MULTIFACTORIAL TESTING OF ROLLER BEARING RELIABILITY

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JEL category: **C0**, **C02**, **C6**, **C63**

Summary:

This paper describes one model of the method of accelerated multi-factorial testing of roller bearing reliability, that represents the result of many years of work and research of the author in the field of technical system effectiveness, and that treats, from the theoretical point of view, a completely new method of multi-factorial reliability testing, which the broader scientific and expert circles are not yet familiar with. The method of accelerated multi-factorial testing of roller bearing reliability, treated by this work, contains significant advantages with respect to all other methods of reliability testing, which have been used so far in the engineering reliability testing practice in the world..

Keywords:

reliability, multifactorial testing, roller bearing, effectiveness

1. Introduction

The basic attributes of the method of accelerated multi-factorial testing of reliability of a mechanical system elements, described in this work, are as described in the text below:

- The function of the diagnostical parameter change in time is attained within the desired scope of the parameter;
- New plans for reliability testing in multi-factorial area (space).
- New statistical method based on regression

analysis.

- Enormous decrease of the number of experiments, and thereby of the costs of reliability testing in multi-factorial space.
- Interpretation of testing results is enabled in the form of Weibull distribution law in the whole defined multi-factorial space.

2. Influential factors

Testing of roller bearing reliability in this example is done in the seven-factor space, where the influential factors are given in the Table 1.

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Table 1. Influential factors

Factors	Factor Type	Abbreviation	Unit of Measure	Bottom Level X_d	Top Level X_g	Variation Interval $W=(X_g - X_d)/2$
X_1	Mean value of radial loadnormal distribution	M_r	kN	2	4	1
X_2	Standard deviation of radial loadnormal distribution	σ_r	kN	0.5	2	0.75
X_3	Mean value of axial loadnormal distribution	M_a	kN	0	2	1
X_4	Standard deviation of axial loadnormal distribution	σ_a	kN	0.5	2	0.75
X_5	Viscosity	k		0.1	4	1.95
X_6	Contamination	η_c		0	1	0.5
X_7	Diagnostic parameter	D	mm ²	3	6	1.5

3. Testing plan

The plan-testing matrix is formed in the following way:

The reliability testing plan matrix in a seven-factor area is arrived at, by taking the seven-factor process as the initial basis, whose complete orthogonal plan 2^7 contains 128 different points. The $2^{7-3} = 2^4$ replica, with the number of 16 experiments, is formed by choosing of the next generator, i.e. the corresponding contrast (Koldžić, 1999), (Stanić, 1990):.

$$\begin{aligned}
 x_5 &= x_1x_2 & J &= x_1x_2x_5 \\
 x_6 &= x_3x_4 & J &= x_3x_4x_6 \\
 x_7 &= x_1x_4 & J &= x_1x_4x_7
 \end{aligned}$$

$$J = x_1x_2x_5 = x_3x_4x_6 = x_1x_4x_7 = x_2x_4x_5x_7 = x_1x_3x_6x_7$$

The testing plan and the results are given in the Table 2, where $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, are the coded values of the factors $X_1, X_2, X_3, X_4, X_5, X_6, X_7$, so that 1 in the Table 2 represents the maximum factor value (the top level in the Table 1), with -1 representing the minimum value (bottom level in the Table 1). The experiments are conducted, obviously, only at maximum and minimum values of influential factors. The experimental results are given in millions of (r) revolutions of roller bearings to failure.

