Efficient Certificateless Signcryption from Bilinear Pairings

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Abstract

Certificateless public key cryptography is an attractive paradigm which combines advantages of both traditional certificate-based and ID-based public key cryptosystems. In this paper, the author formalizes the definition and the security model for certificateless signcryption (CLSC), and proposes a CLSC scheme that is proved to be secure in the random oracle model. The CLSC scheme satisfies such desirable properties as message confidentiality, signature unforgeability, public verifiability and forward secrecy. Moreover, the scheme achieves significant efficiency in that it requires general cryptographic hash functions instead of the MapToPoint function which is inefficient and probabilistic, and the public/private key of a user in the scheme is a single group element, and it is unnecessary to check the validity of the public key, and signcryption algorithm needs no pairing operation while the reverse operation only requires two pairing computations.

Keywords: *certificateless public key cryptography, signcryption, bilinear pairings, security*

1. Introduction

Identity-based public key cryptosystem (ID-PKC) which was introduced by Shamir [1] in 1984 alleviates the burden of certificate management in a traditional public key cryptosystem (PKC). However, there inherently exists a defect named private key escrow problem in ID-PKC because it involves a Private Key Generator (PKG), which can generate a user's private key based on his identity. So, the PKG can literally decrypt any ciphertext and forge any user's signature on any message. To solve the inherent key escrow problem in ID-PKC, Al-Riyami and Paterson [2] introduced a new approach called certificateless public key cryptography (CL-PKC) in 2003. The CL-PKC is intermediate between traditional PKC and ID-PKC. In a CL-PKC, a user's private key is not produced by the PKG alone. Instead, it is composed of a partial private key produced by the PKG and some secret value chosen by the user. As a result, the PKG does not know the user's private key. In such a way, the key escrow problem of ID-PKC can be solved.

Due to the lack of authentication of public keys in CL-PKC, it is difficult to construct a secure and efficient certificateless scheme. Although many schemes [3–15] based on certificateless cryptosystem have been presented following the first work from Al-Riyami and Paterson [2], schemes in [3, 4, 6, 8, 9, 10] are proved to be insecure. Furthermore, most certificateless schemes are inefficient and unpractical with respect to the cost of computation and the length of public/private key. For example, the verification algorithm in certificateless signature schemes [4, 5, 10] requires four expensive pairing computations, and public key in schemes [2, 4, 7, 10] is two group elements and the validity of public key has to be verified, while the verification of public key is costly and requires at least two pairing computations.

The notion of signcryption was presented by Zheng[16] in 1997, and this primitive aims to efficiently perform encryption and signature in a single logical step. The first certificateless signcryption scheme was proposed by Barbosa and Farshim [17] in 2008. Li *et al.* [18] proposed a certificateless hybrid signcryption scheme. Unfortunately, Selvi *et al.* [19] show the scheme is existentially forgeable and then give an improved scheme. Wu *et al.* [20] and Xie *et al.* [21] separately put forward two certificateless signcryption schemes and give their security proofs in the random oracle. However, Selvi *et al.* [22, 23] point out the two schemes are insecure against the Type I adversaries. In 2009, Liu *et al.* [24] constructed a certificateless signcryption scheme in the standard model. Unfortunately, Weng *et al.* [25] point out that Liu *et al.*'s scheme is vulnerable to the Type II adversary's attack. Jing *et al.* [26] proposed a CLSC scheme without bilinear pairings, but Shi *et al.* [28] prove the scheme does not meet the properties of confidentiality and unforgeability.

Contributions. This paper studies signcryption in certificateless public key cryptography. The author proposes a new CLSC scheme and provides detailed security proofs for the scheme. The CLSC scheme satisfies all the current requirements of the signcryption. In addition, the length of public/private key of a user in the scheme is only one group element. Moreover, the scheme never requires public key validity checking. And the scheme achieves significant efficiency in that it requires general cryptographic hash functions instead of the MapToPoint hash function, and signature verification requires only two pairing computations while no pairing operation is needed in signcryption algorithm.

The remaining sections are organized as follows: In the next section, the author gives a brief introduction to bilinear pairings and some mathematical theories. Section 3 provides the framework of CLSC. The author proposes an efficient CLSC scheme and gives its efficiency analysis, and provides its security proof in the random oracle model in Section 4. Conclusions are drawn in the last section.

2. Preliminaries

Let G_1 be a cyclic additive group of prime order p , and G_2 be a cyclic multiplicative group of the same order p . A bilinear pairing is a map $e: G_1 \times G_1 \rightarrow G_2$ which satisfies the following properties:

1) Bilinearity

$$e(aP, bQ) = e(P, Q)^{ab}, \text{ where } P, Q \in G_1, a, b \in \mathbb{Z}_p^*$$

2) Non-degeneracy

There exist $P, Q \in G_1$ such that $e(P, Q) \neq 1$

3) Computability

There is a computable algorithm to get $e(P, Q)$ for all $P, Q \in G_1$.

It is intractable to solve the following problems within polynomial time.

- **k-CAA**: For an integer k , and $s \in \mathbb{Z}_p^*$, $P \in G_1$. Given $\{P, sP, e_1, e_2, \dots, e_k \in \mathbb{Z}_p^*$,

and $\frac{1}{s+e_1}P, \frac{1}{s+e_2}P, \dots, \frac{1}{s+e_k}P\}$, to compute $\frac{1}{s+e}P$, where $e \notin \{e_1, e_2, \dots, e_k\}$.

