The Application of Improved Genetic Algorithm in the Markov State Transition Matrix to SME Technical Efficiency and Scale Revenue Calculation Methods

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Abstract

To solve a state transfer matrix using a Markov model, an improved genetic algorithm approach is proposed. A Markov prediction model is employed to study structures in the SME technical efficiency and scale gains calculation method. The obtained results are compared to the data shown in the China Statistical Yearbooks. It is found that our proposed genetic algorithm approach provides references for optimizing an agriculture SME industrial structure and hence improves the prediction precision.

Keywords: SME technical efficiency returns to scale Improved genetic algorithm

1. Introduction

SMEs are not only economic and social development of an important force, but also China's market economy the most dynamic part. The advantage of SMEs small and fine, small and specialized, small and live, in carrying out technological innovation activities has incomparable advantages of large enterprises, more than 75% of independent innovation products, more than 80% of patents are completed SMEs. China's foreign trade import and export volume of about 60% is provided by the SME, SME export trade to the rapid development of the national economy plays a huge role in promoting. Therefore, strengthening support for SMEs and nurturing for the sustainable development of the national economy has important significance.

As modern prediction approaches are with high scientificity, accuracy and applicability, Markov prediction methods are widely employed in economic and natural science fields. The success of applying these approaches is based on the full information on the mutual transfer probabilities of the possible states of predicted objects. To accurately predict the transfer probability matrix of the Markov states, it is required to enhance the precision of the Markov prediction. Existing methods for solving the transfer probability matrix of the states mainly include statistical estimate approaches, linear system equations approaches, multivariate regression approaches and quadratic programming approaches. Each type of methods has its own advantages and disadvantages. For examples, 1) statistical discrepancies exist and data obtaining procedures are complicated; 2) not all desired transfer probabilities of the states are guaranteed to be satisfied the condition $0 \leq P_{ij} \leq 1$; 3) all obtained regressive equations have to take a significant test. When the significant test is failed, other methods for solving the probability are required and solving the matrix becomes more complicated; and 4) a large amount of calculations is required and the solving process is complicated.

Genetic algorithms (GA) have been drawn a great attention in the artificial intelligence community since the middle of 1980s. At the early 1990s, it is widely studied in the

computing science, information science and optimization communities. Also, it has become one of the crucial technologies for the intelligence computation since the 21st century. Its main advantages include that it is with whole searching strategy, the searches do not require the gradient information and the algorithms do not require continuously differentiable objective functions. The algorithms only require computing the objective functions. Hence, these methods are particularly applicable for solving optimization problems in complicated systems with highly nonlinear, discrete objective functions. Compare to existing methods, this paper proposes an improved genetic algorithm and employed for solving the transfer probability matrix of the states in the Markov model. The obtained results show that the proposed approach is a practical method for achieving high prediction accuracies.

2. Markov Prediction Method

Markov prediction method is to determine the change of the future trend of various states using the Markov chain. It is a stochastic prediction method via exploiting the transfer probabilities of the states of system objects. Therefore, it is a simple method for predicting the event probability. The basic idea is to obtain the transfer probability matrix of the states using raw data sequences, and then estimate the change of the future trend based on the transfer probability matrix of the states. In a dynamical system with random variables, the change of each state is called the state transition. Due to the variable nature of the transfer probabilities of different states of the system, the Markov method is more suitable for short term predictions.

2.1. Markov Process

The main title (on the first page) should begin 1 3/16 inches (7 picas) from the top edge of the page, centered, and in Times New Roman 14-point, boldface type. Capitalize the first letter of nouns, pronouns, verbs, adjectives, and adverbs; do not capitalize articles, coordinate conjunctions, or prepositions (unless the title begins with such a word). Please initially capitalize only the first word in other titles, including section titles and first, second, and third-order headings (for example, “Titles and headings” — as in these guidelines). Leave two blank lines after the title.

2.2. Transfer Probability Matrix of the States

Let the n states of a system be $S_1, S_2, \dots, S_i, \dots, S_n$. For each time, the system could only stay in one of the states S_i ($i=1, 2, \dots, n$). Suppose that the system is in the present state S_i . Denote the probability of transferring to the next state S_j as P_{ij} . After the time Δt , let the number of the transferring possibilities of S_i be n . The table below shows the transferring probabilities of the states.

