FOUNDATIONS
ADVANCES

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Supporting information for article:

On the frequency module of the hull of a primitive substitution tiling

April Lynne D. Say-awen, Dirk Frettlöh and Ma. Louise Antonette N. De Las Peñas

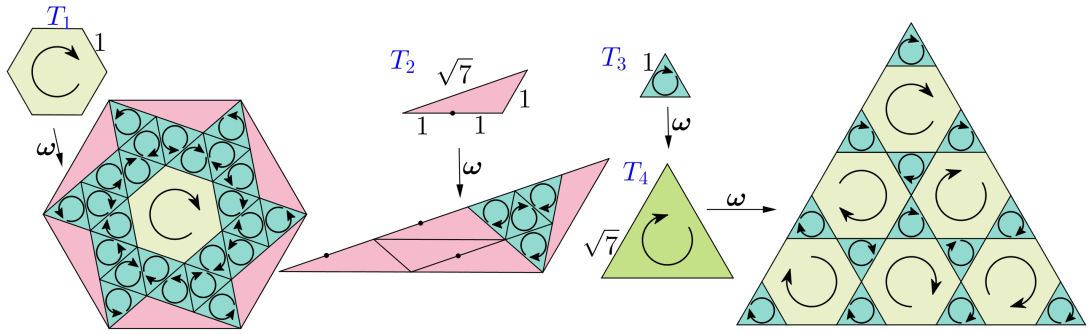


Figure 1 The substitution ω with substitution factor $\sqrt{7}$. The prototiles T_1, T_2, T_3 and T_4 are regular polygons. The circular arrows indicate the orientations of symmetric tiles. The dot at the midpoint on the middle edge of T_2 is a pseudo- vertex.

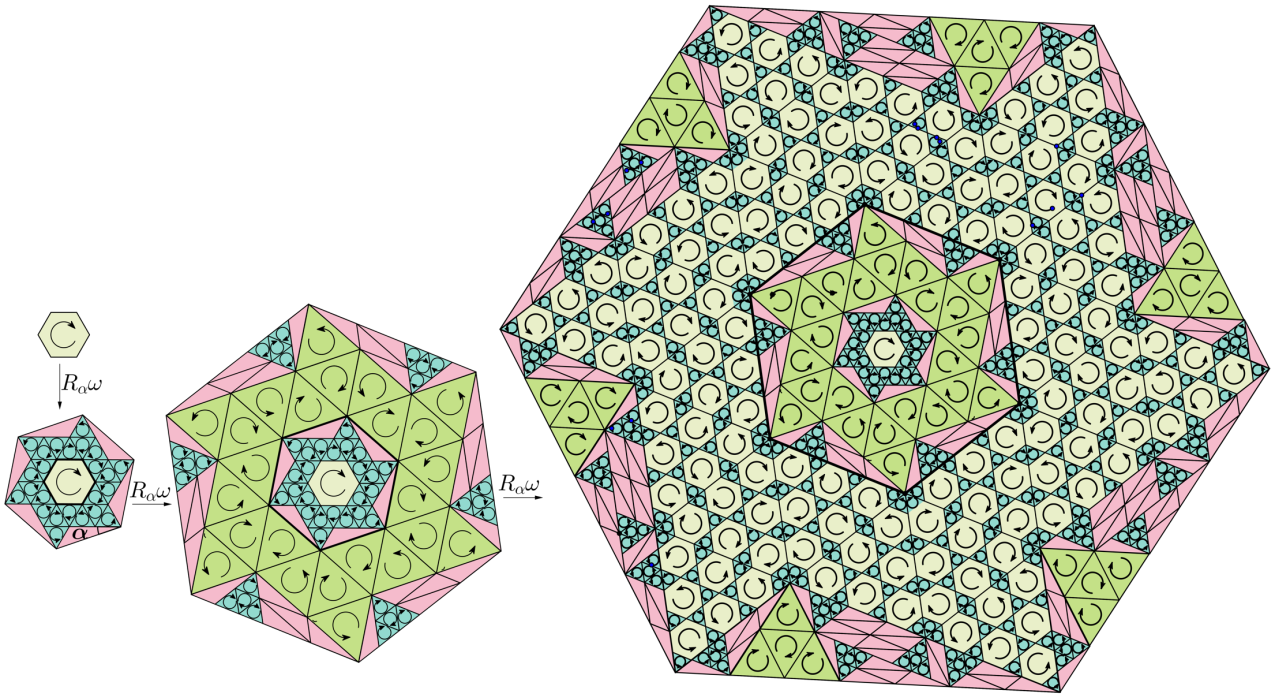


Figure 2 The first four terms of the nested sequence $\left((R_\alpha\omega)^k(T_1)\right)_{k\in\mathbb{N}}$ which converges to a tiling in $\mathbb{X}_\omega$.

