Generalized Fuzzy Time Series Forecasting Model Enhanced with Particle Swarm Optimization

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Abstract

In the past two decades, many forecasting models based on the concepts of fuzzy time series have been proposed for dealing with various problem domains. In this paper, we present a novel model to forecast enrollments and the close prices of stock based on particle swarm optimization and generalized fuzzy logical relationships. After that some concepts of the generalized fuzzy logical relationships and an operation for combining the generalized relationships are introduced in the first part of this paper; we use particle swarm optimization to optimal represents for the given intervals in the universe of discourse to increase the forecasting accuracy. To test the effectiveness of the model, the proposed method is demonstrated on the procedure of forecasting enrollments as well as its experiment on forecasting the close price of Shanghai Stock Exchange Composite Index. Empirical analyses show that the proposed method gets a higher average forecasting accuracy rate than the existing methods.

Keywords: Fuzzy sets; Fuzzy forecasting; Generalized fuzzy logical relationship; particle swarm optimization

1. Introduction

Since fuzzy time series model was proposed by Song and Chissom[1-3] in 1993, there are many forecasting models have been developed to deal with the forecasting problems due to their capability of dealing with the uncertainty and vagueness inherent in the data collected. In the literatures, the experimental analysis existing in many areas, such as forecasting stock price [4-5], tourism demand[6], temperature[7], amount of export [8] and dry bulk shipping index [9], etc.

Most of the exiting methods mainly focused on partitioning the universe of discourse, constructing fuzzy relationships from the fuzzy set, forecasting and defuzzifying the forecasting output. A proper choice of the length of each interval can greatly improve the forecasting results. Huarng [10,11] presented some methods to obtain the proper lengths of the intervals for the fuzzy time series model. Chen proposed the automatic clustering technique to generate clustering-based intervals for some fuzzy time series forecasting models[12,13]. Literatures[14,15] introduced an approach which using an optimization technique with a single-variable constraint to determine an optimal interval length. The particle swarm optimization algorithm and some other algorithms were used to search the appropriate intervals on the fuzzy time series[16-19] as well as genetic algorithms[20]. To improve the fuzzy time series model in constructing fuzzy relationships, forecasting and

defuzzifying the forecasting output, Chen[21] modified the models by using simple algebraic operation in[1-3]. Chen introduced the Fibonacci sequence into the fuzzy time series, and testified the validity of the model[22]. Lee presented some high-order models based on two-factors and genetic simulated annealing techniques[23].

In fuzzy time series forecasting models, the midpoints, the represents of the given intervals, are critical for forecasting and defuzzifying the forecasting output. Although there are many models were proposed for deal with the lengths of intervals by combing with some modern algorithms [11-15] as well as particle swarm optimization[20,16,17]. The intervals are determined by the algorithms and the training data set, the midpoints of the improved intervals are the mean of endpoints of the related intervals. Unfortunately, the issue of how to determine proper represents for the given intervals has not been touched in previous studies. This paper proposes a new fuzzy time series model based on generalized fuzzy logical relationships and particle swarm optimization. The generalized fuzzy time series model was proposed to cope with the instability of the selection for fuzzy sets and improve the forecasted result[4]. At the same time, the particle swarm optimization is used to determine the proper midpoints of the given intervals. The proposed method is demonstrated on the enrollments of the University of Alabama. For the in-depth studies, we also applied the method to the close price of Shanghai Stock Exchange Composite Index (SSECI). The results are compared to the results of existing methods [21,26].

2. Preliminaries

The frame work of Generalized fuzzy time series forecasting model can be found in reference [1-4]. Particle swarm optimization (PSO) is a population based stochastic optimization technique presented by Eberhart and Kennedy in 1995^[24,25], inspired by social behavior of bird flocking or fish schooling. That finding the global best solution by adjusting the trajectory of each individual towards its own best location and towards the best particle of the swarm is the main thought and shining point of PSO which shares many similarities with evolutionary computation techniques such as Genetic Algorithms.