- **Computational Diffie-Hellman Problem (CDHP)**: For unknown random values $a, b \in \mathbb{Z}_p^*$, given aP, bP , computing abP .

- **Computational Diffie-Hellman Inversion Problem (CDHIP)**: For an unknown value $a \in \mathbb{Z}_p^*$, given P, aP , to compute $a^{-1}P$.

- **Modified Computational Diffie-Hellman Inversion problem (MCDHIP)**:

- Given $b \in Z_p^*$, P, aP for unknown value $a \in Z_p^*$, to compute $(a + b)^{-1}P$.
- ***q-Bilinear Diffie-Hellman Inversion Problem (q-BDHIP)***: Given a tuple $(P, \alpha P, \alpha^2 P, \dots, \alpha^q P)$ as input, computing $e(P, P)^{1/\alpha}$.
 - ***Bilinear Diffie-Hellman Inversion problem (BDHIP)***: Given $P, \alpha P$, computing $e(P, P)^{1/\alpha}$.
 - ***Modified Bilinear Diffie-Hellman Inversion problem (MBDHIP)***: Given $b \in Z_p^*$, P, aP for unknown value $a \in Z_p^*$, computing $e(P, P)^{1/(a+b)}$.

3. Frameworks of Certificateless Signcryption

(1) Definition of Certificateless Signcryption

A CLSC scheme is composed of seven algorithms as follows.

Setup: Taking a security parameter l as input and returns the system parameters, $params$ and a master key s .

Partial-Private-Key-Extract: On inputs $params$, s and a user's identity ID , the algorithm returns a partial private key d_{ID} .

Set-Secret-Value: Taking as inputs $params$ and a user's identity ID , the algorithm returns a secret value r .

Set-Private-Key: Taking as inputs $params$, a user's partial private key d_{ID} and his secret value r , and it returns the full private key sk_{ID} .

Set-Public-Key: Taking as inputs $params$ and a user's secret value r , and generates a public key pk_{ID} for the user.

CL-Signcrypt: is a probabilistic algorithm that takes as inputs $params$, a message m , a recipient's identity ID_B and his public key pk_B , and the sender's private key sk_A , and returns a ciphertext $\sigma = CL-Signcrypt(m, sk_A, Q_{ID_B}, pk_B)$.

CL-Unsigncrypt: Taking as inputs $params$, a ciphertext σ , the recipient's private key sk_B , and the sender's identity ID_A and his corresponding public key pk_A . It returns a message-signature pair (m, S) or a distinguished symbol $\perp$ indicating an unsigncryption failure.

(2) Security Model for Certificateless Signcryption

The standard security notion for a signcryption scheme is indistinguishability of signcryption against an adaptive chosen-ciphertext attacker (IND-CCA2) and existentially unforgeable against an adaptive chosen-message attacker (EUF-CMA2).

As defined in security model for certificateless public key cryptography [2], there are two types of adversaries in a CLSC scheme: a Type I adversary A_1 and a Type II adversary A_2 . A Type I adversary does not know the KGC's master key but can replace public keys of arbitrary identities with other public keys of its choice. In contrast, a Type II adversary knows the KGC's master key but is disallowed to replace public key of any user during the game. The formal security models for CLSC schemes were formalized by Barbosa and Farshim [17], I refer the readers to literature [17].

4. An Efficient Certificateless Signcryption Scheme

4.1. The basic scheme

In this section, I present a CLSC scheme which is secure against both Type I and Type

II adversaries in the random oracle model.

Setup: Given a security parameter l , the PKG chooses two groups G_1 and G_2 of same prime order $p > 2^l$ and a modified Weil pairing map $e : G_1 \times G_1 \rightarrow G_2$. P is a generator of group G_1 . Let $g = e(P, P)$, then PKG selects four distinct cryptographic hash functions such that $H_1 : \{0, 1\}^* \rightarrow Z_p^*$, $H_2 : G_1 \rightarrow Z_p^*$, $H_3 : G_2 \rightarrow (0, 1)^l$ and $H_4 : \{0, 1\}^* \times G_2 \rightarrow Z_p^*$. And picks a random $s \in Z_q^*$ as its master key and computes its public key $P_0 = sP \in G_1$. Let E, D be the encryption and decryption algorithms of a secure symmetric cipher. Afterwards, PKG publishes the system public parameters, *params*:

$\{l, G_1, G_2, e, p, P, g, P_0, H_1, H_2, H_3, H_4, E, D\}$, but keeps s secret.

Partial-Private-Key-Extract: Given an identity $ID_i \in (0, 1)^*$, $i \in \{A, B\}$, PKG

computes $Q_i = H_1(ID_i)$, $d_i = \frac{1}{s + Q_i} P$, and sends d_i to a user with identity ID_i as his partial private key by a secure channel.

Set-Secret-Value: The user with identity ID_i picks a random value $x_i \in Z_p^*$ as his secret value.

Set-Public-Key: Taking as inputs *params* and a user's secret value x_i , the algorithm generates the user's public key as $pk_i = x_i(P_0 + Q_i P)$. For simplicity, we set $T_i = P_0 + Q_i P$.

Set-Private-Key: Given *params*, a user's partial private key d_i and his secret value x_i , the

algorithm outputs $sk_i = \frac{1}{(x_i + H_2(pk_i))(s + Q_i)} P$ as the user's private key.

