Controlled Quantum Secure Direct Communication Protocol Based on Extended Three-particle GHZ State Decoy

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Abstract

Extended three-particle GHZ state decoy is introduced in controlled quantum secure direct communication to improve eavesdropping detection probability and prevent correlation-elicitation (CE) attack. Each particle of extended three-particle GHZ state decoy is inserted into sending particles to detect eavesdroppers, which reaches 63% eavesdropping detection probability. And decoy particles prevent the receiver from obtaining the correct correlation between particle 1 and particle 2 before the sender coding on them, so that he can not get any secret information without the controller's permission. In the security analysis, the maximum amount of information that a qubit contains is obtained by introducing the entropy theory method, and two decoy strategies are compared quantitatively. If the eavesdroppers intend to eavesdrop on secret information, the per qubit detection rate of using only two particles of extended three-particle GHZ state as decoy is 58%; while the presented protocol using three particles of extended three-particle GHZ state as decoy reaches per qubit 63%.

Keywords: CQSDC; extend three-particle GHZ state; decoy; eavesdropping detection; security

1. Introduction

One of the great advantages of quantum communication is the security. According to the principle of quantum mechanics, legal communicating parties can detect the existence of eavesdroppers efficiently, which ensures the security of communication[1]. Quantum secure direct communication (QSDC) as a branch of quantum communication has attracted increasing attention[1-34]. The most important characteristic of QSDC is to transmit secret information in quantum communication channel by using the quantum no-cloning theorem, the uncertainty principle and the relevance and non-locality of entanglement particles [1]. "The efficient two-step quantum secure direct communication scheme " was proposed by Long Gui-Lu et al. in 2002, which is the first quantum secure direct communication protocol [2]. After that, Deng Fu-Guo et al. put forward two-step quantum secure direct communication scheme based on dense coding, and discussed the standards of QSDC scheme[3]. Yan Feng-Li et al. proposed QSDC scheme based on quantum teleportation [4]. Yang et al. proposed the concept of generalized Bell state and unitary operations on the basis of logical qubits; based on it, a QSDC scheme was developed, which was immune to collective noise [17]. In 2011, Lin et al. put forward a QSDC schema by using five-qubit entangled states.[20] In 2012, Huang et al. put forward a QSDC schema against collective noise with decoy states based on channel-encryption.[27]

Recently, as a distinctive concept, controlled quantum secure direct communication (CQSDC) [5-8] was pursued. Different from QSDC, CQSDC is used to communicate the secret message with the control of the controller, which is always applied to certain special cryptographic tasks. The receiver receives secret message from the sender through quantum channel, however he can not understand

the message without the controller's permission. Wang Jian et al. introduced multiparty controlled quantum secure direct communication protocol (CQSDC); in this scheme, apart from sender and receiver, there is a controller; the sender sends secret information to receiver based on single-photon in quantum communication channel, while the receiver can not restore the information without controller's permission[5]. And then, the security of CQSDC scheme proposed by Wang Jian et al. was researched and analyzed by Wang Tian-Yin et al.; they put forward a new attack based on teleportation ; by this attack ,receiver could obtain secret message without controller's permission [6]. In 2006, a new CQSDC scheme based on three-particle Greenberger-Horne-Zeilinger (GHZ) state was put forward by Wang Jian et al. ,which achieves a high source capacity[7]. Fei Gao et al. proposed correlation-elicitation (CE) attack based on correlation extractability of GHZ state, and pointed that by this attack Wang Jian 's CQSDC scheme based on GHZ state would release 33.3% secret message[8]. Recently, Li Jian et al. introduced decoy idea in QSDC to ensure security, extended three-particle GHZ state[9] was used as decoys, which brings per qubit 58% eavesdropping detection probability .

In this paper, decoy idea is introduced in CQSDC scheme based on extended three-particle GHZ state called DCQSDC scheme. Each particle of extended three-particle GHZ state is used as decoy particle to detect eavesdropping. By using decoy designed in DCQSDC scheme, per qubit 63% eavesdropping detection probability is reached. Decoy particles prevent receiver from obtaining the correct correlation between particle 1 and particle 2 before sender coding on them, so that he can not get any secret information without controller's permission. Therefore, CE attack is avoided efficiently. Comparing with protocol in [9], DCQSDC scheme improves eavesdropping detection probability. By using the DCQSDC protocol, the secret information can be securely transmitted to the receiver without any useful message being leaked out to the potential eavesdroppers.

