

Optimization Control for Space Manipulator Motion Reliability Considering Multi-Index and Multi-Factor

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Abstract

In order to improve the motion reliability of space manipulator during its long time service on-orbit, multiple indexes which can reflect the motion performance and multiple factors (control variables of manipulator) which have influence on motion reliability are involved in establishing an optimization control model. Thus the problem of motion reliability improvement is expressed as an multiple-objective problem (MOP). In order to solve the MOP, dimension reduction strategy is proposed based on covariance matrix of motion reliability indexes. As a result, the MOP is turned into single objective problem. Then control variables are optimized based on null space of space manipulator, and the best weight matrix can be obtained when the objective is minimal. Simulations are carried out to verify the feasibility and effectiveness of the optimization control strategy. By decreasing the error introduction into the control variables, the motion reliability can be effectively improved.

Keywords: *Space manipulator; Motion reliability; Multi-factor and multi-index*

1. Introduction

Space manipulator [1-3] has been widely used in space station assembling and space exploration. The motion status is monitored during its long time service on-orbit to check whether its performance is adaptable for various operations. In order to evaluate the motion performance of space manipulator, motion reliability[4, 5] is introduced as the reflection of motion status and as the basis for space manipulator optimization control.

The optimization control which aims at improving the motion reliability of space manipulator by adjusting control variables rely on motion reliability evaluation[6, 7]. Many scholars have devoted in the research of reliability evaluation methods. Pandey[8] takes position accuracy as index and considers the factor of clearance to evaluate the motion reliability of manipulator, while Rao[9] analyzes reliability of manipulator by dividing reliability as kinematics and dynamics reliability. In the former work of the author[10], kinematics reliability which only concerns with the pose accuracy of manipulator is discussed. However, single index for motion reliability representation is not enough to reflect the properties of various tasks on-orbit of space manipulator. Multiple indexes should be considered during motion reliability evaluation. Meanwhile, considering the complexity of space environment and space manipulator structure, multi-factor should be involved in reliability analysis.

Thus, the optimization control for motion reliability is represented as MOP considering multiple factors. For solving the MOP, many kinds of multi-objective optimization algorithm (MOA) have been researched, such as MOPSO[11], NSGA-II[12], SPEA2[13]and ant colony algorithm(ACO)[14]. With MOA methods, Pareto solutions can be obtained as a set of optimal solutions. For MOA, there does not exist an unique optimal solution. However, for practical application, it need to choose an

unique solution as the input of control system. Besides, it has huge computation cost with MOA, which is not able to be used in real-time control on-orbit. So in this paper the coupling relationships between indexes and factors is considered and a strategy of dimension reduction is proposed to turn multi-objective problem into single objective problem.

The paper aims at proposing an optimization control strategy for space manipulator motion reliability which considers multi-index and multi-factor, and obtaining the optimal solution via dimension reduction strategy of MOP and null space optimization of control variables. The paper is consist of the following sections. In section 2, the multi-index representation of space manipulator motion reliability is derived. In section 3, optimization control model is established considering multi-factor and multi-index. In section 4, the method for solving the control model is proposed. And the last section is simulation verification.

2. Multi-Index Representation of Space Manipulator Motion Reliability

When considering k motion reliability indexes, each index can be defined as f_1, f_2, \dots, f_k , where ω_i represents the weight for motion reliability of i^{th} index. p_1, p_2, \dots, p_n represents factors having influence on motion reliability, then the index function can be defined as $f_i = f_i(p_1, p_2, \dots, p_n), i=1, \dots, k$, l_1, l_2, \dots, l_k represents the thresholds of indexes. For space manipulator path planning task with s control cycles, the mathematical description of the motion reliability which only considers the index f_i can be represented as:

$$R_i = \Pr \left\{ \bigcap_{j=1}^s \left(\|f_i(p_1, \dots, p_n)\|_j < l_i \right) \right\} \quad (1)$$

