

## FOUNDATIONS

 ADVANCESVolume 78 (2022)
Supporting information for article:

On the combinatorics of crystal structures: number of Wyckoff sequences of given length

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# Supplemental material for the manuscript On the combinatorics of crystal structures: <br> Number of Wyckoff sequences of given length authored by Wolfgang Hornfeck 

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## 1 Mathematical background

### 1.1 MacMahon's formula

In the general case the combinations of a multiset with finite multiplicities are given by (see: Wieder, T. (2011). Progr. Appl. Math. 2, 61-66; Alekseyev, M. (2018). https://mathoverflow.net/questions/293000/ the-combinations-of-a-finite-multiset. Retrieved 26/08/2021)

$$
\begin{equation*}
{ }^{n} W_{k}=\sum_{\mathcal{J} \subseteq\{1,2, \ldots, n\}}(-1)^{|\mathcal{J}|}\binom{n+k-1-|\mathcal{J}|-M_{\mathcal{J}}}{n-1}, \tag{1}
\end{equation*}
$$

in which the sum runs over the powerset of the set of indices $\{1,2, \ldots, n\}$, namely the set of all its possible subsets $\mathcal{J}$, of which there are $2^{n}$ in number, fulfilling the condition $1 \leq i_{1}<i_{2}<\ldots<i_{|\mathcal{J}|} \leq n$ on the indices. The index subsets are used in the binomial coefficient to define the sum $M_{\mathcal{J}}=\sum_{i \in \mathcal{J}} m_{i}$ by extracting the multiplicities $m_{i}$ from their corresponding locations in the same sized set of multiplicities $\left\{m_{1}, m_{2}, \ldots, m_{n}\right\}$.

### 1.2 Algebraic derivation of equation 7

The following section shows how the summation formula given in equation 13 can be derived by algebraic transformations from the generating polynomial of equation 5. Recall that

$$
\begin{equation*}
{ }^{n} W_{k}=\left[x^{k}\right]\left(1+x+\ldots+x^{k}\right)^{\nu}(1+x)^{\varphi} . \tag{2}
\end{equation*}
$$

By the convolution rule for formal power series (see, for instance: Merlini, D., Sprugnoli, R. \& Verri, M. C. (2007). Amer. Math. Monthly 114, 40-57) this becomes

$$
\begin{equation*}
{ }^{n} W_{k}=\sum_{i=0}^{k}\left[x^{i}\right]\left(1+x+\ldots+x^{k}\right)^{\nu} \times\left[x^{k-i}\right](1+x)^{\varphi} . \tag{3}
\end{equation*}
$$

Here, the second factor immediately results from the standard expansion according to the binomial theorem as being

$$
\begin{equation*}
\left[x^{k-i}\right](1+x)^{\varphi}=\binom{\varphi}{k-i} \tag{4}
\end{equation*}
$$

Note, that for the first factor

$$
\begin{equation*}
1+x+\ldots+x^{k}=\frac{1-x^{k+1}}{1-x} \tag{5}
\end{equation*}
$$

as the result of the expansion of a geometric series, and hence

$$
\begin{equation*}
\left(1+x+\ldots+x^{k}\right)^{\nu}=\left(1-x^{k+1}\right)^{\nu}(1-x)^{-\nu} \tag{6}
\end{equation*}
$$

Thus, after a second application of the convolution rule, one has

$$
\begin{equation*}
\sum_{j=0}^{i}\left[x^{j}\right]\left(1-x^{k+1}\right)^{\nu} \times\left[x^{i-j}\right](1-x)^{-\nu} \tag{7}
\end{equation*}
$$

Now, by the binomial theorem again, one finds

$$
\begin{equation*}
\left[x^{j}\right]\left(1-x^{\kappa}\right)^{\nu}=(-1)^{j \kappa}\binom{\nu}{j \kappa} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[x^{i-j}\right](1-x)^{-\nu}=(-1)^{i-j \kappa}\binom{-\nu}{i-j \kappa} . \tag{9}
\end{equation*}
$$

Here, $\kappa=k+1$. Note, that in the latter equation 9 one has to choose $i-j \kappa$ instead of $i-j$, since one can only choose multiples of $\kappa$ in the former equation 8 , and thus has to subtract these choices in the latter equation 9 , correspondingly. Thus,

$$
\begin{equation*}
{ }^{n} W_{k}=\sum_{i=0}^{k}\binom{\varphi}{k-i}(-1)^{i} \sum_{j=0}^{i}\binom{\nu}{j \kappa}\binom{-\nu}{i-j \kappa} . \tag{10}
\end{equation*}
$$

Now, for reasons becoming clear in the following, it proves useful to split the double sum into two parts, ${ }^{n} W_{k}=S_{j=0}+S_{j>0}$, with one part each summing over all the $j=0$ and $j>0$ terms, respectively. It then happens, that the sum for $j=0$ already yields the result stated in equation 13 , namely as

$$
\begin{align*}
S_{j=0} & =\sum_{i=0}^{k}(-1)^{i}\binom{\nu}{0}\binom{-\nu}{i}\binom{\varphi}{k-i} \\
& =\sum_{i=0}^{k}(-1)^{i}\binom{-\nu}{i}\binom{\varphi}{k-i}  \tag{11}\\
& =\sum_{i=0}^{k}\binom{\nu+i-1}{i}\binom{\varphi}{k-i},
\end{align*}
$$

and using an identity for binomial coefficients (see Riordan, J. (1968). Combinatorial Identities, New York: John Wiley \& Sons, page 1) used in the final algebraic transformation step. It remains to prove that $S_{j>0}=0$, where

$$
\begin{equation*}
S_{j>0}=\sum_{i=1}^{k} \sum_{j=1}^{i}(-1)^{i}\binom{\nu}{j \kappa}\binom{-\nu}{i-j \kappa}\binom{\varphi}{k-i} . \tag{12}
\end{equation*}
$$

Here, it suffices to show that the pair of binomial coefficients containing the index $\kappa=k+1$ cannot both be non-zero simultaneously, and thus each individual product term of the sum will vanish independently. For this purpose, recall that a binomial coefficient ${ }^{n} C_{k}$ is non-zero if $n \geq k$. Thus, the binomial coefficients under consideration are both non-zero if and only if both conditions (i) $\nu \geq j \kappa$ and (ii) $-\nu \geq i-j \kappa \Leftrightarrow j \kappa \geq \nu+i$ hold true. However, combining both relations results in $\nu \geq j \kappa \geq \nu+i$, a condition which cannot be fulfilled for both comparative subrelations simultaneously, given that $i>0$.

2 Number of Wyckoff sequences of length $6 \leq k \leq 20$

Table 1: Number of Wyckoff sequences of length $6 \leq k \leq 20$ for the 44 combinatorial types $(\nu, \varphi)$ with $\nu$ non-fixed and $\varphi$ fixed Wyckoff positions.

| type | $k=6$ | $k=7$ | $k=8$ | $k=9$ | $k=10$ | $k=11$ | $k=12$ | $k=13$ | $k=14$ | $k=15$ | $k=16$ | $k=17$ | $k=18$ | $k=19$ | $k=20$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(1,0)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $(1,2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| $(1,4)$ | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| $(1,8)$ | 247 | 255 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 | 256 |
| $(2,0)$ | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| $(2,2)$ | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 76 | 80 |
| $(2,4)$ | 80 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 | 256 | 272 | 288 | 304 |
| $(3,0)$ | 28 | 36 | 45 | 55 | 66 | 78 | 91 | 105 | 120 | 136 | 153 | 171 | 190 | 210 | 231 |
| $(3,2)$ | 85 | 113 | 145 | 181 | 221 | 265 | 313 | 365 | 421 | 481 | 545 | 613 | 685 | 761 | 841 |
| $(3,3)$ | 146 | 198 | 258 | 326 | 402 | 486 | 578 | 678 | 786 | 902 | 1026 | 1158 | 1298 | 1446 | 1602 |
| $(3,4)$ | 248 | 344 | 456 | 584 | 728 | 888 | 1064 | 1256 | 1464 | 1688 | 1928 | 2184 | 2456 | 2744 | 3048 |
| $(4,0)$ | 84 | 120 | 165 | 220 | 286 | 364 | 455 | 560 | 680 | 816 | 969 | 1140 | 1330 | 1540 | 1771 |
| $(4,2)$ | 231 | 344 | 489 | 670 | 891 | 1156 | 1469 | 1834 | 2255 | 2736 | 3281 | 3894 | 4579 | 5340 | 6181 |
| $(4,3)$ | 377 | 575 | 833 | 1159 | 1561 | 2047 | 2625 | 3303 | 4089 | 4991 | 6017 | 7175 | 8473 | 9919 | 11521 |
| $(4,4)$ | 608 | 952 | 1408 | 1992 | 2720 | 3608 | 4672 | 5928 | 7392 | 9080 | 11008 | 13192 | 15648 | 18392 | 21440 |
| $(4,5)$ | 968 | 1560 | 2360 | 3400 | 4712 | 6328 | 8280 | 10600 | 13320 | 16472 | 20088 | 24200 | 28840 | 34040 | 39832 |
| $(4,6)$ | 1520 | 2528 | 3920 | 5760 | 8112 | 11040 | 14608 | 18880 | 23920 | 29792 | 36560 | 44288 | 53040 | 62880 | 73872 |
| $(5,0)$ | 210 | 330 | 495 | 715 | 1001 | 1365 | 1820 | 2380 | 3060 | 3876 | 4845 | 5985 | 7315 | 8855 | 10626 |
| $(5,2)$ | 532 | 876 | 1365 | 2035 | 2926 | 4082 | 5551 | 7385 | 9640 | 12376 | 15657 | 19551 | 24130 | 29470 | 35651 |
| $(5,3)$ | 833 | 1408 | 2241 | 3400 | 4961 | 7008 | 9633 | 12936 | 17025 | 22016 | 28033 | 35208 | 43681 | 53600 | 65121 |
| $(5,4)$ | 1289 | 2241 | 3649 | 5641 | 8361 | 11969 | 16641 | 22569 | 29961 | 39041 | 50049 | 63241 | 78889 | 97281 | 118721 |
| $(5,6)$ | 2972 | 5500 | 9420 | 15180 | 23292 | 34332 | 48940 | 67820 | 91740 | 121532 | 158092 | 202380 | 255420 | 318300 | 392172 |
| $(6,0)$ | 462 | 792 | 1287 | 2002 | 3003 | 4368 | 6188 | 8568 | 11628 | 15504 | 20349 | 26334 | 33649 | 42504 | 53130 |
| $(6,4)$ | 2471 | 4712 | 8361 | 14002 | 22363 | 34332 | 50973 | 73542 | 103503 | 142544 | 192593 | 255834 | 334723 | 432004 | 550725 |
| $(6,5)$ | 3653 | 7183 | 13073 | 22363 | 36365 | 56695 | 85305 | 124515 | 177045 | 246047 | 335137 | 448427 | 590557 | 766727 | 982729 |
| $(6,6)$ | 5336 | 10836 | 20256 | 35436 | 58728 | 93060 | 142000 | 209820 | 301560 | 423092 | 581184 | 783564 | 1038984 | 1357284 | 1749456 |
| $(7,0)$ | 924 | 1716 | 3003 | 5005 | 8008 | 12376 | 18564 | 27132 | 38760 | 54264 | 74613 | 100947 | 134596 | 177100 | 230230 |
| $(7,4)$ | 4396 | 9108 | 17469 | 31471 | 53834 | 88166 | 139139 | 212681 | 316184 | 458728 | 651321 | 907155 | 1241878 | 1673882 | 2224607 |
| $(7,5)$ | 6321 | 13504 | 26577 | 48940 | 85305 | 142000 | 227305 | 351820 | 528865 | 774912 | 1110049 | 1558476 | 2149033 | 2915760 | 3898489 |
| $(7,6)$ | 8989 | 19825 | 40081 | 75517 | 134245 | 227305 | 369305 | 579125 | 880685 | 1303777 | 1884961 | 2668525 | 3707509 | 5064793 | 6814249 |
| $(7,8)$ | 17584 | 41456 | 88720 | 175504 | 325360 | 571312 | 958160 | 1545040 | 2408240 | 3644272 | 5373200 | 7742224 | 10929520 | 15148336 | 20651344 |
| $(8,4)$ | 7380 | 16488 | 33957 | 65428 | 119262 | 207428 | 346567 | 559248 | 875432 | 1334160 | 1985481 | 2892636 | 4134514 | 5808396 | 8033003 |
| $(8,5)$ | 10364 | 23868 | 50445 | 99385 | 184690 | 326690 | 553995 | 905815 | 1434680 | 2209592 | 3319641 | 4878117 | 7027150 | 9942910 | 13841399 |
| $(8,6)$ | 14407 | 34232 | 74313 | 149830 | 284075 | 511380 | 880685 | 1459810 | 2340495 | 3644272 | 5529233 | 8197758 | 11905267 | 16970060 | 23784309 |
| $(9,0)$ | 3003 | 6435 | 12870 | 24310 | 43758 | 75582 | 125970 | 203490 | 319770 | 490314 | 735471 | 1081575 | 1562275 | 2220075 | 3108105 |
| $(9,6)$ | 22180 | 56412 | 130725 | 280555 | 564630 | 1076010 | 1956695 | 3416505 | 5757000 | 9401272 | 14930505 | 23128263 | 35033530 | 52003590 | 75787899 |
| $(10,4)$ | 18238 | 46552 | 108823 | 236522 | 483483 | 937872 | 1738828 | 3099032 | 5334668 | 8904464 | 14459741 | 22907654 | 35490081 | 53880904 | 80304730 |
| $(10,5)$ | 24650 | 64790 | 155375 | 345345 | 720005 | 1421355 | 2676700 | 4837860 | 8433700 | 14239132 | 23364205 | 37367395 | 58397735 | 89370985 | 134185634 |
| $(10,6)$ | 33028 | 89440 | 220165 | 500720 | 1065350 | 2141360 | 4098055 | 7514560 | 13271560 | 22672832 | 37603337 | 60731600 | 95765130 | 147768720 | 223556619 |
| $(10,8)$ | 57799 | 166344 | 432073 | 1030490 | 2286955 | 4772780 | 9446125 | 17852030 | 32398735 | 56730512 | 96220561 | 158611106 | 254831667 | 400030580 | 614859189 |
| $(12,6)$ | 67582 | 204840 | 562263 | 1420406 | 3343899 | 7408752 | 15571660 | 31249128 | 60198156 | 111820016 | 201045213 | 351002010 | 596723937 | 990214584 | 1607261850 |
| $(13,8)$ | 151634 | 511402 | 1550927 | 4300699 | 11047673 | 26564629 | 60297692 | 130098892 | 268366964 | 531832420 | 1016715821 | 1881628273 | 3381401443 | 5916065911 | 10100480866 |
| $(15,6)$ | 169919 | 595395 | 1881510 | 5448670 | 14640774 | 36866574 | 87697730 | 198383370 | 429121650 | 891732586 | 1787261391 | 3466710075 | 6526802575 | 11957753475 | 21366824625 |
| $(19,8)$ | 697480 | 3013560 | 11652910 | 40985850 | 132802324 | 400489068 | 1133504470 | 3031952322 | 7709522536 | 18728264280 | 43650074935 | 97969817325 | 212429838390 | 446250553770 | 910461513255 |

