder as an addition, statistical calculations also proved that there was a *very strong* relationship (correlation coefficient $R^2=0.8219$) between compressive strength and the w/c ratio, and a *medium* relationship ($R^2=0.5455$)

between compressive strength and content indicator x . The correlation between compressive strength and the other concrete composition content indicators – at least when their effects were examined *in isolation* – was quite *weak*, but when the content indicators were considered *together*, even the weakly correlating factors became quite significant.

Performing calculations on the available observations and experimental mixes, we arrive at Eq. (32):

$$\ln f_{c,28} = n_x \cdot \ln c + n_p \cdot \ln p - n_x \cdot \ln (1+x) + n_l \cdot \ln (1-l) + \ln A \quad (32)$$

The same relationship in *multiplication factor exponent* form is Eq. (33):

$$f_{c,28} = A \cdot \frac{\chi_c^{n_x} \cdot p^{n_p}}{(1+x)^{n_x}} \cdot (1+l)^{n_l} \quad (33)$$

where $f_{c,28}$ is the expected compressive strength of a standard cube cured for 28 days [N/mm²],

A is the experimental constant,

χ_c is the volumetric ratio of cement in the paste powder, n_x is the exponent of χ_c ,

p is the volumetric ratio of paste in the concrete, n_p is the exponent of p ,

x is the *volumetric ratio* between the fluid and the paste powder, n_x is the exponent of $(1+x)$,

l the volumetric ratio of air in the concrete, and n_l is the exponent of $(1-l)$.

The parameters of Eq. (33) are summarised in Table 6, based on the observations and experiments carried out with different types of cement, in the analytical ranges laid out in Table 5.

	A	n_x	n_p	n_x	n_l	Note
CEM I 42.5 N	295.371	0.434	0.275	1.665	3.0	ÉMI Non-profit Ltd., 2010
CEM III/B 32.5 N-S	360.055	1.027	0.334	2.084	3.0	
CEM I 42.5 N	585.666	1.190	0.272	2.033	4.0	AUGUSZTIN Betongyártó Ltd., 2008–2010
CEM II A-M (V-LL) 42.5 N	342.302	1.711	-0.240	2.355	3.75	
CEM III/A 32.5	862.337	1.289	0.961	1.860	3.0	

Table 6. The parameters of estimation Eqs. (32) and (33) for the different types of cement investigated during experiments at ÉMI in 2010 and observations at Augusztin Betongyártó Ltd. between 2008–2010.

6. táblázat A (32) illetve (33) becslőképlet paramétereit a vizsgált különféle cementfajták esetében az ÉMI 2010. évi vizsgálatait és az Augusztin Betongyártó Kft. 2008–2010. közötti megfigyelései során

For the 98 mixes made from CEM I 42.5 N and containing limestone powder, referred to in section 2.1., the correlation coefficient of strength estimation made using Eq. (32) or the equivalent Eq. (33) was $R^2=0.8689$, and the individual significance of the concrete composition content indicators can be proved using a *t-test*. Eq. (33) is reminiscent of Eq. (30) of the Feret equation, with the exception that in Eq. (33) the multiplication factors have *independent exponents* and the l/p ratio is separated into independent p and $(1-l)$ factors. Eq. (33) *correlates better with experience* even with high air content, *within the analytical ranges under observation*. Extrapolations can only be made only *hypothetically*.

2.2 Dependence of compressive strength on the w/c ratio and cement content

Ujhelyi and Popovics demonstrated that introducing an additional variable (such as *c* - cement content) to the w/c ratio resulted in improvements to the statistical properties of w/c based estimates [2]. Taking this view as our starting point, we also recalculated our own results, adding to a new variable of *cement content as mass ratio* (*c/R*), while continuing to calculate with air content as volumetric ratio. For the results of the observations and the experiments, we have found the equation in Eq. (34) to be suitable:

$$\ln f_{c,28} = n_c \cdot \ln \left(\frac{c}{R} \right) - n_w \cdot \ln \left(\frac{w}{c} \right) + n_l \cdot \ln (1-l) + \ln A \quad (34)$$

where $f_{c,28}$ is the expected compressive strength of a standard cube cured for 28 days [N/mm²],

A is the experimental constant,

c/R is the mass ratio of cement in the concrete (*R*: concrete density), n_c is the coefficient of *c/R*,

w/c is the traditional water-cement ratio, n_w is the coefficient of *w/c*,

l is the volumetric ratio of air in the concrete, and n_l is the coefficient of *l*.