2. Description of DCQSDC Protocols

Suppose that Bob is the controller, Alice is the sender, and Charlie is the receiver.

Step 1. The preparation stage. Bob and Alice prepare three-particle GHZ states sequences for transmitting secret information and extended three-particle GHZ states for detecting security of quantum communication channel.

S11). **Bob** prepares N ordered GHZ states for transmitting secret information, each of which is in the state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{123}$$

We denote the N three-particle GHZ states by $\{[p_1(S_1), p_1(S_2), p_1(S_3)], [p_2(S_1), p_2(S_2), p_2(S_3)], [p_3(S_1), p_3(S_2), p_3(S_3)] \dots [p_N(S_1), p_N(S_2), p_N(S_3)]\}$, where $p_N(S_1)$ denotes the first particle of the Nth GHZ state, $p_N(S_2)$ denotes the second particle of the Nth GHZ state, $p_N(S_3)$ denotes the third particle of the Nth GHZ state. Sequence composed by $\{p_1(S_1), p_2(S_1), \dots, p_N(S_1)\}$ is called S_1 , sequence composed by $\{p_1(S_2), p_2(S_2), \dots, p_N(S_2)\}$ is called S_2 , sequence composed by $\{p_1(S_3), p_2(S_3), \dots, p_N(S_3)\}$ is called S_3 .

S12). Bob prepares M ordered extended three-particle GHZ states as decoy for detecting eavesdropping when Bob sending particles to Alice, each of which is in the state

$$|\psi_D\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |110\rangle + |111\rangle)_{123}$$

Each extended three-particle GHZ state is called a decoy group, denoted by D_{BA_i} , $i=1,2,\dots,M$, and each particle of D_{BA_i} is called decoy particle, denoted by $D_{BA_{i1}}, D_{BA_{i2}}, D_{BA_{i3}}$, sequence composed by D_{BA_i} is called decoy sequence, denoted by D_{BA} .

S13). According to method in S12) Bob prepares decoy sequence D_{BC} for detecting eavesdropping when Bob sending particles to Charlie.

S14). According to method in S12) Alice prepares decoy sequence D_{AC} for detecting eavesdropping when Alice sending particles to Charlie.

Step 2. The stage of sending messages and detecting eavesdropping.

S21). Bob inserts decoy particles of each group in D_{BA} to S_1 randomly, composing new sequence S_1' . For example, Bob can insert D_{BA} to S_1 like that:

S_1 序列

$$p_1(S_1), p_2(S_1), p_3(S_1), p_4(S_1), p_5(S_1), \dots, p_N(S_1)$$

S_1' 序列

$$p_1(S_1), D_{BA_{11}}, p_2(S_1), D_{BA_{12}}, p_3(S_1), D_{BA_{13}}, p_4(S_1), D_{BA_{21}}, \dots, p_N(S_1)$$

S22). Bob performs Pauli $\{I, \sigma_z, \sigma_x, \sigma_{iy}\}$ operation randomly on each particle in S_2 , generating sequence S_2' . Bob sends S_1' and S_2' to Alice. Bob tells Alice the positions of decoy particles through classical communication channel.

S23). Alice extracts decoy particles according to the positions Bob told her after she receives the particles sequence. Alice reassembles the D_{BA_i} groups by the extracted decoy particles. Alice performs extended three-particle GHZ measurement on each D_{BA_i} group. If there is no eavesdropper, every result should be in the extended three-particle GHZ state. Alice discards decoy particles and recovers S_1 sequence, and then go to the next step. If there is eavesdropper, go back to step1.

S24). Bob inserts decoy particles of each group in D_{BC} to S_3 randomly, composing new sequence S_3' , and sends it to Charlie. The method to insert decoy particles is similar with that in S21). Charlie detects eavesdropping by using method similar with that in S23). If there is no eavesdropper, Bob discards decoy particles and recovers S_3 sequence, then go to the next step. If there is eavesdropper, go back to step1.