CL-Signcrypt. Given a message $m \in (0, 1)^*$, a recipient's identity ID_B and his public key pk_B , and a sender's private key sk_A , the sender with the identity ID_A and his public key pk_A works as follows:

- 1) Pick a random $r \in Z_p^*$, and compute $R = g^r$, set $K = H_3(R)$ and compute $c = E_K(m \parallel ID_A \parallel pk_A)$;
 - 2) Set $h = H_4(m, R, ID_A, ID_B)$;
 - 3) Compute $S = (r + h)sk_A = \frac{(r + h)}{(x_A + H_2(pk_A))(s + Q_A)} P$;
 - 4) Compute $U = r(pk_B + H_2(pk_B)T_B) \in G_1$.
- The ciphertext is $\sigma = (c, U, S)$.

CL-Unsigncrypt. Given a ciphertext $\sigma = (c, U, S)$ and a sender's identity ID_A and his public key pk_A , the recipient with identity ID_B and his public key pk_B acts as follows:

- 1) The recipient uses his private key sk_B to compute R .

$$\begin{aligned} e(U, sk_B) &= e\left(r(x_B T_B + H_2(pk_B)T_B), \frac{P}{(x_B + H_2(pk_B))(s + Q_B)}\right) \\ &= e\left(r((x_B + H_2(pk_B))(s + Q_B)P), \frac{P}{(x_B + H_2(pk_B))(s + Q_B)}\right) \\ &= e(P, P)^r = R. \end{aligned}$$

And sets $K = H_3(R)$, and then recovers $D_K(c) = m \parallel ID_A \parallel pk_A$.

2) Set $h = H_4(m, R, ID_A, ID_B)$.

3) Accept the message m and return the message-signature pair $(m, (h, S))$ if and only if the following equation holds.

$$h = H_4\left(m, e(S, pk_A + H_2(pk_A)T_A) \cdot g^{-h}, ID_A, ID_B\right)$$

The consistency of the scheme is easy to verify as follows:

$$\begin{aligned} & e(S, pk_A + H_2(pk_A)T_A) \cdot g^{-h} \\ &= e\left(\frac{(r+h)}{(x_A + H_2(pk_A))(s+Q_A)}P, pk_A + H_2(pk_A)T_A\right) e(P, P)^{-h} \\ &= e\left(\frac{(r+h)}{(x_A + H_2(pk_A))(s+Q_A)}P, (x_A + H_2(pk_A))(s+Q_A)P\right) e(P, P)^{-h} = e(P, P)^r \\ &= R. \end{aligned}$$

$$\text{Hence, } h = H_4\left(m, e(S, pk_A + H_2(pk_A)T_A) \cdot g^{-h}, ID_A, ID_B\right)$$

Public Verifiability

To prove to a third party that the sender actually signcrypted a message m , the recipient forwards the tuple $(m, (h, S))$ to the third party. Then the third party accepts the proof by verifying whether the following equation holds

$$h = H_4\left(m, e(S, pk_A + H_2(pk_A)T_A) \cdot g^{-h}, ID_A, ID_B\right).$$

4.2. Security

Theorem 1 In the random oracle model, the proposed CLSC scheme is secure against an adaptive chosen-ciphertext attack (IND-CLSC-CCA2) under the q -BDHIP and MBDHIP assumptions.

The theorem follows from Lemmas 1 and 2.

Lemma 1. Assume that an IND-CLSC-CCA2 adversary A_1 has an advantage ε against the above scheme when running in a time t , making q_{h_i} queries to random oracles H_i ($i = 1, 2, 3, 4$), q_{sc} queries to the Signcryption oracle and q_{us} queries to the Unsigncryption oracle. Then there exists an algorithm X to solve the q -BDHIP for $q = q_{h_1}$ with probability

$$\varepsilon' \geq \frac{\varepsilon(q_{h_1}-1)}{q_{h_1}^2(q_{h_3}+2q_{h_4})} \left(1 - q_{sc} \frac{q_{sc}+q_{h_4}}{2^l}\right) \left(1 - \frac{q_{us}}{2^l}\right), \text{ with a time}$$

$t' < t + (O(q^2) + (3q + 3q_{sc} + 2q_{us}))t_{sm} + (q_{h_4} + 2)t_{exp} + (4q_{us} + 1)t_{pr}$, where t_{sm} , t_{exp} and t_{pr} denote respectively the costs of a scalar multiplication in G_1 , an exponentiation in G_2 and a pairing computation.

Proof. Suppose that X is given a challenge: Given $\langle P, sP, s^2P, \dots, s^qP \rangle$, for an unknown value $s \in \mathbb{Z}_p^*$, X 's task is to output $e(P, P)^{1/s}$ after interacting with A_1 . X will interact with A_1 as follows:

Setup: X uses its input to compute a generator $Q \in G_1$ and sets $P_0 = xQ$ for $x = k_I^{-1}s - w$ such that it knows $q-1$ pairs $(w_i, \frac{1}{w_i + x}Q)$ for $i \in \{1, 2, \dots, q-1\}$ as in the proof technique of [29]. X acts as follows:

1) Choose $w_1, w_2, \dots, w_{q-1} \in Z_p^*$ at random and construct a polynomial $f(t) = \prod_{i=1}^{q-1} (t + w_i)$, and expand it to obtain $c_0, c_1, \dots, c_{q-1} \in Z_p^*$ so that $f(t) = \sum_{i=0}^{q-1} c_i t^i$.

