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Macroscopic Quantum Tunnelling of Josephson Vortex in Antiferromagnetic Superconductor

T. KRZYSZTOŃ

Institute of Low Temperature and Structure Research

Polish Academy of Sciences, P.O. Box 1410, 50-950 Wrocław, Poland

The influence of antiferromagnetic order on high temperature superconductor could possibly result in creation of spin-flop domain along the phase core of the Josephson vortex. In this paper we discuss how this feature affects the activation energy of the macroscopic quantum tunnelling of the Josephson vortex.

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1. Introduction

The specific feature caused by the long-range antiferromagnetic order in the mixed state of superconductor might be the creation of spin-flop (or metamagnetic) domain along each vortex core. Let us consider an antiferromagnet with two magnetic sublattices as an example. An infinitesimal magnetic field applied perpendicular to the easy axis makes the ground antiferromagnetic state unstable against the phase transformation to the canted phase (spin-flop, SF). On the contrary, if the magnetic field is applied parallel to the easy axis, the antiferromagnetic configuration is stable up to the thermodynamic critical field H_T . When the field is further increased a canted phase develops in the system. Let us assume that in the antiferromagnetic superconductor the lower critical field fulfils the relation $H_{c1} < \frac{1}{2}H_T$ and that the external field, $H_{c1} < H < \frac{1}{2}H_T$, is applied parallel to the easy axis. Then the superconducting vortices appear in the ground antiferromagnetic state. If the field is increased above $\frac{1}{2}H_T$, the phase transition to the canted phase originates in the vortex core because the field intensity in the core doubles the external one. The spatial distribution of the field around the vortex is a decreasing function of the distance from its centre. Hence the magnetic field intensity in the neighbourhood of the core is less than H_T . Therefore, the rest of the vortex remains in the antiferromagnetic configuration. The radius of spin-flop domain grows as the field is increased. Thus, in the considered model there are two distinct types of vortices. Possible candidate of such system might be $\text{ErBa}_2\text{Cu}_3\text{O}_7$ [1, 2]. This compound has tetragonal unit cell with small orthorhombic distortion

in the ab plane. The Er ions form two sublattices antiferromagnetic structure of magnetic moments lying parallel and antiparallel to the $\mathbf{a}$ direction. The theoretical structure used in the present calculations consists of superconducting layers of thickness d_s and isolating ones of thickness d_i , $d = d_i + d_s$. In the isolating layers, the magnetic moments are running parallel and antiparallel to the $\mathbf{a}$ direction (easy axis). The magnetic field aligned parallel to the conducting planes makes the vortex lattice to accommodate itself to the layer structure so that the vortex cores lie in between the superconducting sheets. A current density j , flowing along the planes perpendicular to the applied magnetic field exerts a Lorentz force on the vortices in the $\mathbf{c}$ direction so that intrinsic pinning barriers are formed on strongly superconducting layers. The experimental evidence of quantum tunnelling is based on the fact that the magnetic moment relaxation rate exhibits two types of behaviour as a function of temperature. Above a characteristic temperature T_0 , in the thermal activation regime, the decay rate is of the Arrhenius type $\Gamma \sim \exp(-U_0/k_B T)$. Below T_0 , the decay rate is believed to be essentially independent of temperature $\Gamma \sim \exp(-S/\hbar)$ and is interpreted as arising from the quantum tunnelling of vortices through intrinsic pinning potential [3, 4].

2. Macroscopic quantum tunnelling

In the following we show a considerable change of tunnelling rate and crossover temperature due to the SF phase transition in the vortex core. Let us consider the vortex line as a straight string-like object of an effective mass per unit length m_{eff} trapped into a metastable state in an intrinsic pinning potential $V(u)$ and exposed to continuous deformation $u(x, t)$ in the $\hat{z}$ direction. The magnetic field is applied in $\hat{x}$ direction. In the semi-classical approximation the quantum decay rate is calculated as a saddle-point solution (bounce) of the Euclidean action S for the string

$$S = \int_{-\infty}^{\infty} dx \int_0^{\hbar\beta} d\tau \left[\frac{1}{2} m_{\text{eff}} \left(\frac{\partial u}{\partial \tau} \right)^2 + \frac{\varepsilon_1}{2} \left(\frac{\partial u}{\partial x} \right)^2 + V(u) - \frac{\eta}{2\pi} \frac{\partial u}{\partial \tau} \int_0^{\hbar\beta} d\tau' \frac{\partial u}{\partial \tau'} \ln \left| \sin \frac{\pi}{\hbar\beta} (\tau - \tau') \right| \right]. \quad (1)$$