Eq. (34) can also be expressed in the form of a multiplication exponent. With regard to the 98 mixes made with CEM I 42.5 N and containing limestone powder, referred to in section 2.1., the correlation coefficient of the estimate is $R^2=0.8668$. The statistical significance of the factors *c/R* and *w/c* and (1-*l*) can be proved separately using a *t*-test.

The parameters of Eq. (34) are summarised in Table 7, based on the observations and experiments carried out with different types of cement, in the analytical ranges laid out in Table 5.

	A	n_c	n_w	n_l	Note
CEM I 42.5 N	54.645	0.236	0.587	3.0	ÉMI Non-profit Ltd., 2010
CEM III/B 32.5 N-S	33.630	0.274	0.922	3.0	
CEM I 42.5 N	43.924	0.197	1.104	4.0	AUGUSZTIN Betongyártó Ltd., 2008-2010
CEM II A-M (V-LL) 42.5 N	12.135	-0.173	1.6670	4.6	
CEM III/A 32.5	238.516	0.996	0.521	2.4	

Table 7. The parameters of estimation Eq. (34) for the different types of cement investigated during experiments at ÉMI in 2010 and observations at Augusztin Betongyártó Ltd. between 2008-2010

7. táblázat A (34) becslőképlet paraméterei a vizsgált különféle cementfajták esetében az ÉMI 2010. évi vizsgálatait és az Augusztin Betongyártó Kft. 2008-2010. közötti megfigyelései során

2.3 Comparison of the methods for compressive strength estimation

The two methods for strength estimation employed with regard to the 98 mixes made with CEM I 42.5 N and containing limestone powder, referred to in section 2.1., can be studied in Fig. 11. At first look, the two methods used in the range of observations appear to be more or less equivalent.

Fig. 11. Comparison of the errors of Eqs. (32) and (34) in estimating compressive strength

11. ábra A nyomószilárdság (32), illetve (34) képletek becslési hibáinak összehasonlítása

We may also compare the two methods by taking a look at the pairs of nomograms on Figs. 12 and 13, and Figs. 14 and 15. The nomograms were made by processing the results of two types of cement investigated during the ÉMI experiments in 2010, by inserting suitable parameters from Tables 6 and 7 into Eqs. (32) and (34), and placing the results in the appropriate analytical ranges in contour diagrams. Bands of the same colour between the contours indicate identical expected levels of compressive strength, and the scales are identical in the pairs of nomograms.

The first observation is that the compressive strengths of concretes with CEM III/B 32.5 N-S approach or even exceed the compressive strengths of similar mixes with CEM I 42.5 N, when cement contents are high and water-cement ratios are low. The nomograms in Figs. 14 and 15 are particularly eloquent: when *w/c*=0.2 and *c/R*=0.22, both cements have an expected compressive strength of ~95 N/mm². This, therefore, provides an explanation for the apparent contradiction mentioned in [5]. The figures reveal that the influences exerted on the strength of cements can be described by characteristics, and not be discretely measured specific properties, and these characteristics depend on the concrete composition content indicators of the mixes, which must not be ignored during mix design.

The second observation is that the estimates of compressive strength based on *w/c-c/R-(1-l)*, when cement content is high and *w/c* is low are progressively less sensitive to the increase in cement content, and extremely sensitive to the reduction in *w/c*. The predicted results in these ranges are unstable. In estimates based on $x \cdot \chi_c \cdot p \cdot l$, this effect appears much more moderate at the extremes of the range.

The third observation is that the *w/c-c/R-(1-l)*-based estimate of strength for identical *w/c* and identical cement content always predicts identical strength, which contradicts the results reported in [3], which stated that compressive strength increases as the dosage of additions increases. At the same time, the effect of additions on strength can be predicted from estimates based on $x \cdot \chi_c \cdot p \cdot l$, as we shall prove in section 3.