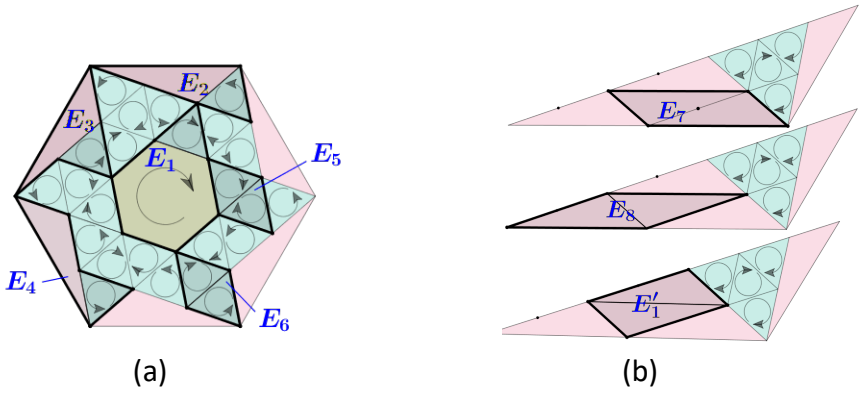


Figure 3 Edge types (a) $E_1, E_2, \ldots, E_6$ in $\omega(T_1)$; and (b) E_7, E_8 and E_1' in $\omega(T_2)$.

