Research on Signal Analysis Method based Wavelet Analysis and Grey Theory

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Abstract

In allusion to the shortcomings of the existing signal analysis method for high-frequency analysis, non-stationary signal analysis and so on, wavelet analysis and grey theory are introduced into the signal analysis, a new signal analysis method based wavelet analysis and grey theory is proposed in this paper. In this method, the wavelet packet is used to nonredundantly, lossless and orthogonally decompose different components of noise signals into different frequency bands with different scales, in order to realize the signal frequency band division with total energy conservation for obtaining the energy feature of each frequency band. Then these energy frequency bands are used to construct the feature vector. And the grey theory is used to analyze the correlation degree between the equipment states and system parameters in order to quickly and accurately determine the position of source. Finally, for a typical signal simulation and analysis, the effectiveness of the signal method is tested and verified.

Keywords: Wavelet packet; grey theory; signal analysis; decomposition; feature vector

1. Introduction

For analyzing non-stationary signal, the traditional signal analysis method of Fourier transform is only difficult to complete analyzing the non-stationary signal. Fourier transform can only obtain the power spectrum density with the average statistical results. And Fourier transform is for the pure frequency domain, it can not describe the local time-frequency characteristics of the signal. For determining the singularity of the non-stationary signal and dynamically analyzing time varying signal and frequency signal, It is difficult to identify the non-period signal from the time domain or frequency domain. It is necessary to finely decompose these time-frequency signals [1]. The emergence of wavelet transform [2] can decompose non-period signal in detail, which is possible. The wavelet analysis is a powerful mathematical tool that provides a time-frequency representation of an analyzed signal in the time domain. It uses a proper mother wavelet to produce a series of child wavelets by the displacement in the timeline and the change of scaling and amplitude. So the wavelet analysis has now been applied to various areas, such as signal processing, image compression, quantum mechanics, classification and recognition, medical imaging and diagnosis, fault diagnosis, data processing and so on. Zhao and Ye [3] proposed one kind of wavelet packet transform based on convolution definition. Its fast decomposition and reconstruction algorithms are given and the outstanding characteristic of this convolution WPT is that no matter how many levels a signal is decomposed. Boix and Cantó [4] described a method for image compression. A new type of global threshold is proposed to improve the wavelet compression technique. The aim is to maintain the retained energy and to increase the compression ratio with respect to other global thresholds commonly

used. Delaunay *et al.* [5] proposed a novel compression scheme with a tunable complexity-rate-distortion trade-off. Modisette *et al.* [6] investigated wavelets in quantum mechanical eigenvalue problems. It is shown that these orthogonal multiresolution functions provide an efficient basis for systems in which the potentials vary strongly in different regions. Patidar and Pachori[7] proposed a new method for classification of cardiac sound signals using constrained tunable-Q wavelet transform (TQWT). González-Carrato et al.[8] proposed a novel pattern recognition approach for a non-destructive test based on macro fibre composite transducers applied in pipes. A fault detection and diagnosis (FDD) method is employed to extract relevant information from ultrasound signals by wavelet decomposition technique. Singh and Khar [9] used Daubechies complex wavelet transform (DCxWT) for image fusion which is approximately shift invariant and provides phase information. A new multimodal medical image fusion using DCxWT at multiple levels is proposed. Du wt al.[10] proposed a novel method based on wavelet leaders multifractal features for rolling element bearing fault diagnosis. The multifractal features, combined with scaling exponents, multifractal spectrum, and log cumulants, are utilized to classify various fault types and severities of rolling element bearing, and the classification performance of each type features and their combinations are evaluated by using SVMs. Chen *et al.* [11] developed a wavelet-based algorithm for proteomic data processing..

So this paper mainly discussed about the vibration signal analysis. The process method of vibration signal based on the wavelet analysis and grey theory is established to lay the theoretical basis for fault diagnosis.

2. Wavelet Analysis and Grey Theory

2.1. Wavelet Analysis

Wavelets are mathematical functions that are used to dissect data into different frequency components and each component is characterized with a resolution appropriate to its scale. The wavelet analysis [2] is a powerful mathematical tool that provides a time-frequency representation of an analyzed signal in the time domain. It uses a proper mother wavelet to produce a series of child wavelets by the displacement in the timeline and the change of scaling and amplitude. Then, these child wavelets compare the displacement of signals in the time line and attain the wavelet coefficients which can represent the similarity between the signals and the wavelets. Thus the information in local time zone can be obtained while fourier analysis is not able to obtain the information. It is due to that the derived wavelet can come up to any small required accuracy and thus conduct accurate measurement of the signals with finite length. Therefore, wavelet analysis is often used for signal denoising. To describe the basic principle of wavelet analysis, the process of wavelet analysis for signal denoising is introduced.

For a continuous time series $x(t)(t \in [-\infty, \infty])$, the function of wavelet $w(g)$ is described as:

$$\psi(\eta) = \psi(\tau, s) = s^{-1/2} \psi\left(\frac{t - \tau}{s}\right) \quad (1)$$

where τ is window function for the time step, t is time, s is wavelet scale. $\psi(\eta)$ should have zero mean and be localized in both time and Fourier space.

