Proc. 13th International School on Theoretical Physics: Symmetry and Structural Properties of Condensed Matter

Study of the Surface Temperature Distribution on Spot Heated Thermal Insulating Foams

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An infrared laser beam has been used to heat a round plate of thermal insulating foam with small pores. The temperature distribution on the opposite surface of the foam has been measured with a thermal imaging camera, as a function of time. The constant laser heating has been realized centrally on the surface. The temperature distribution measurement was then continued during the self-cooling of the sample. We present a method for evaluating thermal conductivity of the insulating foam that is based on two elements: the finite difference method and the least squares approximation.

DOI: 10.12693/APhysPolA.135.1259

PACS/topics: 44.05.+e, 44.10.+i, 02.70.Dh, 02.60.Ed, 07.05.Tp

1. Introduction

From the year of its invention [1], the laser flash method (LFM) for measuring thermal diffusivity has consisted in heating a sample from the front side and detecting the time-dependent temperature from the back side. The initial perfect conditions, i.e. homogeneous and one-dimensional material, homogeneous input energy distribution on the front, an infinitely short pulse, and no heat losses, were soon enriched by radiation and convection losses on the front [2], side heat losses together with transient heat transfer in the sample, and finite-pulse effects [3] (with a correction [4]), [5, 6]. The effects of nonuniform surface heating were taken into consideration in [7]. The actual dimensions of the heat pulse on the front face, and of the temperature sensing area on the back face of the sample were taken into account in [8]. Further improvements of the assumptions were made in [9], where additionally higher-order solutions of the analytical description were applied together with the nonlinear regression routine allowing fitting experimental data to yield thermal diffusivity values with high accuracy. Other authors established analytical solutions by means of the Laplace transform [10] or the Green functions [11]. An application of the gray-body theory for thermal radiation inside a liquid sample at high temperatures [12] was another achievement. The same authors solved their model equations numerically using a finite-difference scheme.

In the presence of so many different factors that influence the heat-conduction process in a sample tested and that are difficult to describe analytically, we decided to take into account those factors by means of the finite-difference method (FDM). The method has been oriented towards testing the thermal diffusivity and conductivity of thermal insulating foam with small pores. FDM allows to take into consideration, except many of the abovementioned factors, also convection and radiation losses with varying temperature on the surface of the sample. We applied the similar approach to investigate the thermal conductivity of a small amount of liquid [13].

2. Method description

An infrared laser beam has been used to heat the center of a round sample of thermal insulating foam, as it shown in Fig. 1. The sample has been covered on its center by a small metal plate with black oxidation and roughly the same diameter as the laser beam. Irradiating the sample from the top has helped avoiding heating the sample parts adjacent to the plate by a convec-

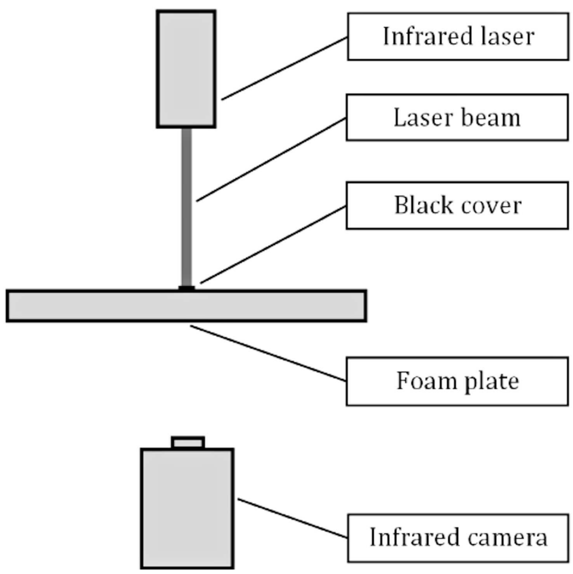

Fig. 1. Diagram of the measuring system.