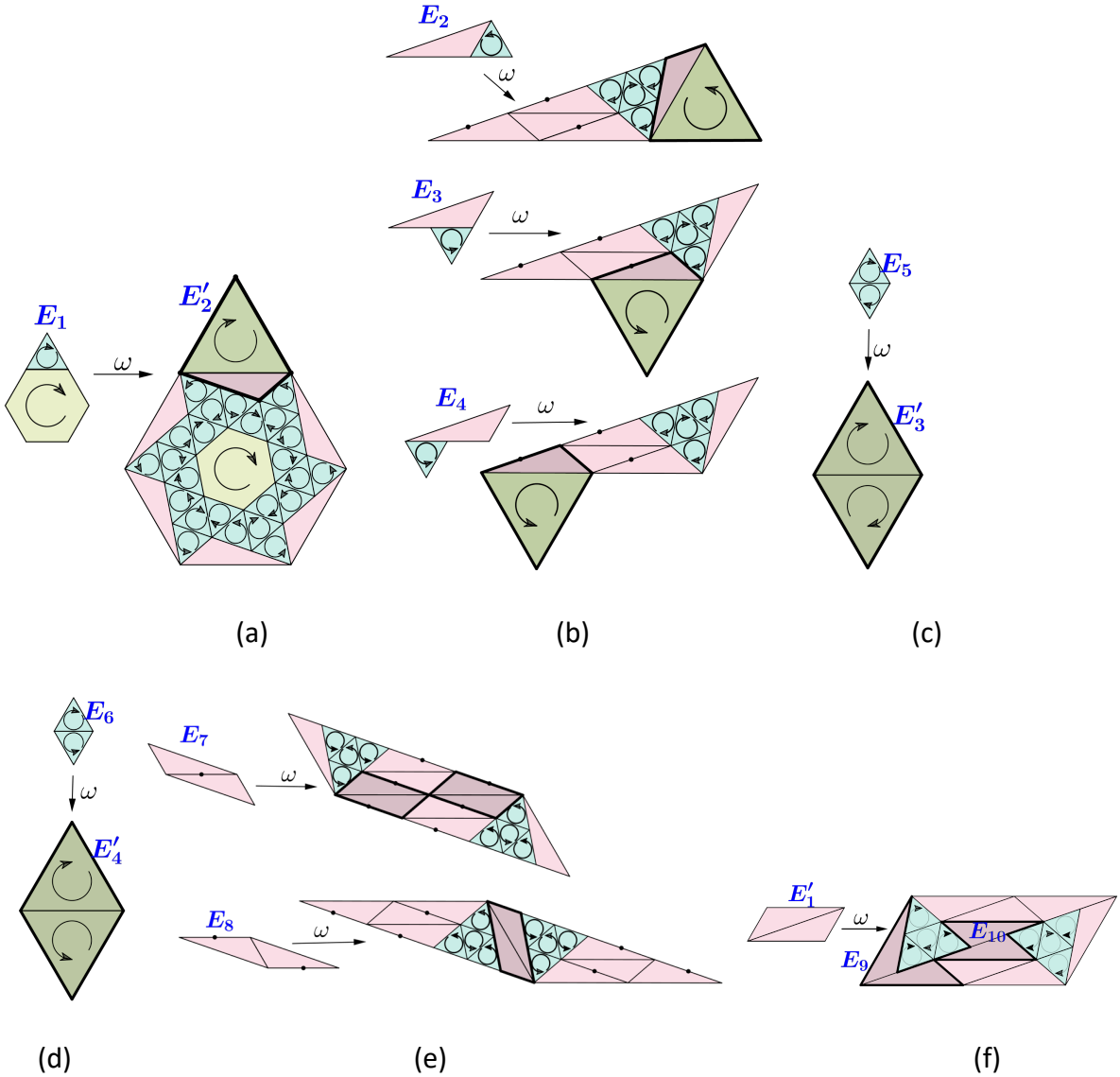


Figure 4 **(a)** $E_2' \subset \omega(E_1)$; **(b)** copies of E_2' in $\omega(E_2), \omega(E_3), \omega(E_4)$; **(c)** $E_3' \subset \omega(E_5)$; **(d)** $E_4' \subset \omega(E_6)$; **(e)** copies of E_1' in $\omega(E_7)$ and $\omega(E_8)$; and **(f)** E_9 and E_{10} in $\omega(E_1')$.

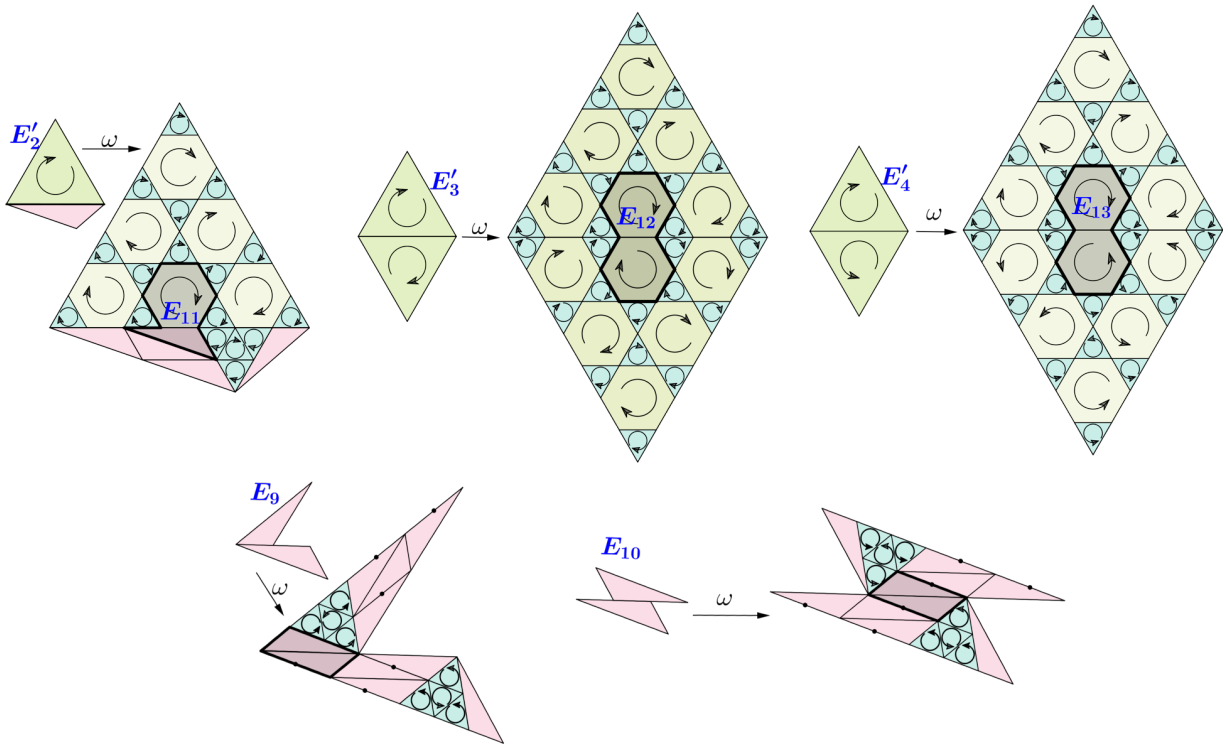


Figure 5 $E_{11} \subset \omega(E'_2)$, $E_{12} \subset \omega(E'_3)$, $E_{13} \subset \omega(E'_4)$, and copies of E'_1 in $\omega(E_9)$ and $\omega(E_{10})$

