12th Symposium of Magnetic Measurements and Modeling SMMM'2016, Częstochowa–Siewierz, Poland, October 17–19, 2016

Study on the Rayleigh Hysteresis Model and its Applicability in Modeling Magnetic Hysteresis Phenomenon in Ferromagnetic Materials

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The following paper presents the basis of a Rayleigh magnetic hysteresis model and the examples of its application for modeling the magnetic characteristics of ferromagnetic materials: ferrites and steels. The presented model allows one to simulate the shape of a magnetic hysteresis loop of the material for the given value of the magnetizing field in the so-called Rayleigh region, as well as to calculate basic magnetic parameters of the material like coercive field, magnetic remanence, and power loss. Four different ferromagnetic materials (two ferrites and two alloy steels) were chosen for the investigation. Each material was investigated within the range of low magnetizing fields, corresponding to the Rayleigh region. On the basis of obtained results, modeling of magnetic characteristics of the investigated materials was performed. Moreover, the range of magnetizing field where the Rayleigh model can be applied for these materials was investigated based on correlation coefficient between experimental results and modeling.

DOI: 10.12693/APhysPolA.131.1244

PACS/topics: 75.30.–m, 75.60.–d, 75.50.Gg, 75.50.Cc, 75.60.Ej, 07.05.Tp

1. Introduction

Modeling magnetic properties of the ferromagnetic materials is a very important issue in modern physical science as well as in technical applications. Ferromagnetic materials are widely utilized in the electronic and energy industry. Inductive components containing ferromagnetic cores, like transformers, filters, chokes, resonant circuits, etc., are present in almost all modern electronic devices [1]. At the same time, investigation of the ferromagnetic materials and their magnetic properties is an intensively developed field of physical science with many new results and discoveries published every year.

There are several approaches to the subject of modeling magnetic characteristics of ferromagnetic materials, especially the magnetic hysteresis phenomenon. Existing models could be divided in two groups. The first group are physical models, which are very complex and connect physical properties of the ferromagnetic material with its magnetic characteristics. The most important of these models are the Jiles–Atherton model [2–4], the Harrison model [5, 6] and the Preisach model [7, 8]. Physical models are relatively hard for implementation because of their complexity and high computing power required to perform the necessary calculations. The second group are simplified technical models describing the magnetic hysteresis phenomenon with simple mathematical functions and not considering the physical properties of the material. For modeling the magnetic hysteresis loop in cores of the transformers, hyperbolic [9] or logarithmic [10] curves are often utilized. Modeling of fluxgate sensors is performed with tri-segmental linear approximation [11] of the magnetic hysteresis loop or approximation based on arctangent function [12]. The common feature of these models is lack of consideration of the physical processes occurring in the material's structure during magnetization, which leads to simplification of the calculations, but limits the possibility of generalization of the model. Therefore these models are mostly utilized in simple technical applications like modeling the characteristics of inductive devices (transformers, fluxgate sensors, filters, etc.) with ferromagnetic cores.

Among the mentioned models, the lack of solution combining the physical approach to the magnetic hysteresis phenomenon with low computational complexity allowing one to perform calculations with a PC rather than a computational cluster, is clearly observable. The possible solution to this issue is utilization of the so-called Rayleigh magnetic hysteresis model [13], which approximates the dependence between the magnetic flux density within the material and the magnetizing field with a parabolic curve. Despite its simplicity, parameters of the Rayleigh model are strongly correlated with the physical properties of the material, which leads to a much more accurate representation of the magnetic hysteresis phenomenon than in the case of technical models. On the other hand, the computational complexity of the model is very low and calculations can be performed even with an ordinary PC. However, the range of applicability of the Rayleigh model is limited to the relatively low values of the magnetizing field. For higher fields the dependence between the magnetic flux density and the magnetizing field loses its initial parabolic character.

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The paper presents the basis of the Rayleigh hysteresis model and the examples of its application for modeling the magnetic characteristics of the ferromagnetic materials: ferrites and steels. All materials were investigated with a special measurement system and, on the basis of the obtained results, modeling was performed. Moreover, the range of the magnetizing field where the Rayleigh model can be applied for these materials was investigated based on the correlation coefficient between experimental results and modeling.