The particle swarm optimization concept consists of changing the velocity each particle toward its fitness value and global version of PSO. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward proper locations. The original process for implementing the local version of PSO is as follows:

- (1) Initialize a population of particles with random positions and velocities on d dimensions in the problem space.
- (2) For each particle, evaluate the desired optimization fitness function in d variables.
- (3) Compare particle's fitness evaluation with particle's $pbest$. If current value is better than $best$, then set $pbest$ value equal to the current value, and the $pbest$ location equal to the current location in d -dimensional space.
- (4) Compare fitness evaluation with the population's overall previous best. If current value is better than $lbest$, then reset $lbest$ to the current particle's array index and value.
- (5) Change the velocity and position of the particle according to equations (1) and (2), respectively:

$$V_{id} = w * V_{id} + C_1 \text{Rand}() * (P_{id} - X_{id}) + C_2 \text{Rand}() * (P_{ld} - X_{id}) \quad (1)$$

$$X_{id} = X_{id} + V_{id} \quad (2)$$

where w denotes the inertia weight factor; X_{id} is the current position of the particle, P_{id} is the position of the particle that experiences the best fitness value; P_{ld} is the location of the

neighborhood best, the neighborhood size is defined as two in this paper; C_1 and C_2 are acceleration values which represent the self-confidence coefficient and the social coefficient, respectively; d denotes the dimension of the problem space; $\text{Rand}()$ denotes a random function which can yield random value in the range of (0, 1) and V_{id} denotes the velocity of the particle.

(6) Loop to step (2) until a criterion was met.

3. Proposed Model

In this subsection, an improved model of fuzzy time series based on particle swarm optimization and generalized fuzzy logical relationship is proposed. Based on the classical procedure of FTS illuminated by Chen [21] and Lee [26], the presented model is given step-by-step as follows.

Step 1: Define the universe of discourse and intervals for rules extraction. Let $U = [\text{starting}, \text{ending}]$, according to the lengths of intervals, U is partitioned into several intervals. For example, $U = \{u_1, u_2, \dots, u_n\}$. In conventional models, m_i is the midpoint of u_i whose corresponding fuzzy set is A_i . To enhance the representativeness, m_i is replaced by the centroid point obtained with particle swarm optimization in the proposed method.

Table 1. Fuzzified Enrollments and Fuzzy Logical Relationship

year	actual value	fuzzified	fuzzy logical relationship
1971	13055	$A_1; A_2$	$A_1 \rightarrow A_2; A_2 \rightarrow A_2$
1972	13563	$A_2; A_1$	$A_2 \rightarrow A_3; A_3 \rightarrow A_3$
1973	13867	$A_3; A_2$	$A_3 \rightarrow A_5; A_2 \rightarrow A_5$
1974	14696	$A_5; A_6$	$A_5 \rightarrow A_9; A_6 \rightarrow A_9$
1975	15460	$A_9; A_8$	$A_9 \rightarrow A_8; A_8 \rightarrow A_8$
1976	15311	$A_8; A_9$	$A_8 \rightarrow A_{10}; A_9 \rightarrow A_{10}$
1977	15603	$A_{10}; A_9$	$A_{10} \rightarrow A_{11}; A_9 \rightarrow A_{11}$
1978	15861	$A_{11}; A_{10}$	$A_{11} \rightarrow A_{14}; A_{10} \rightarrow A_{14}$
1979	16807	$A_{14}; A_{15}$	$A_{14} \rightarrow A_{15}; A_{15} \rightarrow A_{15}$
1980	16919	$A_{15}; A_{14}$	$A_{15} \rightarrow A_{13}; A_{14} \rightarrow A_{13}$
1981	16388	$A_{13}; A_{12}$	$A_{13} \rightarrow A_9; A_{12} \rightarrow A_9$
1982	15433	$A_9; A_8$	$A_9 \rightarrow A_9; A_8 \rightarrow A_9$
1983	15497	$A_9; A_{10}$	$A_9 \rightarrow A_7; A_{10} \rightarrow A_7$
1984	15145	$A_7; A_6$	$A_7 \rightarrow A_7; A_6 \rightarrow A_7$
1985	15163	$A_7; A_8$	$A_7 \rightarrow A_{11}; A_8 \rightarrow A_{11}$
1986	15984	$A_{11}; A_{12}$	$A_{11} \rightarrow A_{15}; A_{12} \rightarrow A_{15}$
1987	16859	$A_{15}; A_{14}$	$A_{15} \rightarrow A_{17}; A_{14} \rightarrow A_{17}$
1988	18150	$A_{17}; A_{18}$	$A_{17} \rightarrow A_{19}; A_{18} \rightarrow A_{19}$
1989	18970	$A_{19}; A_{20}$	$A_{19} \rightarrow A_{21}; A_{20} \rightarrow A_{21}$
1990	19328	$A_{21}; A_{20}$	$A_{21} \rightarrow A_{15}; A_{20} \rightarrow A_{21}$
1991	19337	$A_{21}; A_{20}$	$A_{21} \rightarrow A_{19}; A_{20} \rightarrow A_{19}$
1992	18876	$A_{19}; A_{18}$	$A_{19} \rightarrow *; A_{18} \rightarrow *$