When considering multi indexes f_1, f_2, \dots, f_k , the motion reliability can be represented as:

$$R = \prod_{i=1}^k R_i \quad (2)$$

Combining equation (1) with (2), the mathematical description of the motion reliability considering multi-index can be represented as:

$$R = \prod_{i=1}^k \Pr \left\{ \bigcap_{j=1}^s \left(\|f_i(p_1, \dots, p_n)\|_j < l_i \right) \right\} \quad (3)$$

Where, $\|f_i(p_1, \dots, p_n)\|_j$ represents the value of index f_i in the j^{th} control cycle. Equation (3) represents the intersection of the probability when indexes of motion reliability guarantee the thresholds in each control cycle under the influence of multiple factors. In another way, the motion reliability of space manipulator reflects the statistical law of indexes meeting the thresholds. Via optimization control for motion reliability, each index can be optimized and the probability that each index guarantees the threshold can be improved.

3. Optimization Control Model Considering Multi-Index and Multi-Factor

In order to improve the probability of each index guaranteeing the threshold, the value of each index should be minimized. Thus optimization control of motion reliability aims at minimizing each index, and the control model can be expressed as a MOP problem. In order to derive the representation of control model, the mathematical equations of the constraints and objectives need to be derived respectively.

3.1. Constraints of Optimization Control Model

The constraints mainly mean to guarantee the task completion or limit the parameters of space manipulator into a safe region. For time t during task, in order to limit the parameters of space manipulator, inequality constraints are established, during which constraints of joint angle are expressed as:

$$\begin{aligned} g_1^i(\theta) &= \theta_i - \theta_i^{\max} \\ g_2^i(\theta) &= \theta_i^{\min} - \theta_i \end{aligned} \quad (4)$$

Constraints of joint angular can be expressed as:

$$\begin{aligned} g_3^i(\dot{\theta}) &= \dot{\theta}_i - \dot{\theta}_i^{\max} \\ g_4^i(\dot{\theta}) &= \dot{\theta}_i^{\min} - \dot{\theta}_i \end{aligned} \quad (5)$$

Constraints of joint torque can be expressed as:

$$g_5^i(\tau) = \tau_i - \tau_i^r \quad (6)$$

Wherein, i represents the i^{th} joint of space manipulator. $\theta, \dot{\theta}$ represent joint angle and joint velocity. τ represents joint torque. Superscripts min and max represent the minimum and maximum value of the feasible region of joint variables. τ_i^r represents rated torque of joint i .

In order to guarantee task completion, equality constraints are derived according to different on-orbit operations. When transferring space manipulator with no load, the constraint of guaranteeing task completion mean to track the trajectory during the entire task cycle.

$$h_1(\theta, \dot{\theta}) = |P^a - P^n| \quad (7)$$

When operating space manipulator with load, the force and torque in the operation space should be guaranteed, which is taken as the other task constraint.

$$h_2(\theta, \dot{\theta}, \tau) = |F^a - F^n| \quad (8)$$

Wherein, P, F represent position and force vector in the operation space. And superscript a, n represent their actual value and nominal value, respectively.

3.2. Objective of Optimization Control Model

For space manipulator on-orbit operations, position, velocity and torque in the operation space of space manipulator are the most important parameters considered for space manipulator control^[15]. Thus, position accuracy ΔP , velocity accuracy ΔV , and force accuracy ΔF in the operation space of space manipulator are taken as indexes of motion reliability. The mathematical representations of the indexes are shown as follows:

a. Index of position accuracy in operation space

$$\begin{aligned} \Delta P(t) &= P^a(t) - P^n(t) \\ &= Kin(\theta^a(t)) - Kin(\theta^n(t)) \\ &= Kin(\theta^n(t) + \Delta\theta(t)) - Kin(\theta^n(t)) \end{aligned} \quad (9)$$