Here $\beta = (k_B T)^{-1}$, η is the viscosity coefficient and τ denotes imaginary time. The pinning potential $V(u)$ consists of intrinsic periodic part and the Lorentz potential

$$V(u) = -\frac{\varphi_0 j_c d}{2\pi} \cos \left(\frac{2\pi u}{d} \right) - \varphi_0 j u, \quad (2)$$

where j_c denotes critical depinning current. The last term in (1) is so-called Caldeira–Leggett action [5] which describes ohmic damping. The line tension ε_1 is different for vortices in two different orientations in the ab plane. Those vortices lying parallel to b direction and those ones lying in the a direction but created in the magnetic field less than $\frac{1}{2} H_T$ have the line tension ε_b and those ones lying in the a direction but possessing SF domain and fields above $\frac{1}{2} H_T$ have the line

tension denoted ε_a [6].

$$\varepsilon_b = \epsilon_0 \ln \frac{\lambda_{ab}}{d}; \quad \varepsilon_a = \frac{\varphi_0 H_T}{2} + \frac{9}{128} \epsilon_0 \ln \frac{\varphi_0 \lambda_c^2}{\pi(\mu_0 H_T + M) d^2 \lambda_{ab}^2}, \quad (3)$$

where $\epsilon_0 = \frac{\varphi_0^2}{4\pi\lambda_{ab}\lambda_c\mu_0}$, M is the magnetic moment of the SF domain inside the vortex and H_T is the thermodynamic critical field of spin-flop domain formation. In the semiclassical approximation the decay rate is given by the value of the action on a classical trajectory obtained from the Euler–Lagrange equations of the motion. The trajectory $u_0(x)$ for static solution gives the activation energy in the thermal regime $T > T_0$. Below T_0 a new kind of trajectory, periodic in imaginary time, develops. Therefore, $u(x, \tau)$ can be expanded in the Fourier series with the Matsubara frequencies $u(x, \tau) = u_0(x) + \sum_{n=1}^{\infty} u_n(x) \cos(\omega_n \tau)$, where $\omega_n = \frac{2\pi n}{\hbar\beta}$. Substituting this expansion into the Euler–Lagrange equations and linearising potential around the static solution $u_0(x)$ one obtains in the first step of perturbation

$$-\varepsilon_1 \frac{\partial^2 u_n}{\partial x^2} + V''(u_0) u_n = -(\eta \omega_n - m_{\text{eff}} \omega_n^2) u_n. \quad (4)$$

The above equation has three discrete solutions, the unstable one corresponding to the tunnelling process determines the crossover temperature

$$k_B T_0 = \frac{\hbar \eta}{4\pi m_{\text{eff}}} \left\{ \left[1 + \frac{20\pi \varphi_0 j_c m_{\text{eff}}}{d \eta^2} \left(\frac{j_c - j}{2j_c} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} - 1 \right\}. \quad (5)$$

Below T_0 the predominant mechanism by which the metastable state decays is quantum-mechanical tunnelling. The rate at which this takes place is determined by the action for the bounce trajectory. However, it should be noted that the above results for the saddle-point trajectory of the static solution include the contribution from the elastic and pinning terms alone. To include contributions of effective mass and viscous damping in the bounce action one would have to go beyond the harmonic approximation in the pinning potential as employed in Eq. (4) and substitute for $u(x, \tau)$ the leading terms in the Matsubara frequencies. Then the resulting action of the bounce is given by

$$S = 5.26 d \hbar \beta \left(\frac{\varepsilon_1 d \varphi_0 j_c}{\pi} \right)^{1/2} \left(1 - \frac{j}{j_c} \right)^{5/4} + d^2 \left(\frac{\varepsilon_1 d}{\pi \varphi_0 j_c} \right)^{1/2} \left(5.71 \frac{m_{\text{eff}}}{\hbar \beta} + 2.85 \frac{\eta}{\pi} \right) \left(1 - \frac{j}{j_c} \right)^{3/4}. \quad (6)$$