Step 2: Define fuzzy sets based on the universe of discourse and intervals and fuzzify the historical data. The fuzzy sets with respect to intervals given at Step 1 are defined with triangular fuzzy function and the membership degree of the value x_t at time t in A_i be calculated by formula (3).

$$\mu_{A_i}(t) = \begin{cases} 1 & \text{if } x_t > m_n \text{ or } x_t < m_1 \\ 0 & \text{if } x_t < m_{i-1} \text{ or } x_t > m_{i+1} \\ \frac{x_t - m_{i-1}}{m_i - m_{i-1}} & \text{if } m_{i-1} \leq x_t \leq m_i \\ \frac{m_{i+1} - x_t}{m_{i+1} - m_i} & \text{if } m_i \leq x_t \leq m_{i+1} \end{cases} \quad (3)$$

where x_t is the observed value at time t , m_i is the centroid point of interval u_i which corresponding to main interval of fuzzy set A_i .

Step 3: Establish the fuzzy logical relationships based on the hierarchies of principal fuzzy logical relationship. Given the sample data set and the definition of fuzzy sets, all fuzzy logical relationships between two consecutive data can be established. To forecast the time series, the fuzzy logical relationship matrix must be created at this step based on the fuzzy logical relationships.

Step 4: Forecasting with the fuzzy relation matrices. Let the membership values of $F(t)$ with respect to the given fuzzy sets are $(\mu_1(t), \mu_2(t), \dots, \mu_n(t))$ and $\mu_{t_l}(t)$ is the l th maximum membership degree, respectively. We have the intersection of N fuzzy logical relationship groups, and then the forecasting value $F(t+1)$ is conducted by formula (4).

$$F(t+1) = \bigwedge_N (R^{(1)}(i_1, :), \dots, R^{(l)}(i_l, :), \dots, R^{(N)}(i_N, :)) \circ (m_1, m_2, \dots, m_n)^T \quad (4)$$

where $\circ$ is a composition operation for forecasting with two principles: (1) If the sum of $\bigwedge_N (R^{(1)}(i_1, :), \dots, R^{(l)}(i_l, :), \dots, R^{(N)}(i_N, :))$ equals to 0, then the forecast is m_{t_1} which is the midpoint of the interval corresponding to $A_{t_1}^{(1)}$; (2) Otherwise, the forecasted value is the weighted sum of $m_1, m_2, \dots, m_n$.

Since most of conventional models have been presented for forecasting the historical enrollments of the University of Alabama, in this section, we present stepwise procedures of the proposed method for forecasting the time series data with $N = 2$ and $p = 1$. The well-known data, historical enrollments of the University of Alabama from 1971 to 1992, are shown in Table 1.

Step 1: Partition the universe of discourse U with the automatic clustering technique. The intervals obtained by the algorithm can be listed as $u_1 = [13055, 13354]$, $u_2 = [13354, 13862]$, ..., $u_{21} = [19328, 19337]$. Their midpoints are $m_1 = 13205$, $m_2 = 13608$, ..., $m_{21} = 19332$ respectively.