2. The Rayleigh hysteresis model

The Rayleigh hysteresis model is the first historical approach to modeling a magnetic hysteresis phenomenon. It was proposed by British physicist Lord John Rayleigh in 1887 [14]. Rayleigh was observing the behavior of iron and steel under the influence of low magnetizing fields. The performed experiment allowed him to develop the description of the magnetic hysteresis phenomenon based on the assumption of parabolic dependence between magnetic flux density of the ferromagnetic material (or magnetization of the material [15]) and the magnetizing field. Rayleigh described this dependence with a second order polynomial equation, introducing two coefficients characterizing the ferromagnetic properties of the material: initial relative magnetic permeability μ_i and the so-called Rayleigh coefficient α_{R}.

The initial permeability μ_i is connected with the linear increase of the magnetic flux density within the volume of the material under the influence of an increasing magnetizing field. It is an especially significant phenomenon for relatively low magnetizing fields. On the other hand, for stronger magnetizing fields, the linear dependence is covered by stronger, nonlinear effects. These effects are resulting in deformation of the magnetic flux density waveform from the original shape of the magnetizing field waveform. Rayleigh stated that these effects can be approximated with second order polynomial dependence, where Rayleigh coefficient α_{R} is a parameter of the equation.

Taking all this into account, Rayleigh formulated the second order equation, known as the Rayleigh law of magnetization, describing the initial magnetization curve of the ferromagnetic material [14]:

$$B(H) = \mu_0\mu_i H + \mu_0\alpha_{\mathrm{R}} H^2, \tag{1}$$

where B is the magnetic flux density, H is the magnetizing field and μ_0 is the magnetic permeability of free space. The equation is based on the assumption of linear dependence between relative magnetic permeability μ and magnetizing field [15]:

$$\mu(H) = \mu_i + \alpha_{\mathrm{R}} H. \tag{2}$$

On the basis of Eq. (1), Rayleigh developed the model of the magnetic hysteresis phenomenon, where a hysteresis loop is approximated with the two symmetrical, intersecting parabolic curves. This approximation results in the lenticular shape of the modeled hysteresis loop, with the intersection points of parabolic curves being vertices of the hysteresis loop. The Rayleigh hysteresis loop is described with the system of two second order equations [14]:

$$B(H) = \mu_0[(\mu_i + \alpha_{\mathrm{R}} H_m)H + \frac{\alpha_{\mathrm{R}}}{2}(H_m^2 - H^2)], \tag{3}$$

$$B(H) = \mu_0\ [(\mu_i + \alpha_{\mathrm{R}} H_m)H - \frac{\alpha_{\mathrm{R}}}{2}(H_m^2 - H^2)], \tag{4}$$

where H_m is the maximum value of the magnetizing field (amplitude of the magnetizing waveform). Equation (3) describes upper, decreasing branch of the loop, while Eq. (4) refers to a lower, increasing branch.

The Rayleigh law of magnetization and equations describing the Rayleigh hysteresis loop creates the possibility to calculate values of basic magnetic parameters of the material like maximum value of magnetic flux density B_m, coercive field H_c, magnetic remanence B_r and power losses P_H in certain volume of the material. All calculations are based on the given value of the magnetizing field amplitude H_m.

Maximum magnetic flux density B_m is given with Eq. (1), where the value of the maximum magnetizing field H_m is substituted for the value of the magnetizing field H. This leads to the conclusion that vertices of the Rayleigh hysteresis loop given by the pair of values (H_m, B_m) are consistent with the Rayleigh law of magnetization.