## 3 Frequencies of occurrence $f$ versus Wyckoff sequence length $k$

### 3.1 Overview

The following pages contain plots of the frequencies of occurrence $f$ of Wyckoff sequences of length $k$ for all 230 space group types.

The full listings of the frequencies of occurrence $f$ of Wyckoff sequences of length $k$ is given in ascending order of the space group type number (in boldface) and listed according to the pairs $(k, f)$ in ascending order of $k$. Note, that the space group types of number $78,93,95,96,145,153,154,170,172,179,181,212$ are absent in the listing since no Wyckoff sequence of any length is observed for these space group types.

In the subsequent plots the space group types are denoted by their space group type numbers and augmented by the information about their combinatorial type $(\nu, \phi)$, in which $\nu$ and $\phi$ are the number of non-fixed and fixed sites, respectively. For reasons of clarity both plot ranges share a cutoff at 50 . Potential frequencies, calculated according to the formula

$$
\begin{equation*}
f_{\mathrm{calc}}(k)={ }^{\nu+\varphi} W_{k}=\sum_{i=0}^{k}\binom{\nu+i-1}{i}\binom{\varphi}{k-i} \tag{13}
\end{equation*}
$$

are highlighted with light green bars, while actual frequencies as found in the Pearson's Crystal Data Crystal Structure Database for Inorganic Compounds (Villars, P. \& Cenzual, K. Release 2019/2020. ASM International, Materials Park, Ohio, USA. http://www.crystalimpact.com/pcd/Default.htm) are highlighted by red bars.

Table 2: Frequency of occurrence $f$ of distinct Wyckoff sequences in the PCD for the 230 space group types, denoted by their numbers (\#).

| $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ | $f$ | $\#$ |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 94 | 24 | 5 | 47 | 93 | 70 | 106 | 93 | 0 | 116 | 11 | 139 | 392 | 162 | 44 | 185 | 44 | 208 |  |
| 2 | 1113 | 25 | 24 | 48 | 9 | 71 | 245 | 94 | 5 | 117 | 20 | 140 | 118 | 163 | 70 | 186 | 152 | 209 |  |
| 3 | 38 | 26 | 68 | 49 | 7 | 72 | 75 | 95 | 0 | 118 | 28 | 141 | 130 | 164 | 188 | 187 | 88 | 210 |  |
| 4 | 91 | 27 | 3 | 50 | 12 | 73 | 21 | 96 | 0 | 119 | 44 | 142 | 82 | 165 | 84 | 188 | 16 | 211 |  |
| 5 | 194 | 28 | 20 | 51 | 104 | 74 | 126 | 97 | 9 | 120 | 14 | 143 | 76 | 166 | 487 | 189 | 119 | 212 | 0 |
| 6 | 36 | 29 | 59 | 52 | 65 | 75 | 13 | 98 | 12 | 121 | 49 | 144 | 39 | 167 | 215 | 190 | 75 | 213 | 44 |
| 7 | 71 | 30 | 18 | 53 | 48 | 76 | 23 | 99 | 25 | 122 | 80 | 145 | 0 | 168 | 5 | 191 | 180 | 214 | 9 |
| 8 | 156 | 31 | 119 | 54 | 35 | 77 | 7 | 100 | 26 | 123 | 334 | 146 | 88 | 169 | 18 | 192 | 21 | 215 | 72 |
| 9 | 77 | 32 | 32 | 55 | 208 | 78 | 0 | 101 | 2 | 124 | 15 | 147 | 119 | 170 | 0 | 193 | 77 | 216 | 167 |
| 10 | 112 | 33 | 68 | 56 | 73 | 79 | 18 | 102 | 10 | 125 | 25 | 148 | 277 | 171 | 4 | 194 | 534 | 217 | 85 |
| 11 | 417 | 34 | 34 | 57 | 150 | 80 | 12 | 103 | 2 | 126 | 29 | 149 | 11 | 172 | 0 | 195 | 17 | 218 | 48 |
| 12 | 1240 | 35 | 32 | 58 | 227 | 81 | 29 | 104 | 7 | 127 | 136 | 150 | 56 | 173 | 140 | 196 | 19 | 219 | 24 |
| 13 | 241 | 36 | 157 | 59 | 162 | 82 | 78 | 105 | 6 | 128 | 60 | 151 | 14 | 174 | 61 | 197 | 39 | 220 | 66 |
| 14 | 461 | 37 | 16 | 60 | 155 | 83 | 25 | 106 | 6 | 129 | 122 | 152 | 67 | 175 | 24 | 198 | 56 | 221 | 276 |
| 15 | 704 | 38 | 86 | 61 | 117 | 84 | 27 | 107 | 39 | 130 | 54 | 153 | 0 | 176 | 398 | 199 | 28 | 222 | 8 |
| 16 | 11 | 39 | 23 | 62 | 466 | 85 | 63 | 108 | 18 | 131 | 24 | 154 | 0 | 177 | 3 | 200 | 39 | 223 | 52 |
| 17 | 10 | 40 | 51 | 63 | 507 | 86 | 51 | 109 | 17 | 132 | 20 | 155 | 99 | 178 | 33 | 201 | 24 | 224 | 30 |
| 18 | 75 | 41 | 50 | 64 | 231 | 87 | 175 | 110 | 13 | 133 | 7 | 156 | 93 | 179 | 0 | 202 | 30 | 225 | 287 |
| 19 | 69 | 42 | 30 | 65 | 211 | 88 | 87 | 111 | 18 | 134 | 15 | 157 | 30 | 180 | 45 | 203 | 121 | 226 | 61 |
| 20 | 86 | 43 | 74 | 66 | 70 | 89 | 2 | 112 | 11 | 135 | 37 | 158 | 20 | 181 | 0 | 204 | 83 | 227 | 272 |
| 21 | 27 | 44 | 60 | 67 | 40 | 90 | 14 | 113 | 63 | 136 | 96 | 159 | 78 | 182 | 42 | 205 | 89 | 228 | 11 |
| 22 | 14 | 45 | 20 | 68 | 31 | 91 | 14 | 114 | 54 | 137 | 68 | 160 | 191 | 183 | 12 | 206 | 26 | 229 | 116 |
| 23 | 20 | 46 | 41 | 69 | 102 | 92 | 65 | 115 | 25 | 138 | 44 | 161 | 68 | 184 | 13 | 207 | 2 | 230 | 40 |

### 3.2 Listings (full)

1: $(2,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1),(15,1),(16,1),(17,1)$, $(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1),(30,1),(31,1),(32,1)$, $(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(42,1),(44,1),(45,1),(46,1),(48,1)$, $(49,1),(50,1),(51,1),(52,1),(54,1),(55,1),(56,1),(58,1),(60,1),(61,1),(62,1),(64,1),(66,1),(67,1)$, $(68,1),(72,1),(74,1),(75,1),(76,1),(79,1),(80,1),(82,1),(84,1),(86,1),(90,1),(94,1),(95,1),(96,1)$, $(98,1),(100,1),(102,1),(103,1),(106,1),(112,1),(116,1),(120,1),(124,1),(136,1),(144,1),(160,1)$, $(168,1),(187,1),(210,1),(212,1),(217,1),(224,1),(229,1),(240,1),(248,1),(249,1),(332,1),(408,1) ;$