Table1. Probabilities of System State Transition

State	S_1	S_2	...	S_n
S_1	P_{11}	P_{12}	...	P_{1n}
S_2	P_{21}	P_{22}	...	P_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_n	P_{n1}	P_{n2}	...	P_{nn}

These system states are mutually transferable in certain conditions and the matrix formed by the transferring these probabilities among the states is called the transfer probability matrix of the states. These transitions form a $n \times n$ transfer probability matrix

and it is denoted as P. Hence, the one step transfer probability matrix P can be expressed as:

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

If the system states undergo a number of transitions rather than a single step transition, then it can be described by the k step transition matrix which is written as P(k). Thus, we have:

$$P(k) = P(k-1)P = P^k$$

The k step transition matrix is used for characterizing the transitions where more than one transitions are involved on the basis of the previous transition matrix.

2.3. Markov Prediction Model

Assume that the number of possible states is n in a system. Also, the states are mutually exclusive. The initial state vector of the system S1, S2... Sn, is denoted as:

$$S(0) = (S_1(0), S_2(0), \dots, S_n(0))$$

Here, Si(0) stands for the initial probability of the system in the state i. Since the probability of the system in the state i after the k step transitions is Si(K), the state vector after the K step transitions is:

$$S(k) = (S_1(k), S_2(k), \dots, S_n(k))$$

The relationship between Si(0) and Si(k) is:

$$S(k) = S(k-1)P = S(0)P^k = S(0) \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}^k \quad (1)$$

When the initial state S(0) and the one step transition probability matrix P are known, (1) can be applied to predict the system state at the time instant k. Hence, (1) is called the Markov prediction model. The key point of employing the Markov prediction model is based on the full information of P and the system initial state vector S(0).

2.4. Method for Determining the Transfer Probability of the Markov States

According to the principle of the least square method, let the transfer probability matrix of the estimated states be

$$P' = \begin{bmatrix} P'_{11} & P'_{12} & P'_{13} & P'_{14} \\ P'_{21} & P'_{22} & P'_{23} & P'_{24} \\ P'_{31} & P'_{32} & P'_{33} & P'_{34} \\ P'_{41} & P'_{42} & P'_{43} & P'_{44} \end{bmatrix}$$

Let the fitting error at the time instant t of the system state Sj(j=1,2,...,n) be:

$$e_j(t) = S_j(t) - \sum_{i=1}^n P'_{ij} S_i(t-1) \quad (j=1,2,\dots,n) \quad (2)$$

Within the whole observation period T, let the total square fitting error of the system state Sj be Qj:

$$Q_j = \sum_{t=1}^T [e_j(t)]^2 \quad (j=1,2,\dots,n) \quad (3)$$

Let the total square fitting error of the system state in the whole observation period be Q. Then, we have:

$$Q = \sum_{j=1}^n \sum_{t=1}^T [e_j(t)]^2 \quad (4)$$

The objective is to minimize the total square fitting error of the system state in the whole observation period. Let the obtained value of the transfer probability matrix of the state be P'_{ij} ($i=1,2, \dots, n, j=1,2, \dots, n$). Then, we have the following nonlinear programming model:

$$\begin{aligned} \min Q &= \sum_{j=1}^n \sum_{t=1}^T [S_j(t) - \sum_{i=1}^n P'_{ij} S_i(t-1)]^2 \\ \text{s.t.} \quad &\begin{cases} \sum_{j=1}^n P'_{ij} = 1, & i = 1, 2, \dots, n \\ 0 \leq P'_{ij} \leq 1, & i, j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (5)$$

By solving the above nonlinear programming problem, the transfer probability matrix of the states P' is obtained.

3. Our Proposed Improved Genetic Algorithm

The improved genetic algorithm is based on a binary encoding approach. From the evolution strategy viewpoint, our proposed improved genetic algorithm combines the parent individuals and the child generation individuals produced by crossover together to form a new temporary population and calculate the fitness value of each individual in the new temporary population. The m individuals with the highest fitness values are reserved and the m individuals with the lowest fitness values are mutated. Then, the later m mutants are combined with the previously reserved m individuals to form a new temporary population. By sorting the new population, the m individuals with the highest fitness values are selected and the evolution of the population is completed.

