A Pareto Optimization Applied in Power Control of Wireless Sensor Networks

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Abstract

*In this paper, achievable rate region of two-user interference channel is studied. We use the Pareto-optimal method (e.g., specially in production and consumption) into the wireless sensor nodes **transmit** power allocation strategies in coupled channels. The result shows that when systems in optimality conditions, minim transmit power can arrive maximum channel throughout. What's more, achievable rate of each channel equals and it is only related the channel parameters at the moment.*

Keywords: power allocation strategies, achievable rate, wireless sensor networks, Pareto optimization

1. Introduction

A. Interference in Communication Networks

The wireless sensor network uses universal spectrum reuse, *i.e.*, all source nodes transmit at the same frequency channel to target nodes. In earlier wireless sensor networks, the radius was large and the communication link was mainly range limited, or equivalently noise-limited. Later networks used higher spatial reuse of the spectrum per geographical area, and the transition size decreased to accommodate the decreasing of nodes energy. Therefore, the source nodes density increased and the target nodes got closer to each other. This change shift communication links from being noise-limited to being interference-limited.

B. Capacity of the 2-user Interference Channel

We are interested in the scenario where multiple wireless links operate at the same carrier frequency and mutually interfere with one another. Specifically, we consider two transmitter-receiver pairs TX1 → RX1 and TX2 → RX2 that are located in the proximity of one another, so that RX1 receives significant interference from TX2 and vice versa. We focus on the transmit strategies for TX1 and TX2 that are optimum in the Pareto sense. That is, we look for pairs of transmit strategies such that if any of the strategies is changed so that the transmission rate for the link TX1 → RX1 increases, then the rate for the other link TX2 → RX2 must necessarily decrease, and vice versa. If the interference is sufficiently weak, then it is optimal from a capacity point of view to treat it as additive noise [1]. Hence, Pareto-optimal transmit strategies directly correspond to Pareto-optimal signal-to-interference-and-noise ratio (SINR) pairs.

C. Contributions

Prior references [2-8] have studied many questions about Pareto as assumption that the transmitting nodes and receiving nodes have use amount of complicate techniques, power control cross correlation of pseudo-random codes, *etc.* However, this comes at the expense of sharing messages between the transmitting nodes or between the receiving nodes, and sometimes this might not be a feasible or an efficient strategy. With design dictums of low power consumption and low complexity receivers, no complex successive decoding nor maximum likelihood decoding would be advised[9-12]. This need motivated us to propose an optimization problem about throughput of channel as following assumption. The assumption is that there are no signal cooperation between transmitters (no joint encoding)[13-15], no signal cooperation between receivers (no joint decoding), single input single output communication (SISO), and a frequency flat channel.

2. System Model

The 2-user interference channel is presented in Figure 1 with 2 transmitters and 2 receivers.

Figure1. 2-user Interference Channel Model

The i^{th} transmitter transmits its signal T_i to the intended i^{th} receiver with a power P_i . The receivers have independent additive complex white Gaussian noise with a power noise variance of σ_n^2 . Each transmitter is assumed to have a maximum power constraint of $P_{\max}$. $G_{i,j}$ denotes the channel power gain received at the i^{th} receiver from the j^{th} transmitter, and there are no constraints over the values or distributions of $G_{i,j}$. Therefore $G_{i,i}$ is the channel gain of the i^{th} desired signal, where as $G_{i,j}$, $j \neq i$ represent the interfering channel gains. M_i denotes the maximum reliable rate of communication between the i^{th} transmitter and the i^{th} receiver.

From the view of 2-user physical sense, transmitter-receiver function can be expressed as follows:

$$M_1(P_1, P_2) = \log_2 \left(1 + \frac{kP_1}{1 + lP_2} \right) \quad (1)$$

$$M_2(P_1, P_2) = \log_2 \left(1 + \frac{sP_2}{1 + rP_1} \right) \quad (2)$$

The channel power gains are normalized by the noise variance and k, l, r and s respectively denote as follows :

$$k = \frac{g_{1,1}}{\sigma_n^2} \quad l = \frac{g_{1,2}}{\sigma_n^2} \quad r = \frac{g_{2,1}}{\sigma_n^2} \quad s = \frac{g_{2,2}}{\sigma_n^2}$$

In order to further discussion and use Pareto method, the two variables P_1 and P_2 are expended four variables $P_{1,1}, P_{1,2}, P_{2,1}$ and $P_{2,2}$. The channel power gain and the noise variance will be reset and redefined a symbol, respectively.

$$P_1 = eP_{1,1} + hP_{2,1} \quad ek + hr = 1 \quad (3)$$

$$P_2 = gP_{2,2} + fP_{1,2} \quad gs + fl = 1 \quad (4)$$

Therefore, M_1 and M_2 can be rewritten as follows:

$$M_1(P_{1,1}, P_{1,2}) = \log_2 \left(1 + \frac{P_{1,1}}{1 + P_{1,2}} \right) \quad (5)$$

$$M_2(P_{2,2}, P_{2,1}) = \log_2 \left(1 + \frac{P_{2,2}}{1 + P_{2,1}} \right) \quad (6)$$

Next, from the view of microeconomics, the Pareto optimal thought way is applied to analyze the relation of achievable rate and interference channel parameters.