2: $(1,1),(2,3),(3,5),(4,9),(5,9),(6,9),(7,8),(8,18),(9,20),(10,19),(11,21),(12,21),(13,18)$, $(14,24),(15,24),(16,25),(17,22),(18,23),(19,20),(20,20),(21,27),(22,26),(23,17),(24,20),(25,18)$, $(26,15),(27,18),(28,10),(29,18),(30,16),(31,9),(32,11),(33,10),(34,15),(35,6),(36,8),(37,11)$, $(38,4),(39,8),(40,9),(41,11),(42,10),(43,6),(44,6),(45,7),(46,5),(47,5),(48,5),(49,5),(50,6)$, $(51,6),(52,4),(53,3),(54,7),(55,3),(56,3),(57,4),(58,4),(59,1),(60,3),(61,4),(62,4),(63,2),(64,4)$, $(65,4),(66,5),(67,6),(68,8),(69,5),(70,4),(71,5),(72,3),(73,5),(74,3),(75,7),(76,3),(77,2),(78,2)$, $(79,3),(80,1),(81,5),(82,2),(83,5),(84,3),(85,4),(86,3),(87,4),(89,1),(90,2),(91,1),(92,1),(93,2)$, $(94,1),(95,3),(96,2),(97,2),(98,3),(99,3),(100,2),(101,1),(102,1),(103,2),(104,3),(105,1),(106,1)$, $(107,1),(108,1),(109,2),(110,2),(111,3),(112,3),(113,3),(114,3),(115,1),(116,2),(117,1),(118,3)$, $(119,3),(120,2),(121,3),(122,3),(123,1),(124,1),(125,2),(126,1),(127,3),(128,3),(129,4),(130,2)$, $(131,4),(132,5),(133,2),(134,2),(135,6),(136,1),(137,2),(138,2),(139,4),(140,2),(141,2),(142,3)$, $(143,2),(144,2),(145,2),(146,2),(147,1),(148,2),(149,2),(151,2),(152,1),(153,1),(154,3),(155,1)$, $(156,3),(158,1),(159,3),(160,2),(161,2),(163,1),(164,2),(165,1),(166,3),(168,1),(170,2),(171,3)$, $(172,4),(173,1),(174,1),(175,3),(176,1),(179,1),(180,1),(181,2),(182,1),(183,1),(184,1),(189,1)$, $(193,2),(194,2),(195,3),(196,1),(197,1),(198,1),(199,1),(200,3),(201,1),(203,1),(204,2),(205,1)$, $(206,1),(209,1),(210,1),(211,2),(212,1),(213,1),(214,2),(215,1),(219,1),(220,2),(222,2),(223,2)$, $(224,1),(228,2),(231,1),(232,1),(233,1),(234,1),(236,1),(237,1),(239,1),(241,1),(242,1),(243,1)$, $(245,1),(247,1),(248,1),(249,1),(251,1),(252,1),(253,2),(254,1),(255,1),(258,1),(260,1),(263,1)$, $(264,1),(265,2),(268,1),(271,1),(274,1),(275,1),(278,1),(280,1),(283,1),(284,1),(285,1),(287,1)$, $(288,1),(289,1),(291,1),(293,1),(298,2),(302,2),(303,1),(314,2),(317,1),(320,1),(327,2),(335,2)$, $(338,1),(345,1),(347,1),(350,1),(354,1),(360,1),(363,1),(367,1),(370,1),(375,1),(376,1),(377,1)$, $(379,1),(383,1),(384,1),(391,1),(402,1),(403,1),(408,1),(409,1),(418,2),(424,1),(429,1),(432,1)$, $(435,1),(436,1),(446,1),(461,1),(477,1),(490,1),(491,1),(493,1)$;

3: $(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(13,1),(14,3),(16,2),(20,3),(22,1),(24,1),(28,5)$, $(30,3),(31,1),(33,1),(34,1),(37,1),(42,1),(51,2),(56,2),(62,1),(63,1),(78,1)$;

4: $(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1)$, $(15,1),(16,1),(17,1),(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1)$, $(29,1),(30,1),(31,1),(32,1),(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(42,1)$, $(43,1),(44,1),(45,1),(46,1),(47,1),(48,1),(49,1),(50,1),(51,1),(52,1),(53,1),(54,1),(55,1),(56,1)$, $(57,1),(58,1),(60,1),(61,1),(62,1),(64,1),(67,1),(68,1),(69,1),(72,1),(73,1),(76,1),(80,1),(83,1)$, $(84,1),(86,1),(88,1),(92,1),(94,1),(103,1),(108,1),(113,1),(114,1),(119,1),(141,1),(144,1),(152,1)$, $(167,1),(168,1),(175,1),(198,1),(202,1),(226,1),(235,1),(312,1)$;

5: $(2,1),(3,3),(4,3),(5,3),(6,6),(7,4),(8,5),(9,10),(10,3),(11,6),(12,7),(13,7),(14,7),(15,5)$, $(16,6),(17,8),(18,7),(19,5),(20,5),(21,7),(22,3),(23,1),(24,4),(25,5),(26,3),(27,2),(28,2),(29,3)$, $(30,6),(31,3),(32,4),(33,4),(34,1),(35,1),(36,2),(37,1),(39,3),(41,2),(42,2),(43,1),(44,1),(45,2)$, $(46,2),(49,1),(52,1),(54,2),(56,2),(57,2),(58,2),(60,1),(62,2),(63,1),(65,2),(66,1),(67,1),(69,1)$,
$(75,1),(76,1),(79,1),(84,1),(88,1),(93,1),(96,1),(129,1),(131,1) ;$
6: $(4,1),(5,1),(7,1),(8,2),(9,1),(10,1),(12,2),(13,1),(14,3),(16,1),(17,1),(18,1),(19,2),(20,2)$, $(21,1),(22,1),(24,3),(26,1),(32,2),(34,1),(36,1),(42,1),(51,1),(60,1),(65,1),(120,1),(147,1)$;

7: $(4,1),(5,1),(6,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1),(15,1),(16,1),(17,1),(18,1)$, $(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1),(29,1),(30,1),(31,1),(32,1)$, $(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(44,1),(46,1),(48,1),(50,1),(51,1),(52,1)$, $(54,1),(55,1),(56,1),(60,1),(61,1),(62,1),(63,1),(64,1),(70,1),(72,1),(80,1),(81,1),(84,1),(85,1)$, $(86,1),(87,1),(96,1),(104,1),(106,1),(116,1),(126,1),(138,1),(144,1),(152,1),(168,1),(176,1)$, $(200,1),(314,1),(441,1)$;

8: $(2,1),(3,2),(4,3),(5,2),(6,3),(7,6),(8,4),(9,4),(10,5),(11,4),(12,4),(13,5),(14,7),(15,5)$, $(16,5),(17,5),(18,5),(19,7),(20,3),(21,2),(22,2),(23,4),(24,3),(25,3),(26,2),(27,2),(28,3),(29,1)$, $(30,2),(31,2),(32,2),(33,2),(34,1),(36,1),(37,2),(38,3),(39,2),(40,1),(41,3),(42,4),(43,4),(44,1)$, $(45,1),(47,2),(51,1),(52,1),(53,2),(54,2),(55,2),(57,1),(58,1),(71,1),(74,1),(76,1),(87,1),(91,1)$, $(98,1),(116,1),(155,1),(178,1),(226,1),(263,1)$;

9: $(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1),(15,1)$, $(16,1),(17,1),(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1),(29,1)$, $(30,1),(31,1),(32,1),(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(42,1),(43,1)$, $(44,1),(46,1),(47,1),(48,1),(49,1),(50,1),(51,1),(52,1),(53,1),(55,1),(56,1),(57,1),(58,1),(60,1)$, $(68,1),(71,1),(72,1),(74,1),(75,1),(76,1),(78,1),(80,1),(84,1),(87,1),(90,1),(92,1),(96,1),(107,1)$, $(122,1),(125,1),(137,1),(143,1),(179,1),(183,1),(288,1) ;$

10: $(2,1),(4,5),(5,8),(6,9),(7,5),(8,8),(9,2),(10,7),(11,7),(12,5),(13,6),(14,5),(15,3),(16,2)$, $(17,3),(18,3),(19,2),(20,2),(21,3),(24,2),(25,2),(27,2),(30,3),(32,1),(33,1),(34,2),(35,1),(36,1)$, $(40,2),(42,2),(50,2),(51,1),(53,1),(59,1),(115,1),(148,1)$;

11: $(2,3),(3,1),(4,5),(5,10),(6,8),(7,12),(8,17),(9,14),(10,15),(11,11),(12,19),(13,17),(14,22)$, $(15,17),(16,20),(17,15),(18,17),(19,14),(20,11),(21,15),(22,14),(23,14),(24,9),(25,7),(26,3)$, $(27,3),(28,4),(29,7),(30,2),(31,3),(32,5),(33,3),(34,5),(35,3),(36,5),(37,1),(39,1),(40,1),(42,5)$, $(43,3),(45,1),(46,3),(47,1),(48,4),(49,1),(50,1),(53,1),(54,1),(57,1),(62,1),(63,1),(64,1),(68,2)$, $(69,1),(70,2),(71,1),(72,2),(73,3),(74,1),(75,1),(76,1),(78,3),(79,1),(80,1),(82,1),(83,1),(84,1)$, $(85,1),(87,2),(90,1),(92,1),(105,1),(108,1),(110,1),(117,1),(118,1),(130,1),(137,1),(142,1),(143,1)$, $(227,1),(326,1),(487,1)$;

12: $(1,2),(2,5),(3,17),(4,28),(5,41),(6,51),(7,67),(8,77),(9,61),(10,56),(11,59),(12,62),(13,59)$, $(14,42),(15,66),(16,46),(17,48),(18,42),(19,31),(20,32),(21,21),(22,21),(23,24),(24,22),(25,27)$, $(26,16),(27,19),(28,14),(29,19),(30,12),(31,9),(32,7),(33,8),(34,5),(35,2),(36,3),(37,7),(38,2)$, $(39,2),(40,2),(41,1),(42,2),(43,2),(44,4),(45,3),(46,1),(47,7),(48,3),(49,4),(50,5),(51,3),(52,3)$, $(53,4),(54,3),(55,1),(56,1),(57,4),(59,1),(60,1),(61,1),(62,2),(64,2),(65,1),(68,1),(69,3),(70,1)$, $(71,2),(73,1),(75,1),(76,1),(77,1),(78,2),(80,2),(82,1),(83,2),(87,2),(90,3),(92,2),(95,1),(96,1)$, $(97,1),(98,1),(100,1),(101,1),(102,3),(103,1),(108,1),(109,1),(117,1),(138,2),(139,1),(161,1)$, $(181,1),(277,1),(289,1),(313,1),(428,1)$;

13: $(3,2),(4,7),(5,6),(6,11),(7,8),(8,8),(9,10),(10,13),(11,8),(12,10),(13,6),(14,11),(15,9)$, $(16,8),(17,8),(18,5),(19,4),(20,6),(21,5),(22,2),(23,5),(24,7),(25,5),(26,5),(28,8),(29,3),(30,9)$, $(31,4),(32,2),(33,4),(34,2),(35,1),(36,2),(38,2),(39,1),(40,2),(42,1),(43,1),(44,3),(45,2),(47,2)$, $(48,1),(49,1),(50,2),(51,1),(53,2),(54,1),(60,1),(62,1),(63,2),(80,2),(82,1),(84,1),(85,1),(90,1)$, $(128,1),(132,1),(176,1),(197,1),(209,1)$;

14: $(2,2),(3,6),(4,4),(5,6),(6,8),(7,8),(8,9),(9,9),(10,9),(11,8),(12,8),(13,10),(14,9),(15,10)$,
$(16,7),(17,8),(18,10),(19,9),(20,8),(21,7),(22,9),(23,6),(24,9),(25,5),(26,5),(27,7),(28,4),(29,7)$, $(30,6),(31,4),(32,3),(33,6),(34,7),(35,6),(36,5),(37,5),(38,4),(39,3),(40,4),(41,2),(42,3),(43,3)$, $(44,2),(45,2),(46,2),(47,5),(48,2),(49,1),(50,3),(51,1),(52,3),(53,3),(54,1),(55,3),(56,1),(57,2)$, $(58,1),(59,3),(60,1),(61,1),(62,2),(63,2),(64,1),(65,2),(66,4),(67,2),(68,1),(69,1),(70,1),(71,1)$, $(72,2),(73,2),(74,1),(75,2),(76,1),(77,2),(78,1),(79,1),(80,1),(81,1),(82,1),(83,1),(84,1),(85,1)$, $(86,1),(87,2),(89,2),(90,1),(91,2),(92,1),(94,2),(95,2),(96,1),(97,1),(100,3),(102,1),(103,1)$, $(104,1),(105,2),(106,1),(107,1),(108,1),(109,1),(110,1),(112,2),(114,1),(115,1),(116,1),(117,1)$, $(118,1),(119,1),(120,1),(121,1),(122,1),(123,1),(124,1),(125,1),(126,2),(127,1),(128,1),(129,1)$, $(130,1),(131,1),(132,1),(133,1),(134,2),(135,2),(136,1),(137,3),(138,2),(140,1),(141,1),(142,2)$, $(143,1),(144,1),(145,1),(146,1),(147,1),(148,1),(149,1),(150,1),(151,1),(152,1),(153,1),(154,1)$, $(155,1),(156,1),(157,1),(158,1),(159,1),(161,1),(168,1),(170,1),(181,1),(182,1),(183,1),(185,1)$, $(189,1),(191,1),(194,1),(201,1),(207,2),(209,1),(213,1),(219,1),(223,1),(231,1),(234,1),(237,1)$, $(238,1),(239,1),(240,1),(250,1),(254,1),(255,1),(267,1),(273,1),(284,1),(301,1),(307,1),(316,1)$, $(344,1),(399,1),(403,1),(441,1),(442,1),(443,1),(456,1)$;

