Corrigendum

Direct images of relative pluricanonical bundles

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ABSTRACT

We replace Step 3 of the proof of the local freeness of $f'_*\omega_{X'/Y'}^{\otimes m}$ in Theorem 1.6 in [Direct images of relative pluricanonical bundles, Algebr. Geom. **3** (2016), no. 1, 50–62] because we misused Abramovich–Karu’s lemma on the base change of weakly semistable morphisms.

In this short note, we will freely use the notation in the proof of the local freeness of $f'_*\omega_{X'/Y'}^{\otimes m}$ in Theorem 1.6 in [Fuj16].

OBSERVATION. We put $\Sigma = Y' \setminus U_{Y'}$. Let P be a closed point of Y' . Let H be a general very ample Cartier divisor on Y' passing through P . We assume $\dim Y = n \geq 3$ and that the number of irreducible components of Σ passing through P is equal to n . Then $\text{Supp } \Sigma|_H$ is not a simple normal crossing divisor on H at P . Therefore, we can not apply [AbK00, Lemma 6.2] to the base change of $X^\dagger \rightarrow Y'$ by $H \hookrightarrow Y'$. If the number of the irreducible components of Σ passing through P is less than n , then $\Sigma|_H$ is a simple normal crossing divisor on H since H is general. In this case, by [AbK00, Lemma 6.2], the base change $X_H^\dagger = X^\dagger \times_{Y'} H \rightarrow H$ of $X^\dagger \rightarrow Y'$ by $H \hookrightarrow Y'$ is weakly semistable.

By the above observation, we see that the argument in Step 3 of the proof of the local freeness of $f'_*\omega_{X'/Y'}^{\otimes m}$ in Theorem 1.6 in [Fuj16] is not true as stated (see also the remark below). All we have to do is to replace Step 3 of that proof with the following new argument.

Step 3: Local freeness via the flat base change theorem. We take an arbitrary closed point $P \in Y'$. We take general very ample Cartier divisors $H_1, H_2, \dots, H_{n-1}$, where $n = \dim Y$, such that $C = H_1 \cap H_2 \cap \dots \cap H_{n-1}$ is a smooth projective curve passing through P . We put

$$S_i = H_1 \cap \dots \cap H_i$$

for every $1 \leq i \leq n-1$. Of course, $S_1 = H_1$ and $S_{n-1} = C$. Thus we have a sequence of Cartier divisors

