ALGORITHM FOR ADJUSTING INPUT AND OUTPUT IN A PRODUCTION PROCESS

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Abstract. *In this paper are analyzed a non-differentiable model of production systems aimed at maximizing profit, considering that resources can be procured from multiple markets and goods in their turn can be traded on different markets as well. At the same time, for the described model is proposed a numerical algorithm, based on the generalized gradient method, which iteratively adjusts the inputs of the system at its outputs, obtaining in the limit variant the optimal values of these flows.*

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1. Introduction

The method of mathematical modeling is one of the fundamental methods of economic cybernetics [1]. Production systems occupy a very important place in the range of cybernetic - economic systems. Elementary models, such as linear models, and in other situations by nonlinear models, which also have a nonsmooth character, describe the efficient operation of the production system (enterprise) in some cases.

The corresponding nonlinear patterns are quite complex and can be effectively solved by applying some modifications and generalizations of the non-differentiable optimization methods [2]. The problem-solving algorithm for this purpose uses the subgradients of the function-purpose if the restrictions are fulfilled with a certain approximation, and in case of non-observance, the subgradients of the functions from certain restrictions are used as the direction of movement, the deviation of which exceeds the "permissible" limit. Two distinct cases (A and B) are presented below. Case B represents an important generalization of the case A, while also reflecting the more appropriate situations in which the enterprise might be.

2. Case A

All production factors are purchased from the same market and all goods are marketed in a single market. The model has the following aspect:

$$R(u, x, y) = \sum_{j=1}^n v_j(u_j, y_j) - \sum_{i=1}^m r_i x_i \rightarrow \max(u, x), \quad (1)$$

where: $v_j(u_j, y_j) = c_j \min\{u_j, y_j\} - p_j \max\{0, u_j - y_j\} - q_j \max\{0, y_j - u_j\}$, or:

$$v_j(u_j, y_j) = \begin{cases} c_j u_j & , \text{ if } u_j = y_j \\ c_j u_j - q_j (y_j - u_j) & , \text{ if } u_j < y_j \\ c_j u_j - p_j (u_j - y_j) & , \text{ if } u_j > y_j \end{cases}$$

In Figure 1 is represented graphical interpretation of function $v_j(u_j, y_j)$

Figure 1. Graphical interpretation of function $v_j(u_j, y_j)$.

Remark. $v_j(u_j, y_j)$ represents the revenue that the enterprise would receive, as long as it offers u_j units of product of type j , the request for this product being y_j units.

The model restrictions are:

$$\sum_{j=1}^n a_{ij} u_j \leq b_i + x_i, \quad i = 1, m \quad (2)$$

$$0 \leq \underline{u_j} \leq u_j \leq \overline{u_j} \quad (3)$$

$$0 \leq \underline{x_i} \leq \overline{x_i}, \quad i = \overline{1, m} \quad (4)$$

Here: u_j – the decision variable – the quantity of the offer of the asset j that needs to be determined;

y_j – the demand for the good j ;

v_j – unit income (revenue earned through the sale of a product unit j);

x_i – the amount of resources to be purchased;

r_i – the price of resource i ;

a_{ij} – the technological coefficients;

b_i – resource i availability.

Brief description of model solving algorithm (1) - (4). Initially, the functions are defined:

$$\varphi_i(u, x_i) = \sum_{j=1}^n a_{ij}u_j - b_i - x_i$$

$$\varphi(u, x) = \max\{\varphi_1(u, x_1), \dots, \varphi_m(u, x_m)\}$$

The following groups are considered:

$$U = \{u = (u_1, \dots, u_j, \dots, u_n): \underline{u}_j \leq u_j \leq \overline{u}_j, j = \overline{1, n}\}$$

$$X = \{x = (x_1, \dots, x_i, \dots, x_m): 0 \leq x_i \leq \overline{x}_i, i = \overline{1, m}\}$$

