Nonlinear Controller Design for the Underactuated Crane System

Diantong Liu* and Weiping Guo

*Institute of Computer Science and Technology, Yantai University, Yantai City,
Shandong Province, 264005, People's Republic of China*

diantong.liu@163.com

Abstract

A nonlinear controller is designed for the underactuated crane. In the design process, a nonlinear variable composed with all the system states is defined and then the nonlinear controller is obtained with the Lyapunov stability theory. The system stability is proved via a stability theorem. The simulation results show the system under any initial states is asymptotically stable to the origin and the proposed algorithm is validated.

Keywords: *Underactuated system, Crane, Nonlinear control, Transport control*

1. Introduction

Crane works as a robot in many places such as workshops and harbors to transport all kinds of massive goods. It is desired for the overhead crane to transport its payloads to the required position as fast and as accurately as possible without collision with other equipments. Moreover, the payload swing angle should be kept as small as possible [1].

In the last two decades, the control problems of crane systems have been a focus. Yoshida [2] proposed a saturating control law by using a guaranteed cost control method for a nominal linearized crane dynamics. Giua [3] considered a linearized parameter-varying model of a planar crane and proposed an observer-based control design via Lyapunov equivalence. Liu [4] investigated an adaptive sliding mode fuzzy control approach for a linearized two-dimension overhead crane system. However, these methods based on the linearized crane dynamics may lose the sufficient accuracy and may reduce the performance of the crane control system.

With the development of nonlinear control technology, many nonlinear methods based on the nonlinear model of crane systems have been presented. Yang [5] developed a nonlinear control scheme incorporating parameter adaptive mechanism to ensure the overall closed-loop system stability. Fang and Sun [6, 7] presented a nonlinear coupling control approach. Yang [8] presented a robust control approach based on the wave propagation in a crane cable for a gantry crane system with hoisting. Many sliding mode control algorithms are proposed, such as Wang [9], Choi [10], Ngo [11] and Qian [12].

In this paper, the crane system is transformed into a nonlinear system through a collocated partial feedback linearization. A nonlinear variable that is composed of all the system states is defined and the controller is obtained by a selected Lyapunov function. The system stability is studied via a stability theorem.

2. The Crane System Model

The crane is shown schematically in Figure 1, where, m_1 is the trolley mass, m_2 is the load mass, l is the rope length, θ is the swing angle of the load with respect to the vertical line, x is the trolley position with respect to the origin, f is the force applied to the trolley. Since no rotating torque can drive load swing directly, the crane is a benchmark example of the underactuated mechanical system, which has one control input f and two configuration variables (x, θ) , and its Euler-Lagrange equations of motion [12] can be obtained as

$$\begin{cases} (m_1 + m_2)\ddot{x} + m_2 l \cos(\theta)\ddot{\theta} - m_2 l \sin(\theta)\dot{\theta}^2 = f \\ m_2 l \cos(\theta)\ddot{x} + m_2 l^2 \ddot{\theta} + m_2 g \sin(\theta) = 0 \end{cases} \quad (1)$$

Through the following collocated partial feedback linearization [13],

$$f = (m_1 + m_2 \sin^2 \theta)u - m_2 g \sin \theta \cos \theta - m_2 l \dot{\theta}^2 \sin \theta \quad (2)$$

Figure 1. Crane system configuration

The dynamics can be reduced to

$$\begin{cases} \ddot{x} = u \\ \ddot{\theta} = -(u \cos \theta + g \sin \theta) / l \end{cases} \quad (3)$$

3. The Nonlinear Controller Design

A nonlinear variable that includes all the system states is defined for the nonlinear underactuated crane system as

$$s = k_1 \theta + k_2 x + k_3 (m_2 l \dot{x} \cos \theta + m_2 l^2 \dot{\theta}) \quad (4)$$

where, k_1, k_2 and k_3 are some positive constants to assure the system stability. The derivative of the nonlinear variable can be obtained

$$\dot{s} = k_1 \dot{\theta} + k_2 \dot{x} + k_3 (m_2 l \ddot{x} \cos \theta + m_2 l^2 \ddot{\theta} - m_2 l \dot{x} \dot{\theta} \sin \theta) \quad (5)$$

Form the equation (1), it can be obtained

$$\dot{s} = k_1\dot{\theta} + k_2\dot{x} - k_3(m_2gl\sin\theta + m_2l\ddot{\theta}\sin\theta) \quad (6)$$

In order to obtain a nonlinear controller, the following scalar positive definite Lyapunov function is selected as

$$V_1 = \frac{1}{2}s^2$$

Its derivative is

$$\dot{V}_1 = s\dot{s}$$

To make $\dot{V}_1$ be a negative definite function, let

$$\dot{s} = e - b_1s \quad (7)$$

i.e.,

$$e = \dot{s} + b_1s = k_1\dot{\theta} + k_2\dot{x} - k_3(m_2gl\sin\theta + m_2l\ddot{\theta}\sin\theta) + b_1s \quad (8)$$