15: $(1,2),(2,7),(3,5),(4,12),(5,17),(6,21),(7,22),(8,22),(9,20),(10,20),(11,20),(12,21),(13,25)$, $(14,24),(15,13),(16,12),(17,21),(18,16),(19,12),(20,16),(21,15),(22,15),(23,15),(24,14),(25,7)$, $(26,9),(27,10),(28,7),(29,13),(30,9),(31,7),(32,5),(33,6),(34,9),(35,3),(36,8),(37,4),(38,5),(39,9)$, $(40,4),(41,4),(42,5),(43,5),(44,5),(45,4),(46,5),(47,3),(48,4),(49,4),(50,3),(51,3),(52,3),(53,4)$, $(54,1),(55,1),(56,3),(57,2),(59,4),(60,4),(61,2),(62,3),(64,3),(65,4),(66,1),(67,2),(68,2),(69,1)$, $(70,2),(71,1),(72,2),(73,3),(74,3),(75,3),(76,1),(77,2),(78,3),(81,2),(82,2),(83,1),(86,2),(87,1)$, $(88,3),(89,1),(91,1),(92,2),(94,1),(95,1),(96,1),(97,1),(98,1),(99,1),(100,1),(101,1),(102,2)$, $(103,2),(105,1),(106,1),(107,1),(108,4),(109,1),(110,1),(111,1),(112,2),(113,4),(116,1),(117,1)$, $(118,1),(121,1),(122,1),(124,1),(126,1),(127,1),(129,1),(131,1),(132,2),(134,2),(135,2),(136,1)$, $(137,2),(138,1),(140,2),(142,2),(143,2),(144,2),(145,1),(146,2),(147,2),(148,1),(151,1),(155,1)$, $(162,1),(166,1),(180,1),(192,1),(197,1),(203,1),(204,1),(219,2),(221,1),(224,1),(323,1),(331,2)$, $(332,1),(341,1),(357,1),(419,1)$;

16: $(6,1),(9,2),(10,1),(11,1),(20,2),(26,1),(31,1),(37,1),(64,1)$;
17: $(3,1),(4,1),(5,1),(10,1),(11,2),(14,1),(22,1),(25,1),(27,1)$;
18: $(1,1),(3,1),(4,2),(5,4),(6,3),(7,3),(8,4),(9,5),(10,2),(11,4),(12,4),(13,2),(14,3),(15,5)$, $(16,1),(17,1),(18,3),(19,2),(20,1),(21,1),(22,1),(23,1),(24,2),(26,1),(27,2),(28,1),(29,1),(30,3)$, $(32,1),(33,1),(35,1),(38,1),(42,1),(43,1),(49,1),(55,1),(62,1),(102,1),(106,1)$;

19: $(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1)$, $(15,1),(16,1),(17,1),(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1)$, $(29,1),(30,1),(31,1),(32,1),(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(42,1)$, $(45,1),(46,1),(47,1),(49,1),(51,1),(52,1),(56,1),(57,1),(60,1),(61,1),(64,1),(65,1),(66,1),(68,1)$, $(70,1),(72,1),(74,1),(76,1),(89,1),(99,1),(100,1),(119,1),(123,1),(133,1),(140,1),(157,1),(158,1)$;

20: $(3,2),(4,4),(5,2),(6,4),(7,5),(8,7),(9,3),(10,6),(11,7),(12,3),(13,3),(14,5),(15,1),(17,3)$, $(18,2),(19,2),(20,3),(21,2),(22,1),(23,2),(24,2),(26,1),(30,1),(32,1),(33,1),(34,1),(35,1),(37,1)$, $(38,1),(42,1),(43,1),(47,1),(49,1),(53,1),(70,1),(98,1),(128,1),(204,1)$;

21: $(2,1),(3,2),(4,1),(5,1),(6,1),(7,1),(8,2),(9,1),(10,1),(11,2),(12,3),(14,1),(16,1),(17,1)$, $(18,1),(19,3),(29,1),(30,1),(44,1),(55,1)$;

22: $(2,1),(6,2),(7,2),(8,4),(11,2),(20,2),(46,1)$;
23: $(3,2),(4,3),(5,1),(6,1),(8,2),(9,1),(10,1),(12,1),(14,2),(15,1),(16,1),(22,1),(43,1),(44,1)$, $(46,1)$;

24: $(8,1),(14,1),(16,1),(18,1),(27,1)$;
25: $(2,1),(4,2),(5,1),(6,2),(7,1),(8,1),(10,1),(11,1),(12,2),(14,1),(18,1),(20,2),(22,1),(26,1)$, $(32,1),(34,1),(36,1),(44,1),(45,1),(47,1)$;

26: $(4,2),(5,2),(6,4),(7,4),(8,3),(9,4),(10,3),(11,2),(12,6),(13,2),(14,3),(15,3),(16,3),(17,1)$, $(18,2),(20,3),(21,1),(22,1),(23,2),(24,3),(31,2),(34,1),(35,1),(36,1),(37,1),(41,1),(44,1),(45,1)$, $(46,1),(47,1),(52,1),(62,1),(84,1)$;

27: $(16,1),(29,1),(36,1)$;
28: $(2,1),(4,3),(6,2),(7,1),(8,1),(9,1),(12,1),(13,1),(15,1),(16,1),(17,1),(20,1),(23,1),(25,1)$, $(27,1),(38,1),(46,1)$;

29: $(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1),(15,1)$, $(16,1),(17,1),(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(26,1),(27,1),(28,1),(29,1),(30,1)$, $(31,1),(32,1),(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(43,1),(44,1),(45,1)$, $(46,1),(49,1),(50,1),(51,1),(52,1),(54,1),(61,1),(62,1),(73,1),(76,1),(79,1),(84,1),(90,1),(131,1)$, $(142,1),(162,1),(238,1)$;

30: $(4,1),(6,1),(7,1),(10,1),(13,2),(14,1),(16,1),(17,1),(18,1),(19,1),(20,1),(29,1),(30,1)$, $(34,1),(53,1),(55,1),(57,1)$;

31: $(2,1),(3,2),(4,2),(5,3),(6,3),(7,5),(8,4),(9,5),(10,4),(11,4),(12,5),(13,4),(14,4),(15,4)$, $(16,5),(17,2),(18,2),(19,4),(20,3),(21,4),(22,5),(23,1),(24,3),(26,1),(27,1),(28,4),(29,2),(30,3)$, $(31,2),(32,2),(33,2),(34,2),(35,1),(36,2),(37,1),(38,1),(40,1),(41,3),(42,1),(43,3),(44,2),(57,1)$, $(68,1),(75,1),(142,1),(157,1),(177,1)$;

32: $(4,1),(6,2),(9,2),(10,2),(12,1),(13,3),(14,2),(15,1),(17,1),(23,2),(26,1),(31,1),(33,1)$, $(39,2),(42,2),(43,1),(44,1),(45,1),(53,1),(56,1),(84,1),(108,1),(112,1)$;

33: $(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(14,1),(15,1)$, $(16,1),(17,1),(18,1),(19,1),(20,1),(21,1),(22,1),(23,1),(24,1),(25,1),(26,1),(27,1),(28,1),(29,1)$, $(30,1),(31,1),(32,1),(33,1),(34,1),(35,1),(36,1),(37,1),(38,1),(39,1),(40,1),(41,1),(42,1),(43,1)$, $(46,1),(47,1),(48,1),(51,1),(56,1),(59,1),(62,1),(64,1),(66,1),(68,1),(70,1),(71,1),(72,1),(73,1)$, $(74,1),(75,1),(76,1),(77,1),(78,1),(79,1),(80,1),(82,1),(85,1),(92,1),(94,1),(144,1)$;

34: $(2,1),(3,1),(6,2),(7,3),(8,3),(9,2),(10,1),(13,1),(14,1),(15,1),(17,2),(18,3),(19,1),(20,2)$, $(21,1),(23,1),(26,1),(28,1),(30,1),(35,1),(36,1),(41,1),(44,1),(459,1)$;

35: $(2,1),(4,1),(6,3),(7,1),(8,4),(10,2),(11,3),(12,1),(13,1),(14,1),(15,1),(16,2),(17,2),(18,1)$, $(19,1),(20,1),(21,1),(22,2),(29,1),(34,1),(36,1)$;

36: $(2,1),(3,3),(4,3),(5,4),(6,5),(7,6),(8,6),(9,6),(10,6),(11,5),(12,8),(13,8),(14,5),(15,8)$, $(16,9),(17,5),(18,5),(19,5),(20,2),(21,3),(22,5),(23,4),(24,3),(25,5),(26,3),(27,2),(29,6),(30,1)$, $(31,2),(32,1),(33,1),(34,2),(35,2),(36,2),(46,1),(48,1),(51,2),(52,1),(54,1),(55,2),(62,1),(63,1)$, $(79,1),(81,1),(83,1),(90,1),(101,1)$;

37: $(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(11,1),(12,2),(18,1),(19,1),(22,1),(25,1),(26,1),(41,1)$, $(48,1)$;

38: $(3,1),(4,5),(5,4),(6,5),(7,5),(8,7),(9,5),(10,6),(11,3),(12,5),(13,3),(14,1),(15,4),(16,1)$, $(17,3),(19,1),(20,1),(23,2),(25,2),(26,1),(27,3),(28,1),(29,1),(30,2),(31,1),(35,2),(39,2),(40,3)$, $(43,1),(58,1),(64,1),(82,1),(117,1),(152,1)$;

39: $(4,2),(5,2),(6,1),(8,4),(10,3),(11,2),(13,1),(14,1),(15,1),(17,1),(19,1),(20,1),(24,1)$, $(29,1),(45,1)$;

40: $(2,1),(3,2),(4,4),(5,3),(6,3),(7,3),(8,3),(9,5),(11,2),(12,3),(13,2),(15,3),(16,1),(17,1)$, $(18,2),(20,1),(21,3),(22,1),(23,1),(26,1),(28,1),(29,2),(30,1),(55,1),(58,1)$;

41: $(2,2),(3,3),(4,2),(5,4),(6,1),(7,1),(8,4),(10,4),(11,1),(12,2),(13,4),(14,1),(15,2),(16,2)$, $(17,2),(19,1),(24,4),(26,3),(27,1),(28,1),(29,1),(32,1),(33,1),(39,1),(45,1)$;

42: $(2,1),(4,1),(5,1),(6,2),(7,2),(8,3),(9,2),(10,1),(11,2),(12,1),(13,1),(14,2),(15,3),(17,1)$, $(20,1),(22,1),(26,1),(28,1),(37,1),(39,1),(41,1)$;