The algorithm consists in determining two strings $\{u^k\}$ and $\{x^k\}$ in accordance with the following rules:

$$\begin{aligned} u^{k+1} &= P_U(u^k + h_k g_u^k), & \text{where } g_u^k &= \text{grad } R_u(u^k, x^k, y) \text{ if } \varphi(u^k, x^k) \leq \delta_k, \\ x^{k+1} &= P_X(x^k + h_k g_x^k), & \text{where } g_x^k &= \text{grad } R_x(u^k, x^k, y) \text{ if } \varphi(u^k, x^k) \leq \delta_k, \end{aligned}$$

$$\begin{aligned} u^{k+1} &= P_U(u^k - h_k g_u^k), & \text{where } g_u^k &= \text{grad } \varphi_u(u^k, x^k) \text{ if } \varphi(u^k, x^k) > \delta_k, \\ x^{k+1} &= P_X(x^k - h_k g_x^k), & \text{where } g_x^k &= \text{grad } \varphi_x(u^k, x^k) \text{ if } \varphi(u^k, x^k) > \delta_k. \end{aligned}$$

Moreover, the j -th component of the vector $g_u^k = \text{grad } R_u(u^k, x^k, y)$ is calculated by the rule:

$$(g_u^k)_j = \begin{cases} c_j, & \text{if } u_j^k = y_j, \\ c_j + q_j, & \text{if } u_j^k < y_j, \\ -p_j, & \text{if } u_j^k > y_j. \end{cases}$$

i -th component of vector $g_x^k = \text{grad } R_x(u^k, x^k, y)$ is equal to r_i , j -th component of vector $g_u^k = \text{grad } \varphi_u(u^k, x^k)$ is equal to $a_{i_k j}$ where i_k is the index, for which the maximum functions $\varphi(u, x)$ in point (u^k, x^k) , are realized, and i -th component of vector $g_x^k = \text{grad } \varphi_x(u^k, x^k)$ is equal to the value - 1, if $i = i_k$ and is equal to zero otherwise.

Regarding numerical strings $\{h_k\}$ and $\{\delta_k\}$ [3], the following assumed conditions:

$$h_k > 0, h_k \rightarrow 0, \delta_k > 0, \delta_k \rightarrow 0$$

$$\sum_{k=0}^{\infty} h_k \delta_k = \infty, h_k / \delta_k \rightarrow 0$$

The compliance of these conditions will ensure the convergence of the strings $\{u^k\}$ and $\{x^k\}$ described earlier, to optimal input and corresponding optimal output.

3. Case B

Each factor can be purchased from multiple markets of a given number of m_i markets. The j -type product can be sold in some of the n_j markets (Figure 2).

Figure 2. Scheme of supplying the enterprise with resources and distribution of goods sales

Considering the conditions that are refined in case B, the mathematical model (1) - (4) takes the following form (1') - (5'):

$$R(u, x, y) = \sum_{j=1}^n \sum_{l=1}^{n_j} v_j^l(u_j^l, y_j^l) - \sum_{i=1}^m \sum_{s=1}^{m_i} r_i^s x_i^s \rightarrow \max(u, x) \quad (1')$$

where $v_j^l(u_j^l, y_j^l) = c_j^l \min\{u_j^l, y_j^l\} - p_j^l \max\{0; u_j^l - y_j^l\} - q_j^l \max\{0; y_j^l - u_j^l\}$.

In this case:

$$\begin{aligned} u &= (u_1^1, \dots, u_1^{n_1}; \dots; u_j^1, \dots, u_j^{n_j}; \dots; u_n^1, \dots, u_n^{n_n}) \\ x &= (x_1^1, \dots, x_1^{m_1}; \dots; x_i^1, \dots, x_i^{m_i}; \dots; x_m^1, \dots, x_m^{m_m}) \\ y &= (y_1^1, \dots, y_1^{n_1}; \dots; y_i^1, \dots, y_i^{n_j}; \dots; y_n^1, \dots, y_n^{n_n}) \end{aligned}$$

Restrictions in the new model are as follows:

$$\sum_{j=1}^n \sum_{l=1}^{n_j} a_{ij} u_j^l \leq b_i + \sum_{s=1}^{m_i} x_i^s, i = 1, 2, \dots, m \quad (2')$$

$$0 \leq \underline{u}_j \leq \sum_{l=1}^{n_j} u_j^l \leq \bar{u}_j, j = \overline{1, n} \quad (3')$$

$$0 \leq u_j^l \leq \bar{u}_j \quad (4')$$

$$0 \leq x_i^s \leq \bar{x}_i^s, s = \overline{1, m_i}; i = \overline{1, m} \quad (5')$$