2) Pick values $k_I \in Z_p^*$ for $I \in \{1, 2, \dots, q_{h_1}\}$ and $w \in Z_p^*$, where $w \notin \{w_1, w_2, \dots, w_{q-1}\}$. Suppose $x = k_I^{-1}s - w$, X computes the following:

$$Q = f(x)P = \sum_{i=0}^{q-1} c_i (x^i P) = \sum_{i=0}^{q-1} c_i (k_I^{-1}s - w)^i P = \sum_{i=0}^{q-1} c_i' (s^i P), \quad \text{and}$$
 compute $sQ = sf(x)P = \sum_{i=0}^{q-1} c_i' (s^{i+1} P)$.

3) Set $P_0 = k_I^{-1} \cdot sQ - wQ = xQ$ and $g = e(Q, Q)$.

X sends $params = \{G_1, G_2, Q, P_0, g, H_1, H_2, H_3, H_4, E, D\}$ to A_1 .

H_1 -Queries: X maintains a hash list H_1^{list} of tuple (ID_i, Q_i) ($i \in \{1, 2, \dots, q_{h_1}\}$) as explained below. The list is initially empty. When A_1 makes a hash oracle query on ID_i , X returns the previously defined value if it exists. Otherwise, X acts as follows: If $ID_i = ID_I$, X sets $Q_i = w$ and returns Q_i to A_1 . Otherwise, X answers $Q_i = w_i$ for $w_i \in \{w_1, w_2, \dots, w_{q-1}\}$. In both cases, X inserts (ID_i, Q_i) into H_1^{list} .

Partial-Private-Key-Extraction Queries: X maintains a list E^{list} of tuple (ID_i, Q_i, d_i) is initially empty. For any given identity ID_i , X recovers the corresponding tuple (ID_i, Q_i) from the list H_1^{list} , if $ID_i \neq ID_I$, then returns $d_i = \frac{1}{x + Q_i}Q$ to A_1 and adds (ID_i, Q_i, d_i) to the E^{list} . Otherwise, X aborts and outputs “failure” (denote the event by E_1).

Public-Key-Extraction Queries: X maintains a list pk^{list} of tuple (ID_i, Q_i, pk_i, x_i) which is initially empty. For on input ID_i , X picks a random $x_i \in Z_p^*$, computes $pk_i = x_i(P_0 + Q_iQ) = x_iT_i$ and returns pk_i . Then adds (ID_i, Q_i, pk_i, x_i) to the list pk^{list} .

H_2 -Queries ; For a query on input ID_i and pk_i , If $ID_i = ID_I$, X recovers the corresponding tuple (ID_i, Q_i, pk_i, x_i) from the list pk^{list} and sends $h_{2i} = k_I - x_i \in Z_p^*$ to A_1 (If the identity ID_I 's public key pk_I has been replaced by A_1 , we assume that C can additionally obtain the secret value corresponding to the replaced public key). Otherwise, X picks a random $h_{2i} \in Z_p^*$ and returns it to A_1 . In both cases, X adds tuple (ID_i, pk_i, h_{2i}) to the list H_2^{list} .

Private-Key-Extraction Queries: For a query on input ID_i , If $ID_i = ID_I$, X stops and outputs “failure” (denotes the event by E_2). Otherwise, X performs as follows:

- If lists E^{list} , pk^{list} and H_2^{list} contain the corresponding tuple (ID_i, Q_i, d_i) , (ID_i, Q_i, pk_i, x_i) and (ID_i, pk_i, h_{2i}) respectively, X sets $sk_i = \frac{d_i}{(x_i + h_{2i})}$ and sends it to

A_1 .

- Otherwise, X makes a Partial Private Key Extraction query, a Public Key Extraction query and a H_2 query on ID_i , then simulates as the above process and sends $sk_i = \frac{d_i}{(x_i + h_{2i})}$ to A_1 .

Public-Key-Replacement Queries: When A_1 queries on input (ID_i, pk'_i) , X checks whether an element (ID_i, Q_i, pk_i, x_i) is contained in the pk^{list} . If it does, X sets $pk_i = pk'_i$ and adds $(ID_i, Q_i, pk_i, x_i = \perp)$ to the pk^{list} . Otherwise, X executes public key extraction to generate (ID_i, Q_i, pk_i, x_i) , then sets $pk_i = pk'_i$ and adds $(ID_i, Q_i, pk_i, x_i = \perp)$ to the pk^{list} .

H_3 -Queries: For an input $R \in G_2$, X picks a random value $h_3 \in Z_p^*$ and returns it to A_1 , and inserts (R, h_3) into the list H_3^{list} .

H_4 -Queries: For an input (m, R, ID_u, ID_v) with identities ID_u / ID_v of sender/recipient, X answers the defined value if it exists and a random value $h_4 \in Z_p^*$ otherwise. Additionally, X computes $\delta = R \cdot e(Q, Q)^{h_4}$ and inserts a record $(m, R, ID_u, ID_v, h_4, \delta)$ into a list H_4^{list} .