Table 2. Fuzzy Logical Relationships Groups when $N = 2$

Group number $l = 1l = 2$
Group 1 $A_1 \rightarrow A_2(1) \ A_1 \rightarrow A_3(1)$
Group 2 $A_2 \rightarrow A_3(1) \ A_2 \rightarrow A_2(1), A_5(1)$
Group 3 $A_3 \rightarrow A_5(1) \ A_6 \rightarrow A_7(1), A_9(1)$
Group 4 $A_5 \rightarrow A_9(1) \ A_8 \rightarrow A_8(1), A_9(1), A_{11}(1)$
Group 5 $A_7 \rightarrow A_7(1), A_{11}(1) \ A_8 \rightarrow A_{10}(1), A_{11}(1)$
Group 6 $A_8 \rightarrow A_{10}(1) \ A_{10} \rightarrow A_7(1), A_{14}(1)$
Group 7 $A_9 \rightarrow A_7(1), A_8(1), A_9(1) \ A_{12} \rightarrow A_9(1), A_{15}(1)$
Group 8 $A_{10} \rightarrow A_{11}(1) \ A_{14} \rightarrow A_{13}(1), A_{17}(1)$
Group 9 $A_{11} \rightarrow A_{14}(1), A_{15}(1) \ A_{15} \rightarrow A_{15}(1)$
Group 10 $A_{13} \rightarrow A_9(1) \ A_{18} \rightarrow A_{19}(1)$
Group 11 $A_{14} \rightarrow A_{15}(1) \ A_{20} \rightarrow A_{19}(1), A_{21}(2)$
Group 12 $A_{15} \rightarrow A_{13}(1), A_{17}(1)$
Group 13 $A_{17} \rightarrow A_{19}(1)$

Group 14 $A_{18} \rightarrow A_{21}(1)$
Group 15 $A_{21} \rightarrow A_{19}(1), A_{21}(1)$

Step 2: Fuzzify each datum in the training data set into a fuzzy set defined in Step 1. With triangular-shaped membership function defined as formula(4), the fuzzy sets and all observations are defined as the second column of Table 1 by formula(4). For example, the datum is 13055 in 1971, the membership degrees then are (1.0000, 0.3147, 0, ..., 0, ..., 0). With $N = 2$, the maximum value is 1 which is the membership degree of A_1 and the second maximum value is 0.3147 which is the membership degree of A_2 , therefore, the datum of 1971 be fuzzified as A_1 and A_2 . For $N = 2$, all of the training data are fuzzified with this method and listed in the third and fourth column of Table 1.

Step 3: Construct the fuzzy logical relationships based on the fuzzified training data obtained in Step 2 and create the fuzzy relation matrix on the basis of fuzzy logical relationship groups.

From Table 2, the obtained fuzzy logical relationships are listed in the fifth and sixth column of the Table. The fuzzy logical relationships are grouped and listed in Table 2. The weights of the fuzzy logical relationship are counted by the recurrent fuzzy relationships with using the method of Lee's [25].

Table 3. Forecasts of Enrollment with $N = 2$

Year	Actual	$p = 1 \quad p = 2 \quad p = 3 \quad p = 4 \quad p = 5 \quad p = 6$					
value							
1971	13055	♠	♠	♠	♠	♠	♠
1972	13563	13608	13481	13608	13545	13608	13566
1973	13867	14014	13938	13913	13900	13893	13887
1974	14696	14696	14846	14696	14621	14696	14646
1975	15460	15457	15384	15457	15494	15457	15481
1976	15311	15347	15402	15217	15293	15296	15299
1977	15603	15732	15668	15646	15635	15629	15625
1978	15861	15923	15892	15882	15876	15873	15871
1979	16807	16747	16835	16826	16821	16818	16816
1980	16919	16863	16891	16900	16905	16908	16910
1981	16388	17269	16238	16388	16313	16388	16338
1982	15433	15457	15384	15457	15421	15457	15433
1983	15497	15347	15402	15457	15494	15515	15481
1984	15145	15154	15150	15217	15147	15147	15147
1985	15163	15154	15159	15160	15161	15161	15162
1986	15984	15923	16010	16001	15997	15994	15993
1987	16859	16863	16835	16863	16849	16863	16854
1988	18150	17269	18300	18150	18075	18150	18100
1989	18970	18923	18947	18954	18958	18961	18962
1990	19328	19333	19335	19330	19329	19329	19329
1991	19337	19128	19117	19336	19336	19336	19336
1992	18876	19128	19117	18892	18888	18885	18884