Its derivative is

$$\dot{e} = k_1\ddot{\theta} + k_2\ddot{x} - k_3(m_2gl\dot{\theta}\cos\theta + m_2l\ddot{\theta}\sin\theta + m_2l\dot{\theta}\ddot{\theta}\sin\theta + m_2l\dot{\theta}^2\cos\theta) + b_1\dot{s}$$

From the equation (2), it can be obtained

$$\begin{aligned} \dot{e} = & (k_2 - k_3m_2l\dot{\theta}\sin\theta - k_1\cos\theta/l + k_2m_2\dot{x}\sin\theta\cos\theta)u \\ & - g\sin\theta(k_1 - k_2m_2l\dot{x}\sin\theta)/l - k_2(m_2gl\dot{\theta}\cos\theta + m_2l\dot{\theta}^2\dot{x}) + b_1\dot{s} \end{aligned} \quad (9)$$

The Lyapunov function is reselected to

$$V = \frac{1}{2}s^2 + \frac{1}{2}e^2 \quad (10)$$

Using (7) and (9), the time derivative $\dot{V}$ is given by

$$\begin{aligned} \dot{V} = & s\dot{s} + e\dot{e} \\ = & -b_1s^2 + (s + \dot{e})e \\ = & -b_1s^2 + (s + (k_2 - k_3m_2l\dot{\theta}\sin\theta - k_1\cos\theta/l + k_2m_2\dot{x}\sin\theta\cos\theta)u \\ & - g\sin\theta(k_1 - k_2m_2l\dot{x}\sin\theta)/l - k_2(m_2gl\dot{\theta}\cos\theta + m_2l\dot{\theta}^2\dot{x}\cos\theta) + b_1\dot{s})e \end{aligned} \quad (11)$$

We note that the variable u enters the right hand side of equation (11). In order to make the $\dot{V}$ be negative definite, we can make the following equation hold

$$\begin{aligned} & (k_2 - k_3m_2l\dot{\theta}\sin\theta - k_1\cos\theta/l + k_2m_2\dot{x}\sin\theta\cos\theta)u \\ & - g\sin\theta(k_1 - k_2m_2l\dot{x}\sin\theta)/l - k_2(m_2gl\dot{\theta}\cos\theta + m_2l\dot{\theta}^2\dot{x}\cos\theta) + b_1\dot{s} + s = -b_2e \end{aligned} \quad (12)$$

where b_2 is a design constant, Such that

$$\dot{V} = -b_1s^2 - b_2e^2 \quad (13)$$

Therefore, the control law can be obtained from (12) as

$$u = (k_2 - k_3 m_2 l \dot{\theta} \sin \theta - k_1 \cos \theta / l + k_2 m_2 \dot{x} \sin \theta \cos \theta)^{-1} \\ (g \sin \theta (k_1 - k_2 m_2 l \dot{x} \sin \theta) / l + k_2 (m_2 g l \dot{\theta} \cos \theta + m_2 l \dot{\theta}^2 \dot{x} \cos \theta) - b_1 \dot{s} - s - b_2 e) \quad (14)$$

Theorem 1: the crane system described by equation (1) is asymptotically stable to its equilibrium point $(x, \dot{x}, \theta, \dot{\theta}) = (0, 0, 0, 0)$ under the control input (2) with (14) and some positive constants k_1, k_2, k_3, b_1, b_2 .

Proof:

The design process of the nonlinear controller has proved: the time derivative $\dot{V}$ of the chosen positive definite Lyapunov function V is negative definite.

Integrating both sides of (13) yields

$$\int_0^t \dot{V} d\tau = \int_0^t (-b_1 s^2 - b_1 e^2) d\tau$$

Then

$$V(t) - V(0) = \int_0^t (-b_1 s^2 - b_1 e^2) d\tau$$

We find that

$$V(t) = \frac{1}{2} s^2 + \frac{1}{2} e^2 = V(0) + \int_0^t (-b_1 s^2 - b_1 e^2) d\tau \leq V(0) < \infty$$

Therefore it can be obtained that $s \in L_\infty$ and $e \in L_\infty$, i.e.

$$\sup |s| = \|s\|_\infty < \infty, \quad \sup |e| = \|e\|_\infty < \infty$$

At the same time, from (13)

$$\dot{V} = s\dot{s} + e\dot{e} = -b_1 s^2 - b_1 e^2 < \infty$$

It is obvious that $\dot{s} \in L_\infty$ and $\dot{e} \in L_\infty$, i.e.

$$\sup |\dot{s}| = \|\dot{s}\|_\infty < \infty, \quad \sup |\dot{e}| = \|\dot{e}\|_\infty < \infty$$

Now let

$$s_1 = \theta + \frac{k_3 m_2 l^2}{k_1} \dot{\theta} \\ s_2 = x + \frac{k_3 m_2 l \cos \theta}{k_2} \dot{x}$$