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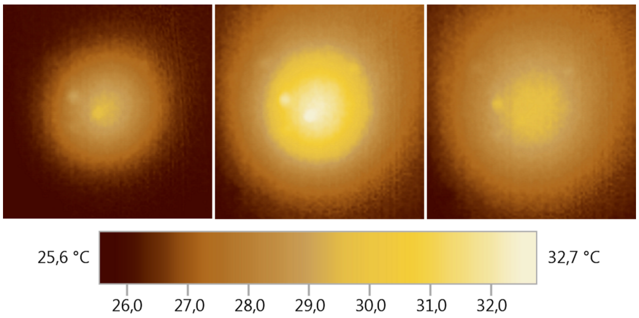

Fig. 2. Three measurement results, for $t = 74$, 300, 358 s.

tion. The temperature distribution over the sample has been measured as a function of time at the bottom of the sample by an infrared camera. The entire measuring device has been placed in a temperature stabilized zone. The laser heating was realized constantly from the beginning until the temperature distribution measured by the camera reached approximately a stationary state. For the sample investigated, the heating time defined in this way has turned out to be equal to about 300 s, and we have assumed exactly such amount of time for the constant heating. The temperature distribution measurement was then continued for the same time during the self-cooling of the sample. The measurement results are presented in Fig. 2.

The heat flow through the sample as well as the surrounding space has been modeled by the finite-difference method (FDM) to calculate the temperature distribution at the bottom vs. time. The following FDM assumptions has been taken for granted:

- the following input parameters are known:
  - the constant ambient temperature T_0, that is also the initial temperature of the whole measuring system
  - the density ρ and specific heat c, of the anisotropic sample tested
  - the density ρ_c, specific heat c_c, thermal conductivity λ_c, and thermal emissivity ε (equal to 1), of the cover plate
  - the time t_h of constant heating
  - the cylindrical sample diameter and height, D and H, respectively
  - the cover plate diameter and height, D_c and H_c, respectively
- the following output parameters are initially assumed, though they are changed by the fitting method to achieve finally the values sought:
  - the heat flux q_0 of the laser radiation, in W/m^2
  - the laminar-convection conductivity h for a horizontal sample plate
  - the thermal conductivity λ of the sample investigated (the main parameter sought)
- the cylindrical foam plate is divided into coaxial rigs with the various radius r, the constant thickness Δr, and height Δz

The heat equation in cylindrical coordinates

$$\lambda\left(\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c\frac{\partial T}{\partial t}+q, \quad (1)$$

is transformed by the finite-difference method to the following equation for a thin coaxial ring element of the sample with radius r:

$$T(t+\Delta t) = \quad (2)$$

$$T(t)+\frac{\lambda\Delta t}{\rho c\Delta z^2}\left[T(z+\Delta z)-2T(z)+T(z-\Delta z)\right]$$
$$+\frac{2\lambda\Delta t}{\rho c r\Delta r}\left[T(r+\Delta r)-T(r-\Delta r)\right]$$
$$+\frac{\lambda\Delta t}{\rho c\Delta r^2}\left[T(r+\Delta r)-2T(r)+T(r-\Delta r)\right]$$
$$+\frac{\Delta t}{\rho c\Delta z}q(t)-\frac{h\Delta t}{\rho c\Delta z}\left[T-To\right]-\frac{\varepsilon\sigma\Delta t}{\rho c\Delta z}\left[T^4-To^4\right],$$

where z is the down-sense vertical coordinate of the thin ring, $q(t)$ is the time-dependent heat flux, $q(t) = q_o \neq 0$ for $0 \leq t \leq t_h$, σ is the Stefan–Boltzmann constant, ε is the thermal emissivity of the sample surface. For the purpose of simplification, the following denotations of temperature dependence upon coordinates r, z, t are identical: $T(r,z,t) \equiv T(r) \equiv T(z) \equiv T(t) \equiv T$. The first term in the lowest line of Eq. (2), that corresponds to laser heating, is realized only for the thin rings located on the black cover plate. The last two terms in the lowest line of Eq. (2), that correspond to convection and radiation, are realized for the thin rings located on the surface of the sample.