The continuous wavelet transform(*CWT*)of arbitrary signal $x(t)$ is a linear operation that consists in projecting the signal $x(t)$ onto a wavelet basis. Mathematically, the *CWT* is defined as follows:

$$W(\tau, s) = s^{-1/2} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t - \tau}{s} \right) dt \quad (2)$$

where ψ^* is representative for complex conjugate function of ψ . One can construct a two dimensional picture of wavelet power by smoothly varying both τ and s . $W(\tau, s)$ is the frequency of peaks in the spectrum of $x(t)$. In order to obtain wavelet power spectrum and wavelet phase at the same time, one always uses the complex-valued basic wavelet.

The *CWT* calculation necessitates a large amount of computation time and resources. The discrete wavelet transform (*DWT*) requires less computation time and is simpler to implement than the *CWT*. The *DWT* scales and positions are usually based on powers of the dyadic scales and positions. The modified wavelet representation is described as follows:

$$\psi_{m,n} \left(\frac{t - \tau}{s} \right) = s_0^{-m/2} \psi \left(\frac{t - n\tau_0 s_0^m}{s_0^m} \right) \quad (3)$$

where m is to control the scale, n is to control the time, and m and n are integers. τ_0 is the location parameter ($\tau_0 > 0$), and s_0 is a specified fixed dilation step ($s_0 > 0$). In general, $\tau_0 = 1$ and $s_0 = 2$.

For a discrete time series x_i , the dyadic wavelet transform is become:

$$\psi_{m,n} = 2^{-m/2} \sum_{i=0}^{N-1} \psi(2^{-m} i - n) x_i \quad (4)$$

where $\psi_{m,n}$ is the wavelet coefficient for the discrete wavelet of scale $s = 2^m$ and location $\tau = 2^m n$, $N = 2^m$.

2.2. Grey Theory

Grey theory is proposed by Deng in 1989 [12]. It is used to address uncertain problems that are characterized by the existence of discrete data and incomplete information [13]. Grey theory has now been applied to various areas, such as systems analysis, data processing, modeling, prediction, decision-making and control, which are neither deterministic nor totally unknown, but are partially known. Upon its introduction, grey theory received positive attention from domestic and foreign academic circles and from many workers. Up to now, the grey forecasting model has been widely applied in various fields for forecasting, including economy and industry [14,15], energy and environment [16-18], medical and health [19,20], and risk assessment [21-22].

The grey forecasting model uses the operations of accumulated generation to reduce the variation in the original data series and to build differential equations by linearly transforming the data series. GM (1,1), which is the simplest and most widely applied model of the grey forecasting theory, indicates one variable and one order grey forecasting model. The general procedure for a grey forecasting model is derived as follows:

Step 1: Suppose the observed series, i.e., the original series of $x^{(0)}$ is expressed by the following equation:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) = x^{(0)}(k) \quad (5)$$

where $k = 1, 2, 3, 4, \dots, n (n \geq 2)$.

Step 2: Ensure the class ratio $\sigma^{(0)}(k)$ of $x^{(0)}$ is within the interval $(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$, where $\sigma^{(0)}(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)}, k = 2, 3, 4, \dots, n$

Step 3: A new sequence $x^{(1)}$ is generated by the accumulated generation operation.

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) = x^{(1)}(k) \quad (6)$$

where $k = 1, 2, 3, 4, \dots, n (n \geq 2)$ and $x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$.

Step 4: Establish a first-order differential equation and is denoted as follows:

$$x^{(0)}(k) + \alpha z^{(1)}(k) = b, k = 2, 3, 4, \dots, n \quad (7)$$

$$z^{(1)}(k) = \alpha x^{(1)}(k) + (1 - \alpha)x^{(1)}(k-1) \quad (8)$$

where $z^{(1)}(k)$ is the whitened value, and $\alpha (0 < \alpha < 1)$ is are developing coefficient, b is grey input.

Step 5: According to Step 4 and by the Least Ordinary Square Method, coefficient of $\hat{\theta}$ becomes:

$$\hat{\theta} = \begin{pmatrix} \alpha \\ b \end{pmatrix} \quad (9)$$

Step 4: The expression in Step4. can be denoted by another type (i.e., grey difference equation) through white process as follows:

$$\frac{dx^{(1)}(k)}{dk} + \alpha x^{(1)}(k) = b \quad (10)$$

The discrete sequence $x^{(1)}$ whitening equations is used to solve the expression (9):

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{b}{\alpha}]e^{-\alpha k} + \frac{b}{\beta} \quad (11)$$