The index of position accuracy represents position deviation between the actual and nominal value during the task cycle of space manipulator (namely $t \in [t_0, t_e]$, t_0, t_e represent the initial and terminal time of task, respectively). The index is a function which taking time as the independent variable. And $Kin(\cdot)$ represents positive kinematics function of space manipulator, $\theta^a(t)$ and $\theta^n(t)$ represent actual joint angle and nominal joint angle at moment t . Taking Taylor expansion on equation (9) and reserve the first order term, then

$$\begin{aligned} \Delta P(t) &= Kin(\theta^n(t)) + Kin'(\theta^n(t)) \cdot \Delta\theta(t) - Kin(\theta^n(t)) \\ &= Kin'(\theta^n(t)) \cdot \Delta\theta(t) \end{aligned} \quad (10)$$

b. Index of velocity accuracy in operation space

$$\Delta V(t) = V^a(t) - V^n(t) = J^{-1} \cdot \Delta \dot{\theta}(t) \quad (11)$$

The index of velocity accuracy represents velocity deviation between the actual and nominal velocity during the task cycle of space manipulator.

c. Index of torque accuracy in operation space

$$\Delta F(t) = F^a(t) - F^n(t) = J^T \cdot \Delta \tau(t) \quad (12)$$

The index of torque accuracy indicates the deviation between the actual and nominal value during the task cycle of space manipulator. According to equations (10)-(12), the linear relationships between three mentioned indexes and control variable joint angle $\theta(t)$, joint velocity $\dot{\theta}(t)$, joint torque $\tau(t)$ are established, respectively. Via adjusting the error introduction into the control variables, the indexes can be optimized.

Combining with equations (10)-(12), the optimization control model of motion reliability can be expressed as:

$$\begin{aligned} \min \quad & Z(\theta, \dot{\theta}, \tau) = [\Delta P, \Delta V, \Delta F]^T \\ \text{s.t.} \quad & g_j^i(\theta, \dot{\theta}, \tau) \leq 0 \\ & h_k(\theta, \dot{\theta}, \tau) < \varepsilon_k \\ & i = 1, \dots, n; j = 1, \dots, 5; k = 1, 2. \end{aligned} \quad (13)$$

Where ε_k represents the threshold of P and F .

4. Calculation for Optimization Control Model of Motion Reliability

In order to solve the MOP in equation (13), although it can be solved by MOA such as MOPSO to obtain the Pareto front, how to choose the unique solution which conforms to the practical requirement need to be discussed. Besides, it brings about high computation cost in solving the MOP with many times' iterations, and usually falls into local optimal solution. Thus via decoupling and carrying out dimensionless on the relationships between motion reliability indexes, a dimension reduction method based on covariance matrix is proposed in this section, with which the multiple indexes can be turned into a single objective. Then by optimizing errors during control variables, the single objective can be successfully minimized.

4.1. The Dimension Reduction Strategy of Multi-Objective Problem Based On Covariance Matrix

Aiming at space manipulator path planning tasks with s control cycles, the set of each index $\Delta P, \Delta V, \Delta F$ can be obtained as $\{\Delta P\}_s, \{\Delta V\}_s, \{\Delta F\}_s$ during task cycle. The mean value of each set can be expressed as

$$\Delta \bar{P} = \frac{\sum_{i=1}^s \Delta P_i}{s}, \quad \Delta \bar{V} = \frac{\sum_{i=1}^s \Delta V_i}{s}, \quad \Delta \bar{F} = \frac{\sum_{i=1}^s \Delta F_i}{s} \quad (14)$$

Average processing is an approach to implement the data dimensionless. Based on the mean value of ΔP and ΔV , their covariance can be expressed as

$$\text{Cov}(\Delta P, \Delta V) = \frac{\sum_{i=1}^s (\Delta P_i - \Delta \bar{P})(\Delta V_i - \Delta \bar{V})}{s-1} \quad (15)$$

In equation (15), $s-1$ is determined as denominator instead of s , which is called unbiased estimate. In the same way, the covariance between arbitrary two indexes can be obtained. Then the covariance matrix of three motion reliability indexes can be expressed as

$$C = \begin{bmatrix} Cov(\Delta P, \Delta P) & Cov(\Delta P, \Delta V) & Cov(\Delta P, \Delta F) \\ Cov(\Delta V, \Delta P) & Cov(\Delta V, \Delta V) & Cov(\Delta V, \Delta F) \\ Cov(\Delta F, \Delta P) & Cov(\Delta F, \Delta V) & Cov(\Delta F, \Delta F) \end{bmatrix} \quad (16)$$