Coercive field H_c for the given value of the magnetizing field amplitude H_m is given as a positive root of the second order polynomial function describing the increasing branch of the Rayleigh hysteresis loop (Eq. (4)):

$$B(H = H_c) = \mu_0[(\mu_i + \alpha_{\mathrm{R}} H_m)H_c - \frac{\alpha_{\mathrm{R}}}{2}(H_m^2 - H_c^2)] = 0. \tag{5}$$

The value of the coercive field H_c is given with the equation

$$H_c = \frac{-(\mu_i + \alpha_{\mathrm{R}} H_m) + \sqrt{(\mu_i + \alpha_{\mathrm{R}} H_m)^2 + \alpha_{\mathrm{R}}^2 H_m^2}}{\alpha_{\mathrm{R}}}. \tag{6}$$

Magnetic remanence B_r for the given value of the magnetizing field amplitude H_m can be calculated by substitution of the zero value of the magnetizing field ($H = 0$) to the equation describing decreasing branch of the Rayleigh hysteresis loop (Eq. (3)), which leads to the equation

$$B_r = \frac{\alpha_{\mathrm{R}}}{2}\mu_0 H_m^2. \tag{7}$$

Power loss P_H in the certain volume of the material is connected with the surface area of the hysteresis loop [15]:

$$P_H = f V_e \int_0^{B_m} H\,\mathrm{d}B, \tag{8}$$

where f is frequency of the magnetizing field waveform and V_e is an effective volume of the ferromagnetic material. The integral operation is usually performed with numerical algorithms. However, as in the Rayleigh model functions describing the hysteresis loop are analytically integrable, it is possible to determine analytical formula describing the dependence between power loss and amplitude of the magnetizing field

$$P_H = \frac{4}{3} f V_e \mu_0 \alpha_{\rm R} H_m^3. \quad (9)$$

While for a high magnetizing field, the Rayleigh model is not satisfactory because of low correlation between hysteresis loop and parabolic curves, this model is a very good approximation of the hysteresis loop obtained for low magnetizing fields, in many cases lower than the saturation coercive field. For such fields, the hysteresis loop exhibits a lenticular shape, which could be well approximated with two intersecting parabolic curves.

3. Investigated materials

During the performed experiment, four ferromagnetic materials were investigated. Two of them were ferrites, ceramic materials chemically composed of iron oxide Fe_2O_3 and one or more metallic elements [16]. The metallic components of investigated ferrites were nickel (Ni) and zinc (Zn). Chemical composition of the materials as well as their magnetic parameters in saturation region (measured for the magnetizing field amplitude from saturation region) are presented in Table I.

TABLE I

Chemical compositions and their Magnetic parameters of the investigated ferrite materials: $Ni_{0.36}Zn_{0.64}Fe_2O_4$ (A) and $Ni_{0.36}Zn_{0.67}Fe_{1.97}O_4$ (B) measured in the saturation region.

Material	A	B
average density [mg/m^3]	5.08	5.05
maximum magnetic flux density [T]	0.325	0.260
coercive field [A/m]	45	95
magnetic remanence [T]	0.120	0.087
initial magnetic permeability [–]	600	250

TABLE II

Investigated alloy steels with international symbols [17] and magnetic parameters measured in the saturation region.

	X30Cr13	13CrMo4-5
alloying elements [-]	Cr	Cr, Mo
maximum magnetic flux density [T]	1.280	1.590
coercive field [A/m]	785	670
magnetic remanence [T]	0.940	1.245
initial magnetic permeability [–]	100	80

The second group of investigated materials were constructional alloy steels utilized in the energy industry. In Table II, investigated steels are listed with their international symbols according to the [17] and basic magnetic parameters in the saturation region.

All investigated materials were formed into ring-shaped cores. On the basis of their geometric dimensions, the values of the effective magnetic flow path of the magnetic flux within the core and effective cross-sectional area of the core were calculated for each sample, which was necessary to properly measure their magnetic characteristics and parameters. On each core, two sets of windings were made: magnetizing and sensing, which allowed us to perform the investigation.

4. Experimental results and modeling

The investigation was performed with hysteresisgraph HB-PL30 developed at Warsaw University of Technology. Several magnetic hysteresis loops were measured with the increasing value of the magnetizing field amplitude H_m for each material. The frequency of the magnetizing field for ferrite materials was 1.0 Hz and for steel materials 0.1 Hz. On the basis of obtained initial magnetization curves, coefficients of the Rayleigh equation were determined for each investigated material by a second order polynomial curve fitting given with the general equation

$$B_m = a H_m^2 + b H_m, \quad (10)$$

where a and b are coefficients of the equation. This fitting allows one to calculate values of the Rayleigh coefficient $\alpha_{\rm R}$ and initial relative magnetic permeability μ_i:

$$\alpha_{\rm R} = \frac{a}{\mu_0}, \mu_i = \frac{b}{\mu_0}. \quad (11)$$

The determined coefficients are presented in Table III. All calculations were performed with free, open-source GNU Octave 4.0.0 software.