3. Signal Feature Extraction based on Wavelet Packet and Grey Theory

3.1. The Idea of Signal Feature Extraction

For the different mechanical vibration signals, some faults have certain sensitive bands. When the fault occurs, the vibration signal in this band will change greatly. So the wavelet packet and grey theory are introduced to analyze and compare each frequency band of vibration signals under different operation condition in order to effectively monitor the occurrence and development of the fault, and make the corresponding prediction of the long-run trend. The wavelet packet is regarded as an effective method for non-stationary signal analysis, it has been widely applied to the field of vibration signal for fault diagnosis. The wavelet packet is used to nonredundantly, lossless and orthogonally decompose different components of noise signals into different frequency bands with different scales, and realizes the signal frequency band division and the total energy is conserved. Because the sampling frequency and bandwidth are halved after each decomposition, the sampling frequency of bandpass signal will depend on its bandwidth. The wavelet packet decomposition does not cause the loss of information in the decomposition process. Therefore, the complexity of vibration signals with wavelet packet decomposition is reasonable. The grey theory can deal with problems that are characterized by incomplete information. It can eliminate the defects of mathematical

statistical methods in systematic analysis and can provide a solution of system in which the model is unsure or the information is incomplete. So the grey theory is regarded as a judgment index to describe the complex degree or irregular degree of the vibration signal in the different scale and different frequency band in order to extract the non-stationary characteristics of fault signal.

3.2. Extract I Feature Extraction of Fault Vibration Signal

The vibration signal is composed of the normal vibration signal and fault vibration signal. Some output sampling data are selected from the vibration signal. And the random signals are regarded as the input excitation signals. Then the wavelet packet analysis is used to establish a characteristic vector for displaying vibration signal states in order to use the grey theory for comparing and determining the defect detection of fault vibration signal.

The first, the spectrum characteristics of vibration signal is analyzed briefly. Under the same condition, when the vibration signal mixed with noise, the different frequency waveform, amplitude frequency and power spectrum characteristics will have some change with different degrees. The amplitude frequency and power spectrum of the normal signal is shown in Figure 1. And the amplitude frequency and power spectrum of the fault signal is shown in Figure 2.

Figure 1. The Amplitude Frequency and Power Spectrum for Normal Signal

Figure 2. The Amplitude Frequency and Power Spectrum for Fault Signal

As can be seen from the Figure 1 and Figure 2, when the vibration signal mixes the noise, the amplitude frequency and power spectrum characteristics exist the obvious changes, which have obvious suppression function for some frequency-power components and enhancement function for other frequency- power components. It has the different inhibiting and augmenting functions for the input signal of different frequency.

According to the above analysis, the fault vibration signal in the fault diagnosis is mainly analyzed. When the mixed signal is obtained, it will be decomposed into

three layers using wavelet packet analysis method. The feature vector is composed of two signal features from low frequency to high frequency in the first layer, four signal features from low frequency to high frequency in the second layer and eight signal features from low frequency to high frequency in the third layer. The decomposed tree structure is shown in Figure 3.

Figure 3. The Decomposed Tree based on Wavelet Packet

4. The Experiment Analysis of Fault Signal Feature Extraction

In this method, the fault is saved in the computer in advance. The corresponding visualization method is used to display the obtained fault. According to the above theory, the normal vibration signal and the vibration signal with noise are analyzed by using the wavelet packet in order to obtain the amplitude-power spectrum waveform. The normal vibration signal, the fault vibration signal and the mixed vibration signal are respectively decomposed. The amplitude frequency and power spectrum are shown in Figure 4-Figure 7.

Figure 4. The Amplitude Frequency and Power Spectrum for Normal Signal in the First Layer

Figure 5. The Amplitude Frequency and Power Spectrum for Fault Signal in the First Layer

Figure 6. The AF and Power Spectrum for the Normal Signal in the Second Layer

Figure 7. The Amplitude Frequency and Power Spectrum for Fault Signal in the Second Layer

As can be seen from the Figure 4 to Figure 7, the waveform results of fault and normal signal are obtained by using wavelet packet decomposition. Generally, it can

find out the difference between the normal vibration signal and fault vibration signal by comparing the waveform and the power spectrum.

5. Conclusion

The wavelet analysis is a powerful mathematical tool that provides a time-frequency representation of an analyzed signal in the time domain. A proper mother wavelet is used to produce a series of child wavelets in the timeline and the change of scaling and amplitude. Grey theory is used to address uncertain problems that are characterized by the existence of discrete data and incomplete information. It received positive attention from domestic and foreign academic circles and from many workers. So the wavelet packet and grey theory are introduced to analyze and compare each frequency band of vibration signals under different operation condition in this paper. The goal is to effectively monitor the occurrence and development of the fault and make the corresponding prediction of the long-run trend. In here, the wavelet packet is regarded as an effective method for non-stationary signal analysis and grey theory can deal with problems that are characterized by incomplete information in order to eliminate the defects of mathematical statistical methods in systematic analysis and provide a solution of system. In the actual application, the system faults have generated large influence on the signal energy in the each band. The analyzed results show that the noise signal analysis method based wavelet analysis and grey theory has a strong ability to describe the complexity of the vibration signal and a very good effect for judging the running states of mechanical equipments, and can be regarded as an effective analysis method for monitoring and forecasting the fault of mechanical equipment.

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