43: $(2,2),(3,2),(4,3),(5,2),(6,4),(7,3),(8,4),(9,3),(10,2),(11,5),(12,2),(13,2),(14,3),(15,3)$, $(16,2),(17,3),(18,1),(19,3),(20,4),(21,1),(23,2),(24,2),(25,1),(26,1),(28,1),(29,1),(31,1),(32,2)$, $(34,1),(35,1),(38,1),(45,1),(56,1),(59,1),(60,2),(129,1)$;

44: $(2,1),(3,2),(4,3),(5,4),(6,6),(7,8),(8,3),(9,4),(10,6),(11,3),(12,1),(13,2),(14,1),(16,2)$, $(17,1),(19,2),(21,1),(22,2),(23,1),(30,1),(35,1),(36,2),(39,1),(40,1),(64,1)$;

45: $(3,1),(5,1),(6,3),(7,1),(10,1),(12,3),(14,1),(18,1),(37,1),(42,1),(63,1),(64,1),(65,1)$, $(67,1),(69,1),(76,1)$;

46: $(3,1),(4,2),(5,1),(6,3),(7,4),(8,4),(9,4),(10,2),(11,2),(12,1),(13,2),(14,1),(15,3),(16,2)$, $(17,1),(18,1),(23,1),(28,1),(41,1),(42,1),(72,1),(104,1),(110,1)$;

47: $(2,1),(3,2),(4,4),(5,8),(6,3),(7,5),(8,9),(9,12),(10,13),(11,8),(12,4),(13,7),(14,3),(16,2)$, $(18,6),(19,1),(20,1),(28,1),(38,2),(41,1)$;

48: $(3,1),(5,2),(8,1),(9,1),(16,1),(40,1),(41,1),(42,1)$;
49: $(4,2),(6,3),(11,1),(36,1)$;
50: $(4,1),(5,1),(6,3),(7,2),(8,1),(13,1),(14,2),(18,1) ;$
51: $(2,2),(3,3),(4,3),(5,7),(6,11),(7,3),(8,9),(9,8),(10,7),(11,4),(12,8),(13,4),(14,3),(15,6)$, $(16,4),(17,9),(18,1),(19,3),(20,1),(21,1),(23,1),(29,1),(31,1),(32,1),(35,1),(37,1),(38,1)$;

52: $(4,6),(5,3),(6,5),(7,7),(8,6),(9,3),(10,4),(11,5),(12,1),(13,1),(14,3),(15,2),(17,1),(18,4)$, $(19,3),(20,2),(21,2),(22,1),(23,1),(26,1),(27,1),(40,2),(88,1)$;

53: $(3,5),(4,3),(6,3),(7,2),(8,2),(9,4),(10,3),(11,5),(12,3),(13,4),(14,5),(15,4),(17,1),(19,1)$, $(26,1),(29,1),(30,1)$;

54: $(2,1),(3,1),(4,1),(5,4),(6,3),(7,2),(8,1),(9,1),(10,4),(11,1),(15,1),(16,1),(17,1),(18,2)$, $(19,1),(21,3),(23,1),(27,1),(31,1),(39,1),(40,1),(41,1),(42,1)$;

55: $(2,1),(3,1),(4,7),(5,6),(6,13),(7,17),(8,14),(9,15),(10,17),(11,14),(12,14),(13,5),(14,10)$, $(15,6),(16,7),(17,5),(18,6),(19,3),(20,3),(21,5),(22,3),(23,3),(24,1),(25,2),(26,4),(27,1),(28,1)$, $(29,1),(30,1),(31,1),(33,2),(35,2),(37,2),(40,1),(41,1),(42,1),(44,1),(50,1),(55,1),(56,2),(58,1)$, $(60,2),(61,1),(71,1),(78,1),(97,1)$;

56: $(3,1),(4,2),(5,4),(6,7),(7,4),(8,4),(9,4),(10,5),(11,5),(12,3),(13,2),(14,4),(15,4),(16,3)$, $(17,3),(18,3),(19,1),(20,1),(21,1),(23,1),(24,2),(26,1),(29,1),(31,1),(34,1),(39,2),(40,1),(54,1)$, $(69,1)$;

57: $(1,1),(2,2),(3,2),(4,6),(5,7),(6,11),(7,13),(8,11),(9,11),(10,12),(11,7),(12,7),(13,10)$, $(14,8),(15,3),(16,3),(17,1),(18,4),(19,4),(20,2),(21,1),(22,3),(23,4),(24,1),(26,2),(27,1),(30,2)$, $(33,1),(34,2),(35,1),(42,1),(53,1),(59,1),(60,1),(67,1),(78,1),(93,1)$;

58: $(2,2),(3,4),(4,7),(5,11),(6,17),(7,13),(8,15),(9,5),(10,8),(11,11),(12,16),(13,14),(14,9)$, $(15,11),(16,12),(17,6),(18,2),(19,6),(20,10),(21,4),(22,2),(23,2),(24,2),(25,1),(26,2),(27,1)$, $(28,3),(29,2),(30,2),(32,1),(33,1),(35,1),(36,1),(39,2),(42,1),(43,1),(44,2),(45,3),(47,1),(48,1)$, $(51,2),(53,2),(55,1),(60,1),(61,1),(63,1),(80,1),(101,1),(128,1),(174,1)$;

59: $(2,3),(3,4),(4,8),(5,7),(6,10),(7,17),(8,9),(9,11),(10,10),(11,6),(12,7),(13,5),(14,3)$, $(15,7),(16,1),(17,5),(18,2),(19,3),(20,1),(21,3),(23,4),(24,1),(25,1),(26,3),(29,1),(30,1),(31,3)$, $(33,2),(36,1),(37,1),(38,5),(39,5),(40,2),(44,1),(45,1),(47,1),(49,1),(53,1),(55,1),(62,1),(70,1)$, $(103,1),(141,1)$;

60: $(2,2),(3,3),(4,5),(5,5),(6,6),(7,7),(8,6),(9,6),(10,7),(11,7),(12,3),(13,5),(14,7),(15,6)$, $(16,5),(17,7),(18,6),(19,4),(20,6),(22,4),(23,4),(24,3),(25,4),(26,5),(27,1),(28,3),(30,2),(31,2)$, $(32,1),(33,1),(35,1),(36,2),(37,2),(38,2),(39,2),(42,1),(43,1),(45,2),(46,1),(48,1),(63,2),(64,1)$, $(65,1),(71,1),(127,1),(166,1)$;

61: $(2,2),(3,2),(4,2),(5,2),(6,3),(7,2),(8,4),(9,3),(10,2),(11,3),(12,3),(13,3),(14,3),(15,4)$, $(16,2),(17,3),(18,4),(19,3),(20,2),(21,2),(22,2),(23,2),(24,3),(25,1),(26,1),(27,2),(28,1),(29,2)$, $(30,2),(31,2),(32,1),(33,1),(34,1),(35,1),(36,1),(37,2),(38,1),(39,1),(40,1),(42,1),(43,1),(44,1)$, $(45,1),(46,1),(48,1),(50,1),(51,1),(52,1),(53,1),(54,1),(60,1),(61,1),(66,1),(67,1),(68,2),(70,1)$, $(71,1),(80,1),(99,1),(101,1),(104,1),(107,1),(110,1),(114,1),(116,1),(122,1),(135,1),(136,1)$, $(138,1),(144,1),(145,1),(149,1),(265,1) ;$

62: $(1,1),(2,1),(3,6),(4,6),(5,7),(6,10),(7,14),(8,14),(9,16),(10,15),(11,21),(12,19),(13,19)$, $(14,15),(15,18),(16,20),(17,14),(18,14),(19,12),(20,12),(21,11),(22,10),(23,13),(24,12),(25,8)$, $(26,11),(27,4),(28,5),(29,4),(30,6),(31,4),(32,6),(33,4),(34,1),(35,4),(36,2),(37,2),(38,4),(39,4)$, $(40,5),(41,7),(42,5),(43,6),(44,6),(45,4),(46,3),(47,7),(48,2),(49,4),(50,4),(51,5),(52,1),(53,1)$, $(54,1),(55,2),(56,3),(59,1),(60,2),(61,1),(63,1),(67,1),(68,2),(75,2),(76,2),(78,1),(79,2),(81,1)$, $(83,2),(84,1),(85,1),(86,1),(97,1),(98,1),(99,1),(104,1),(113,1),(114,2),(120,1),(144,1),(234,1)$;

63: $(1,1),(2,4),(3,13),(4,26),(5,30),(6,42),(7,47),(8,46),(9,44),(10,34),(11,29),(12,21),(13,16)$, $(14,11),(15,13),(16,6),(17,13),(18,9),(19,9),(20,14),(21,8),(22,3),(23,7),(24,4),(25,2),(26,5)$, $(27,1),(28,4),(29,6),(30,3),(31,4),(32,2),(33,3),(34,3),(35,5),(38,1),(41,1),(45,3),(46,1),(51,1)$, $(55,1),(57,1),(71,1),(73,1),(85,1),(94,2),(99,1),(119,1),(140,1),(315,1),(321,1)$;

64: $(1,1),(2,4),(3,9),(4,18),(5,19),(6,19),(7,15),(8,14),(9,20),(10,22),(11,10),(12,10),(13,11)$, $(14,4),(15,7),(16,6),(17,3),(18,4),(19,4),(20,1),(21,3),(22,3),(23,2),(24,1),(25,2),(28,1),(29,2)$, $(32,1),(36,1),(37,1),(38,1),(41,1),(42,1),(43,1),(45,1),(49,1),(117,1),(130,1),(170,1),(241,1)$, (249, 1), (252, 1), (257, 1);

65: $(1,1),(2,6),(3,8),(4,17),(5,16),(6,22),(7,19),(8,21),(9,21),(10,11),(11,7),(12,7),(13,3)$, $(14,5),(15,4),(16,11),(17,9),(18,5),(19,3),(20,4),(21,1),(22,2),(23,1),(25,1),(26,1),(34,1),(37,1)$, $(39,1),(70,1),(376,1)$;

66: $(2,1),(4,2),(5,3),(6,7),(7,5),(8,6),(9,10),(10,5),(11,2),(12,5),(13,5),(14,4),(15,2),(16,1)$, $(17,1),(19,1),(21,1),(22,1),(23,1),(24,1),(28,2),(31,2),(60,1),(100,1)$;

67: $(2,2),(3,3),(4,7),(5,2),(6,2),(7,3),(8,1),(9,4),(10,2),(12,1),(13,1),(15,1),(16,1),(17,1)$, $(18,2),(19,3),(26,1),(31,1),(32,1),(37,1)$;

68: $(2,1),(3,1),(4,1),(5,5),(6,3),(7,4),(8,4),(9,1),(10,2),(11,1),(12,2),(14,1),(16,1),(18,1)$, $(21,1),(22,2)$;

69: $(1,3),(2,3),(3,5),(4,7),(5,8),(6,16),(7,14),(8,5),(9,9),(10,5),(11,3),(12,3),(13,1),(14,1)$, $(15,2),(17,2),(18,3),(19,3),(20,1),(21,1),(23,2),(26,1),(27,1),(43,2),(45,1)$;

70: $(1,1),(2,6),(3,10),(4,13),(5,13),(6,12),(7,8),(8,5),(9,6),(10,1),(11,4),(12,1),(13,1),(14,2)$, $(16,1),(18,1),(19,3),(20,5),(21,1),(24,1),(26,1),(27,1),(29,1),(31,1),(40,1),(60,1),(63,1),(65,1)$, $(68,1),(70,1),(76,1)$;

71: $(1,1),(2,5),(3,13),(4,32),(5,20),(6,27),(7,19),(8,13),(9,16),(10,8),(11,9),(12,9),(13,10)$, $(14,6),(15,11),(16,2),(17,9),(18,5),(19,2),(20,6),(21,3),(22,1),(23,2),(24,2),(26,3),(27,2),(28,1)$, $(30,1),(38,1),(39,1),(44,1),(46,1),(49,1),(51,1),(53,1)$;