Calculated coefficients allow one to model the shape of the hysteresis loop for a given value of the magnetizing

TABLE III

Coefficients of the Rayleigh equation determined for investigated materials.

Material	μ_i [–]	$\alpha_{\rm R}$ [m/A]
$Ni_{0.36}Zn_{0.64}Fe_2O_4$	644.2	10.45
$Ni_{0.36}Zn_{0.67}Fe_{1.97}O_4$	285.1	1.90
X30Cr13	69.1	0.16
13CrMo4-5	63.7	0.79

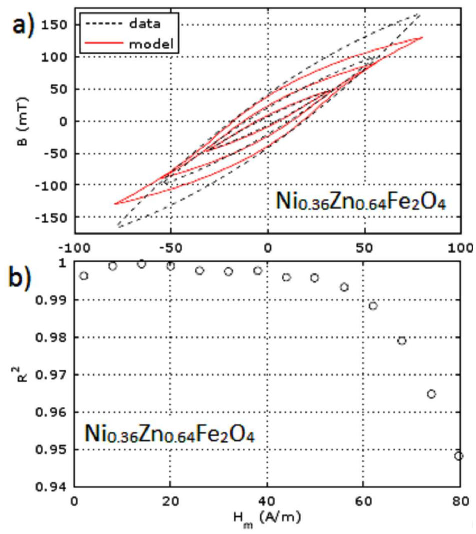

Fig. 1. Measurement and modeling results obtained for $Ni_{0.36}Zn_{0.64}Fe_2O_4$ ferrite material: (a) selected hysteresis loops (H_m = 32, 56, and 80 A/m), (b) R^2 determination coefficient dependence on magnetizing field amplitude H_m.

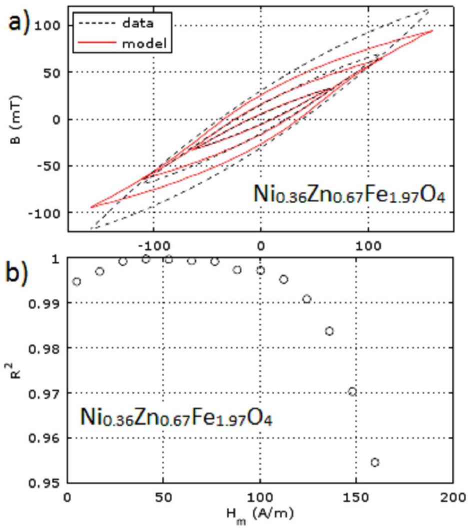

Fig. 2. Measurement and modeling results obtained for $Ni_{0.36}Zn_{0.67}Fe_{1.97}O_4$ ferrite material: (a) selected hysteresis loops (H_m = 64, 112, and 160 A/m), (b) R^2 determination coefficient dependence on magnetizing field amplitude H_m.

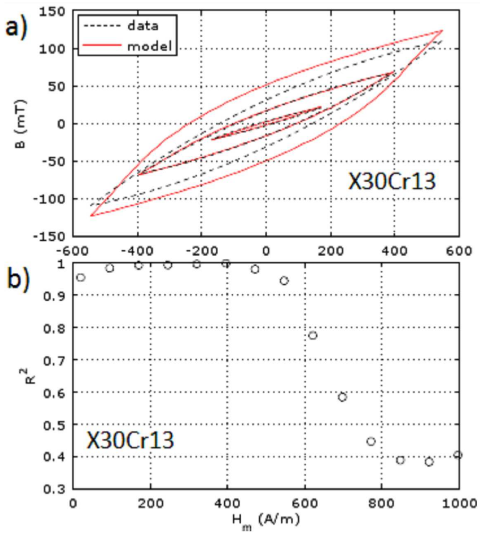

Fig. 3. Measurement and modeling results obtained for X30Cr13 steel: (a) selected hysteresis loops (H_m = 170, 396, and 546 A/m), (b) R^2 determination coefficient dependence on magnetizing field amplitude H_m.