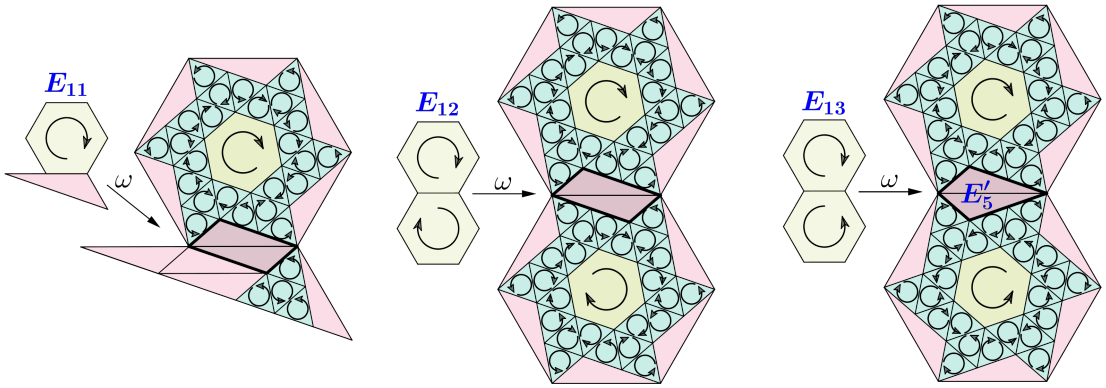


Figure 6 Copies of E'_1 in $\omega(E_{11})$ and $\omega(E_{12})$, and $E'_5 \subset \omega(E_{13})$.

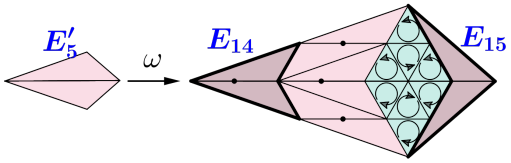


Figure 7 E_{14} and E_{15} in $\omega(E'_5)$.

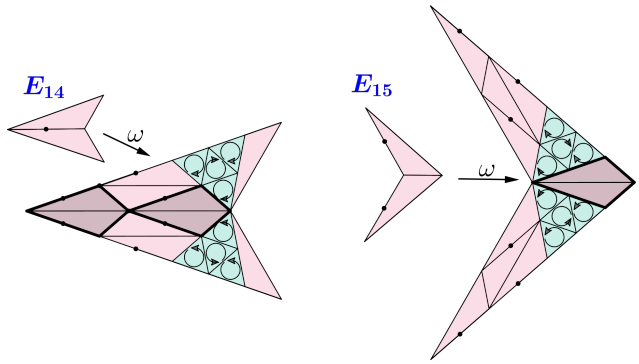


Figure 8 Copies of E_5' in $\omega(E_{14})$ and $\omega(E_{15})$.

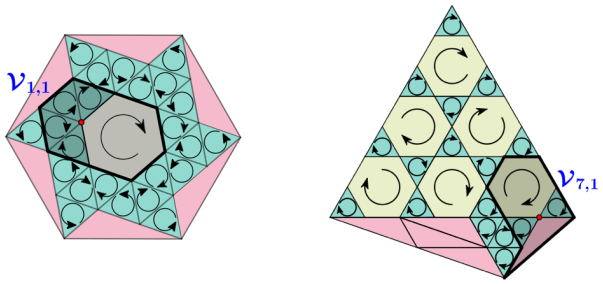


Figure 9 $\omega(T_1)$ contains $\mathcal{V}_{1,1}$ and $\omega(E_2')$ contains $\mathcal{V}_{7,1}$.