Signcrypt Queries: When a Signcrypt query on a message m for sender/recipient with identity ID_u / ID_v for $u, v \in \{1, \dots, q_{h_1}\}$ and corresponding public key pk_u / pk_v is coming, X acts as follows:

- If $ID_u = ID_I$, X does not know the sender's private key sk_u , but he knows the recipient's private key sk_v since $ID_v \neq ID_I$ (If the recipient's public key pk_v has been replaced by A_1 , we assume that X can keep track of the secret value generated by A_1). X finds the corresponding tuple (ID_I, pk_I, h_{2I}) and the tuple (ID_v, pk_v, h_{2v}) from H_2^{list} , and chooses two random values $a, h_4 \in Z_p^*$, computes $U = a(pk_I + h_{2I}T_I) - h_4(pk_v + h_{2v}T_v)$, $S = a \cdot sk_v$, $R = e(a(pk_I + h_{2I}T_I), sk_v) / e(Q, Q)^{h_4}$ and then backpatches to define the value $H_4(m, R, ID_u, ID_v)$ as h_4 (X aborts in the unlikely event that $H_4(m, R, ID_u, ID_v)$ is already defined, denote the event by E_3). Then, X finds the corresponding tuple (R, h_3) from H_3^{list} and computes $c = E_{h_3}(m \| ID_I \| pk_I)$. At last, X outputs a ciphertext $\sigma = (c, U, S)$.

- Otherwise, X has the sender's private key and can answer the query according to the specification of the Signcrypt algorithm.

Unsigncrypt Queries: Suppose the request is on a ciphertext $\sigma = (c, U, S)$ for identities (ID_u, ID_v) for $u, v \in \{1, \dots, q_{h_1}\}$ and with corresponding public key (pk_u, pk_v) . C acts as follows:

- If $ID_v = ID_I$, X has not the recipient's private key sk_v , but he knows the sender's private key sk_u since $ID_u \neq ID_I$. Obviously, the equation

$e(U, sk_u) = e(pk_I + h_{2I}T_I, S - h_4sk_u)$ holds, where $H_4(m, R, ID_u, ID_v)$ is the hash value obtained in the Signcrypt algorithm. X computes $\delta = e(pk_u + h_{2u}T_u, S)$ and recovers the tuple $(m, R, ID_u, ID_v, h_4, \delta)$ from H_4^{list} (If this tuple does not exist, $\sigma = (c, U, S)$ is rejected, denote the event by E_4) and further examines whether the following equality holds:

$$e(U, sk_u) / e(pk_I + h_{2I}T_I, S) = e(pk_I + h_{2I}T_I, -h_4sk_u).$$

If the equation holds, X outputs the message-signature pair $(m, (h_4, S))$, otherwise, $\sigma = (c, U, S)$ is rejected.

- If $ID_v \neq ID_I$, X knows the recipient's private key and can normally run the Unsigncrypt algorithm.

Finally, A_1 outputs messages (m_0, m_1) and identities (ID_S^*, ID_R^*) with public keys (pk_S^*, pk_R^*) , where the private key of identity ID_R^* was not corrupted. Here note ID_R^* does not have been submitted to both oracles Public-Key-Replacement and Partial-Private-Key-Extract. If $ID_R^* \neq ID_I$, X outputs "failure" and aborts (denote the event by E_5). Otherwise, X chooses $\lambda \in Z_p^*$ and computes $U^* = \lambda Q$, picks randomly $c^* \in \square$ ($\square$ denotes the ciphertext space) and $S^* \in G_1$ to return the challenge $\sigma^* = (c^*, S^*, U^*)$. Here if we define $\lambda = rs$ for $r \in Z_p^*$ and we can check that $U^* = \lambda Q = rsQ = r(P_0 + Q_I Q)(x_I + h_{2I}) = r(pk_I + h_{2I}T_I)$.

A_1 then makes a new series of queries but is not allowed to ask the unsigncrypt oracle of $\sigma^* = (c^*, S^*, U^*)$ for the combination (ID_S^*, ID_R^*) and (pk_S^*, pk_R^*) . Moreover, no Private-Key-Extraction query can be made on ID_R^* at any time and ID_R^* can not be submitted to both oracles Public-Key-Replacement and Partial-Private-Key-Extract.

To omit A_1 's eventual output, X fetches tuple (R, h_3) or (m, R, h_4, c, δ) from list H_3^{list} or H_4^{list} , the chosen tuple will contain the right element $R = e(Q, Q)^r = e(P, P)^{f(x)^2 \lambda / s}$. Then the q -BDHIP solution can be extracted. Since

$$\begin{aligned} R &= e(Q, Q)^r = e(Q, Q)^{\lambda / s} = e((1/s)f(x)P, f(x)P)^\lambda \\ &= e\{(1/s) \sum_{i=0}^{q-1} c'_i(s^i P), \sum_{i=0}^{q-1} c'_i(s^i P)\}^\lambda \\ &= e\{(c'_0/s)P, c'_0 P\}^\lambda e\{c'_0 P + \sum_{i=0}^{q-1} c'_i(s^i P), \sum_{i=1}^{q-1} c'_i(s^{i-1} P)\}^\lambda. \end{aligned}$$

Hence, X can successfully compute and output

$$e(P, P)^{1/s} = \left\{ R / e\{c'_0 P + \sum_{i=0}^{q-1} c'_i(s^i P), \sum_{i=1}^{q-1} c'_i(s^{i-1} P)\}^\lambda \right\}^{c'_0{}^{-2} \lambda^{-1}}$$

as a solution to A_1 's challenge.

Now analyze the advantage of X in this game. The responses to A_1 's H_i for $i \in \{1, 2, 3, 4\}$ queries are indistinguishable from the real life. We note that X only fails in providing a consistent simulation because one of the events E_1, E_2, E_3, E_4, E_5 . Hence, if none of the events E_1, E_2, E_3, E_4 and E_5 happens, X can solve the q -BDHIP successfully.