Step 4: Calculate the forecasts. To carry out the calculations by the proposed principles, we

illustrate the forecasting process of enrollment of 1972 and 1991 as follows. With the definition of membership function (4), the membership degrees of enrollment of 1972 is $(0.5573, 0.9441, 0.4427, 0, \dots, 0)$. The two maximum membership attributes are of the second and the first fuzzy sets. Then, the second row of $R^{(1)}$ is $(0, 0, 1, 0, \dots, 0)$ and the first row of $R^{(2)}$ is $(0, 0, 1, 0, \dots, 0, \dots, 0)$. We have $\bigwedge_N (R_{1972_1}^{(1)}, R_{1972_2}^{(1)}) = (0, 0, 1, \dots, 0)$. According to the second principle $F(1973)$ then is m_3 , i.e. 14014, which is the middle point of A_3 . Table 3 shows the actual enrollments and the forecasted enrollments obtained by using the proposed method with $p = 1, 2, \dots, 6$.

Table 4. A Comparison of MSE of the Proposed Method with the Existing Methods

Year	Actual	value	[21]	[10]	[18]	[19]	[12]	$p=1$	$p=5$	$p=1$	$p=1$
								$N=1$	$N=5$		
1971	13055										
1972	13563	14000	14000	14230	14242	13608	13608	13608	13608		
1973	13867	14000	14000	14230	14242	14014	13892	14014	13893		
1974	14696	14000	14000	14230	14242	14696	14696	14696	14696		
1975	15460	15500	15500	15541	15474	15467	15467	15457	15457		
1976	15311	16000	15000	15541	15474	15455	15314	15347	15296		
1977	15603	16000	16000	15541	15474	15455	15628	15732	15629		
1978	15861	16000	16000	16196	15474	15923	15873	15923	15873		
1979	16807	16000	16000	16196	16147	16863	16818	16747	16818		
1980	16919	16833	17500	16196	16988	17768	17012	16863	16908		
1981	16388	16833	16000	17507	16988	16388	16388	16388	16388		
1982	15433	16833	16000	16196	16147	15467	15412	15457	15457		
1983	15497	16000	16000	15541	15474	15455	15521	15457	15515		
1984	15145	16000	15500	15541	15474	15455	15146	15154	15147		
1985	15163	16000	16000	15541	15474	15247	15179	15154	15161		
1986	15984	16000	16000	15541	15474	16037	15994	15923	15994		
1987	16859	16000	16000	16196	16147	16863	16863	16863	16863		
1988	18150	16833	17500	17507	16988	17768	18150	18150	18150		
1989	18970	19000	19000	18872	19144	19149	19005	18923	18961		
1990	19328	19000	19000	18872	19144	19333	19328	19333	19329		
1991	19337	19000	19500	18872	19144	19128	19336	19128	19336		
1992	18876	19000	19000	18872	19144	19128	18885	18923	18885		
MSE	407507	229516	261162	228920	56580	712	5072	248			
RMSE	638	479	511	478	238	27	71	16			
MAE	499	398	429	386	143	16	47	11			
MAPE	0.03110	0.02483	0.02660	0.02396	0.00857	0.00103	0.00290	0.00072			

4. Empirical Analysis

For that it is simple and easy to display the process of the forecasting models, enrollment data of University of Alabama is forecasted in this paper and widely served the experiment of fuzzy timeseries studies. Since time-series models have been used to make predictions in the areas of stock price forecasting for many years, the daily Shanghai Stock Exchange Composite Index closing prices covering the period from 1997 to 2006 are adopted as the training dataset for further analysis.

In statistics, the Mean Squared Error (MSE), the Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) are some typical criteria for quantifying the difference between values implied by an estimator and the actual values of the quantity being estimated. MSE is a risk function for measures the average of the squares of the difference. For an unbiased estimator, the MSE is the variance, and the RMSE is the square root of the variance known as the standard error. Furthermore, RMSE has over MSE because its scale is the same as the forecasts. As an average of the absolute percent errors, MAPE is another criterion for the comparisons of forecasting results in the paper.