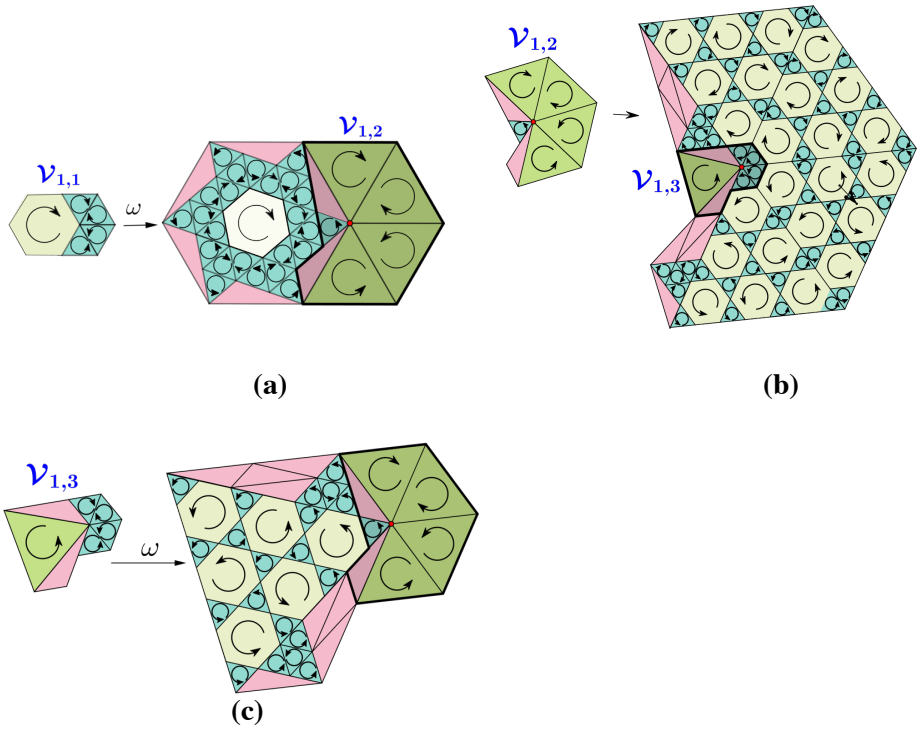


Figure 10 (a) $\mathcal{V}_{1,1}$ yields the vertex star $\mathcal{V}_{1,2}$; (b) $\mathcal{V}_{1,2}$ yields the vertex star $\mathcal{V}_{1,3}$; and (c) $\mathcal{V}_{1,3}$ yields an equivalent copy of $\mathcal{V}_{1,2}$.

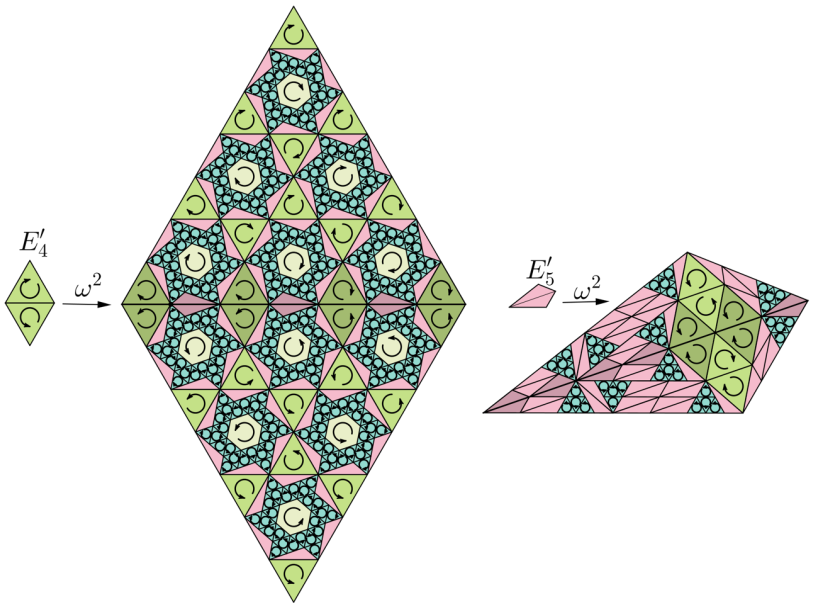


Figure 11 Every edge type along the 2-order super-edge of $\omega^2(E_4')$ or $\omega^2(E_5')$ is either equivalent to E_4' or E_5'.