Now we assess the probability for these events. From the description above, we have $\Pr[E_1] = 1/q_{h1}$, we can observe that $\neg E_5$ means $\neg E_2$ and we also know that

$$\Pr[E_3] \leq q_{sc}(q_{sc} + q_{h4}) / 2^l, \Pr[E_4] \leq q_{us} / 2^l \text{ and } \Pr[\neg E_5] = 1/q_{h1}.$$

So, we can compute as follows:

$$\Pr(\neg E_1 \wedge \neg E_3 \wedge \neg E_4 \wedge \neg E_5) \geq \left(\frac{q_{h_1}-1}{q_{h_1}}\right) \left(1 - q_{sc} \frac{q_{sc} + q_{h_4}}{2^l}\right) \left(1 - \frac{q_{us}}{2^l}\right) \frac{1}{q_{h_1}}.$$

We obtain the announced bound by noting that X fetches the correct element from list H_3^{list} or H_4^{list} with probability $1/(q_{h_3} + 2q_{h_4})$.

Taking the above analysis on these events, we know that challenger X's advantage

$$\varepsilon' \geq \frac{\varepsilon(q_{h_1}-1)}{q_{h_1}^2(q_{h_3} + 2q_{h_4})} \left(1 - q_{sc} \frac{q_{sc} + q_{h_4}}{2^l}\right) \left(1 - \frac{q_{us}}{2^l}\right).$$

From the above description of X, we can conclude that the running time of X is bound by

$$t' < t + (O(q^2) + (3q + 3q_{sc} + 2q_{us}))t_{sm} + (q_{h_4} + 2)t_{exp} + (4q_{us} + 1)t_{pr}.$$

Lemma 2. Assume that an IND-CLSC-CCA2 adversary A_2 has an advantage ε against our scheme when running in time t , asking q_{hi} queries to random oracles H_i ($i = 1, 2, 3, 4$), q_{sc} queries to signcryption oracle and q_{us} queries to the unsigncryption oracle. Then there is an algorithm X to solve the *MBDHIP* with probability

$$\varepsilon' \geq \frac{\varepsilon}{q_{h_1}(q_{h_3} + 2q_{h_4})} \left(1 - q_{sc} \frac{q_{sc} + q_{h_4}}{2^l}\right) \left(1 - \frac{q_{us}}{2^l}\right)$$

within a time $t' < t + (3q_{h_1} + 3q_{sc} + q_{us})t_{sm} + (q_{h_4} + q_{sc} + q_{us})t_{exp} + 2q_{us} \cdot t_{pr}$, where t_{sm} , t_{exp} and t_{pr} also denote respectively the costs of a scalar multiplication in G_1 , an exponentiation in G_2 and a pairing computation.

Proof. Suppose X is given a challenge of a random instance of the *MBDHIP*:

Given $h_{2l} \in Z_p^*$, P , aP and $(a+h_{2l})P \in G_1$, where a is unknown to X. X's goal is to output $e(P, P)^{1/(a+h_{2l})}$ by interacting with adversary A_2 .

Setup: X runs algorithm Setup, randomly picks a value $s \in Z_p^*$ as the system master key, sets $g = e(P, P)$ and $P_0 = sP$, and picks an identity ID_i at random as the challenge ID in this game, and gives *params* $\{P, g, P_0, H_1, H_2, H_3, H_4\}$ and the system master key s to A_2 .

H_1 -Queries: X maintains a hash list H_1^{list} of tuple (ID_i, Q_i) as explained below. The list is initially empty. When A_2 makes a hash oracle query on ID_i , if the query ID_i has already appeared on the H_1^{list} , then the previously defined value is returned. Otherwise, X randomly picks a value $Q_i \in Z_p^*$ and returns it to A_2 . Then, adds (ID_i, Q_i) to H_1^{list} .

Public Key Extraction Queries: X maintains a list pk^{list} of tuple (ID_i, Q_i, pk_i, x_i) which is initially empty. When A_2 queries on input ID_i , X checks whether pk^{list} contains a tuple for this input. If it does, the previously defined value is returned. Otherwise, X works as follows:

- If $ID_i = ID_i$, X finds the tuple (ID_i, Q_i) in H_1^{list} and sets $pk_i = saP + Q_i aP$ and sends pk_i to A_2 , and inserts $(ID_i, Q_i, pk_i, \perp)$ into pk^{list} .

- Otherwise, X recovers the tuple (ID_i, Q_i) from H_1^{list} and picks a random $x_i \in Z_p^*$, computes $pk_i = x_i(P_0 + Q_i P)$ and returns pk_i and adds (ID_i, Q_i, pk_i, x_i) into the pk^{list} .

H_2 -Queries ; For query on input ID_i and pk_i , X picks a random $h_{2i} \in Z_p^*$ and returns it to A_2 , then adds tuple (ID_i, pk_i, h_{2i}) to the list H_2^{list} .

Private Key Extraction Queries: For query on input ID_i , If $ID_i = ID_i$, X stops and outputs "failure". Otherwise, X performs as follows:

- If the lists pk^{list} and H_2^{list} contain the corresponding tuple (ID_i, Q_i, pk_i, x_i) and (ID_i, pk_i, h_{2i}) respectively, X sets $sk_i = \frac{P}{(x_i + h_{2i})(s + Q_i)}$ and

sends it to A_2 .