72: $(2,2),(3,2),(4,12),(5,11),(6,5),(7,5),(8,3),(9,7),(10,4),(11,2),(12,4),(13,5),(14,2),(15,3)$, $(16,5),(27,1),(40,1),(73,1)$;

73: $(2,1),(5,1),(6,1),(7,1),(8,5),(11,1),(12,1),(13,3),(17,1),(19,1),(21,1),(26,1),(35,1),(40,1)$,
$(345,1)$;
74: $(1,1),(2,1),(3,6),(4,11),(5,15),(6,14),(7,14),(8,10),(9,4),(10,7),(11,13),(12,5),(13,6)$, $(14,4),(15,1),(17,3),(18,1),(24,1),(25,2),(26,1),(28,1),(31,1),(32,2),(34,2)$;

75: $(10,1),(11,1),(12,2),(18,1),(22,3),(24,2),(26,1),(42,2)$;
76: $(4,1),(5,1),(8,1),(9,1),(10,1),(11,1),(12,1),(13,1),(16,1),(17,1),(22,1),(23,1),(24,1)$, $(25,1),(26,1),(27,1),(28,1),(37,1),(41,1),(48,1),(49,1),(52,1),(54,1)$;

77: $(1,1),(2,1),(11,1),(12,2),(53,1),(200,1)$;
79: $(3,1),(4,1),(5,2),(6,2),(7,5),(8,1),(10,1),(14,1),(17,1),(18,1),(24,1),(29,1) ;$
80: $(4,1),(6,1),(11,1),(12,1),(14,2),(16,1),(19,1),(21,1),(37,1),(38,1),(104,1)$;
81: $(2,1),(4,1),(5,1),(6,4),(7,1),(8,2),(9,2),(10,3),(11,1),(12,1),(14,2),(16,2),(17,1),(19,1)$, $(20,1),(23,1),(25,3),(124,1)$;

82: $(3,5),(4,6),(5,3),(6,8),(7,5),(8,8),(9,5),(10,2),(11,4),(12,4),(13,1),(14,3),(15,1),(16,2)$, $(17,1),(18,2),(20,2),(22,1),(23,1),(24,3),(25,1),(26,3),(36,1),(41,1),(45,1),(46,1),(52,1),(68,1)$, $(113,1)$;

83: $(4,3),(5,1),(6,1),(7,2),(8,3),(9,2),(11,1),(13,2),(14,2),(16,1),(17,1),(26,1),(31,1),(89,1)$, $(98,1),(101,1),(107,1)$;

84: $(2,1),(4,4),(5,5),(6,4),(7,3),(8,1),(10,2),(11,2),(12,1),(14,1),(27,1),(30,1),(47,1)$;
85: $(3,2),(4,4),(5,6),(6,4),(7,6),(8,4),(9,4),(10,4),(11,3),(12,3),(13,2),(14,3),(15,1),(17,2)$, $(19,2),(20,1),(21,2),(23,1),(25,1),(27,1),(31,1),(32,1),(40,1),(43,1),(72,1),(136,1),(275,1)$;

86: $(3,1),(4,1),(5,6),(6,3),(7,2),(8,3),(9,4),(10,2),(11,3),(12,3),(13,2),(14,2),(15,2),(16,2)$, $(17,2),(19,1),(21,1),(23,1),(30,1),(31,1),(38,1),(39,1),(44,1),(50,1),(60,1),(62,1),(67,1),(69,1)$;

87: $(2,2),(3,5),(4,11),(5,23),(6,20),(7,14),(8,14),(9,12),(10,11),(11,13),(12,7),(13,6),(14,6)$, $(15,4),(16,3),(17,2),(18,1),(19,4),(22,2),(25,1),(33,1),(35,1),(38,1),(43,1),(44,1),(49,1),(50,1)$, $(52,2),(53,2),(54,1),(62,1),(72,1)$;

88: $(2,2),(3,6),(4,6),(5,6),(6,6),(7,6),(8,5),(9,8),(10,5),(11,3),(12,5),(13,3),(14,1),(15,3)$, $(16,2),(18,1),(19,1),(20,1),(21,1),(22,2),(24,1),(25,1),(26,2),(27,1),(29,1),(30,1),(32,1),(33,1)$, $(40,1),(47,1),(53,1),(66,1),(114,1)$;

89: $(11,1),(26,1)$;
90: $(6,2),(8,4),(9,1),(10,2),(11,2),(12,3)$;
91: $(3,1),(5,1),(6,2),(7,1),(8,2),(9,1),(13,1),(14,1),(16,1),(19,1),(26,1),(39,1)$;
92: $(1,1),(2,2),(3,3),(4,2),(5,3),(6,2),(7,4),(8,3),(9,3),(10,3),(11,3),(12,4),(13,2),(14,3)$, $(15,1),(16,2),(17,3),(18,1),(19,1),(20,2),(21,1),(22,2),(24,1),(25,1),(26,1),(28,3),(29,1),(30,1)$, $(31,1),(32,1),(45,1),(50,1),(71,1),(73,1)$;

94: $(4,1),(8,1),(13,1),(56,1),(91,1)$;
97: $(3,1),(4,2),(5,3),(6,1),(10,1),(11,1) ;$
98: $(2,1),(4,1),(5,2),(6,3),(14,1),(20,1),(22,1),(23,1),(24,1)$;
99: $(2,1),(3,2),(4,4),(5,5),(6,2),(7,1),(8,2),(10,1),(11,2),(13,2),(14,1),(20,1),(22,1)$;
100: $(4,1),(5,2),(6,1),(7,3),(8,1),(9,5),(10,4),(11,5),(12,1),(14,2),(58,1)$;
101: $(7,1),(14,1)$;
102: $(2,1),(4,1),(5,2),(6,2),(8,1),(10,1),(13,1),(14,1)$;
103: $(2,1),(13,1)$;
104: $(5,3),(6,1),(8,1),(9,1),(38,1)$;
105: $(5,1),(7,1),(8,2),(10,1),(11,1)$;
106: $(4,1),(5,1),(13,1),(18,1),(25,1),(135,1)$;

107: $(2,1),(4,2),(5,5),(6,6),(7,5),(8,3),(9,2),(11,3),(12,2),(13,3),(15,1),(17,2),(21,1),(23,1)$, $(24,1),(25,1)$;

108: $(3,2),(4,1),(5,3),(6,1),(7,1),(8,1),(9,1),(11,1),(12,2),(15,1),(17,1),(18,1),(19,1),(21,1)$;
109: $(2,1),(3,2),(4,5),(5,1),(6,2),(7,1),(8,3),(9,1),(10,1)$;
110: $(3,1),(4,1),(6,2),(7,1),(8,1),(9,1),(11,1),(13,1),(22,1),(23,2),(24,1)$;
111: $(2,1),(3,2),(4,3),(5,2),(6,1),(7,2),(11,1),(12,1),(13,2),(16,1),(19,1),(152,1)$;
112: $(3,4),(4,3),(5,1),(9,1),(11,1),(13,1)$;
113: $(3,4),(4,5),(5,3),(6,6),(7,9),(8,7),(9,4),(10,2),(11,6),(12,1),(13,1),(14,1),(15,3),(22,1)$, $(24,1),(39,1),(41,4),(42,2),(47,1),(49,1)$;

114: $(2,1),(3,5),(4,5),(5,3),(6,3),(7,4),(8,2),(9,4),(10,3),(11,2),(12,2),(13,1),(14,1),(15,3)$, $(16,1),(17,1),(19,2),(20,2),(21,1),(22,2),(26,1),(27,1),(28,1),(31,1),(102,1),(114,1)$;

115: $(3,3),(4,3),(5,1),(7,4),(8,1),(12,1),(13,2),(15,1),(19,2),(20,2),(21,1),(25,2),(28,1)$, $(86,1)$;

116: $(4,1),(5,1),(6,1),(8,1),(9,1),(16,1),(19,1),(20,1),(28,1),(45,1),(48,1)$;
117: $(4,2),(5,3),(6,2),(7,5),(8,3),(9,1),(10,2),(16,1),(106,1)$;
118: $(2,1),(3,1),(5,2),(6,2),(7,2),(8,3),(9,1),(10,3),(13,1),(14,1),(18,1),(21,1),(22,2),(23,3)$, $(26,1),(34,1),(50,1),(52,1)$;

119: $(2,2),(3,3),(4,3),(5,7),(6,6),(7,1),(8,3),(10,7),(12,3),(16,2),(19,1),(22,1),(24,1),(25,1)$, $(26,1),(28,1),(40,1)$;

120: $(3,1),(4,3),(5,1),(6,2),(7,1),(8,1),(9,2),(10,2),(14,1)$;
121: $(2,1),(3,3),(4,7),(5,10),(6,7),(7,3),(8,2),(9,3),(10,4),(11,1),(12,1),(13,2),(19,1),(27,1)$, $(28,1),(41,1),(73,1)$;

122: $(2,2),(3,9),(4,8),(5,5),(6,6),(7,9),(8,6),(9,5),(10,4),(11,11),(12,1),(13,3),(15,2),(16,3)$, $(18,1),(24,1),(44,2),(45,1),(60,1)$;

123: $(2,5),(3,12),(4,14),(5,17),(6,37),(7,49),(8,51),(9,49),(10,39),(11,13),(12,13),(13,11)$, $(14,2),(16,1),(17,6),(18,4),(19,4),(21,2),(22,1),(24,1),(25,2),(30,1)$;

124: $(2,1),(3,4),(4,3),(7,6),(11,1)$;
125: $(2,1),(3,3),(4,4),(5,3),(6,3),(7,5),(8,2),(9,3),(21,1)$;
126: $(4,5),(5,1),(6,1),(7,1),(8,1),(11,2),(18,1),(19,1),(21,1),(22,2),(23,5),(24,3),(25,2)$, $(26,2),(28,1)$;

127: $(2,1),(3,6),(4,16),(5,16),(6,13),(7,13),(8,18),(9,10),(10,15),(11,2),(12,2),(13,4),(14,2)$, $(15,1),(16,1),(17,4),(19,1),(21,1),(22,4),(26,1),(31,2),(36,1),(39,1),(101,1)$;

128: $(3,1),(4,3),(5,10),(6,6),(7,9),(8,6),(9,3),(10,1),(11,1),(12,1),(13,2),(14,4),(15,2),(16,3)$, $(17,2),(18,2),(19,1),(20,2),(49,1)$;

129: $(2,3),(3,7),(4,18),(5,17),(6,16),(7,20),(8,9),(9,3),(10,5),(11,5),(12,2),(13,3),(14,5)$, $(15,2),(16,2),(17,2),(18,1),(23,1),(24,1)$;

130: $(3,4),(4,6),(5,8),(6,10),(7,5),(8,5),(10,3),(11,2),(12,1),(13,1),(14,1),(15,2),(18,2)$, $(30,1),(31,1),(32,1),(40,1)$;

131: $(2,2),(3,7),(4,2),(5,4),(6,1),(7,1),(10,1),(11,1),(12,1),(15,1),(18,1),(19,1),(91,1)$;
132: $(4,4),(5,1),(6,1),(8,5),(9,4),(10,2),(11,1),(13,1),(28,1)$;
133: $(4,1),(6,2),(7,1),(11,1),(12,1),(136,1)$;
134: $(2,1),(4,1),(5,2),(6,4),(8,3),(9,1),(13,1),(14,1),(16,1)$;
135: $(1,1),(3,1),(4,3),(5,5),(6,2),(7,1),(8,1),(9,6),(10,5),(11,3),(12,2),(13,2),(15,1),(18,2)$, $(21,1),(25,1)$;