field amplitude H_m with the system of Eqs. (3) and (4). In Fig. 1 and Fig. 2 the results of measurements and modeling for both investigated ferrite materials are presented. For each material a family of selected measured hysteresis loops is presented along with modeling results (Fig. 1a and Fig. 2a). For each measured value of the magnetizing field amplitude H_m, the R^2 determination coefficient between measurement results and modeling results was calculated. The dependence of the R^2 coefficient on the magnetizing field amplitude H_m for each ferrite material is also presented in Fig. 1b and Fig. 2b. For both discussed materials it is clearly seen that above some value

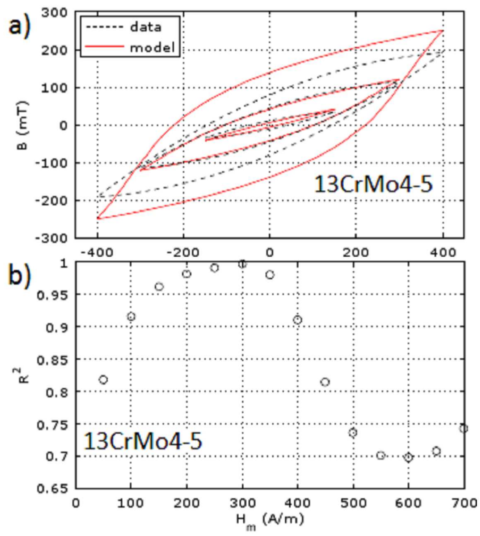

Fig. 4. Measurement and modeling results obtained for 13CrMo4-5 steel: (a) selected hysteresis loops (H_m = 150, 300, and 400 A/m), (b) R^2 determination coefficient dependence on magnetizing field amplitude H_m.

of the H_m, the model is no longer satisfactory, which is expressed by the rapidly decreasing value of the R^2 coefficient. However, for low values of magnetizing field amplitude H_m the Rayleigh model approximates hysteresis loop well.

The measurement and modeling results obtained for the investigated steel materials are presented in Fig. 3 and Fig. 4. Again it can be observed that for higher values of the magnetizing field amplitude H_m there is a significant decrease of the R^2 coefficient value while for lower fields the value of the coefficient remains on the satisfactory level above 0.9.

On the basis of the determined coefficients of the Rayleigh model, it is also possible to model the basic magnetic parameters of ferromagnetic materials. Coercive field H_c, magnetic remanence B_r and power loss in the certain volume of the material P_H are calculated according to Eqs. (6), (7), and (9), respectively. Figure 5 presents the results of measurement and modeling of basic magnetic parameters for both investigated ferrite materials. Calculated curves are characterized by a relatively high value of R^2 coefficient, which confirms the applicability of the model.

Figure 6 presents results of measurement and modeling of the basic magnetic parameters of the investigated steel materials. Obtained values of R^2 determination coefficient are slightly lower than for ferrite materials but still remain on the satisfactory level over 0.9 in most cases. Thus it can be stated that the Rayleigh model is also applicable for steel materials.

For each investigated material, the range of the magnetizing field amplitude H_m, where the Rayleigh model can be applied, was determined on the basis of values of an R^2 determination coefficient between experimental results and modeling calculated for Rayleigh hysteresis loops of increasing amplitude (presented in Fig. 1b, Fig. 2b, Fig. 3b, and Fig. 4b). The value of magnetizing field amplitude, where a significant decrease of R^2 coefficient starts to occur, was treated as the limit of magnetizing field H_L of the Rayleigh model applicability. For each investigated material, determined values of the limit of magnetizing field H_L were compared with the values of the saturation coercive field H_{CS} given in Table I and Table II. The ratio of the limit of the magnetizing field H_L to the saturation coercive field H_{CS} was calculated. Results are presented in Table IV.