3. Discussion of the results

The effective mass and viscosity coefficient of the vortex were estimated in [6] $\eta = \varepsilon_1 \frac{4\mu_0 \sqrt{3} \kappa^2}{\pi \rho_N \ln \kappa}$ and $m = \varepsilon_1^2 \frac{9\lambda_{ab}^2 m_e \mu_0}{\varphi_0^2 \pi^2 \varepsilon_F (\ln \kappa)^2}$, where m_e denotes the mass of the electron, ε_F is the Fermi energy and κ — the Ginzburg–Landau parameter. One can easily see that the crossover temperature Eq. (5) is proportional to the line tension of the vortex, hence

$$\frac{T_{0b}}{T_{0a}} = \frac{\varepsilon_a}{\varepsilon_b}. \quad (7)$$

In order to evaluate Eq. (7) it is necessary to calculate H_T and M in Eq. (3). To find the thermodynamic critical field H_T , and then to calculate M the following argumentation is proposed. At low fields, in the vicinity of the lower critical field H_{c1} , the intensity of the field in the vortex core is $2H_{c1}$ [7]. When the external field is increased, the field intensity in the vortex core increases because of the superposition of the fields of the surrounding vortices. The field intensity in the core must reach H_T in order to originate a transition to the SF phase. Thus, taking into account only z nearest neighbours we can write for the nonunilateral triangular lattice [8]

$$\varphi_0 H_T = 2\varphi_0 H_{c1} + z\varepsilon_b \left(\ln \frac{\lambda_{ab}}{d} \right)^{-1} \left[K_0 \left(\frac{c}{\lambda_{ab}} \right) + 2K_0 \left(\frac{c}{2\lambda_{ab}} \sqrt{\frac{3\lambda_c}{\lambda_{ab}}} \right) \right], \quad (8)$$

where c is the lattice constant. Since $\varphi_0 H_{c1} = \varepsilon_b$ we obtain

$$\frac{\varepsilon_a}{\varepsilon_b} = 1.6 + \frac{9}{128} \left[\ln \left(\frac{\varphi_0 \lambda_c^2}{\pi(\mu_0 H_T + M)d^2 \lambda_{ab}^2} \right) / \ln \left(\frac{\lambda_{ab}}{d} \right) \right] \approx 1.7, \quad (9)$$

because the fraction in Eq. (9) give approximately 1.5 for the following parameters: $c \approx \lambda_{ab}$; $B_T = \mu_0 H_T + M \approx 0.37$ T calculated for $5.5\mu_B$ per Er ion in $\text{ErBa}_2\text{Cu}_3\text{O}_7$; $\mu_0 H_{c2}^{ab} \approx 150$ T; $d = 1$ nm; $d/\xi_c = 1$; $\lambda_c = 125$ nm; $\lambda_c/\lambda_{ab} = 5$. From these considerations one can draw the conclusion that crossover temperature for the vortex with spin-flop domain is approximately twice lower than that of vortex without such domain which suggests that at constant temperature we can change flux creep regime from quantum to thermal, and vice versa, by changing magnetic field direction or intensity alone. The activation energy calculated from Eq. (6) for given temperature and current density is almost 30% higher for the vortex possessing spin-flop domain in the core than that of the vortex that does not possess magnetic domain. Another important effect is that the action, and hence activation energy, is rendered temperature dependent so that the quantum tunnelling rate becomes temperature dependent below the crossover temperature. Moreover, including the damping and inertial mass contributions reduces the quantum tunnelling rate.

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