S25). Alice encrypts her secure message by randomly performing eight Pauli-pairs operation on S_1 and S_2' sequence, and forms coding sequence C_1 and C_2' . The Pauli-pairs are:

$$\begin{aligned} \{U_1 = \sigma_z \otimes \sigma_z, U_2 = I \otimes \sigma_z, U_3 = i\sigma_y \otimes \sigma_z, U_4 = \sigma_x \otimes \sigma_z, \\ U_5 = I \otimes \sigma_x, U_6 = \sigma_x \otimes \sigma_z, U_7 = \sigma_x \otimes \sigma_x, U_8 = i\sigma_y \otimes \sigma_x\} \end{aligned}$$

S26). Alice inserts decoy particles of each group in D_{AC} to C_1 and C_2' randomly, and sends them to Charlie. The method to insert decoy particles is similar with that in S21). Charlie detects eavesdropping by using method similar with that in S23). If

there is no eavesdropper, Charlie discards decoy particles and recovers C_1 and C_2' , then go to the next step. If there is eavesdropper, go back to step1.

S27). Bob tells Charlie the Pauli operations that he performed on S_2 through classical channel. Charlie performs corresponding Pauli operations on C_2' , and obtained C_2 . Charlie performs GHZ measurement on C_1 , C_2 and S_3 , then gets the information Alice sent to him.

3. Security Analysis

3.1. Analysis of Eavesdropping Detection Probability

Suppose that Eve is eavesdropping When Bob sends particles sequence mixed with decoy particles to Alice, because Eve does not know which particle is decoy particle, he can only performs the same attack operation on all particles. On assumption that the attack operation Eve performs on particles is E , after performing the attack operation E , the original $|0\rangle$ state becomes $|0'\rangle = E \otimes |0x\rangle = \alpha|0x_0\rangle + \beta|1x_1\rangle$; and the original $|1\rangle$ state becomes $|1'\rangle = E \otimes |1x\rangle = m|0y_0\rangle + n|1y_1\rangle$; $|\alpha|^2 + |\beta|^2 = 1$, $|m|^2 + |n|^2 = 1$.

After being attacked by Eve, the state of the composed system becomes:

$$\begin{aligned} |\psi\rangle_{Eve} &= E \otimes E \otimes E \left[\frac{1}{\sqrt{3}} (|0x0x0x\rangle + |1x1x0x\rangle + |1x1x1x\rangle) \right] \\ &= \frac{1}{\sqrt{3}} [(\alpha|0x_0\rangle + \beta|1x_1\rangle) \otimes (\alpha|0x_0\rangle + \beta|1x_1\rangle) \otimes (\alpha|0x_0\rangle + \beta|1x_1\rangle) \\ &\quad + (m|0y_0\rangle + n|1y_1\rangle) \otimes (m|0y_0\rangle + n|1y_1\rangle) \otimes (\alpha|0x_0\rangle + \beta|1x_1\rangle) \\ &\quad + (m|0y_0\rangle + n|1y_1\rangle) \otimes (m|0y_0\rangle + n|1y_1\rangle) \otimes (m|0y_0\rangle + n|1y_1\rangle)] \\ &= \frac{1}{\sqrt{3}} (\alpha^3|0x_00x_00x_0\rangle + \alpha^2\beta|0x_00x_01x_1\rangle + \alpha\beta\alpha|0x_01x_10x_0\rangle + \alpha\beta^2|0x_01x_11x_1\rangle \\ &\quad + \beta\alpha^2|1x_10x_00x_0\rangle + \beta\alpha\beta|1x_10x_01x_1\rangle + \beta^2\alpha|1x_11x_10x_0\rangle + \beta^3|1x_11x_11x_1\rangle \\ &\quad + m\alpha\alpha|0y_00y_00x_0\rangle + m\alpha\beta|0y_00y_01x_1\rangle + m\alpha\alpha|0y_01y_10x_0\rangle + m\alpha\beta|0y_01y_11x_1\rangle \\ &\quad + m\alpha\alpha|1y_10y_00x_0\rangle + m\alpha\beta|1y_10y_01x_1\rangle + m\alpha\alpha|1y_11y_10x_0\rangle + m\alpha\beta|1y_11y_11x_1\rangle \\ &\quad + m^3|0y_00y_00y_0\rangle + m^2n|0y_00y_01y_1\rangle + mn\alpha|0y_01y_10y_0\rangle + mn^2|0y_01y_11y_1\rangle \\ &\quad + nm^2|1y_10y_00y_0\rangle + nm\alpha|1y_10y_01y_1\rangle + n^2m|1y_11y_10y_0\rangle + n^3|1y_11y_11y_1\rangle) \end{aligned}$$