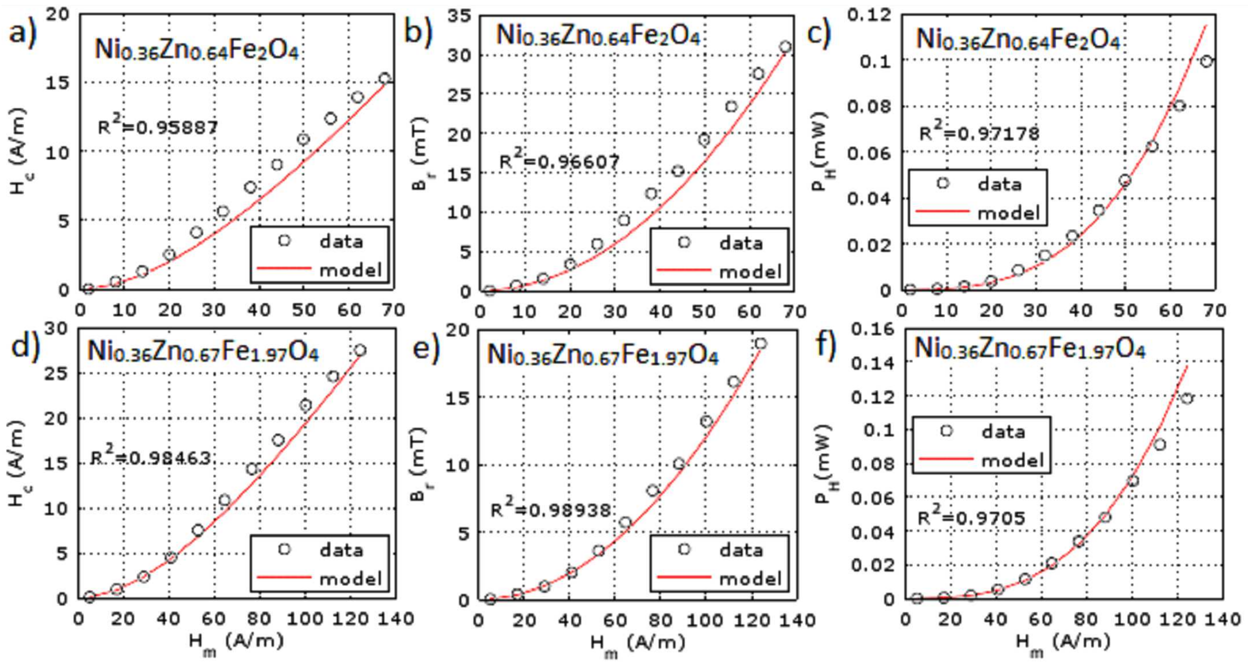

Fig. 5. Results of measurement and modeling of magnetic parameters of investigated ferrite materials: (a) coercive field H_c, (b) magnetic remanence B_r and (c) power loss P_H of the $Ni_{0.36}Zn_{0.64}Fe_2O_4$ ferrite, (d) coercive field H_c, (e) magnetic remanence B_r and (f) power loss P_H of the $Ni_{0.36}Zn_{0.67}Fe_{1.97}O_4$ ferrite.