Via diagonalizing covariance matrix C , the relationships between motion reliability indexes can be decoupled. Because covariance matrix is a symmetric matrix, there must exist an orthogonal matrix M which meets $M^T C M = A$. Wherein, A means diagonalizable matrix of covariance matrix C and M means the eigenvector matrix of covariance matrix C . Then the MOP can be turned into single objective problem:

$$Z = |M \cdot [\Delta P \quad \Delta V \quad \Delta F]^T| \quad (17)$$

So far, during the process of obtaining the covariance matrix of motion reliability indexes, dimensionless is achieved for each index. Then based on the eigenvector matrix of covariance matrix which is obtained via diagonalizing covariance matrix, the motion reliability indexes are decoupled. As a result, the MOP of space manipulator motion reliability optimization is transferred into single objective problem.

4.2. Objective Solution Based on Null Space Optimization

In order to achieve minimization of the objective shown in (17), it need to find the optimal control variables in (10)-(12). Since the control variables such as joint angle, joint velocity and joint torque have strong coupling relationships, for joint i , it has:

$$\begin{aligned} \Delta \dot{\theta}_i(t + \Delta t) &= \frac{\Delta \theta_i(t + \Delta t) - \Delta \theta_i(t)}{\Delta t} \\ \Delta \tau_i(t + \Delta t) &= I_i' \cdot \frac{\Delta \dot{\theta}_i(t + \Delta t) - \Delta \dot{\theta}_i(t)}{\Delta t} \end{aligned} \quad (18)$$

I_i' represents rotational inertia of joint i . In practical applications, errors of joint angle are comprehensively influenced by assembling errors, transmission errors, frictional wear, and so on. Kinds of errors influence joint angle indirectly via errors of joint velocity and torque. Here the function which represents errors of joint angle is defined as:

$$\Delta \theta_i = v(\theta_i) \quad (19)$$

Wherein, θ_i represents the angle of joint i , and $\Delta \theta_i$ represents the error at the current angle. Thus when joint moves to θ_i , joint angle obeys normal distribution $\theta_i \sim (\theta_i, \Delta \theta_i)$. During the feasible region $\theta_i \in [\theta_i^{\min}, \theta_i^{\max}]$, there exists $\tilde{\theta}_i$, making $\Delta \theta_i$ minimal, namely $\tilde{\theta}_i = v^{-1}(\Delta \theta_i^{\min})$. Thus the vector $\tilde{\theta} = [\tilde{\theta}_1 \quad \dots \quad \tilde{\theta}_i \quad \dots \quad \tilde{\theta}_n]^T$ can be constructed considering each joint, and the optimizing index can be established as:

$$z_\theta = \frac{1}{2} (\theta - \tilde{\theta})^T W^T W (\theta - \tilde{\theta}) \quad (20)$$

In equation (20), W means the diagonal weight matrix. With the optimizing index, the optimal vector for joint angle can be defined as $h(t) = -\nabla z_\theta(t) = -W(\theta(t) - \tilde{\theta})$. Then during the control cycle Δt , the variation of joint angle can be expressed as:

$$\Delta \theta(t) = J^{-1} V(t) + (I - J^T J) h(t) \quad (21)$$

Based on equation (21), equations (10)-(12) can be transformed as:

$$\begin{aligned} \Delta P(t) &= Kin'(\theta^n) \cdot \Delta \theta(t) \\ \Delta V(t) &= J^{-1} \cdot \frac{\Delta \theta(t) - \Delta \theta(t - \Delta t)}{\Delta t} \\ \Delta F(t) &= J^T \cdot I^r \cdot \frac{\Delta \dot{\theta}(t) - \Delta \dot{\theta}(t - \Delta t)}{\Delta t} \\ t &\in [t_0 + \Delta t, t_e] \end{aligned} \tag{22}$$

Where $\Delta \theta = [\Delta \theta_1, \dots, \Delta \theta_n]^T$; $\Delta \dot{\theta} = [\Delta \dot{\theta}_1, \dots, \Delta \dot{\theta}_n]^T$; $I^r = [I_1^r, \dots, I_n^r]^T$. $\Delta \theta$ and $\Delta \dot{\theta}$ can be obtained by equations (18) and (21).