Table 2 The testing plan and results

lan items	Plan matrix X_s							Experimental results Y				
								* $y=10^{-1} \ln r$, $\bar{y} = \sum y / 4$				
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	y_2	y_3	y_4	$\bar{y}$
1	1	1	1	1	1	1	1	0.7	1.4	2.1	2.3	1.625
2	1	1	1	-1	1	-1	-1	0.05	1	1.4	2.2	1.1125
3	1	1	-1	1	1	-1	1	0.9	1.4	2.2	2.8	1.825
4	1	1	-1	-1	1	1	-1	0.9	1.3	2	2.8	1.975
5	1	-1	1	1	-1	1	1	1.2	2	2.3	3	2.125
6	1	-1	1	-1	-1	-1	-1	1.3	2	2.6	3.2	2.275
7	1	-1	-1	1	-1	-1	1	1.3	2.1	2.7	3.1	2.325
8	1	-1	-1	-1	-1	1	-1	5.4	6	6.8	7.4	6.4
9	-1	1	1	1	-1	1	-1	1.5	2.2	2.5	3.4	2.4
10	-1	1	1	-1	-1	-1	1	2	2.5	3	3.9	2.85
11	-1	1	-1	1	-1	-1	-1	2.1	3	3.8	4.3	3.3
12	-1	1	-1	-1	-1	1	1	6.4	7	7.6	8.3	7.325
13	-1	-1	1	1	1	1	-1	3	3.5	4.3	4.9	3.9
14	-1	-1	1	-1	1	-1	1	4	4.6	5.5	5.8	4.975
15	-1	-1	-1	1	1	-1	-1	1	1.3	2	2.8	1.77
16	-1	-1	-1	-1	1	1	1	7	8	8.4	9	8.1

* r – number of revolution till failure

4. Testing results processing

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_2x_4	x_2x_3	x_1x_6	x_2x_6	x_4x_5	x_3x_5	x_5x_6
$X_s =$ (16×15)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	-1
	1	1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1
	1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	1
	1	1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	-1	-1
	1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	-1	1
	1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1
	1	1	-1	-1	-1	-1	1	-1	1	1	1	-1	1	1	-1
	1	-1	1	1	1	-1	1	-1	1	1	-1	1	-1	-1	-1
	1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	-1	1
	1	-1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1
	1	-1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	1	-1
	1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	1	1
	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	-1	1	-1
	1	-1	-1	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1
	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1

$$\bar{Y} = \begin{pmatrix} 1.625 \\ 1.113 \\ 1.825 \\ 1.975 \\ 2.125 \\ 2.275 \\ 2.325 \\ 6.4 \\ 2.40 \\ 2.85 \\ 3.3 \\ 7.325 \\ 3.9 \\ 4.975 \\ 1.77 \\ 8.1 \end{pmatrix} \quad \hat{Y} = X_s \cdot B = \begin{pmatrix} 1.56 \\ 1.05 \\ 1.89 \\ 2.04 \\ 2.06 \\ 2.21 \\ 2.39 \\ 6.46 \\ 2.46 \\ 2.91 \\ 3.24 \\ 7.26 \\ 3.96 \\ 5.04 \\ 1.71 \\ 8.04 \end{pmatrix} \quad B = 16^{-1} X_s' \bar{Y} = \begin{pmatrix} 3.39 \\ -0.93 \\ -0.59 \\ -0.73 \\ -0.98 \\ -0.23 \\ 0.84 \\ 0.50 \\ 0.47 \\ -0.07 \\ -0.27 \\ -0.31 \\ 0.10 \\ 0.48 \\ -0.10 \end{pmatrix} \quad \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_{24} \\ b_{23} \\ b_{16} \\ b_{26} \\ b_{45} \\ b_{35} \\ b_{56} \end{pmatrix}$$

Table 3 The testing results

$\begin{aligned} e_1 &= y_1 - \hat{y} \\ e_2 &= y_2 - \hat{y} \\ e_3 &= y_3 - \hat{y} \\ e_4 &= y_4 - \hat{y} \\ e &= \bar{y} - \hat{y} \end{aligned}$						$m = \frac{\bar{e}}{ \bar{e} }$ $m = \frac{\bar{e}_1}{\bar{e}_j}$	$\begin{aligned} \mathbf{E} \\ E_1 &= e_1 m \\ E_2 &= e_2 m \\ E_3 &= e_3 m \\ E_4 &= e_4 m \end{aligned}$			
$\hat{y}$	$\bar{e}$	e_1	e_2	e_3	e_4	m	E_1	E_2	E_3	E_4
1.564	0.061	-0.86	-0.16	0.54	0.74	1	-0.86	-0.16	0.54	0.74
1.051	0.061	-1.00	-0.05	0.35	1.15	1	-1.00	-0.05	0.35	1.15
1.886	-0.061	-0.99	-0.49	0.31	0.91	-1	0.99	0.49	-0.31	-0.91
2.036	-0.061	-1.14	-0.74	-0.04	0.76	-1	1.14	0.74	0.04	-0.76
2.064	0.061	-0.86	-0.06	0.24	0.94	1	-0.86	-0.06	0.24	0.94
2.214	0.061	-0.91	-0.21	0.39	0.99	1	-0.91	-0.21	0.39	0.99
2.386	-0.061	-1.09	-0.29	0.31	0.71	-1	1.09	0.29	-0.31	-0.71
6.461	-0.061	-1.06	-0.46	0.34	0.94	-1	1.06	0.46	-0.34	-0.94
2.461	-0.061	-0.96	-0.26	0.04	0.94	-1	0.96	0.26	-0.04	-0.94
2.911	-0.061	-0.91	-0.41	0.09	0.99	-1	0.91	0.41	-0.09	-0.99
3.239	0.061	-1.14	-0.24	0.56	1.06	1	-1.14	-0.24	0.56	1.06
7.264	0.061	-0.86	-0.26	0.34	1.04	1	-0.86	-0.26	0.34	1.04
3.961	-0.061	-0.96	-0.46	0.34	0.94	-1	0.96	0.46	-0.34	-0.94
5.036	-0.061	-1.04	-0.44	0.46	0.76	-1	1.04	0.44	-0.46	-0.76
1.709	0.061	-0.71	-0.41	0.29	1.09	1	-0.71	-0.41	0.29	1.09
8.039	0.061	-1.04	-0.04	0.36	0.96	1	-1.04	-0.04	0.36	0.96