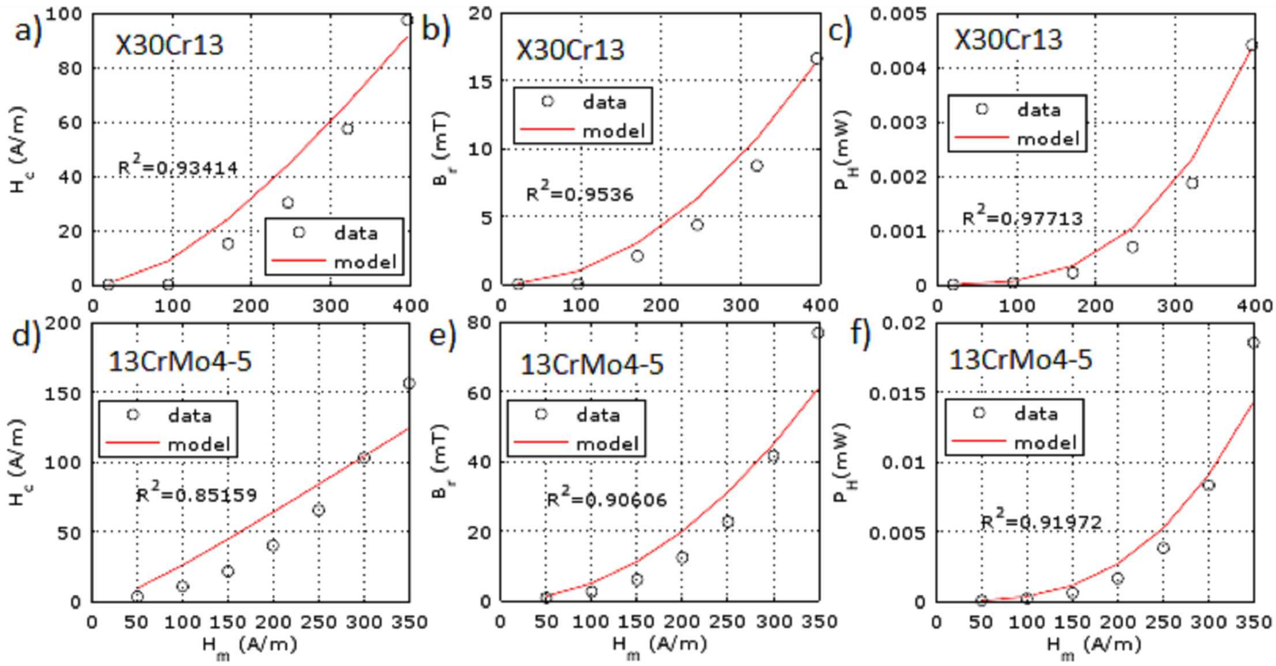

Fig. 6. Results of measurement and modeling of magnetic parameters of investigated steel materials: (a) coercive field H_c, (b) magnetic remanence B_r and (c) power loss P_H of the X30Cr13 steel, (d) coercive field H_c, (e) magnetic remanence B_r and (f) power loss P_H of the 13CrMo4-5 steel.

TABLE IV

Evaluation of the Rayleigh model applicability for modeling magnetic characteristics of investigated ferromagnetic materials: H_L — limit of magnetizing field, $R^2{\sim}H_L$ — correlation coefficient R^2 corresponding to H_L, H_{CS} — saturation coercive field.

Material	H_L [A/m]	$R^2 \sim H_L$ [–]	H_{CS} [A/m]	H_L/H_{CS} [–]
$Ni_{0.36}Zn_{0.64}Fe_2O_4$	68	0.991	45	1.51
$Ni_{0.36}Zn_{0.67}Fe_{1.97}O_4$	124	0.979	95	1.31
X30Cr13	472	0.976	785	0.60
13CrMo4-5	350	0.983	670	0.52

On the basis of presented results it can be observed that for ferrites the range of applicability of the Rayleigh model is significantly higher than the saturation coercive field ($H_L = (1.3 \div 1.5)H_{CS}$). For steel the range is much lower than the saturation coercive field ($H_L = (0.5 \div 0.6)H_{CS}$).

5. Conclusion

The presented model allows to simulate the shape of the magnetic hysteresis loop of the material for a given value of the magnetizing field, as well as to calculate basic magnetic parameters of the material. The results of modeling are characterized by high correlation with experimental data expressed by a high value of the R^2 determination coefficient between measurement results and modeling.

During the performed experiment, it was confirmed that for a high magnetizing field the Rayleigh model is not satisfactory because of low correlation between the hysteresis loop and parabolic curves. However, for low magnetizing fields, about $(1.3 \div 1.5)$ of saturation coercive field for ferrite materials and $(0.5 \div 0.6)$ of a saturation coercive field for steel materials, the Rayleigh model is a very accurate approximation of the hysteresis loop.

The obtained results presented in the paper indicate the possibility of application of the Rayleigh hysteresis model in certain technical applications, like SPICE modeling of the inductive components with ferromagnetic cores.

Acknowledgments

This work was partially supported by the statutory founds of Institute of Metrology and Biomedical Engineering, Warsaw University of Technology (Poland).

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