The functions are defined in the similar way:

$$\varphi_i(u, x_i^1, \dots, x_i^{m_i}) = \sum_{j=1}^n \sum_{l=1}^{n_j} a_{ij} u_j^l - b_i - \sum_{s=1}^{m_i} x_i^s, i = 1, 2, \dots, m$$

To apply the algorithm, it is also necessary to define the following functions:

$$\underline{\Psi}_j(u_j^1, \dots, u_j^{n_j}) = \underline{u}_j - \sum_{l=1}^{n_j} u_j^l$$

$$\overline{\Psi}_j(u_j^1, \dots, u_j^{n_j}) = \sum_{l=1}^{n_j} u_j^l - \overline{u}_j, j = \overline{1, n}$$

Obviously, in the allowable version, the values of all these functions must be less than or equal to zero:

$$\underline{\Psi}_j(u_j) \leq 0; \overline{\Psi}_j(u_j) \leq 0, j = 1, 2, \dots, n \quad (6')$$

Further functions are determined:

$$\Psi(\cdot) = \max\{\underline{\Psi}_1(\cdot), \dots, \underline{\Psi}_n(\cdot); \overline{\Psi}_1(\cdot), \dots, \overline{\Psi}_n(\cdot)\},$$

$$\varphi(\cdot) = \max\{\varphi_1(\cdot), \dots, \varphi_m(\cdot); \Psi(\cdot)\}.$$

To solve problem (1') - (5'), an algorithm developed for case A can be applied, taking into account the fulfillment of constraints (6') with the same tolerance threshold δ_k . In this case, the sets U and X are defined by inequalities (4') and (5'), respectively.

Remark. The above algorithms are developed based on the assumption that the components of a set are known in advance, for example, when a company produces according to the concluded agreements. But, of course, there are also interesting cases in which these components are random or uncertain [4], and the objective function is expressed in terms of average profit:

$$\max_{u, x} [M_y(R(u, x, y))],$$

or the worst profit indicator (Wald criterion):

$$\max_{u, x} \min_y R(u, x, y),$$

or minimizing Savage's regret function:

$$\min_{u, x} [\max_y (\max_{u, x} R(u, x, y) - R(u, x, y))],$$

or maximization, in the sense of the criterion of realism of Hurwitz:

$$\min_{u, x} [a \min_y R(u, x, y) + (1-a) \max_y R(u, x, y)],$$

where the parameter $\alpha \in [0;1]$ expresses the decision maker's tendency toward pessimism.

4. Conclusions

This paper proposes and analyzes two non-differentiable models of production systems. As controllable factors serve the vectors of inputs (inputs) and vectors of final products (outputs). At the same time, the models also include a number of uncontrollable factors such as demand for products. For the case where the application is a priori known, based on the generalized gradient method, there are developed the numerical algorithms, which ensure the optimal determination of the optimal input and output variants. An important idea in the algorithm is the application of the tolerance ceiling in relation to the restrictions describing the consumption of the resources (2'), as well as the restrictions that express the distribution of the output streams (3') to different markets. The flexible use of the tolerance ceiling offers the possibility for effective solving of practical problems even in situations where the above-mentioned restrictions do not respect the Slater condition. Researches has shown that for cases where the threshold of tolerance is ignored, the effective solving of problems is practically impossible. Described model can be extended and applied in research of optimal behavior of production systems, for different market structures, such as perfect competition, monopoly, duopoly or oligopoly.

References

- [1] Emil Scarlat , Nora Chirița. Cibernetica sistemelor economice. București 2003.
- [2] Шор Н. З. Методы минимизации недифференцируемых функций и их приложения, Киев, "Наукова Думка", 1979.
- [3] Godonoagă Anatol, Baractari Anatolie. Modele economice nediferențiabile. Aspecte decizionale. Editura ASEM, Chișinău , 2011.
- [4] Taha Hamdy A. Operations Research an Introduction. Third Edition, New York, London, 1982.