The radiation term in Eq. (2), proportional to $(T^4 - T_0^4)$, can be extended in the Taylor series near $T = T_0$ with an accuracy of the first power, i.e. $T^4 - T_0^4 \approx 4T_0^3(T-T_0)$, because the sample temperature differs from the ambient one by a dozen or so degrees centigrade at the most, as it turns out from the experiment. Thus, the two terms in Eq. (2), relating to convection and radiation, can be joined, which reduces three unknown parameters h, ε, σ to the effective laminar-convection conductivity $h' = h + 4T_0^3\varepsilon\sigma$ in the least squares fitting (LSF). Without this reduction, LSF would be divergent since the parameters h, ε, and σ, that are varied by LSF, have the similar logical influence upon the temperature calculated, that is they all stand at the terms that are proportional to the difference between sample and ambient temperatures.

The temperature on the opposite surface of the foam sample, $T_b(r,t) = T(r, z = H, t)$, vs. radius and time, evaluated from the FDM, has been used as a fitting nonlinear function in LSF method to get the foam thermal conductivity, λ, among other fitting parameters, i.e. the heat flux q of the laser radiation and the effective laminar-convection conductivity h'. The fitting function $T_b(r,t)$ is defined in the two-dimensional domain, so that the LSF method fits a curved surface to the three-dimensional measuring points. The time range for temperature measurement must be wide enough to include the time of self-cooling so that the measuring curve features more characteristics, as it is shown in Fig. 3. The partial derivatives, $\partial T_b/\partial\lambda$, $\partial T_b/\partial q$, and $\partial T_b/\partial h$, needed during LSF calculations for each time and for each radius, have been evaluated numerically, which requires double computing the whole finite-difference procedure for each derivative, i.e. for two close values of λ, q or h'.

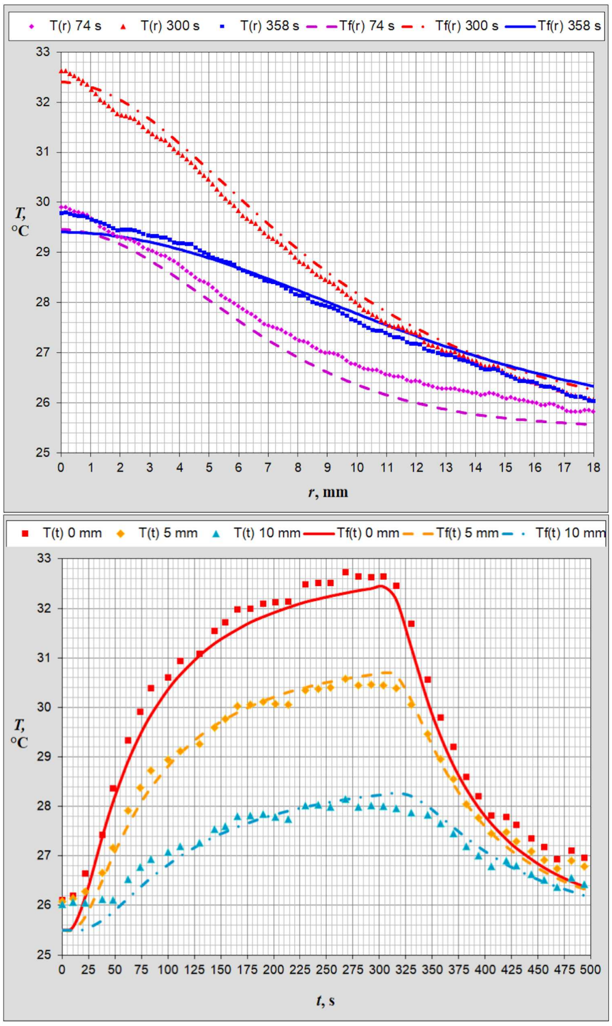

Fig. 3. Exemplary results $T(r)$ for three times t = 74, 300, 358 s, and $T(t)$ for three radii r = 0, 5, 10 mm. Measurement results are denoted by points, and FDM-LSF results are denoted by lines. The heating time $t_h = 300$ s. The number of measuring points $n = 125 \times 40 = 5000$.