Thus

$$x_1 = k_1 s_1 + k_2 s_2$$

From our research result (Theorem 1 in [14]), it can be known that $s_1 \in L_2$, $s_2 \in L_2$ and $s_1 \in L_\infty$, $\dot{s}_1 \in L_\infty$, $s_2 \in L_\infty$, $\dot{s}_2 \in L_\infty$. According to Barbalat's lemma, $\lim_{t \rightarrow \infty} s_1 = 0$, $\lim_{t \rightarrow \infty} s_2 = 0$, i.e. when $t \rightarrow \infty$,

$$\theta + \frac{k_3 m_2 l^2}{k_1} \dot{\theta} = 0 \quad (15)$$

$$x + \frac{k_3 m_2 l \cos \theta}{k_2} \dot{x} = 0 \quad (16)$$

For (15), a positive definite Lyapunov function $V_2 = \frac{1}{2} \theta^2$ is selected and $\dot{V}_2 = \theta \dot{\theta} = -\frac{k_3 m_2 l^2}{k_1} \dot{\theta}^2$ is negative definite. For (16), a positive definite Lyapunov function $V_3 = \frac{1}{2} x^2$ is selected and $\dot{V}_3 = x \dot{x} = -\frac{k_3 m_2 l \cos \theta}{k_2} \dot{x}^2$ is negative definite when $\theta \in (-\pi/2, \pi/2)$. Therefore, the crane system is asymptotically stable to the equilibrium point $(x, \dot{x}, \theta, \dot{\theta}) = (0, 0, 0, 0)$.

4. Simulation Studies

In this section, the validity of the proposed nonlinear controller by the transport control problem of an overhead crane system. The control objective of the transport control is to transport the load to the required position as fast and as accurately as possible with no free swings. In our simulations, the physical parameters of the overhead crane system in Figure 1 are determined as $m_1 = 1$, $m_2 = 0.1$, $l = 1$ and gravitational acceleration $g = 9.8$. The parameters of the proposed nonlinear controller is selected as $k_1 = 1$, $k_2 = 15$, $k_3 = 100$, $b_1 = 5$, $b_2 = 5$. The simulation results are shown in Figure 2 and Figure 3.

Figure (a). Position and velocity of the cart

Figure (b). Angle and angle velocity of the pendulum

Figure (c). Control force

Figure (d). Responses of the nonlinear state variable s and e

Figure 2. Simulation results under system initial state $(x, \dot{x}, \theta, \dot{\theta}) = (1, 0, 0, 0)$

Figure (a). Position and velocity of the cart

Figure (b). Angle and angle velocity of the pendulum

Figure (c). Control force

Figure (d). Responses of the nonlinear state variable s and e

Figure 3. Simulation results under system initial state $(x, \dot{x}, \theta, \dot{\theta}) = (0, 0, -\pi/3, 0)$

The simulation results in Figure 2 are obtained under the initial state $(x, \dot{x}, \theta, \dot{\theta}) = (1, 0, 0, 0)$ and the simulation results in Figure 3 are obtained under the initial state $(x, \dot{x}, \theta, \dot{\theta}) = (0, 0, -\pi/3, 0)$. It can be seen from the simulation results and many other

simulations that the crane system is asymptotically stable under any initial states with the proposed control algorithm. At the same time, it is shown that both the defined nonlinear state variable s in (4) and the e defined in (8) are asymptotically stable. On the other hand, the control performance can be improved through adjusting the parameters of the proposed controller. Lots of simulation experiments show that the parameters k_1, k_2 respectively correspond to the system states θ, x , therefore it is easy to adjust the parameters for an improved system performance.

5. Conclusions

Using a collocated partial feedback linearization, the underactuated crane is transformed to a nonlinear system that is used in the control design process. A nonlinear variable is defined and a nonlinear control algorithm is obtained with a selected Lyapunov function. The system stability is studied via a stability theorem and the simulation results show the validity of the proposed control algorithm.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (61175086) and A Project of Shandong Province Higher Educational Science and Technology Program (No. J13LN25).

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Authors

Diantong Liu

He received the B.S. degree in Electrical engineering from Shandong Institute of Technology, China, in 1994, the M.S. degree in Mechanical Engineering from Tianjin University, China, in 2001, and the Ph.D. degree in Control Engineering from Institute of Automation, Chinese Academy of Sciences, China, in 2004. Currently he is a professor of Institute of Computer Science and Technology, Yantai University. His main research interests include intelligent control, nonlinear system control and mechanical system control.

Weiping Guo

He received the B.S. degree in Electrical engineering from Shandong Institute of Technology, China, in 1996, and the M.S. degree in Control Engineering from Beijing Institute of Technology, China, in 2003. Currently she is a lecturer of Institute of Computer Science and Technology, Yantai University. Her main research interests include chaos and nonlinear system control.