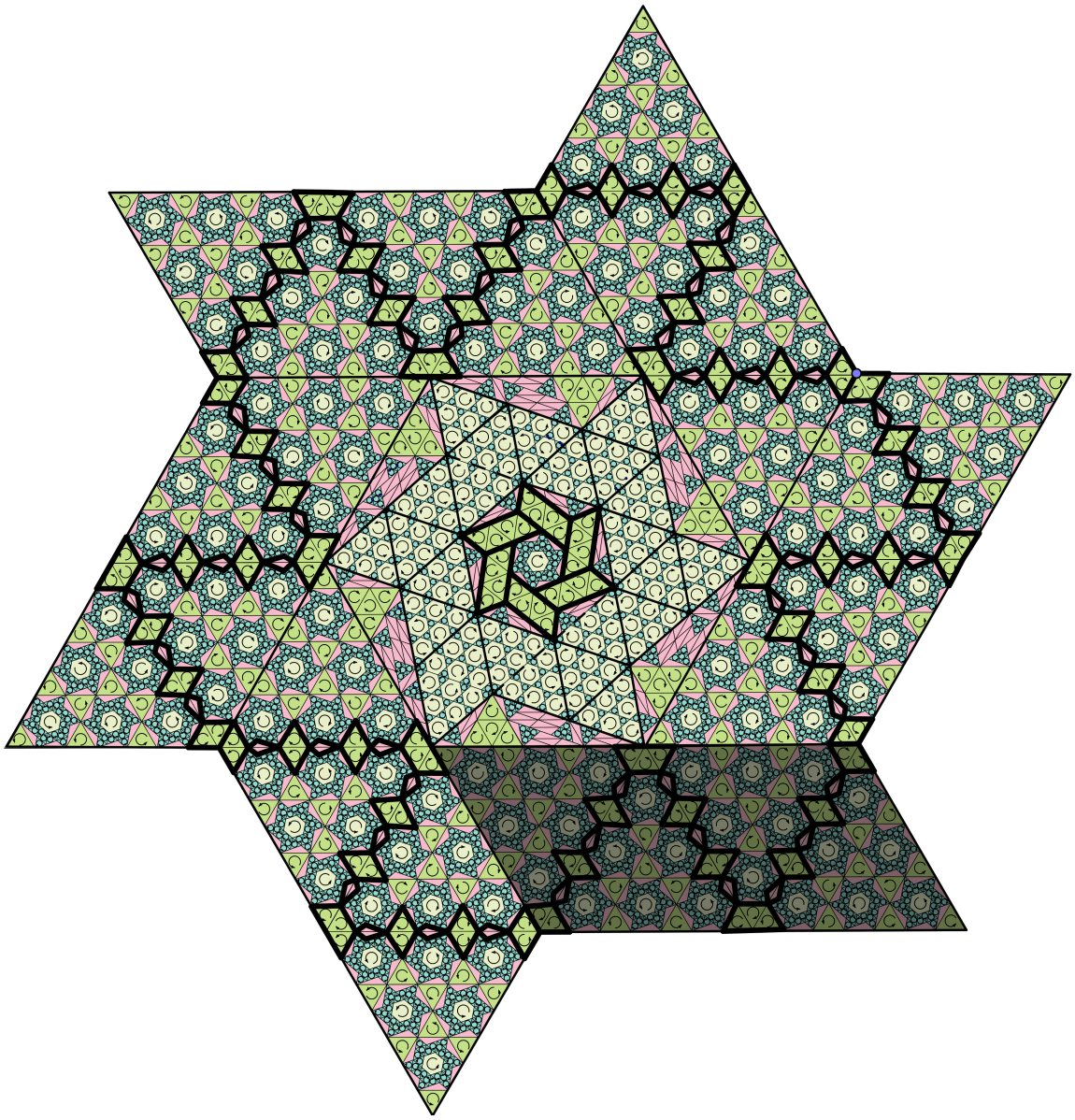


Figure 12 A portion of the partition $\mathcal{H}_1$ of $\mathcal{T}_\omega$. A patch in $\mathcal{H}_1$ consisting of more than one tile is enclosed by thick black edges. The partition of a patch equivalent to $\omega^2(\mathcal{P}_7)$ is shaded.