Figure 1 The testing results

Table 4. The testing results

*The remainder E is lined in one column by the rising sequence, and in another one by the declining one.											$Y'=\ln(1-n/N)^{-1}$			

<i>n</i>	<i>*E</i>	<i>*E</i>	$\hat{y}_s$	<i>X'</i>	<i>X''</i>	1- <i>n</i> / <i>N</i>	(1- <i>n</i> / <i>N</i>) ⁻¹	<i>Y</i>	<i>Y</i>	(<i>X'</i>) ²	(<i>X''</i>) ²	<i>X'·Y</i>	<i>X''·Y</i>
53	0.96	-0.86	2.06	3.02	2.92	0.17	5.82	1.76	0.57	9.13	8.55	1.71	1.65
54	0.96	-0.86	2.06	3.02	2.92	0.16	6.40	1.86	0.62	9.13	8.55	1.87	1.81
55	0.99	-0.86	2.06	3.05	2.92	0.14	7.11	1.96	0.67	9.28	8.55	2.05	1.97
56	0.99	-0.91	2.06	3.05	2.97	0.13	8.00	2.08	0.73	9.28	8.84	2.23	2.18
57	1.04	-0.91	2.06	3.10	2.97	0.11	9.14	2.21	0.79	9.59	8.84	2.46	2.36
58	1.04	-0.94	2.06	3.10	3.00	0.09	10.67	2.37	0.86	9.59	8.99	2.67	2.58
59	1.06	-0.94	2.06	3.12	3.00	0.08	12.80	2.55	0.94	9.74	8.99	2.92	2.81
60	1.06	-0.94	2.06	3.12	3.00	0.06	16.00	2.77	1.02	9.74	8.99	3.18	3.06
61	1.09	-0.99	2.06	3.15	3.05	0.05	21.33	3.06	1.12	9.90	9.29	3.52	3.41
62	1.09	-1.00	2.06	3.15	3.06	0.03	32.00	3.47	1.24	9.93	9.37	3.92	3.80
63	1.14	-1.04	2.06	3.20	3.10	0.02	64.00	4.16	1.43	10.22	9.60	4.56	4.42
64	1.15	-1.14	2.06	3.21	3.20	0.00	#DIV/0!	#DIV/0!	2	10.30	10.23	6.42	6.40
Σ				136.9	126.8				-32.8	325.8	284.2	-17.3	-12.7

5. Testing result interpretation

On the basis of the obtained data it follows that:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6 + b_7x_7 + b_{24}x_2x_4 + b_{23}x_2x_3 + b_{16}x_1x_6 + b_{26}x_2x_6 + b_{45}x_4x_5 + b_{35}x_3x_5 + b_{56}x_5x_6.$$

$$\hat{y} = 3.39 - 0.93x_1 - 0.59x_2 - 0.73x_3 - 0.98x_4 - 0.23x_5 + 0.84x_6 + 0.5x_7 + 0.47x_2x_4 - 0.07x_2x_3 - 0.27x_1x_6 - 0.31x_2x_6 + 0.1x_4x_5 + 0.48x_3x_5 - 0.1x_5x_6 \quad (1)$$