In our opinion, the use of estimates of compressive strength based on the single variable *w/c* in modern concrete engineering

Fig. 12. x - χ_c - p - l -based nomogram for mixes with stone powder and CEM III/B 32.5 N-S, based on experiments at ÉMI in 2010, ($p = 0.280$; $l = 0.010$)

12. ábra Az x - χ_c - p - l -alapú becslés nomogramja CEM III/B 32,5 N-S cementtel készült kölsztes keverékekre, az ÉMI 2010. évi kísérletek feldolgozása alapján, $p=0,280$ és $l=0,010$ esetén

Fig. 13. x - χ_c - p - l -based nomogram for mixes with stone powder and CEM I 42.5 N, based on experiments at ÉMI in 2010, ($p = 0.280$; $l = 0.010$)

13. ábra Az x - χ_c - p - l -alapú becslés nomogramja CEM I 42,5 N cementtel készült kölsztes keverékekre, az ÉMI 2010. évi kísérletek feldolgozása alapján, $p=0,280$ és $l=0,010$ esetén

Fig. 14. w/c - c/R -(1- l)-based nomogram for mixes with stone powder and CEM III/B 32.5 N-S, based on experiments at ÉMI in 2010, ($l = 0.010$)

14. ábra A w/c - c/R -(1- l)-alapú becslés nomogramja CEM III/B 32,5 N-S cementtel készült kölsztes keverékekre, az ÉMI 2010. évi kísérletek feldolgozása alapján, $l=0,010$ esetén

Fig. 15. w/c - c/R -(1- l)-based nomogram for mixes with stone powder and CEM I 42.5 N, based on experiments at ÉMI in 2010, ($l = 0.010$)

15. ábra A w/c - c/R -(1- l)-alapú becslés nomogramja CEM I 42,5 N cementtel készült kölsztes keverékekre, az ÉMI 2010. évi kísérletek feldolgozása alapján, $l=0,010$ esetén

is of concern. These methods can be significantly improved by introducing one additional variable (e.g. cement content), but there is still a need - disregarding *air content* - to include another variable (e.g. χ_c) in order to describe the influence of the content of additions. This brings us to the point we already reached with the application of concrete composition content indicators in section 2.1.: *in general*, four independent variables are necessary for estimating the compressive strength.

3. The effect of additions on compressive strength

For illustration, let us produce two concrete mixes containing 350 kg/m³ cement, with a water-cement ratio $w/c=0.5$, and containing 1% (v/v) air ($l=0.01$). Mix no. 1 contains no additions ($\chi_c=0.99$), while mix no. 2 contains enough for $\chi_c=0.55$. If we calculate the expected compressive strengths of the two mixes using Eq. (32), on the basis of the parameters acquired from the ÉMI experiments, which are in the first two rows of Table 6, then the results are as given in Table 8.

Table 8 clearly shows that it is possible to predict, using the x - χ_c - p - l -based estimation, the change in the compressive strength expected from an increase in addition content. It can

ser. no.	recipe		content indicators					$f_{c,28, estimated}$ [N/mm ²]	
	Material	kg/ m ³	χ_c	x	p	l	λ_{AD}	CEM I 42.5 N	CEM III/B 32.5 N
1.	cement	350	0.990	1.485	0.293	0.010	0.057	44.7	34.4
	additions	3							
	aggregates	1840							
	water	168							
	admixtures	7							
2.	cement	350	0.550	0.825	0.387	0.010	0.031	62.6	39.3
	additions	258							
	aggregates	1592							
	water	168							
	admixtures	7							
			compressive strength increase					40%	14%

Table 8. Two concrete mixes with a cement dosage of 350 kg/m³ and a water-cement ratio of $w/c=0.5$, but with different dosages of additions, and their estimated compressive strengths calculated from the concrete composition content indicators, equation (32) and the parameters contained in Table 6 (based on ÉMI experiments, 2010)

8. táblázat 350 kg/m³ cementadagolású, 0,5 víz-cement tényezőjű betonkeverék megvalósítása két különböző kiegészítőanyag-adagolás mellett, és azok becsült szilárdságai a betonösszetételei állapotjelzőkből, a (32) képletből és a 6. táblázat paramétereiből (ÉMI kísérletek, 2010. alapján)

be observed that in mix no. 2 the figure for x fluid-powder volumetric ratio was significantly diminished (from $x=1.485$ to $x=0.825$), which exerts a strengthening effect. Nevertheless, there are some types of cement and some ranges of concrete composition content indicators where strength does not show an increase, but rather a stagnation or even a decrease. This example was only shown to highlight the important role played by the *characteristic behaviour* of concrete constituents, which depends not only on the quality of the materials, but also on the concrete composition content indicators.