Figure 1. Elite Group Keeping Genetic Algorithm Evolution Strategy Program Block Diagram

Base on elitism reservation methods found in previous literatures, this method results to good qualities of population. Meanwhile, the excellent individuals are reserved among the individuals in the parent population and the individuals produced by crossover and mutation. Our proposed method can successfully prevent the occurrence of poor individuals generated by the crossover and mutation operations during the evolution process. In the mutation operation, the m individuals with the lowest fitness values among the parent and the child generations produced by the crossover operation are mutated. In such method, the occurrence of the poor individuals is avoided while the mutation probability is increased. The proposed method enables to produce more new individuals in which it increases the population diversity, enhances the population periodicity and enable the escape from the local optimal solutions.

4. Computer Numerical Simulation Results

The section takes the structure of the SMEs agriculture industry in the Heilongjiang province as an example to illustrate the proposed method. The structure of the agriculture industry involves the constitution of all sections inside the agriculture industry as well as their interrelationships. It actually includes the structures of the industry and internal relations among the agriculture, forestry, animal husbandry and fishery industries. Heilongjiang province is one of the most important agricultural provinces in China. Its advantageous agricultural production conditions together with the scientific forecast of the proportional values of the provincial agriculture industry will benefit the province. This will also facilitate the rapid and sustainable developments of both the agriculture industry and rural economy. Furthermore, it provides a theoretical support for making

rational decisions on adjusting the structure of the agriculture industry in the province. By adopting the Markov theory to the forecast of the structural problems arising in the agriculture industry, the agriculture, forestry, animal husbandry and fishery industries are connected each others. Also, by using the proposed improved genetic algorithm to solve the transfer matrix of the states in the Markov model, the calculation is much simpler and the obtained result is more accurate.

The data of the agriculture, forestry, animal husbandry and fishery industries of SMEs in the Heilongjiang province between 2005 and 2013 are presented in the China Statistical Yearbooks are summarized in Table 2.

Table 2. Agricultural Production Values of Various Industries in Heilongjiang Province Unit (Billion Yuan)

Year	agriculture	forestry	animal husbandry	fishery
2005	502.9	59.1	294.2	23.1
2006	620.2	65.8	400.7	25.0
2007	718.6	67.3	461.2	27.4
2008	787.4	68.0	480.7	30.1
2009	971.9	79.0	585.0	25.1
2010	1142.3	89.6	813.1	36.0
2011	1206.8	85.2	870.2	45.2
2012	1369.2	95.5	965.8	53.7
2013	1801.8	110.2	1189.9	58.9

The proportional values of the agriculture, forestry, animal husbandry and fishery industries in the Heilongjiang province are calculated based on the data listed in Table 2 are presented in Table 3.

Table 3. Agricultural Production Value Proportion of Various Industries in Heilongjiang Province

Year	agriculture	forestry	animal husbandry	Fishery
2005	0.5720	0.0672	0.3345	0.0263
2006	0.5579	0.0592	0.3604	0.0225
2007	0.5638	0.0528	0.3618	0.0215
2008	0.5764	0.0498	0.3518	0.0220
2009	0.5851	0.0476	0.3522	0.0151
2010	0.5489	0.0431	0.3907	0.0173
2011	0.5467	0.0386	0.3942	0.0205
2012	0.5511	0.0385	0.3888	0.0216
2013	0.5700	0.0349	0.3765	0.0186

Based on the data listed in Table 3, take the annual proportional values of the agriculture, forestry, animal husbandry and fishery industries in the Heilongjiang province as the state and the total square fitting error of the system state in the whole observation period (There are 8 years between 2006 and 2013.) as the optimization objective. The objective function is as follows:

$$\min Q = \sum_{j=1}^4 \sum_{t=1}^8 [S_j(t) - \sum_{i=1}^4 p_{ij} S_i(t-1)]^2 \quad (6)$$

$$s.t. \begin{cases} \sum_{j=1}^4 p'_{ij} = 1, & i = 1, 2, 3, 4 \\ 0 \leq p'_{ij} \leq 1, & i, j = 1, 2, 3, 4 \end{cases}$$