The thermal diffusivity of the foam investigated, i.e. boron-containing non-flammable polyurethane foam [14], has been estimated by the fitting procedure at $\alpha = (0.4101 \pm 0.0022) \times 10^{-6}$ m^2/s. The thermal conductivity together with its uncertainty $\lambda \pm u_0(\lambda)$ can be computed by the formula $\lambda = \alpha\rho c$ and equals $\lambda = (0.06238 \pm 0.00033)$ W/(m K). The uncertainty $u_0(\alpha)$, and hence $u_0(\lambda)$, has been calculated by the fitting procedure from the measuring-points scattering, and from the numerical model ability to adapt to the measuring points. Since the input parameters to the joined FDM and LSF methods, such as sample or cover diameter, thickness, density, or specific heat, are measured with finite accuracy, the uncertainty $u_0(\lambda)$ must be enlarged under the law of uncertainty propagation to the following expression for the total uncertainty $u(\lambda)$:

$$u(\lambda) = \sqrt{u_0^2(\lambda) + \sum_i \left(\frac{\partial\lambda}{\partial a_i}u(a_i)\right)^2} =$$

$$\lambda\sqrt{\left(\frac{u_0(\lambda)}{\lambda}\right)^2 + \sum_i \left(\frac{\partial\lambda}{\partial a_i}\frac{a_i}{\lambda}\frac{u(a_i)}{a_i}\right)^2} =$$

$$\lambda\sqrt{\left(\frac{u_0(\lambda)}{\lambda}\right)^2 + \sum_i \left(k_i\frac{u(a_i)}{a_i}\right)^2} =$$

$$\lambda\sqrt{v_0^2 + \sum_i (k_i v_i^2)^2}, \quad (3)$$

where a_i and $u(a_i)$ are the i-th input parameter and its uncertainty, adequately, v_i is the i-th input parameter relative uncertainty, k_i is the coefficient of influence of relative uncertainty of the i-th parameter on the total uncertainty thermal conductivity. The partial derivatives in Eq. (3) have been calculated numerically by repeating the whole FDM-LSF procedure for two close values of each input parameter. The k_i coefficients are shown in Table I.

TABLE I

The relative uncertainty of thermal conductivity v_0 and the coefficients k_i for some input parameters

v_0	D	H	D_c	H_c	T_0	t_h
0.0053	0.033	0.083	0.0062	0.023	15.4	1.97

The coefficient k, definition of which follows from Eq. (3), expresses the character of influence of the given input parameter relative uncertainty on the output parameter relative uncertainty $u(\lambda)$ in the FDM-LSF procedure used. For example, if the output parameter depends on the m-th power of the input one, then $k = m$. Since the coefficients are dimensionless, the character of the above influence is size independent, in a certain range. Therefore, the values of the coefficients obtained here show the quality of the method itself and may be used with different values of the input parameters or their uncertainties.

The k_i values, shown in Table I, together with the specific values of the input parameters and their uncertainties, shown in Table II, have been used to calculate by Eq. (3) the thermal-conductivity total uncertainty $u(\lambda) = 0.00047$ W/(m K).

TABLE II

The values of the input parameters and their uncertainties

D [mm]	H [mm]	D_c [mm]	H_c [mm]	T_0 [K]	t_h [s]
60.0	10.0	4.0	1.0	298.5	300
0.2	0.2	0.05	0.05	0.1	0.1

3. Conclusions

The thermal conductivity of the insulating foam can be calculated by the joined methods of finite differences and least squares. The thermal-conductivity relative uncertainty v_0, achieved during fitting the numerical heat-transfer model to experimental data, shows the quality of the model itself. Such accuracy is achieved when every input parameter is measured with relative uncertainty that is a few times less than v_0/k_i. Otherwise, the thermal-conductivity uncertainty increases. After taking into account all input uncertainties, the thermal conductivity takes on the value $\lambda = (0.06238 \pm 0.00047)$ W/(m K). Other results of the model are the heat flux $q_0 = (3.657 \pm 0.015)$ mW/mm^2, and the effective laminar-convection conductivity $h' = (4.807 \pm 0.036)$ W/(m^2 K).

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