- Otherwise, X makes public key extraction query and H_2 query on ID_i , then simulates as the above process and sends $sk_i = \frac{P}{(x_i + h_{2i})(s + Q_i)}$ to A_2 .

H_3 -Queries: For an input $R \in G_2$, X returns the defined value if it exists and a random $h_3 \in Z_p^*$ otherwise, and then inserts (R, h_3) into the list H_3^{list} .

H_4 -Queries: For an input (m, R) with identities ID_u / ID_v of sender/recipient, X answers the defined value if it exists and a random $h_4 \in Z_p^*$ otherwise. Additionally, X recovers the list H_3^{list} and finds $h_3 = H_3(R) \in Z_p^*$ (If h_3 does not exist, X simulates H_3 oracle on its own to obtain $h_3 = H_3(R) \in Z_p^*$) and inserts the elements $(m, R, h_4, c, \delta = R \cdot e(P, P)^{h_4})$ into the list H_4^{list} .

Signcrypt Queries: When A_2 asking a signcrypt query on a message m for sender/recipient with identities ID_u / ID_v for $u, v \in \{1, \dots, q_{h_i}\}$ and corresponding public key pk_u / pk_v , X acts as follows:

If $ID_u = ID_I$, X does not know the sender's private key sk_u , but he knows the recipient's private key sk_v since $ID_u \neq ID_I$. X randomly chooses $\varphi \in Z_p^*$ and $\psi \in Z_p^*$ and computes $U = \varphi(pk_I + h_{2I}T_I) - \psi(pk_v + h_{2v}T_v)$, $S = \varphi \cdot sk_v$, where $T_i = P_0 + Q_i P (i \in \{I, v\})$ and backpatches to define the value $H_4(m, R)$ as ψ (X aborts in the unlikely event that $H_4(m, R)$ is already defined). The ciphertext is $\sigma = (c = E_{h_3}(m \| Q_I), U, S)$.

- Otherwise, X knows the sender's private key and can answer the query according to the specification of signcrypt algorithm.

Unsigncrypt Queries: Suppose the request is on a ciphertext $\sigma = (c, U, S)$ for identities (ID_u, ID_v) for $u, v \in \{1, \dots, q_{h_i}\}$ and with corresponding public key (pk_u, pk_v) . X acts as follows:

- If $ID_v = ID_I$, X has not the recipient's private key sk_v , but he knows the sender's private key sk_u since $ID_u \neq ID_I$. Obviously, the equation $e(U, sk_u) = e(pk_I + h_{2I}T_I, S - h_4 sk_u)$ holds, where $h_4 = H_4(m, R)$ is the hash value obtained in the signcrypt algorithm. X computes $\delta = e(pk_u + h_{2u}T_u, S)$ and recovers the tuple (m, R, h_4, c, δ) from H_4^{list} (If this tuple does not exist, $\sigma = (c, U, S)$ is rejected) and further examines whether the following equality holds: $e(U, sk_u) / e(pk_I + h_{2I}T_I, S) = e(pk_I + h_{2I}T_I, -h_4 sk_u)$.

If the equation holds, X outputs the message-signature pair $(m, (h_4, S))$, otherwise, $\sigma = (c, U, S)$ is rejected.

- If $ID_v \neq ID_I$, X knows the recipient's private key and can normally run the unsigncrypt algorithm.

Finally, A_2 outputs messages (m_0, m_1) and identities (ID_S^*, ID_R^*) with public keys (pk_S^*, pk_R^*) . Here ID_R^* may not have been submitted to Private-Key-Extract query.

If $ID_R^* \neq ID_I$, X outputs “failure” and aborts. Otherwise, X chooses $\lambda \in Z_p^*$ and computes $U^* = \lambda(P_0 + Q_I P)$, picks randomly $c^* \in \mathcal{C}$ ($\mathcal{C}$ denotes the ciphertext space) and $S^* \in G_1$, and returns the challenge $\sigma^* = (c^*, S^*, U^*)$. Here if we define $\lambda = r(a + h_{2I})$ for $r \in Z_p^*$ and we can check that

$$U^* = \lambda(P_0 + Q_I P) = r(a + h_{2I})(P_0 + Q_I P) = r(pk_I + h_{2I}T_I).$$

A_2 then makes a new series of queries but is not permitted to ask for the unsigncrypt oracle of $\sigma^* = (c^*, S^*, U^*)$ for the combination (ID_S^*, ID_R^*) and (pk_S^*, pk_R^*) . Moreover no private key exposure query can be made on ID_R^* at any time.

To ignore A_2 's eventual output, X fetches tuple (R, h_3) or (m, R, h_4, c, δ) from list H_3^{list} or H_4^{list} , the chosen tuple will contain the right element $R = e(P, P)^r = e(P, P)^{\lambda/(a+h_{2I})}$. Hence, X can successfully compute $e(P, P)^{1/(a+h_{2I})} = R^{\lambda^{-1}}$ and output $R^{\lambda^{-1}}$ as a solution to A_2 's challenge. So, X breaks *MBDHIP*. The analysis of X's advantage is similar to that of Lemma 1. This completes my proof.

Theorem 2. In the random oracle model, the proposed CLSC scheme is existentially unforgeable against adaptive chosen messages attacks (EUF-CLSC-CMA2) under the *k*-CAA and *MCDHIP* assumptions.