4. Investigation of deformations during the laboratory experiments

During the *laboratory experiments* at ÉMI Nonprofit Ltd., deformations were also tested by a Graf-Kaufman apparatus. The components of the concrete mixes were the same as those referred to in Table 5. Basalt powder was used as an addition. The concrete composition content indicators of the mixes evaluated are laid out in Table 9.

Three prismatic specimens were prepared from each concrete mix, with nominal dimensions of $40 \times 40 \times 160$ mm, and brass measuring spikes were inserted into the ends while the concrete was still fresh. The prisms were kept for 1 day in moulds covered with a damp cloth, and at the age of 1 day they were removed from the moulds. The initial masses of the prisms were measured, as well as the precise *lengths for comparison* (nominal 160 mm + protruding measuring spikes). The specimens were stored under water until day 7, after which they were stored under laboratory conditions. The temperature and humidity were constantly monitored and recorded. Changes in the mass and length of the prisms were measured at regular intervals, that is at days 1, 7, 14, 21, 28, 56, 112, ..., from which the *relative* values for drying and deformation were *calculated*. Changes in mass and dimension were expressed as a percentage, marked for +/- (*negative: mass reduction and shrinkage; positive: mass increase and expansion*).

Fig. 16. The deformation process of samples made with CEM I 32.5 N over a logarithmic timescale, with the dimensions of the prisms on day 112 regarded as 100%. The arrows indicate the possible values for deformation projected to day 1000, by continuing the trends. The question mark is there because all projections of this nature need to be validated through real measurements

16. ábra A CEM I 32,5 N cementből készült hasábkok alakváltozási folyamata logaritmikus időskálán szemlélítve, ha a hasábkok 112 napos méreteit tekintjük 100%-nak. A nyilakkal az eddigi trend folytatódása esetén az 1000 napos korra prognosztizálható alakváltozás lehetséges értékeit jelöljük. A „?” utalás arra, hogy minden prognózist csak valós mérési eredmények hitelesíthetnek

Results are demonstrated here up to the 112 days of age measurements. It can be interesting to know, however, *which stage of the deformation process has the concrete reached by day 112 in its unrestrained deformation process?* A clear answer to the question can only be given when further results are available, so the measurements are continuing. The picture emerging from the measurements can be seen in Fig. 16. The diagram shows the deformations measured on prisms made with CEM I 42.5 N at days 1, 14, 21, 28, 56 and 112, but in the diagram the *base for comparison (100%) is the length measured on day 112*, and not on day 1, and the relative lengths of the prisms have been calculated and shown relative to this base. By necessity, the curves all intersect at a single point (100%) on day 112, revealing quite spectacularly the diffuse dynamics of the deformation process that has gone before, and hinting at the possible values of deformation to be expected subsequently,

Types of cement used for deformation tests conducted by ÉMI	Content (structural) indicators						Other parameters and data						
	p	x	x _c	λ _{AD}	l	Traditional w/c	a*	f _a ** m ² /m ³	f _z *** m ² /m ³	f _z /f _a	c/R****	D _m mm	
CEM I 42.5 N 27 experiments	MIN.	0.098	0.690	0.379	0.000	0,000	0.251	0.495	1587	1.186×106	111	4.66%	4
	MAX.	0.472	2.633	0.988	0.049	0.202	1.624	0.769	12168	1.352×106	747	22.26%	8
	AVG.	0.293	1.430	0.762	0.016	0.043	0.665	0.665	7753	1.248×106	186	12.48%	8
	STD.	0.082	0.572	0.214	0.019	0.036	0.315	0.066	1855	5.522×104	121	4.76%	1
CEM III/B 32.5 N-S 25 experiments	MIN.	0.203	0.684	0.084	0,000	0.000	0.252	0.522	3145	1.352×106	119	2.54%	4
	MAX.	0.465	2.583	0.988	0.051	0.064	3.758	0.757	12168	1.442×106	452	21.69%	8
	AVG.	0.294	1.454	0.657	0.014	0.038	1.177	0.668	7990	1.388×106	183	9.97%	8
	STD.	0.072	0.555	0.32	0.019	0.016	0.981	0.063	1458	2.773×104	60	5.67%	1

Table 9. Concrete composition content indicators of mixes evaluated as part of the deformation tests conducted by ÉMI in 2010 (p: paste ratio in the concrete, x: volumetric ratio of liquid compared to the paste powder, x_c : volumetric ratio of cement in the paste powder, λ_{AD} : volumetric ratio of admixtures to paste powder, l: air volumetric ratio in the concrete), and other data (*a: volumetric ratio of aggregates in the concrete mix, ** f_a : volumetric specific surface area of the aggregates [$\phi > 0.063$ mm] calculated using Kausay's method [6], *** f_z : volumetric specific surface area of the paste powder [$\phi < 0.063$ mm], ****c/R: the mass ratio of cement in the concrete)