If the extended three-particle GHZ measurement is performed on the decoy particles, the probability that Alice gets correct measurement results is just the probability without eavesdroppers, which can be calculated as:

$$p(|\psi_E\rangle) = \frac{1}{3} (|\alpha^3|^2 + |m^2\alpha|^2 + |m^3|^2 + |\beta^2\alpha|^2 + |n^2\alpha|^2 + |n^2m|^2 + |\beta^3|^2 + |n^2\beta|^2 + |n^3|^2)$$

Suppose $|\alpha|^2 = a, |\beta|^2 = b, |m|^2 = s, |n|^2 = t$, because $a + b = 1, s + t = 1$, the lower bound of the probability that every qubit is eavesdropped on and is detected is:

$$p(|\psi_E\rangle) = \frac{1}{3} (a^3 + a(1-t)^2 + (1-t)^3 + (1-a)^2a + t^2a + t^2(1-t) + (1-a)^3 + t^2(1-a) + t^3)$$

$$= \frac{1}{3}(a^3 - a - 2at + at^2 + 2 - 3t + 5t^2 + a^2 - t^3)$$

It can be considered that the Shannon entropy of a binary channel is equal to the maximum amount of information that a qubit contains. If Bob sends a particle in state $|0\rangle$ to Alice, then the maximum amount of information containing in the $|0\rangle$ state qubit is:

$$I_{|0\rangle} = -a \log_2 a - (1-a) \log_2 (1-a) = H(a)$$

If Bob sends a particle in state $|1\rangle$ to Alice, then the maximum amount of information containing in the $|1\rangle$ state qubit is:

$$I_{|1\rangle} = -t \log_2 t - (1-t) \log_2 (1-t) = H(t)$$

Because the particle that Bob sends to Alice is considered in either of the states $|0\rangle$ or $|1\rangle$ with equal probability $p=0.5$, we can get $a=t$. So for a qubit, the total amount of information that will be eavesdropped by Eve is:

$$I = I_{|0\rangle} + I_{|1\rangle} = \frac{1}{2}[H(a) + H(t)] = H(a)$$

$$\text{Because } a=t, \text{ then } p(|\psi_E\rangle) = \frac{1}{3}(a^3 + 4a^2 - 4a + 2)$$

$$d_{low} = 1 - p(|\psi_E\rangle) = 1 - \frac{1}{3}(a^3 + 4a^2 - 4a + 2)$$

When $I=1$, $d_{low}=0.63$. That is, the detection probability is more than 63%, when Eve intends to eavesdrop on the information contained in a qubit.

In lecture [9] only two particles of extended three-particle GHZ state are used as decoy, which brought per qubit 58% eavesdropping detection probability. However in this DCQSDC scheme, three particles of extended three-particle GHZ state are used to design decoy, which reaches per qubit 63% eavesdropping detection probability.

3.2 CE Attack

In lecture [8], CQSDC scheme based on GHZ state is pointed out existing CE attack. That is, if Charlie can obtain the correlation between particle 1 and 2 before Alice's coding operations, he can get 33.3% information about this secret without Bob's permission. In this DCQSDC scheme, Bob inserts decoy particles to sequence S_1 randomly before sending it to Alice. Because Charlie does not know which particle is the decoy particle, he can not get the correlation between particle 1 and 2 before Alice's coding operations, he consequently can not obtain the secret information without Bob's permission. Furthermore, each particle's sending accompanied by decoy particles, and the eavesdropping detection probability of per qubit is more than 63%, so Charlie will be easily found if he eavesdrops.

4. Conclusions

In the DCQSDC protocol presented in this paper, each particle of extended three-particle GHZ state is used as decoy particle to detect eavesdroppers, which reaches per qubit 63% eavesdropping detection probability. At the same time randomly introducing decoy particle in CQSDC scheme prevents the secret information from CE attack efficiently. Comparing with protocol in [9], DCQSDC scheme improves eavesdropping detection probability. By using the DCQSDC protocol, the secret

information can be securely transmitted to the receiver without any useful message being leaked out to the potential eavesdroppers.

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