136: $(1,1),(2,3),(3,8),(4,14),(5,8),(6,8),(7,9),(8,5),(9,5),(10,6),(11,6),(12,1),(13,1),(14,1)$, $(15,2),(16,2),(17,1),(18,1),(19,1),(21,2),(22,3),(24,1),(27,1),(31,1),(32,2),(33,1),(36,1),(57,1)$; 137: $(2,4),(3,5),(4,11),(5,7),(6,8),(7,4),(8,2),(9,6),(10,4),(11,3),(12,4),(15,1),(17,1),(18,1)$, $(23,1),(24,1),(26,1),(29,1),(42,1),(52,1),(72,1)$;

138: $(1,1),(2,1),(3,2),(4,2),(5,4),(6,3),(7,3),(8,2),(9,1),(11,1),(12,8),(13,1),(16,1),(17,1)$, $(22,1),(23,1),(24,1),(25,1),(27,1),(35,1),(36,1),(38,1),(40,1),(41,2),(213,1),(215,1) ;$

139: $(1,2),(2,6),(3,24),(4,33),(5,50),(6,49),(7,42),(8,63),(9,33),(10,37),(11,12),(12,6),(13,3)$, $(14,7),(15,2),(16,3),(17,4),(18,1),(19,2),(22,1),(23,1),(24,1),(25,2),(26,1),(27,1),(29,1),(32,1)$, $(35,1),(39,1),(40,1),(43,1)$;

140: $(2,4),(3,12),(4,20),(5,21),(6,14),(7,15),(8,7),(9,5),(10,2),(11,3),(12,3),(13,2),(14,4)$, $(15,1),(16,2),(17,1),(18,1),(22,1)$;

141: $(1,1),(2,9),(3,19),(4,24),(5,14),(6,20),(7,14),(8,13),(9,3),(10,3),(11,3),(12,1),(13,4)$, $(16,1),(19,1)$;

142: $(1,1),(2,4),(3,5),(4,5),(5,10),(6,7),(7,6),(8,7),(9,12),(10,8),(11,2),(12,1),(13,3),(14,1)$, $(15,1),(16,1),(17,1),(32,1),(41,1),(49,1),(51,1),(61,1),(73,1),(77,1)$;

143: $(5,2),(6,3),(7,1),(8,1),(9,3),(10,4),(11,2),(12,5),(13,4),(14,2),(15,3),(16,3),(17,2)$, $(18,3),(20,1),(22,3),(23,3),(24,2),(26,3),(27,1),(28,3),(29,1),(31,1),(32,1),(33,1),(34,1),(35,1)$, $(36,1),(37,1),(38,1),(39,1),(42,1),(46,2),(54,1),(57,1),(100,1),(128,1),(142,1),(162,1),(186,1)$, $(208,1),(325,1)$;

144: $(2,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1),(12,1),(13,1),(14,1),(15,1),(16,1),(18,1)$, $(23,1),(24,1),(27,1),(28,1),(30,1),(33,1),(36,1),(37,1),(40,1),(42,1),(44,1),(45,1),(52,1),(55,1)$, $(57,1),(58,1),(59,1),(60,1),(67,1),(89,1),(96,1),(100,1),(111,1),(117,1),(125,1) ;$

146: $(2,1),(3,1),(4,4),(5,3),(6,4),(7,2),(8,6),(9,5),(10,4),(11,2),(12,4),(13,2),(14,3),(15,4)$, $(16,5),(18,2),(19,1),(20,1),(21,1),(22,1),(24,4),(25,2),(26,3),(28,1),(29,1),(30,1),(31,1),(33,2)$, $(35,1),(36,2),(44,2),(51,1),(52,1),(56,1),(59,1),(60,1),(62,1),(66,1),(68,2),(69,1),(75,1),(98,1)$; 147: $(3,3),(4,5),(5,12),(6,10),(7,9),(8,7),(9,3),(10,8),(11,8),(12,7),(13,4),(14,5),(15,4)$, $(16,1),(17,1),(18,3),(19,3),(21,1),(22,4),(23,2),(24,3),(25,2),(26,1),(27,1),(28,1),(31,2),(32,1)$, $(34,1),(35,1),(38,1),(71,1),(158,1),(175,1),(259,1),(271,1)$;

148: $(1,1),(2,4),(3,7),(4,9),(5,16),(6,21),(7,16),(8,16),(9,14),(10,17),(11,14),(12,15),(13,11)$, $(14,13),(15,8),(16,9),(17,6),(18,8),(19,7),(20,4),(21,7),(22,5),(23,3),(24,1),(26,5),(27,1),(28,3)$, $(29,3),(30,2),(32,2),(33,2),(34,1),(35,2),(37,1),(38,2),(39,1),(41,1),(44,1),(45,1),(46,2),(47,2)$, $(48,2),(52,1),(60,1),(63,1),(65,1),(72,1),(103,1),(104,1),(136,1),(178,1),(185,1),(276,1)$;

149: $(4,3),(5,1),(6,4),(7,2),(9,1)$;
150: $(2,1),(3,1),(4,4),(5,4),(6,2),(7,3),(8,2),(9,4),(10,2),(11,6),(12,3),(13,5),(14,2),(15,5)$, $(17,1),(21,2),(24,2),(25,1),(42,2),(54,1),(70,1),(71,1),(74,1)$;

151: $(3,1),(4,1),(5,2),(7,1),(8,1),(9,3),(11,1),(12,1),(14,1),(26,1),(36,1)$;
152: $(1,2),(2,2),(3,3),(4,2),(5,3),(6,5),(7,1),(8,3),(9,5),(10,3),(11,1),(12,1),(13,6),(14,2)$, $(16,3),(17,1),(18,1),(19,6),(20,2),(21,2),(22,1),(25,1),(26,1),(29,3),(33,2),(34,1),(35,1),(37,1)$, $(40,1),(54,1)$;

155: $(2,2),(3,5),(4,1),(5,7),(6,7),(7,9),(8,6),(9,11),(10,8),(11,5),(12,5),(13,8),(14,7),(15,2)$, $(16,2),(17,1),(18,1),(23,2),(25,2),(26,1),(27,2),(28,2),(29,1),(31,1),(83,1)$;

156: $(3,2),(4,2),(5,2),(6,2),(7,3),(8,3),(9,5),(10,4),(11,1),(12,3),(15,2),(16,4),(18,3),(19,1)$, $(20,4),(21,2),(24,4),(27,1),(28,3),(29,1),(30,1),(32,3),(33,1),(35,1),(36,4),(38,1),(39,1),(40,2)$, $(42,1),(44,2),(45,1),(46,1),(48,2),(51,1),(52,2),(54,1),(56,1),(57,1),(60,1),(63,1),(64,2),(66,1)$,
$(68,1),(72,2),(73,1),(75,1),(80,1),(87,1),(88,1),(128,1)$;
157: $(4,1),(5,3),(6,4),(7,1),(8,2),(10,3),(11,1),(12,1),(13,1),(14,2),(15,1),(16,1),(18,1)$, $(19,1),(22,1),(28,1),(33,1),(40,1),(51,1),(56,1),(62,1)$;

158: $(2,1),(6,1),(7,2),(9,1),(11,1),(14,2),(16,2),(18,3),(19,1),(27,1),(29,1),(32,2),(34,1)$, $(110,1)$;

159: $(4,2),(5,4),(6,4),(7,2),(8,3),(9,5),(10,2),(11,4),(12,1),(13,4),(14,4),(15,1),(16,5),(17,2)$, $(18,1),(19,1),(20,3),(21,2),(22,3),(23,1),(24,1),(25,1),(26,4),(27,1),(30,2),(31,1),(33,2),(36,1)$, $(38,1),(43,1),(45,1),(46,1),(47,1),(49,1),(52,2),(59,1),(76,1),(97,1)$;

160: $(2,3),(3,5),(4,3),(5,7),(6,5),(7,9),(8,9),(9,8),(10,8),(11,3),(12,7),(13,5),(14,5),(15,4)$, $(16,1),(17,2),(18,5),(19,3),(20,2),(21,3),(22,2),(24,2),(25,1),(26,1),(27,2),(28,2),(30,1),(32,1)$, $(33,2),(34,2),(36,2),(37,1),(38,1),(39,1),(40,1),(42,2),(43,3),(44,3),(45,3),(46,4),(47,2),(48,4)$, $(49,1),(50,3),(51,5),(52,5),(53,2),(54,2),(55,1),(56,1),(57,1),(58,3),(60,1),(62,1),(64,2),(68,1)$, $(70,1),(72,1),(74,1),(76,1),(80,2),(84,1),(85,1),(86,1),(88,1),(94,1),(95,1),(96,1),(98,1),(100,1)$, $(108,1),(116,1),(126,1),(174,1),(262,1),(274,1),(346,1)$;

161: $(2,1),(3,2),(4,3),(5,4),(6,5),(7,4),(8,5),(9,4),(10,5),(11,2),(12,3),(14,1),(15,2),(16,1)$, $(17,1),(18,3),(19,3),(20,4),(21,4),(22,3),(23,1),(24,1),(25,1),(35,2),(36,1),(72,1),(73,1)$;

162: $(2,1),(3,4),(4,8),(5,13),(6,8),(7,6),(8,2),(16,1),(53,1)$;
163: $(2,1),(3,3),(4,10),(5,13),(6,12),(7,8),(8,3),(9,3),(10,3),(11,4),(12,2),(13,1),(14,1)$, $(15,1),(18,2),(22,1),(24,1),(90,1)$;

164: $(2,3),(3,10),(4,14),(5,18),(6,23),(7,15),(8,15),(9,20),(10,13),(11,13),(12,9),(13,7),(14,3)$, $(15,2),(17,2),(18,2),(19,4),(20,3),(21,2),(22,2),(23,2),(26,1),(28,1),(31,1),(32,1),(37,1),(38,1)$; 165: $(2,1),(3,1),(4,6),(5,8),(6,7),(7,2),(8,6),(9,9),(10,6),(11,5),(12,8),(13,3),(14,3),(15,2)$, $(16,5),(17,1),(18,3),(19,2),(22,2),(24,1),(25,1),(28,1),(43,1)$;

166: $(1,2),(2,7),(3,20),(4,32),(5,35),(6,48),(7,45),(8,35),(9,32),(10,24),(11,36),(12,30)$, $(13,23),(14,13),(15,17),(16,10),(17,7),(18,12),(19,19),(20,10),(21,5),(22,2),(23,5),(24,2),(25,1)$, $(28,2),(30,1),(31,1),(32,2),(33,1),(34,1),(37,1),(38,1),(39,1),(43,1),(45,1),(47,1),(53,1)$;

167: $(1,1),(2,4),(3,13),(4,15),(5,20),(6,23),(7,22),(8,17),(9,16),(10,15),(11,11),(12,8),(13,6)$, $(14,4),(15,7),(16,4),(17,3),(18,5),(19,3),(20,1),(21,4),(22,2),(23,1),(25,2),(27,2),(29,1),(30,1)$, $(31,1),(40,1),(45,1),(122,1)$;

168: $(10,1),(12,1),(22,1),(38,1),(74,1)$;
169: $(5,1),(6,1),(7,1),(8,1),(9,1),(12,1),(15,1),(16,1),(17,1),(18,1),(23,1),(28,1),(29,1)$, $(30,1),(32,1),(34,1),(39,1),(59,1)$;

171: $(7,1),(11,1),(13,1),(40,1)$;
173: $(2,1),(3,2),(4,4),(5,8),(6,7),(7,4),(8,9),(9,10),(10,3),(11,8),(12,7),(13,5),(14,7),(15,8)$, $(16,6),(17,2),(18,6),(19,5),(20,1),(21,4),(22,3),(23,2),(24,3),(25,2),(26,3),(27,1),(28,2),(29,1)$, $(30,3),(31,1),(32,2),(34,1),(35,1),(36,1),(40,2),(46,1),(68,1),(74,1),(75,1),(79,1)$;