9. táblázat Az ÉMI 2010. évi alakváltozási vizsgálataiban értékelésbe vont keverékek betonösszetétele állapotjelzői (p: péparány a betonban, x: folyadék térfogataránya a pépporhoz képest, x_c : cement térfogataránya a pépporban, λ_{AD} : adalékszer térfogataránya a pépporhoz, l: levegő térfogataránya a betonban), és egyéb adatai (*a: a betonkeverékben lévő adalékanyag térfogataránya, ** f_a : az adalékanyag [$\phi > 0,063$ mm] térfogati fajlagos felülete Kausay eljárása szerint [2] számítva, *** f_z : a péppor [$\phi < 0,063$ mm] térfogati fajlagos felülete, ****c/R: a cement tömegaránya a betonban)

on this logarithmic timescale. Based on the illustration, *at least* two thirds of the process of shrinkage that is projected for the 1000-day period has already developed by the day 112, and there is a *suspicion* that *the deformation - all other conditions being equal - could be proportional to the time logarithm* (or to the logarithm raised to the power of x), which shall be confirmed, or disconfirmed, by statistical analysis of the results. The analyses of the results gained after 3 years period is just being under processing. The present paper merely deals with the 112 day results.

5. Factors that influence concrete deformation

Detailed literature review is outside the scope of the present paper. According to the traditional view, the kind of concrete deformation that we have tested can be classified as *drying shrinkage*, and Fig. 17 indicates one representative example from the technical literature [7].

Fig. 17. Depiction of shrinkage with influencing parameters [7]
17. ábra A zsugorodást befolyásoló egyes tényezők hatása [7]

We would like to quote here two statements, the first by János Ujhelyi [8]: "Concerning the composition of concrete, *shrinkage increases* when the dosage of cement and water increases, and *the quantity of fine particles in the aggregate increases* (italics added by the author of the present paper). But even more important than these effects are the external conditions [9]: the method and conditions of storage." The second quotation is by Attila Erdélyi, [10]: "The reduction in the w/c ratio is therefore the best method of moderating drying shrinkage ϵ_{cds} , - together with retention of the cement content. If $w/c = 0.3-0.35$ and $c = 450 \text{ kg/m}^3$, then the predicted total drying shrinkage will be $0.3-0.44 \text{ ‰}$ for time $t = \infty$ ".

Our own experiments have only dealt with a part of the complete range, and have inevitably investigated the effects of just a few concrete composition materials, but our aim was to use freshly measured factual data to seek confirmation of the statements given in the literature. We were also interested in whether it was possible to demonstrate the effects of concrete composition content indicators on deformation, and if so, how could these effects best be described. These expectations were based on a relatively large number of experimental settings, in addition to which great care was taken to ensure identical conditions in the method of storing the samples and in the external circumstances.

6. Evaluation of the deformation results

Two different approaches were applied in the evaluation of the test results, as was the case with compressive strength: calculations were made using the concrete composition content indicators (p , x , χ_c), and again using the traditional, effective w/c -ratio (the latter with the c/R cement-mass ratio and the χ_c variables).

Furthermore, we evaluated the effect of the *fine particle content* of the aggregates, therefore an additional variable was included in both cases: the f_z/f_a ratio, which is the ratio between the *volumetric specific surface areas of the paste powder* and of the aggregates in the mixture. The *volumetric specific surface areas* of the aggregates were calculated from the *size distribution of particles* by Kausay's method [6], with the difference being that the form factor was given as 5. For the paste powders, the results of the Blaine procedure were used to calculate the volumetric specific surface areas. The *ratios of the specific surface areas* have a practical significance, which we would like to emphasise. In the case of a paste powder of cement and additions with approximately identical levels of fineness, and aggregates where $D_{\max} \leq 8 \text{ mm}$, if $f_z/f_a < 140$, then the aggregate is of *high sand content*, if $140 \leq f_z/f_a < 190$, then it is of *medium sand content*, if $190 \leq f_z/f_a < 450$, then it is of *low sand content*, and if $450 \leq f_z/f_a$, then it is *sand deficient*. The ratio of the *volumetric specific surface areas* can play an important role as a factor not only in shrinkage, but also in the effect on *fresh concrete consistency*.