So far, equation (17) is transformed into the optimization problem which taking single control variable θ as decision variable and single index Z as optimization objective. By the given search region of the elements in W , a sequence of joint angles can be determined to achieve the minimization of optimization objective.

5. Simulations Verification

5.1. Research Object

This paper takes 7 degrees of freedom (DOF) manipulator as a research object. The coordinate system of the manipulator is shown in figure 1, while the DH parameters are listed in Table 1.

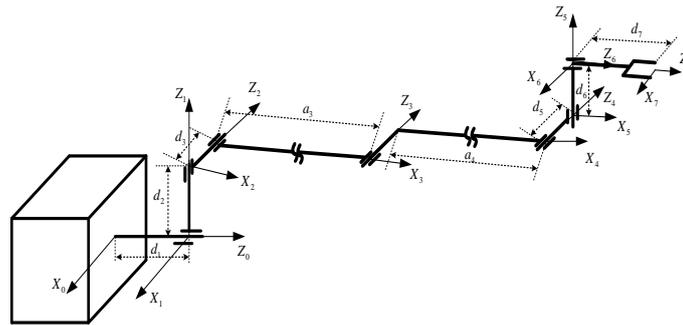


Figure 1. Coordinate Systems of 7-DOF Manipulator

Table 1 DH Parameters of Manipulator

i	$\theta_i(^{\circ})$	$d_i(\text{m})$	$a_{i-1}(\text{m})$	$\alpha_{i-1}(^{\circ})$
1	0	0.6	0	90
2	90	0.5	0	-90
3	0	0.5	5	0
4	0	0.5	5	0
5	0	0.5	0	90
6	-90	0.5	0	-90
7	0	0.6	0	0

5.2. Objective Function Minimization and Optimal Coefficient Calculation

Carry out space manipulator path planning task, and set the initial configuration and target pose as $[-50, -170, 150, -60, 130, 170, 0]^{\circ}$ and $[9.6\text{m}, 0\text{m}, 3\text{m}, -1\text{rad}, -0.5\text{rad}, -2\text{rad}]$, respectively. The execute time of task is 20s, acceleration time is 5s, and control cycle is 50ms. Assume the function between joint angle and joint angle error as:

$$v(\theta_i) = \frac{0.03}{270^2} \theta_i^2 + 0.005 \tag{23}$$

i represents the i^{th} joint of manipulator. Assume errors introduced into each joint of manipulator obey the same error function. And the joint angle related to the minimal error introduction is $\tilde{\theta}_i = 0$. Then assume the elements of diagonal matrix are the same, which is represented as w , thus $W = w \cdot I$, where I is identity matrix. At this condition, the optimal weight for each joint is the same. Then w can be changed to find the minimal value of objective function. Since it will bring pose error to the end-effector of manipulator when introducing the optimal vector into path planning task, the coefficient w should be limited, making the actual position guarantee the position accuracy threshold. Set the maximal threshold of position accuracy as 0.05m, then the feasible region of w is obtained as $[-3e^{-5} \ 3e^{-5}]$. And the corresponding range of objective function can be obtained as the black region in Figure 2.