Definition.1[4]. A rate pair $(M_1^*, M_2^*) \in R$ is Pareto optimal if there is no other rate pair $(M_1, M_2) \in R$ with $(M_1, M_2) \geq (M_1^*, M_2^*)$. (The inequality is component-wise. For notational brevity, power variable symbol can be redefined as:

$$P_{1,1} = u, \quad P_{2,1} = v, \quad P_{2,2} = w, \quad P_{1,2} = x \quad (7)$$

By inserting Eq.(7) into Eq.,(3),(4), (5) and (6) we derive the following equations.

$$M_1(s, t) = \log_2 \left(1 + \frac{u}{1 + v} \right) \quad (8)$$

$$M_2(x, y) = \log_2 \left(1 + \frac{w}{1 + x} \right) \quad (9)$$

$$P_1 = eu + hx \quad (10)$$

$$P_2 = gw + fv \quad (11)$$

In mathematical optimization, the method of Lagrange multipliers provides a strategy for finding the maxima of a function subject to constraints.

Maximize Eq.(8)

Subject to Eq. (9), (10) and (11)

Lagrange multiplier λ , σ and μ are introduced and the Lagrange function is constructed as Eq.(12).

$$Z = M_1(u, v) + \lambda[\bar{M}_2 - M_2(w, x)] + \sigma(P_1 - eu - hx) + \mu(P_2 - gw - fv) \quad (12)$$

The objective function Z differentiates with respect to s, t, x and y , respectively.

$$\frac{\partial Z}{\partial u} = \frac{\partial M_1}{\partial u} - \sigma e = 0 \rightarrow \frac{\partial M_1}{\partial u} = \sigma e \quad (13)$$

$$\frac{\partial Z}{\partial t} = \frac{\partial C_1}{\partial t} - \mu b = 0 \rightarrow \frac{\partial C_1}{\partial t} = \mu b \quad (14)$$

$$\frac{\partial Z}{\partial w} = -\lambda \frac{\partial M_2}{\partial w} - \mu g = 0 \rightarrow \frac{\partial M_2}{\partial w} = -\frac{\mu g}{\lambda} \quad (15)$$

$$\frac{\partial Z}{\partial y} = -\lambda \frac{\partial C_2}{\partial y} - \sigma d = 0 \rightarrow \frac{\partial C_2}{\partial y} = -\frac{\sigma d}{\lambda} \quad (16)$$

In economics, a relation is known as the fact that the Marginal Rate of Technical Substitution (MRTS) at a Pareto optimal resource allocation is the same for all consumers (here transmitter-receiver pairs). Along an isoquant, the MRTS shows the rate at which one input (*e.g.*, transmit power of different channel) may be substituted for another, while maintaining the same level of output (*e.g.*, achievable rate). The MRTS can also be seen as the slope of an isoquant at the point in question.

$$MRTS_{P_1 P_2}^{M_1} = \frac{\frac{\partial M_1}{\partial u}}{\frac{\partial M_1}{\partial v}} = \frac{\sigma e}{\mu f} \quad (17)$$

$$MRTS_{P_1 P_2}^{M_2} = \frac{\frac{\partial M_2}{\partial w}}{\frac{\partial M_2}{\partial x}} = \frac{\mu g}{\sigma h} \quad (18)$$

Reviewing the Eq.(9), $M_2(x, y)$ differentiates with respect to w and x , respectively.

$$\frac{\partial M_2}{\partial w} = \frac{1}{\ln 2 \cdot (1 + w + x)} \quad (19)$$

$$\frac{\partial M_2}{\partial x} = \frac{(-w)}{\ln 2 \cdot (1 + w + x)(1 + x)} \quad (20)$$

By inserting Eq.(19) and Eq.(20) into Eq.(18), Eq.(21) is derived as follows:

$$\frac{\mu g}{\sigma h} = -\frac{1+x}{w} \Rightarrow \frac{\mu}{\sigma} = \left(\frac{h}{g}\right) \cdot \left(-\frac{1+x}{w}\right) \quad (21)$$

By Eq. (19),(20)and (21), Eq.(17)and (18) can further simplify as follows:

$$MRTS_{P_1 P_2}^{M_1} = \frac{\frac{\partial M_1}{\partial u}}{\frac{\partial M_1}{\partial v}} = \frac{eg}{fh} \cdot \left(\frac{-w}{1+x}\right) \quad (22)$$

$$MRTS_{P_1 P_2}^{M_2} = \frac{\frac{\partial M_2}{\partial w}}{\frac{\partial M_2}{\partial x}} = \frac{1+x}{-w} \quad (23)$$

$$MRTS_{P_1 P_2}^{M_1} = MRTS_{P_1 P_2}^{M_2} \quad (24)$$

Eq.(24)is the Pareto optimal condition. It is the different Pareto point (here achievable rate) with respect to different transmit power. The conclusion suggests that the tangent is the minimum cost line. That is, minimum power can arrive maximum channel throughput.