In view of the fact that measurements were made on the same prisms at different ages - it was given that the age of the concrete (t [days]) was also considered as a *variable* - even if only to check the suspicion expressed in connection with Fig. 16 above.

6.1 Relationship between deformation and concrete composition content indicators

Eq. (35) was found to be suitable for estimating deformation (drying shrinkage) of the prisms made from the experimental mixes (only the *multiplication factor exponential formula* is represented):

$$\epsilon_{cds}(t) = A \cdot \frac{(1+x)^{n_x}}{\chi_c^{n_\chi}} \cdot \left(\frac{p}{a}\right)^{n_p} \cdot \left(\frac{f_z}{f_a}\right)^{n_f} \cdot (\ln t)^{n_t} \quad (35)$$

where $\epsilon_{cds}(t)$ is the expected value, as a percentage, of the drying deformation of the prisms, at the age of $28 \leq t \leq 112$ days,

A is the experimental constant,

x is the liquid-powder volumetric ratio in the paste, and n_x is the coefficient of $(1+x)$,

χ_c is the cement volumetric ratio in the paste powder, and n_χ is the coefficient of χ_c ,

p/a is the volumetric ratio of paste and aggregates in the concrete, and n_p is the coefficient of p/a ,

f_z/f_a is the ratio of the paste powder and aggregate volumetric surface area, and n_f is the coefficient of f_z/f_a , and

t [days] is the age of the concrete in the range $28 \leq t \leq 112$ days, and n_t is the coefficient of $\ln t$.

Table 10 summarises the parameters of Eq. (35) for two types of cement, in the ranges given in Table 9.

6.2 Relationship between deformation and the w/c ratio and cement content

If x is replaced by w/c , and p/a is replaced as a variable by the cement ratio c/R , then we reach, from an evaluation of the results, the multiplication factor exponential equation (36):

$$\varepsilon_{cds}(t) = A \cdot \left(\frac{w}{c}\right)^{n_w} \cdot \left(\frac{c}{R}\right)^{n_c} \cdot \left(\frac{f_z}{f_a}\right)^{n_f} \cdot (\ln t)^{n_t} \quad (36)$$

where w/c is the traditional water-cement ratio, and n_w is the coefficient of w/c ,

c/R is the cement mass ratio in the concrete (R : concrete density), n_c is the coefficient of c/R ,

and the others parameters are the same as in Eq. (35).

Table 10 summarises the parameters of Eq. (36) for two types of cement, in the ranges given in Table 9.

6.3 Comparison of the methods for drying shrinkage estimation

The two methods of estimation described above are virtually equivalent in their statistical reliability, as can be seen from the data in Table 10.

If we plot the estimated and measured values for deformation at days 28, 56 and 112 for all the prisms, there is hardly any significant difference between the two methods (see Fig. 18). From a statistical point of view, there is no visible difference between the two methods.

Fig. 18. Deformation of the prisms with CEM III/B 32.5 N-S at days 28, 56 and 112, comparing the values estimated using Eqs. (35) and (36) and the values actually measured, during the ÉMI experiments of 2010.

18. ábra A CEM III/B 32,5 N-S cementből készült próbatestek 28, 56 és 112 napos alakváltozásainak (35) illetve (36) képletekből becsült értékei és a ténylegesen mért értékek között, az ÉMI 2010. évi kísérletei során

7. Discussion of results

If estimation nomograms (Fig. 19) are made for the ranges – as of Table 9 – of the mixes made with CEM III/B 32.5 N-S, where the bands of the same colour between the contours represent (approximately) equal degrees of deformation, it can be seen that the x - χ_c - p/a -based estimation, in the case of low χ_c values, is progressively less sensitive to an increase in the x liquid-powder volumetric ratio, but more sensitive to a decrease in χ_c , that is, an increase in the content of additions. The contours of the nomogram in this range are running close together, indicating the instability of the estimation. To estimate deformation, therefore, the estimate based on w/c - c/R - χ_c appears more stable (Fig. 20). This creates a strange "reversed analogy" with the estimate for compressive strength, where density among the contours of the nomograms was observed in the w/c - c/R -based estimates, and in that instance the p - x - χ_c -based estimate appeared to be the more stable.