The theorem follows from Lemma 3 and lemma 4.

Lemma 3. In the random oracle model, the CLSC scheme is secure against any EUF-CLSC-CMA2 adversary A_1 if the *k*-CAA problem is hard in G_1 .

Proof. Suppose that X receives a challenge: Given $\{P, sP, e_1, e_2, \dots, e_k \in Z_p^*,$ and $\frac{1}{s+e_1}P, \frac{1}{s+e_2}P, \dots, \frac{1}{s+e_k}P\}$, for an unknown value $s \in Z_p^*$, X's task is to compute $\frac{1}{s+e_0}P$ for some $e_0 \notin \{e_1, e_2, \dots, e_k\}$. X will interact with A_1 as follows:

Setup: X sets $g = e(P, P)$ and $P_0 = sP$, and picks an identity ID_I at random as the challenge ID in this game, and gives *params* $\{P, g, P_0, H_1, H_2, H_3, H_4\}$ to A_1 .

H_1 -Queries: X maintains a hash list H_1^{list} of tuple (ID_i, Q_i) $i \in \{1, \dots, q_h\}$ as explained below. The list is initially empty. When A_1 makes a hash oracle query on ID_i , X returns the previously defined value if it exists. Otherwise, X acts as follows: If $ID_i = ID_I$, X sets $Q_i = e_0$ and returns Q_i to A_1 . Otherwise, X answers $Q_i = e_i$ for $e_i \in \{e_1, e_2, \dots, e_k\}$. In both cases, X inserts (ID_i, Q_i) into H_1^{list} .

-Partial Private Key Extraction Queries, Public Key Extraction Queries, H_2, H_3, H_4 -Queries, Private Key Extraction Queries, Public Key Replacement, Signcrypt Queries and Unsigncrypt Queries: The responses to these queries are similar to the corresponding answers in Lemma 1.

Eventually, A_1 outputs a ciphertext $\sigma^* = (c^*, U^*, S^*)$ for identities (ID_S^*, ID_R^*) with public keys (pk_S^*, pk_R^*) , where the private key of identity ID_S^* was not corrupted. Here note ID_S^* does not have been submitted to both oracles Public-Key-Replacement and Partial-Private-Key-Extract. If $ID_S^* \neq ID_I$, X outputs “failure” and aborts. Otherwise, X computes $R^* = e(U^*, sk_R^*)$ and finds the tuple (m, R^*, h_4^*) in H_4^{list} . Then by replays of X

with the same random tape but different choices of H_4 , X can obtain another valid signature (R^*, h_4^*, S^*) for $h_4^* \neq h_4^*$. Then, we can get

$$S^* = (r^* + h_4^*)sk_s^* = \frac{(r^* + h_4^*)}{(x_s^* + h_2^*)(s + e_0)} P \text{ and}$$

$$S^* = (r^* + h_4^*)sk_s^* = \frac{(r^* + h_4^*)}{(x_s^* + h_2^*)(s + e_0)} P.$$

$$S^* - S^* = \frac{h_4^* - h_4^*}{x_s^* + h_2^*} \cdot \frac{1}{s + e_0} P.$$

As a result, X can output $\frac{1}{s + e_0} P = \frac{(x_s^* + h_2^*)(S^* - S^*)}{h_4^* - h_4^*}$ as a solution to A_1 's

challenge.

Lemma 4. In the random oracle model, the CLSC scheme is secure against any EUF-CLSC-CMA2 adversary A_2 if the *MCDHIP* is hard in G_1 .

Lemma 4 can be formally proven in the same way as that of Lemma 3. So, I omit it here.

4.3. Efficiency Analysis

Here notations Sm , Ex and Pr denote the cost of computing a scalar multiplication in G_1 , an exponentiation in G_2 and one pairing computation, respectively. If we also select the base group G_1 and the bilinear map from elliptic curves [29], which results in a group of 160 bits size. So, the public/private key size of our scheme is about 160 bits, and message expansion is 320 bits length which is the same as that of Zheng's scheme [16].

The hash function maps an identity to an element in G_1 used by these schemes [17, 18, 27] usually requires a MapToPoint operation [30], which is inefficient and probabilistic. Our scheme avoids this problem since it does not require mapping an identity onto a cyclic group. ("Pr*" denotes the pairing can be pre-computed.)

Table 1. Comparison between My Scheme and Other Schemes

CLSC	Signcrypt	Unsigncrypt	Public-Key-Size	Private-Key-Size	Message Expansion
[17]	$4Sm+1Pr+1Ex$	$5Pr+1Sm$	160bits	320bits	320bits
[18]	$4Sm+1Pr+1Ex$	$5Pr+1Sm$	160bits	320bits	320bits
[21]	$2Sm+1Ex$	$4Sm+2Ex+2Pr$	160bits	320bits	480bits
[27]	$3Sm+1Ex$	$2Sm+1Pr+2Pr^*$	320bits	320bits	480bits
My Scheme	$3Sm+1Ex$	$1Ex+1Sm+2Pr$	160bits	160bits	320bits

5. Conclusions

Signcryption is a useful tool and it can achieve confidentiality, authentication, integration and non-repudiation in an efficient and secure manner. This paper proposes a CLSC scheme that is proved to be secure in the random oracle. The proposed scheme satisfies all the current requirements of the signcryption schemes and it achieves significant efficiency and its ciphertext expansion is short such that it can be used in environments with bandwidth constraints.

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