Table 5. Comparison of Forecasting Enrollment

	N	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
RMSE	N = 1	317.55	213.96	58.47	41.20	32.34	33.38
	N = 2	287.92	102.22	33.89	32.42	15.75	21.51
	N = 3	282.82	87.79	33.89	32.42	15.75	21.51
	N = 4	71.22	87.79	33.89	32.42	15.75	21.51
	N = 5	71.22	87.79	33.89	32.42	15.75	21.51
MAE	N = 1	190.36	127.04	30.64	29.43	18.23	21.87
	N = 2	146.18	76.88	23.03	22.52	10.96	14.63
	N = 3	136.43	66.51	23.03	22.52	10.96	14.63
	N = 4	47.28	66.51	23.03	22.52	10.96	14.63
	N = 5	47.28	66.51	23.03	22.52	10.96	14.63
MAPE	N = 1	0.0115	0.0075	0.0020	0.0019	0.0012	0.0014
	N = 2	0.0086	0.0047	0.0015	0.0014	0.0007	0.0009
	N = 3	0.0081	0.0041	0.0015	0.0014	0.0007	0.0009
	N = 4	0.0029	0.0041	0.0015	0.0014	0.0007	0.0009
	N = 5	0.0029	0.0041	0.0015	0.0014	0.0007	0.0009

In Table 4, this study makes a comparison of the forecasted results of the proposed method with Chen's method[3], Cheng *et al.*'s methods[8,10], Huarng's method[15], Chen's method[6] with $p = 1$ and $p = 5$. From the Table 4, we can see that the proposed method has a forecasting error than Chen's method, Cheng *et al.*'s methods on the basis of the four criteria of evaluation. With the same p , the proposed method also has a smaller forecasting error than that of Chen's. In other words, the proposed method gets a higher average forecasting accuracy rate than these counterparts.

Table 6. Comparison of Forecasting SSECI of 1997

	N	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
RMSE	N = 1	11.57	9.18	8.34	4.20	3.40	3.50
	N = 2	9.60	4.76	3.39	2.33	2.70	1.83
	N = 3	9.47	4.76	3.39	2.33	2.70	1.83
	N = 4	8.83	4.68	3.36	2.22	2.64	1.74
	N = 5	8.83	4.68	3.36	2.22	2.64	1.74

MAE	N = 1	7.04	3.81	2.83	1.61	1.16	1.26
	N = 2	4.52	1.87	1.26	0.72	0.75	0.43
	N = 3	4.33	1.87	1.26	0.72	0.75	0.43
	N = 4	3.36	1.77	1.22	0.63	0.70	0.38
	N = 5	3.36	1.77	1.22	0.63	0.70	0.38
MAPE	N = 1	0.0057	0.0031	0.0023	0.0013	0.0009	0.0010
	N = 2	0.0037	0.0015	0.0010	0.0006	0.0006	0.0003
	N = 3	0.0035	0.0015	0.0010	0.0006	0.0006	0.0003
	N = 4	0.0028	0.0014	0.0010	0.0005	0.0006	0.0003
	N = 5	0.0028	0.0014	0.0010	0.0005	0.0006	0.0003

To further investigate the effect of the hierarchies of fuzzy logical relationships on the forecasting results, we apply the proposed method to handle forecasting enrollments with different p and N . Table 5 shows the forecasted RMSE, MAE and MAPE obtained by using the proposed method with $p = 1, 2, \dots, 6$ and $N = 1, 2, \dots, 5$.

From the table, we can arrive at two conclusions: firstly, the forecasting errors will reduce while N is increasing for a given p ; secondly, the proposed model can obtain higher forecast forecasting accuracy rate with an increasing p for a given N .