The emphasis could be shifted to express the above as follows: increasing the content of additions leads to an increase in shrinkage. This is particularly clear from Fig. 19: for extremely low χ_c values (and therefore very high content of additions) the degree of deformation at day 112 is rather considerable, even as much as -0.10%. In the case of mixes made using CEM I 42.5 N, an increase in the content of additions also demonstrably increases the shrinkage.

The effect of sand content is also interestingly revealing. Figs. 21 and 22 show that if the sand content is significantly increased (compared with the mixes featured in Figs. 19 and 20), then if the ratio of $f_z/f_a = 183$ is decreased to $f_z/f_a = 119$, then the shrinkage of mixes with CEM III/B 32.5 N-S increase to a significant extent; for extremely low χ_c values (and therefore high content of additions) the degree of deformation can be as high as -0.15%. Meanwhile, shrinkage of mixes with CEM I 42.5 N – in the ranges investigated – is only minimally influenced by a change in sand content, which is also expressed by the value for the exponential n_f falling close to zero.

Comparing the deformation properties of the two types of cement investigated, it is worth looking at the mixes with practically no additions ($\chi_c = 0.99$) and medium sand content ($f_z/f_a = 180-190$) in the range for which measurements could be made for both types of cement (see Figs. 23 and 24). It is clear that, in this range, mixes with CEM III/B 32.5 N-S display less shrinkage than those with CEM I 42.5 N.

Parameters of the Eqs. (35) and (36)								Statistical data	
Estimation on the basis of x - χ - p /Eq. (35)/	Types of cement	A	n_t	n_x	n_x	n_p	n_i	R^2	Standard Error of the Estimate
	CEM I 42.5 N	-9.8393×10^{-5}	0.8570	0.3432	0.2234	0.4424	0.0700	0.8126	0.0053%
	CEM III/B 32.5 N-S	-7.1735×10^{-3}	0.7172	0.3841	0.2849	0.1056	-0.8021	0.8170	0.0108%
Estimation on the basis of w/c - c/R /Eq. (36)/	Types of cement	A	n_t	n_x	n_w	n_c	n_i	R^2	Standard Error of the Estimate
	CEM I 42.5 N	-3.6727×10^{-4}	0.8579	0.5359	0.3942	0.5425	0.0313	0.7890	0.0057%
	CEM III/B 32.5 N-S	-1.0940×10^{-2}	0.6673	0.2797	0.2881	0.2235	-0.6932	0.8194	0.0107%

Table 10. Parameters of estimation Eqs. (35) and (36) for the two types of cement investigated
10. táblázat A (35) és (36) becslőképletek paraméterei a vizsgált két cementfajtára

Fig. 19. Nomogram of deformation estimates based on $x-\chi_c-p/a$ for mixes made with CEM III/B 32,5 N-S at the age of $t=112$ days, when $p=0.294$ and $f_z/f_a=183$ (medium sand content aggregate)

19. ábra CEM III/B 32,5 N-S cementtel készült keverékek $x-\chi_c-p/a$ -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $p=0.294$ és $f_z/f_a=183$ (közepes homoktartalmú adalékanyag) esetén

Fig. 20. Nomogram of deformation estimates based on $w/c-c/R-\chi_c$ for mixes made with CEM III/B 32,5 N-S at the age of $t=112$ days, when $\chi_c=0.99$ and $f_z/f_a=183$ (neat cement paste powder, medium sand content aggregate)

20. ábra CEM III/B 32,5 N-S cementtel készült keverékek $w/c-c/R-\chi_c$ -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0.99$ és $f_z/f_a=183$ (tisztá cementes péppor, közepes homoktartalmú adalékanyag) esetén

Fig. 21. Nomogram of deformation estimates based on $x-\chi_c-p/a$ for mixes with CEM III/B 32,5 N-S at the age of $t=112$ days, when $p=0.294$ and $f_z/f_a=119$ (medium sand content aggregate)

21. ábra CEM III/B 32,5 N-S cementtel készült keverékek $x-\chi_c-p/a$ -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $p=0.294$ és $f_z/f_a=119$ (homokdús adalékanyag) esetén

Fig. 22. Nomogram of deformation estimates based on $w/c-c/R-\chi_c$ for mixes with CEM III/B 32,5 N-S at the age of $t=112$ days, when $\chi_c=0.99$ and $f_z/f_a=119$ (pure cement paste powder, high sand content aggregate)

22. ábra CEM III/B 32,5 N-S cementtel készült keverékek $w/c-c/R-\chi_c$ -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0.99$ és $f_z/f_a=119$ (homokdús adalékanyag) esetén

If, however, 15% v/v (inert stone powder) addition is added to the paste powder, and the aggregate sand content is also increased ($f_z/f_a=110-120$), then the "tables are turned": while the difference in shrinkage of the two types of cement is slight if the cement content is high, but if the dosage of cement is lower then the mixes with CEM I 42.5 N display demonstrably less shrinkage (see Figs. 25 and 26).