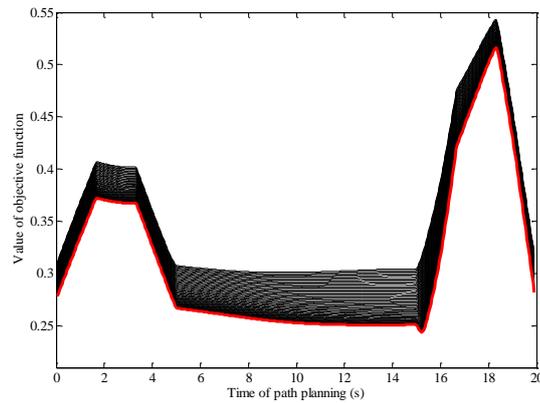


Figure 2. Region of Objective Function Corresponding To the Feasible Region Of w

In Figure 2, the value of objective function is a result contributed by three sub-objectives, and the value changes with the path planning task executes. The minimal value at each control cycle is lightened with red color in Figure 2. It can be known that the optimal coefficient related to each control cycle is not constant. The law of optimal coefficient during the entire task is shown in Figure 3.

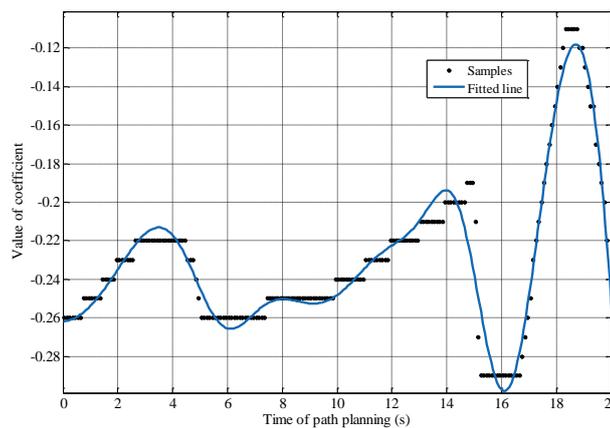


Figure 3. The Optimal Coefficient during the Entire Task

Based on six order Gaussian fitting, the function of optimal coefficient can be fitted as:

$$w = \sum_{i=1}^6 a_i \exp\left(-\left(x - b_i\right) / c_i\right)^2 \quad (24)$$

Wherein, $a = [-0.2586, -0.06748, -0.6629, 0.1702, -0.2419, -0.3499]^T$;
 $b = [-0.1949, 5.763, 22.15, 14.62, 9.561, 15.51]^T$;
 $c = [5.258, 1.673, 2.122, 1.526, 4.656, 2.47]^T$.

5.3. Verification of Optimization Control Strategy in Improving Motion Reliability

With equation (24), the objective function can be minimized during the entire path planning task, and the joint angle and errors introduced into the angle can be obtained. The angle and joint error of joint 2 is shown in Figure 4.