174: $(4,1),(5,4),(6,3),(7,2),(8,4),(9,6),(10,1),(11,1),(12,3),(13,1),(14,3),(15,2),(16,3),(17,6)$, $(18,2),(19,2),(20,2),(21,1),(22,2),(26,2),(27,2),(28,2),(29,1),(31,1),(32,1),(45,1),(52,1),(132,1)$; 175: $(3,1),(4,1),(5,2),(7,3),(8,1),(9,3),(10,1),(11,4),(12,1),(15,1),(22,1),(24,1),(25,1),(29,1)$, $(33,1),(76,1)$;

176: $(2,3),(3,8),(4,19),(5,31),(6,34),(7,28),(8,42),(9,49),(10,30),(11,21),(12,20),(13,13)$, $(14,9),(15,9),(16,8),(17,10),(18,8),(19,6),(20,4),(22,1),(23,2),(24,4),(26,1),(27,1),(28,4),(29,2)$, $(30,2),(32,1),(33,3),(34,1),(35,1),(37,3),(40,1),(46,2),(48,1),(49,1),(52,1),(53,1),(61,1),(63,1)$, $(66,1),(68,2),(72,1),(91,1),(96,1),(104,1),(109,1),(116,1),(129,1),(131,1)$;

177: $(6,1),(9,1),(13,1)$;
178: $(1,1),(2,1),(3,2),(4,2),(6,2),(7,2),(8,3),(9,4),(10,5),(11,3),(12,3),(13,2),(16,1),(17,1)$, $(19,1)$;

180: $(2,4),(3,6),(4,8),(5,4),(6,4),(7,2),(8,1),(11,3),(13,2),(17,1),(18,2),(19,3),(21,1),(22,1)$, $(23,2),(25,1)$;

182: $(2,1),(3,2),(4,14),(5,5),(6,5),(7,4),(8,1),(9,2),(10,2),(12,1),(13,1),(14,1),(17,1),(19,1)$, (20, 1);

183: $(3,2),(4,1),(7,2),(8,1),(9,3),(10,1),(22,1),(25,1)$;
184: $(6,1),(7,1),(8,1),(9,2),(10,2),(11,4),(14,1),(18,1)$;
185: $(2,1),(4,1),(5,4),(6,1),(7,5),(8,4),(9,2),(10,6),(11,4),(13,2),(15,1),(16,2),(18,1),(19,2)$, $(20,1),(21,3),(23,2),(24,1),(32,1)$;

186: $(2,3),(3,4),(4,8),(5,11),(6,11),(7,15),(8,19),(9,13),(10,11),(11,11),(12,5),(13,3),(14,6)$, $(15,6),(16,9),(17,1),(18,4),(20,3),(23,2),(26,1),(27,2),(30,1),(31,1),(41,2)$;

187: $(2,1),(3,5),(4,6),(5,10),(6,9),(7,7),(8,4),(9,7),(10,3),(11,4),(12,2),(13,2),(14,2),(15,1)$, $(16,2),(17,2),(18,4),(19,1),(20,2),(21,2),(24,1),(25,1),(26,1),(27,5),(29,1),(35,2),(79,1)$;

188: $(2,1),(3,1),(4,1),(5,2),(6,2),(8,3),(10,1),(12,2),(13,1),(14,1),(15,1)$;
189: $(3,5),(4,9),(5,14),(6,19),(7,18),(8,12),(9,7),(10,6),(11,2),(12,1),(14,1),(15,4),(16,2)$, $(17,5),(18,4),(19,2),(20,1),(21,1),(23,1),(24,2),(26,1),(28,2)$;

190: $(4,8),(5,11),(6,6),(7,13),(8,7),(9,6),(10,2),(11,2),(12,8),(13,1),(15,3),(18,1),(19,1)$, $(20,2),(21,1),(22,1),(107,1),(108,1)$;

191: $(1,1),(2,7),(3,13),(4,25),(5,26),(6,15),(7,13),(8,12),(9,8),(10,6),(11,4),(12,6),(13,6)$, $(14,4),(15,2),(16,6),(17,3),(18,4),(19,3),(20,7),(21,6),(23,3)$;

192: $(5,2),(6,5),(7,2),(8,5),(9,1),(10,3),(11,3)$;
193: $(2,1),(3,7),(4,16),(5,10),(6,9),(7,6),(8,4),(9,5),(10,6),(11,2),(12,1),(13,4),(14,1),(18,1)$, $(21,1),(25,1),(30,1),(37,1)$;

194: $(1,3),(2,13),(3,27),(4,43),(5,70),(6,44),(7,50),(8,36),(9,34),(10,31),(11,40),(12,27)$, $(13,20),(14,17),(15,16),(16,9),(17,4),(18,10),(19,4),(20,6),(21,4),(22,2),(23,4),(24,2),(25,1)$, $(26,1),(29,1),(36,1),(39,3),(42,1),(43,1),(44,1),(47,1),(51,3),(52,3),(53,1)$;

195: $(3,1),(4,1),(6,2),(8,2),(9,2),(10,4),(12,1),(14,1),(19,1),(20,1),(35,1)$;
196: $(4,1),(5,4),(7,4),(8,1),(10,1),(13,2),(14,1),(16,2),(20,1),(23,1),(25,1)$;
197: $(3,2),(4,5),(5,4),(6,4),(7,2),(9,3),(10,6),(12,1),(13,2),(14,1),(15,3),(16,1),(27,1),(29,1)$, $(30,1),(37,1),(38,1)$;

198: $(2,2),(3,3),(4,3),(5,5),(6,4),(7,5),(8,4),(9,2),(10,3),(11,2),(12,5),(13,2),(14,2),(16,3)$, $(18,1),(19,1),(22,2),(26,2),(30,1),(33,1),(47,1),(49,1),(51,1)$;

199: $(1,1),(2,2),(3,3),(4,2),(5,5),(6,1),(7,1),(8,2),(9,2),(10,2),(11,1),(12,1),(13,1),(14,1)$, $(16,1),(18,1),(20,1)$;

200: $(3,6),(4,4),(5,4),(6,2),(7,3),(8,2),(9,1),(11,1),(13,1),(14,5),(15,4),(16,1),(17,2),(41,1)$, $(42,1),(43,1)$;

201: $(3,1),(4,1),(5,6),(6,6),(7,3),(9,1),(11,1),(16,2),(18,1),(22,1),(24,1)$;
202: $(2,1),(3,2),(4,5),(5,2),(6,3),(7,1),(8,2),(9,1),(10,1),(11,3),(12,1),(13,1),(14,2),(20,1)$, $(59,1),(62,1),(77,1),(83,1)$;

203: $(5,3),(6,3),(7,4),(8,3),(9,4),(10,10),(11,10),(12,12),(13,9),(14,11),(15,8),(16,10),(17,8)$, $(18,6),(19,5),(20,3),(21,1),(22,1),(24,3),(26,1),(30,1),(31,1),(32,1),(33,1),(35,1),(49,1) ;$

204: $(2,4),(3,7),(4,10),(5,8),(6,12),(7,6),(8,6),(9,8),(10,6),(11,3),(12,1),(13,4),(14,4),(18,1)$, $(20,1),(64,1),(70,1)$;

205: $(1,2),(2,4),(3,7),(4,6),(5,7),(6,9),(7,5),(8,5),(9,4),(10,6),(11,2),(12,1),(13,3),(14,4)$, $(15,2),(17,2),(18,2),(19,1),(20,2),(21,1),(22,1),(24,1),(26,1),(29,1),(31,1),(32,1),(33,1),(34,2)$, $(35,1),(37,1),(50,1),(54,1),(83,1)$;

206: $(1,1),(3,4),(4,4),(5,6),(6,4),(7,3),(10,1),(26,1),(27,1),(31,1)$;
207: $(3,1),(7,1)$;
208: $(6,1),(8,1)$;
209: $(3,1),(11,1),(12,2)$;
210: $(3,1),(4,1),(5,1),(6,1),(7,2)$;
211: $(2,1)$;
213: $(2,2),(3,1),(4,4),(5,4),(6,8),(7,6),(8,2),(9,6),(10,5),(11,1),(12,1),(13,1),(14,1),(15,1)$, $(18,1)$;

214: $(3,2),(4,1),(5,1),(6,1),(8,1),(11,1),(15,1),(24,1)$;
215: $(2,1),(3,4),(4,3),(5,6),(6,7),(7,12),(8,20),(9,8),(10,6),(12,2),(13,1),(19,1),(23,1)$;
216: $(2,3),(3,8),(4,10),(5,22),(6,20),(7,20),(8,9),(9,6),(10,6),(11,7),(12,2),(13,5),(14,4)$, $(15,4),(16,3),(17,12),(18,5),(19,7),(20,5),(22,1),(25,1),(26,1),(28,1),(29,1),(37,1),(108,1)$, $(118,1),(367,1)$;

217: $(2,2),(3,6),(4,15),(5,8),(6,17),(7,8),(8,7),(9,2),(10,6),(11,2),(12,4),(13,1),(18,1),(19,1)$, $(20,1),(21,1),(24,1),(32,1),(77,1)$;

218: $(3,3),(4,5),(5,3),(6,9),(7,5),(8,4),(9,8),(10,4),(11,2),(12,1),(13,2),(22,1),(31,1)$;
219: $(3,1),(4,2),(5,3),(6,7),(7,2),(8,2),(10,1),(11,1),(12,1),(13,1),(14,1),(21,1),(22,1)$;
220: $(1,2),(2,3),(3,4),(4,6),(5,8),(6,11),(7,9),(8,3),(9,3),(10,6),(11,2),(12,1),(13,2),(19,2)$, $(21,2),(23,1),(48,1)$;

221: $(1,1),(2,8),(3,34),(4,25),(5,14),(6,16),(7,26),(8,31),(9,30),(10,30),(11,22),(12,9),(13,7)$, $(14,8),(15,4),(16,2),(17,2),(19,2),(20,2),(22,1),(23,2)$;

222: $(4,1),(5,1),(6,2),(10,1),(12,1),(13,1),(14,1)$;
223: $(2,3),(3,4),(4,10),(5,9),(6,5),(7,7),(8,4),(9,4),(10,1),(12,2),(13,1),(25,1),(28,1)$;
224: $(2,7),(3,2),(4,3),(5,4),(6,4),(7,5),(8,4),(10,1)$;
225: $(1,1),(2,14),(3,28),(4,48),(5,44),(6,44),(7,30),(8,24),(9,14),(10,4),(11,5),(12,1),(13,10)$, $(14,1),(15,1),(16,1),(21,1),(23,3),(24,2),(25,1),(26,3),(27,1),(28,2),(29,1),(32,2),(36,1)$;

226: $(2,1),(3,1),(4,5),(5,6),(6,1),(7,5),(8,5),(9,5),(10,5),(11,5),(12,6),(13,4),(14,4),(15,3)$, $(16,2),(17,1),(19,1),(58,1)$;

227: $(1,2),(2,8),(3,13),(4,37),(5,31),(6,23),(7,23),(8,21),(9,22),(10,21),(11,13),(12,11)$, $(13,11),(14,5),(15,10),(16,7),(17,3),(18,1),(19,1),(20,4),(21,2),(23,1),(26,1),(48,1)$;

228: $(2,1),(3,1),(5,3),(6,1),(10,1),(11,1),(12,1),(13,1),(31,1)$;
229: $(1,2),(2,9),(3,13),(4,15),(5,12),(6,8),(7,10),(8,4),(9,6),(10,5),(11,3),(12,6),(13,2)$, $(14,6),(15,5),(16,3),(17,1),(21,1),(30,1),(44,1),(45,1),(49,1),(54,1)$;

230: $(2,3),(3,6),(4,7),(5,7),(6,6),(7,2),(8,1),(9,4),(10,2),(11,1),(22,1)$;

### 3.3 Plots $(f \leq 50, k \leq 50)$




























no data





| no data |
| :---: |
|  |
|  |




\#97 $(7,4)$















$\square$
\#155
$(4,2)$

\#157 $(4,0)$








## 48

no data

















no data