Based on the proposed improved genetic algorithm method, the nonlinear programming problem in (6) is solved. That is, Q is minimized and the transfer probability matrix of the state P' is obtained. The parameters of the improved genetic algorithm are selected based on the following. The intervals of the variables are selected as [0, 1]. The precisions of the variables are selected as 4. The population size is selected as 40. The crossover probability is selected as 0.75 and the mutation probability is selected as 0.05. When the difference of the objective functional values between the best and the worst individuals in the population is less than 10⁻⁴, the algorithm is terminated. The obtained Q value is Q=0.002230 and the transfer probability matrix of the state P' is:

$$P' = \begin{bmatrix} 0.7261 & 0.0166 & 0.2573 & 0 \\ 0.1467 & 0.7304 & 0 & 0.1229 \\ 0.3427 & 0 & 0.6196 & 0.0377 \\ 0.9956 & 0.0044 & 0 & 0 \end{bmatrix}$$

According to the obtained transfer probability matrix of the state, (1) is employed to predict the proportional values of the agriculture, forestry, animal husbandry and fishery industries in the Heilongjiang province between 2014 and 2015. The forecast is shown in Table 4.

Table 4. Forecast Result and Forecast Precision

<i>industries</i>	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Agriculture	actual value	0.5579	0.5638	0.5764	0.5851	0.5489	0.5467	0.5511		
	forecast value	0.5660	0.5597	0.5626	0.5683	0.5576	0.5560	0.5681	0.5606	0.5630
	absolute error	0.0081	0.0041	0.0138	0.0168	0.0187	0.0093	0.0070	0.0094	
	relative error	1.45%	0.73%	2.39%	2.87%	3.41%	1.70%	1.27%	1.65%	
	actual value	0.0592	0.0528	0.0498	0.0476	0.0431	0.0386	0.0385		
	forecast value	0.0587	0.0526	0.0480	0.0460	0.0445	0.0407	0.0374	0.0373	0.0367
Forestry	absolute error	0.0005	0.0002	0.0018	0.0016	0.0014	0.0021	0.0011	0.0024	
	relative error	0.84%	0.38%	3.61%	3.36%	3.25%	5.44%	2.86%	6.88%	
	actual value	0.3604	0.3618	0.3518	0.3522	0.3907	0.3942	0.3888		
	forecast value	0.3545	0.3669	0.3693	0.3663	0.3688	0.3833	0.3849	0.3827	0.3813
	absolute error	0.0059	0.0051	0.0175	0.0141	0.0219	0.0109	0.0039	0.0062	
	relative error	1.64%	1.41%	4.97%	4.00%	5.61%	2.77%	1.00%	1.65%	
animal husbandry	actual value	0.0225	0.0215	0.0220	0.0151	0.0173	0.0205	0.0216		
	forecast value	0.0225	0.0215	0.0220	0.0151	0.0173	0.0205	0.0216		
	absolute error	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
	relative error	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
	actual value	0.0225	0.0215	0.0220	0.0151	0.0173	0.0205	0.0216		
	forecast value	0.0225	0.0215	0.0220	0.0151	0.0173	0.0205	0.0216		

forecast	0.02	0.02	0.02	0.01	0.01	0.02	0.01	0.01	0.01	0.01
value	09	09	01	94	91	00	96	94	94	90
absolute	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
error	16	06	19	43	18	05	20	08		
relative	7.11	2.79	8.64	28.4	10.4	2.44	9.26	4.30		
error	%	%	%	8%	0%	%	%	%		

According to the data listed in Table 4, by using our proposed improved genetic algorithm based on the Markov prediction model, the transfer probability matrix of the state of the structural value of the agriculture industry in the Heilongjiang province exhibits a higher prediction precision. The average relative error is 4.33%.

5. Conclusion

The transfer probability matrix of the state based on a Markov prediction model is studied in this paper. By considering the relationships among the agriculture, forestry, animal husbandry and fishery industries of SMEs, a least square method is used to construct the Markov prediction model. The Markov prediction method is adopted to forecast the structural values of the agriculture, forestry, animal husbandry and fishery industries of SMEs in the Heilongjiang province between 2014 and 2015. The prediction results on the optimization of the structure of the agriculture industry in the Heilongjiang province are very useful. Meanwhile, they can be employed for the formulation of rational and effective policies for developing a structure of the agriculture industry. The result shows that the proposed improved genetic algorithm generates a good quality of population and it can be used for solving the transfer probability matrix of the state via a Markov model. It can overcome the shortcomings of existing methods such as statistical discrepancies, the requirement of large amount of calculations and complicated solving process. The proposed improved genetic algorithm for solving the transfer matrix of the state via the Markov model has high prediction precision. It is a practical method for short term forecasting applications.

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