If we compare the statements from the literature with our results, there is essentially very close agreement, in addition to which some *novel* facts have come to light regarding the influence of additions on deformation, and the quantification of the effects of the aggregate sand content.

The effect of concrete composition content indicators on the deformation of hardened concretes can be clearly verified, although it will be necessary to make further measurements in the future to discover, with a degree of certainty that meets the

requirements of the modern age, the way in which concrete components of varying properties (cements, additions, aggregates, shrinkage-reducing agents, etc.) and concrete mixes made from them influence deformation.

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Fig. 23. Nomogram of deformation estimates based on w/c - c/R - χ_c for mixes with CEM III/B 32.5 N-S at the age of $t=112$ days, when $\chi_c=0.99$ and $f_2/f_1=183$ (pure cement paste powder, medium sand content aggregate)

23. ábra CEM III/B 32,5 N-S cementtel készült keverékek w/c - c/R - χ_c -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0,99$ és $f_2/f_1=183$ (tisztá cementes péppor, közepes homoktartalmú adalékanyag) esetén

Fig. 24. Nomogram of deformation estimates based on w/c - c/R - χ_c for mixes with CEM I 42.5 N at the age of $t=112$ days, when $\chi_c=0.99$ and $f_2/f_1=186$ (pure cement paste powder, medium sand content aggregate)

24. ábra CEM I 42,5 N cementtel készült keverékek w/c - c/R - χ_c -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0,99$ és $f_2/f_1=186$ (tisztá cementes péppor, közepes homoktartalmú adalékanyag) esetén

Fig. 25. Nomogram of deformation estimates based on w/c - c/R - χ_c for mixes with CEM III/B 32.5 N-S at the age of $t=112$ days, when $\chi_c=0.85$ and $f_2/f_1=119$ (15% v/v additions, high sand content aggregate)

25. ábra CEM III/B 32,5 N-S cementtel készült keverékek w/c - c/R - χ_c -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0,85$ és $f_2/f_1=119$ (15 tf% kiegészítő, homokdús adalékanyag) esetén

Fig. 26. Nomogram of deformation estimates based on w/c - c/R - χ_c for mixes with CEM I 42.5 N at the age of $t=112$ days, when $\chi_c=0.85$ and $f_2/f_1=111$ (15% v/v additions, high sand content aggregate)

26. ábra CEM I 42,5 N cementtel készült keverékek w/c - c/R - χ_c -alapú alakváltozás-becslő nomogramja $t=112$ napos korban, $\chi_c=0,85$ és $f_2/f_1=111$ (15 tf% kiegészítő, homokdús adalékanyag) esetén

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Betonkeverékek egyszerűsített alapmodellje és alkalmazása.

2. rész: Nyomószilárdságot és alakváltozást befolyásoló tényezők

A dimenzió nélküli betonösszetéti állapotjelzők bevezetésével a betonkeverékek strukturális összetétele szemléletesen leírható, lehetőséget kínálva a friss és megszilárdult betonkeverékek teljesítményjellemzőit befolyásoló hatások eddigiektől eltérő megközelítésű elemzésére. Cikkünkben megvizsgáltuk a nyomószilárdságot befolyásoló tényezőket és az elemzés révén közelebb jutottunk a szakirodalomban említett két, eddig nem értelmezett probléma magyarázatához. A laboratóriumi kísérletek során készült kisméretű próbahasábok alakváltozásainak vizsgálati eredményeit is ismertettük, és az elemzés során összefüggéseket keresünk a betonösszetéti állapotjelzők és a megszilárdult betonok alakváltozása között. Érdekes párhuzam mutatkozik a szilárdságot és az alakváltozást befolyásoló hatásokban, és lehetőség nyílik arra, hogy az alakváltozás is előre becsülhető a betonösszetéti állapotjelzőkből.

Kulcsszavak: betontechnológia, betonösszetétel tervezése, betonösszetéti állapotjelzők