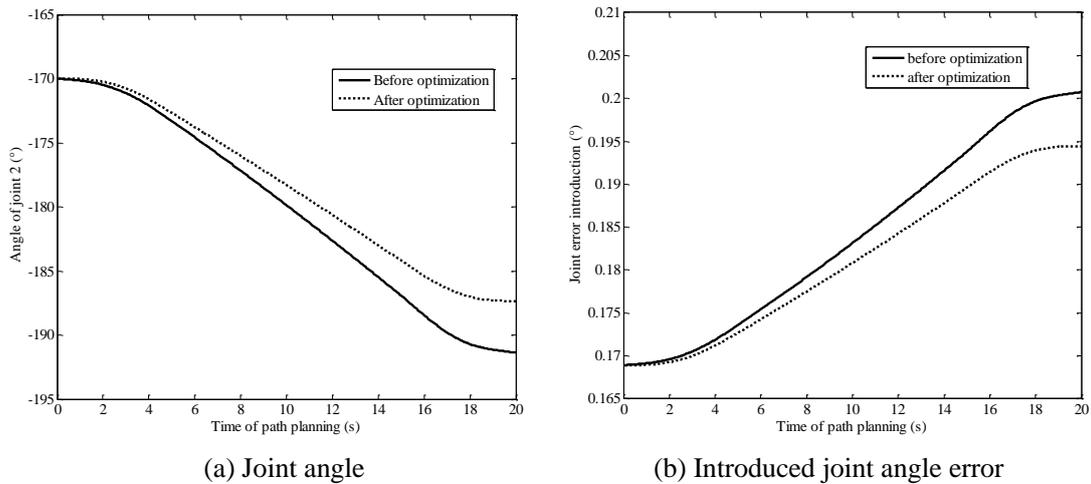


Figure 4. The Angle and Angle Error of Joint 2 after Optimization

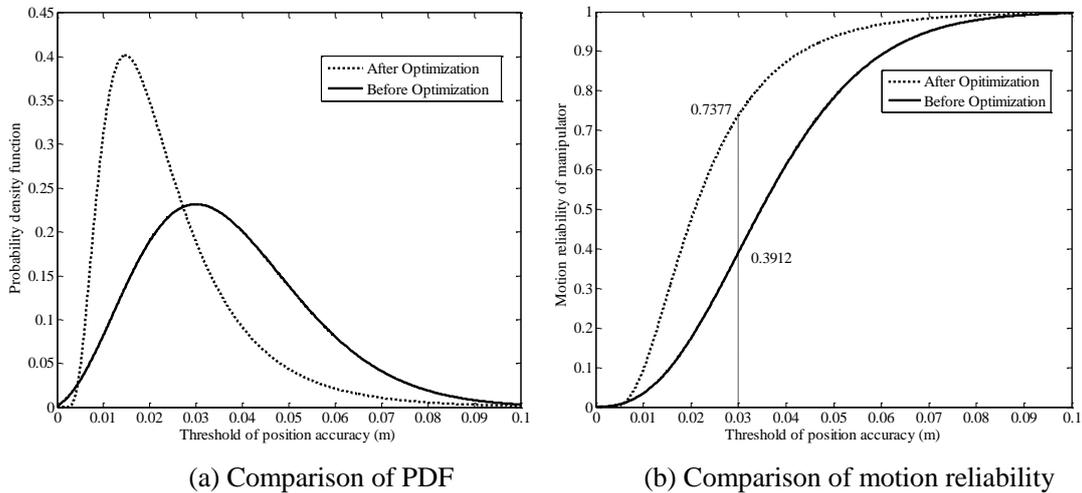


Figure 5. Comparison of Space Manipulator Motion Reliability

In Figure 4, it can be found that after optimization the angle of joint 2 becomes nearer to $\tilde{\theta}_2$ and the expectation of angle error becomes smaller. The other joints have the same trend. Considering the errors introduced into the joint, based on the method proposed in [6], the motion reliability of space manipulator is calculated. 500 times' simulations are carried out with the errors' introduction in Figure 4(b). Then the

probability density function (PDF) and motion reliability are obtained in Figure 5. It can be found that with the proposed optimization control strategy, the motion reliability is improved at different position accuracy thresholds. For example, when the threshold of position accuracy is set at 0.03m, the motion reliability is 39.12% before optimization and becomes 73.77% after optimization. The motion reliability is improved 34.65%, which verifies the effectiveness and feasibility of the optimization control method.

6. Conclusion

This paper aims at improving the motion reliability during space manipulator on-orbit operations. Taking the variety of manipulator operations into account, an optimization control model considering multi-index and multi-factor is established for motion reliability improvement. Considering the coupling relationships of indexes and complexity in solving multi-objective problem, dimensionality reduction strategy based on covariance matrix of motion reliability indexes is proposed to turn MOP into single objective problem. Then by optimizing the introduced errors in joint variables, the optimal weight matrix can be obtained.

The proposed method is adapted for various path planning task, such as trajectory tracking and point-to-point path planning. For trajectory tracking task, since trajectory should be guaranteed during the entire task, it has some limits on the adjustable region of the joints. While for point-to-point path planning which only concerns with target position, the trajectory shape can be adjusted in the workspace of space manipulator to achieve higher motion reliability improvement.

Acknowledgements

This work is supported by the National Key Basic Research Program of China (2013CB733000), the National Natural Science Foundation of China (61403038) and the National Natural Science Foundation of China (61175080).

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