$$Y' =: S_0 \supset S_1 \supset \dots \supset S_{n-1} = C.$$

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We put $X_{S_i}^\dagger = X^\dagger \times_{Y'} S_i$ for every i . Then, by adjunction, $X_{S_i}^\dagger$ is Gorenstein for every i . Note that $X^\dagger$ is Gorenstein and that $X_{S_{i+1}}^\dagger$ is a Cartier divisor on $X_{S_i}^\dagger$ for every i . By [AbK00, Lemma 6.2], the morphism $X_{S_i}^\dagger \rightarrow S_i$ is weakly semistable outside P for every $1 \leq i \leq n-1$. In particular, $X_{S_i}^\dagger$ is normal outside $(f^\dagger)^{-1}(P)$ for every $1 \leq i \leq n-1$. Since $X_{S_i}^\dagger$ is Gorenstein and $\text{codim}_{X_{S_i}^\dagger}(f^\dagger)^{-1}(P) \geq 2$ for every $1 \leq i \leq n-2$, the scheme $X_{S_i}^\dagger$ is normal for every $1 \leq i \leq n-2$. Note that $f^\dagger: X^\dagger \rightarrow Y'$ is equidimensional. By [Kar00, Lemma 2.12(2)], the morphism $X_{S_{n-1}}^\dagger \rightarrow S_{n-1}$ is weakly semistable because $\text{Supp } \Sigma|_{S_{n-1}} = P$. Therefore, $X_{S_{n-1}}^\dagger$ is normal. Thus, $X_{S_i}^\dagger$ is normal for every i . We note that $X_{S_i}^\dagger$ has only rational Gorenstein singularities outside $(f^\dagger)^{-1}(P)$ for every i because $X_{S_i}^\dagger \rightarrow S_i$ is weakly semistable outside P for every i . We consider the pair $(X_{S_{n-2}}^\dagger, X_{S_{n-1}}^\dagger)$. Note that $X_{S_{n-1}}^\dagger$ has only rational Gorenstein singularities because $X_{S_{n-1}}^\dagger \rightarrow S_{n-1}$ is weakly semistable (see [AbK00, Lemma 6.1]). We also note that a normal variety has only rational Gorenstein singularities if and only if it has only canonical Gorenstein singularities. By the inversion of adjunction (see [KM98, Theorem 5.50]), the pair $(X_{S_{n-2}}^\dagger, X_{S_{n-1}}^\dagger)$ is plt in a neighborhood of $X_{S_{n-1}}^\dagger$. Therefore, $X_{S_{n-2}}^\dagger$ has only canonical Gorenstein singularities. Apply the same argument to the pair $(X_{S_{i-1}}^\dagger, X_{S_i}^\dagger)$ for $n-2 \geq i \geq 2$ inductively. Then $X_{S_i}^\dagger$ has only canonical Gorenstein singularities for every i . We put $\tilde{X}_{S_i} = \tilde{X} \times_{Y'} S_i$ for every i . Since $(X^\dagger, X_{S_1}^\dagger)$ is plt by the inversion of adjunction as above, $(\tilde{X}, \tilde{X}_{S_1})$ is also plt by the negativity lemma (see, for example, [KM98, Proposition 3.51]). Thus, $\tilde{X}_{S_1}$ is normal (see [KM98, Proposition 5.51]). By the negativity lemma and adjunction again, $\tilde{X}_{S_1}$ has only canonical singularities. By repeating this process $n-1$ times, we obtain that $\tilde{X}_C = \tilde{X} \times_{Y'} C$ has only canonical singularities. Since $\tilde{X}_C \rightarrow C$ is a dominant morphism from a normal irreducible variety $\tilde{X}_C$ onto a smooth curve C , it is flat. In particular, $\tilde{X}_C \rightarrow C$ is equidimensional. Therefore, $\tilde{f}: \tilde{X} \rightarrow Y'$ is equidimensional by the choice of C . Since $\tilde{X}$ is Cohen–Macaulay and Y' is smooth, $\tilde{f}$ is flat (see [Har77, Chapter III, Exercise 10.9] and [AIK70, Chapter V, Proposition 3.5]). Moreover, $\mathcal{O}_{\tilde{X}}(mK_{\tilde{X}})$ is flat over Y' for every integer m since $\mathcal{O}_{\tilde{X}}(mK_{\tilde{X}})$ is Cohen–Macaulay by [Fuj16, Lemma 2.4] and $\tilde{f}$ is equidimensional (see [AIK70, Chapter V, Proposition 3.5]). By applying [Fuj16, Lemma 4.1] and the base change theorem (see [Har77, Chapter III, Theorem 12.11]) to $\tilde{X}_C \rightarrow C$, we obtain that

$$\dim H^0(\tilde{X}_y, \mathcal{O}_{\tilde{X}}(mK_{\tilde{X}/Y'})|_{\tilde{X}_y})$$

is independent of $y \in Y'$ for every positive integer m . By the base change theorem (see [Har77, Chapter III, Corollary 12.9]), we obtain that $f_*\omega_{X'/Y'}^{\otimes m} \simeq f_*\mathcal{O}_{\tilde{X}}(mK_{\tilde{X}/Y'})$ is locally free for every positive integer m .

Remark. Let k be the number of the irreducible components of Σ passing through P . Then $X_{S_i}^\dagger \rightarrow S_i$ is not weakly semistable for $n+1-k \leq i \leq n-2$ because $\text{Supp } \Sigma|_{S_i}$ is not a simple normal crossing divisor on S_i for $n+1-k \leq i \leq n-2$. On the other hand, we always have $\text{Supp } \Sigma|_{S_{n-1}} = P$. Therefore, $X_{S_{n-1}}^\dagger \rightarrow S_{n-1}$ is weakly semistable by [Kar00, Lemma 2.12(2)].

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