Table 7. Comparison of Forecasting SSECI of 1998

	$Np = 1p = 2p = 3p = 4p = 5p = 6$						
RMSE	N = 1	11.57	9.18	8.34	4.20	3.40	3.50
	N = 2	9.60	4.76	3.39	2.33	2.70	1.83
	N = 3	9.47	4.76	3.39	2.33	2.70	1.83
	N = 4	8.83	4.68	3.36	2.22	2.64	1.74
	N = 5	8.83	4.68	3.36	2.22	2.64	1.74
MAE	N = 1	7.04	3.81	2.83	1.61	1.16	1.26
	N = 2	4.52	1.87	1.26	0.72	0.75	0.43
	N = 3	4.33	1.87	1.26	0.72	0.75	0.43
	N = 4	3.36	1.77	1.22	0.63	0.70	0.38
	N = 5	3.36	1.77	1.22	0.63	0.70	0.38
MAPE	N = 1	0.0057	0.0031	0.0023	0.0013	0.0009	0.0010
	N = 2	0.0037	0.0015	0.0010	0.0006	0.0006	0.0003
	N = 3	0.0035	0.0015	0.0010	0.0006	0.0006	0.0003
	N = 4	0.0028	0.0014	0.0010	0.0005	0.0006	0.0003
	N = 5	0.0028	0.0014	0.0010	0.0005	0.0006	0.0003

Table 8. Comparison of Mean forecasting SSECI

	$Np = 1p = 2p = 3p = 4p = 5p = 6$						
RMSE	N = 1	13.48	8.19	6.15	5.30	4.72	3.91
	N = 2	10.93	5.65	4.28	3.74	3.30	2.86
	N = 3	10.84	5.64	4.28	3.74	3.30	2.86
	N = 4	8.17	4.97	4.03	3.16	2.93	2.59
	N = 5	8.17	4.97	4.03	3.16	2.93	2.59

MAE	N = 1	8.47	3.77	2.41	1.95	1.56	1.21
	N = 2	5.87	2.25	1.43	1.17	0.95	0.73
	N = 3	5.76	2.24	1.43	1.17	0.95	0.73
	N = 4	3.59	1.87	1.30	0.92	0.78	0.64
	N = 5	3.58	1.87	1.30	0.92	0.78	0.64
MAPE	N = 1	0.0058	0.0026	0.0017	0.0013	0.0011	0.0008
	N = 2	0.0040	0.0015	0.0010	0.0008	0.0006	0.0005
	N = 3	0.0040	0.0015	0.0010	0.0008	0.0006	0.0005
	N = 4	0.0025	0.0013	0.0009	0.0006	0.0005	0.0004
	N = 5	0.0025	0.0013	0.0009	0.0006	0.0005	0.0004

To test the universality of the conclusions, the proposed model has also been used to forecast the close price of SSECI covering the ten years period from 1997 to 2006. Table 6 and 7 show the forecasted RMSE, MAE and MAPE of SSECI in 1997 and 1998. Table 8 illustrates the mean forecasted RMSE, MAE and MAPE of SSECI from 1997 to 2006. From these three tables, we can easy to accept the above conclusions as true.

Some example of actual values and forecasts of 1997 are depicted in Figures 1 and 2. Figure 1 tells us that the performance of the proposed model is improving a lot with the increasing N in the same number of intervals. Figure 2 illustrates that the proposed model obtains the better forecasts when the number of intervals are increasing. All of these conclusions have also been depicted by Table 6,7 and 8 with the three criteria.

Figure 1. Forecasts of 1997 with Different N

Figure 2. Forecasts of 1997 with Different p

5. Conclusion

It is critical to determine the proper represents of the given intervals besides the determination of effective lengths of intervals for forecasting in fuzzy time series. The objective of this study is to provide a method for proper represents of intervals to improve forecasting with generalized fuzzy time series model. In the proposed method, the proper represents of intervals are obtained by using local version of standard particle swarm optimization to find the minimum value of the forecasted errors. The new proposed method is applied to well-known enrollment data as well as the close price of SSECI. Obtained results are compared with other methods proposed in the literature.

To sum up, the empirical analyses for different applications show that the proper represents of intervals outperformed most of the other midpoints of intervals. Hence, except for the superiority of generalized fuzzy time series model, this study also suggests that finding an optimal represents for given intervals can be applied to improve fuzzy time series forecasting.

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