On the basis of Table 4 and the method of least squares, and after further processing of the results, as describe in literature (Koldžić, 1999), using transformation equitation (2) (Koldžić, 1999), (Stanić, 1990),

$$x = \frac{X - X_o}{w}, \quad X_o = \frac{X_g + X_d}{2} \quad (2)$$

it is obtained:

$$\alpha = 1.6$$

$$\ln \beta = 9.48 - 0.66M_r - 1.3\sigma_r - 1.1M_a - 2.44\sigma_a + 0.39k + 4.5\eta_c + 0.33D - 0.84\sigma_r\sigma_a - 0.1\sigma_rM_a - 0.54M_r\eta_c - 0.83\sigma_r\eta_c - 0.07\sigma_a k + 0.24M_a k - 0.1k\eta_c. \quad (3)$$

The data from Table 4 are entered into the probability chart in Figure 1 and the line, which corresponds to these data.

The obtained reliability indicators relate to the whole defined multi-factorial space, from minimum to maximum values of influential factors, on the basis of which can, therefore,

reliability in each point of the experimental space be calculated on the basis of the expression:

$$R_{(r)} = e^{-(r/\beta)^\alpha}$$

where - $R(r)$ reliability depends on the r number of revolutions.

For example, if it has been determined that a roller bearing will operate in the operating conditions where

$$M_r = 2.4; \quad \sigma_r = 1; \quad M_a = 1.1; \quad \sigma_a = 1; \quad k = 3.5; \quad \eta = 0.8 \quad (4)$$

and for the limit value of diagnostic parameter $D = 6$, so that for the given operational conditions the values of $\alpha = 1.6$ and $\beta = 963$ are obtained, reliability for some numbers of revolution is as given in Table 5.

Table 5 Reliability and number of revolutions

Reliability $R(r)$ [%]	Number of revolutions r
90 (0.90)	2280x10 ⁶
95 (0.95)	1505x10 ⁶
99 (0.99)	540 x 10 ⁶

On the basis of the (1) functional dependence, the functional dependence of diagnostical parameter on the number of revolutions can be arrived at. Namely, for any known operation conditions the (1) expression looks like this:

$$\hat{y} = \ln \hat{r} = A + 0.5x_7$$

and (Draper & Smith, 1966), (Koldžić, 1999):

$$y = \ln r = A + 0.5x_7 + \varepsilon \quad (5)$$

where A represent constant value, while ε is a random quantity for the observed population.

After entering of transformation equation (2) into the (5) expressions, the results are obtained in the form of:

$$D = C + 3 \ln r \quad (6)$$

This is obviously the case of a logarithmic random function, since C is a random quantity for one population and for one level of the diagnostical parameter. This random function can be approximately replaced, within the scope of the diagnostical parameter from $D_{min} = 3$ to $D_{max} = 6$ by a linear random function.

The obtained data regarding the diagnostical parameter enable forming of the most promising and most economical models of technical systems maintenance - the models of preventive maintenance according to condition. The experimental results interpreted in this way, from the point of view of diagnostic parameter enable predicting of operating time to failure (of the number of revolutions till failure in this example), on the basis of only one check-up of the state. Namely, on the basis of the results of the mentioned check-up of the state, the C quantity in the (6) expression can be obtained, so that

this random function after one diagnostic becomes deterministic.

CONCLUSION

Briefly stated, the advantages of this method are as follows:

- 1) Enormous decrease of the number of experiments, and thereby of the costs of reliability testing in a multi-factorial space. As can be seen on the Table 2, the experiments have been conducted four times in each point of the plan, on four elements (roller bearings). Since, according to the testing plan matrix, the experiments are conducted at 16 points, the total number of elements in the sample is $4 \times 16 = 64$. The number of analytically reached data relevant to reliability in defined multi-factorial space, in this example is $64 \times 16 = 1024$.
- 2) Interpretation of testing results is enabled in the form of Weibull distribution law in the whole defined multi-factorial space;
- 3) The function of the diagnostical parameter change in time is attained within the desired scope of the parameter;
- 4) This method does not comprise any casual improvisations so that results are absolutely accurate.

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