From Eq.(24), Eq.(25) can be derived.

$$\frac{1+x}{-w} = \frac{eg}{fh} \cdot \left(\frac{-w}{1+x}\right) \Rightarrow \frac{w}{1+x} = \sqrt{\frac{fh}{eg}} \quad (25)$$

By Eq.(25) inserting Eq.(9) , Eq.(26) can be derived.

$$\sqrt{\frac{fh}{ef}} = 2^{M_2} - 1 \quad (26)$$

Based on the same principle , we treat Eq.(9) as objective function and treat Eq.(8),(10)and(11)as constraint conditions. Similar conclusion can be derived as follows:

$$\sqrt{\frac{fh}{eg}} = 2^{M_1} - 1 \quad (27)$$

We can draw a perfect conclusion from Eq.(26)and(27)

3. Experimental Results and Analysis

Power control (PC) mechanism have been elaborated as follows. The model for power control uses the Carrier to Interferer (C/I) ratio at the receiver as well as the receiving information power level as shown in Figure 2.

Figure 2. Power Control Mechanism

The model considers the interference caused by alien systems as well as the intra-system interference. The control algorithm compares the C/I value at the receiver with the minimum required and the maximum allowed C/I value. In order to keep the received C/I in its fixed boundaries the transmission power is controlled (if possible). Consequently the most important value during power control is the C/I. If the C/I is in the required scope, the transmission power is varied to keep the received power in its fixed boundaries, too.

Figure 3 shows an example of the power algorithm. The axis of ordinate contains the C/I threshold and the axis of abscissa contains the C-thresholds. The two straight lines include all possible values for C/I for a received interference power I_1 and I_2 . The area defined by the thresholds is marked with grey. The control of the corresponding station's transmission power should be the point on the straight line across the marked area. Regarding the interference I_1 , the transmission power must be pulled up until the

minimum receiving power is reached. The upper C/I threshold demand cannot be fulfilled here. Concerning I_2 , the grey marked area can be reached.

Figure 3. An Example of the Power Control

We will applied the following system parameters to analyze the experiment result.

Table 1. System Parameters

Variable Signal	Parameter (Unit)	Value
R	Femtocell Radius (m)	30
f	FAP carrier frequency(GHz)	2
$P_{\max}$	Femtocell user transmission power(W)	10
M	pseudo random cycle number By Monte Carlo method	100
K	power iterative number	100
T	FAP number in two-tier networks	20
N	femtocell user number in a FAP area	10
F(x)	Whether femtocell user transmission signal or not	Bernoulli distribution
n	Number of every femtocell user transmission signal	10
q	Failure possibility of transmission signal	0.3
k	Failure Number of transimission signal of every user in one FAP area	4
$\Gamma_{\max}$	Max SINR threshold	$\Gamma^* + 2\text{dB}$
$\Gamma_{\min}$	Min SINR threshold	$\Gamma^* - 2\text{dB}$
β_i	cost factor	0.1
σ^2	additive gaussian white noise power (W)	1×10^{-4}

	FDD LTE simulation frequency (GHz)	2
	FDD LTE UL lowest frequency(MHz)	1920
	FDD LTE UL highest frequency(MHz)	1980
	FDD LTE DL lowest frequency(MHz)	2110
	FDD LTE DL highest frequency(MHz)	2170
	LTE bandwidth (MHz)	15
	Number of RBs per LTE UE	12

Pareto optimality of the interference channel communication means that when two transmission powers are given, there is maximum value to overall channels achievable rate. It is only related to two inference channel parameters. When reaching the Pareto conditions, two channel capacity achieve a balance at the same time.

Figure 4. Max Achievable Rate and Pareto Point

In Figure 4, M_1 -coordinate and M_2 -coordinate show throughput of channel 1 and throughput of channel 2, respectively. At the point F, when throughput of channel 1 is preliminarily given, throughput of channel 2 can arrive the maximum Y_1 and vice versa. If one channel achievable rate increase, throughput of the other channel surely decrease. Slope of a curve shows the ratio of throughput from one channel to the other channel. It just shows the Pareto optimal thought. On the point E, the achievable rate of two channels equals and the sum of transmit power of two channels is minimum. So, the overall system is optimal at the moment. Eq.(28) use mathematical language to describe the physical meaning of point E.

$$M_1 = M_2 = \log_2 \left(1 + \sqrt{\frac{fh}{eg}} \right) \quad (28)$$

4. Conclusions

We use the Pareto-optimal solution to analyze the achievable rate of the two-user interference channel. Our main contribution has derived a characterization of channel. When achievable rate of each channel equals, it is only related the channel parameters. The objective of this paper is to provide a theoretical method for simplifying more complicate technique that is used to mitigate interference and improve transmission rate. Our hope is that the results will be useful for future research.

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