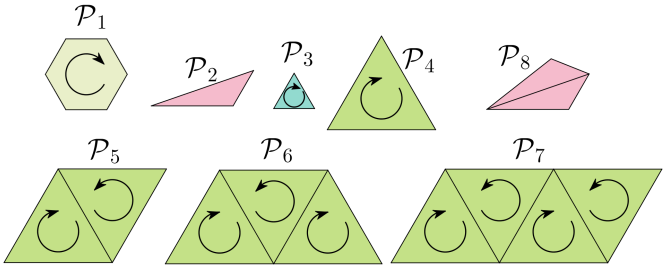


Figure 13 The complete list of non-equivalent patches in the partition $\mathcal{H}_1$ of $\mathcal{T}_\omega$.

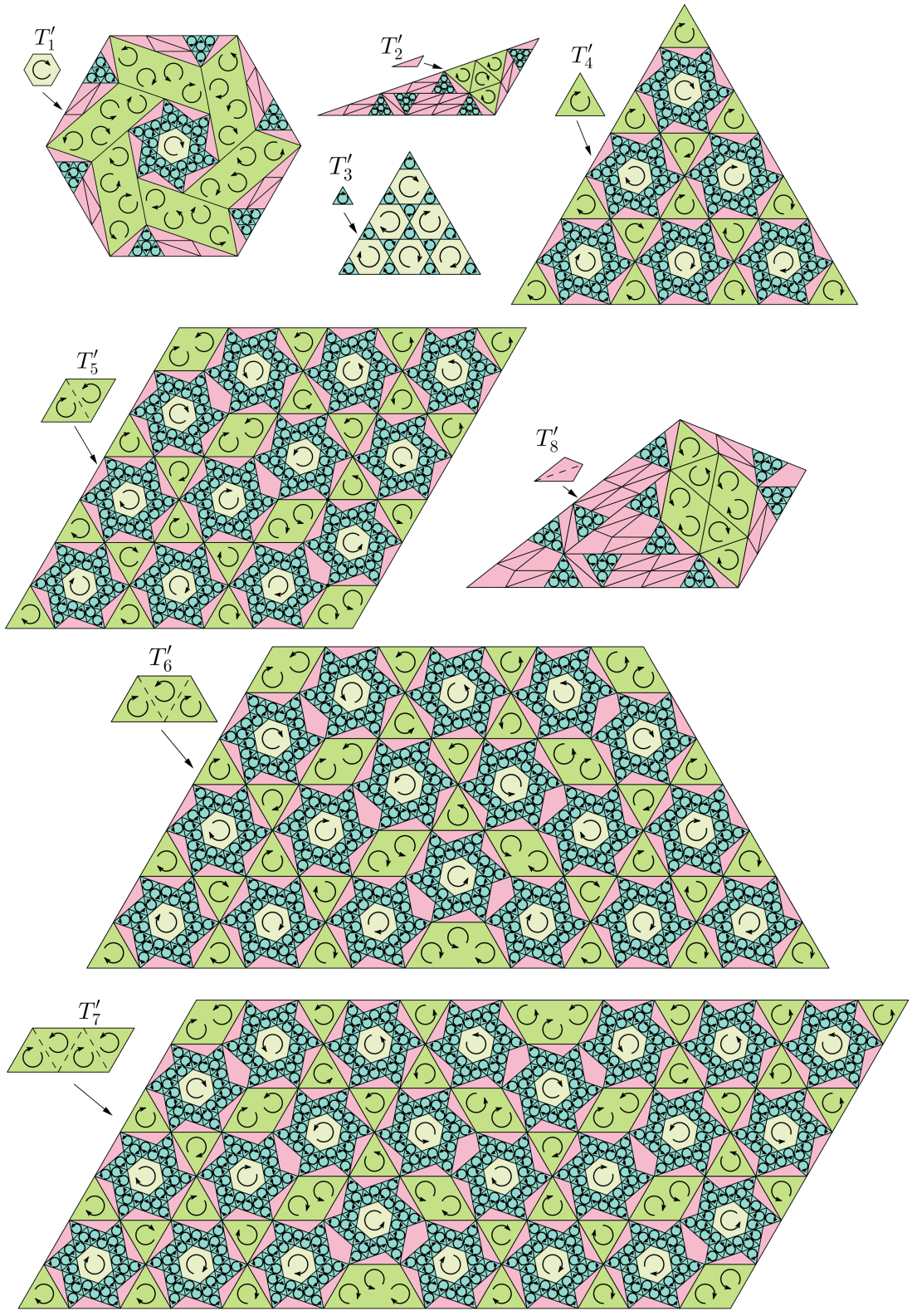
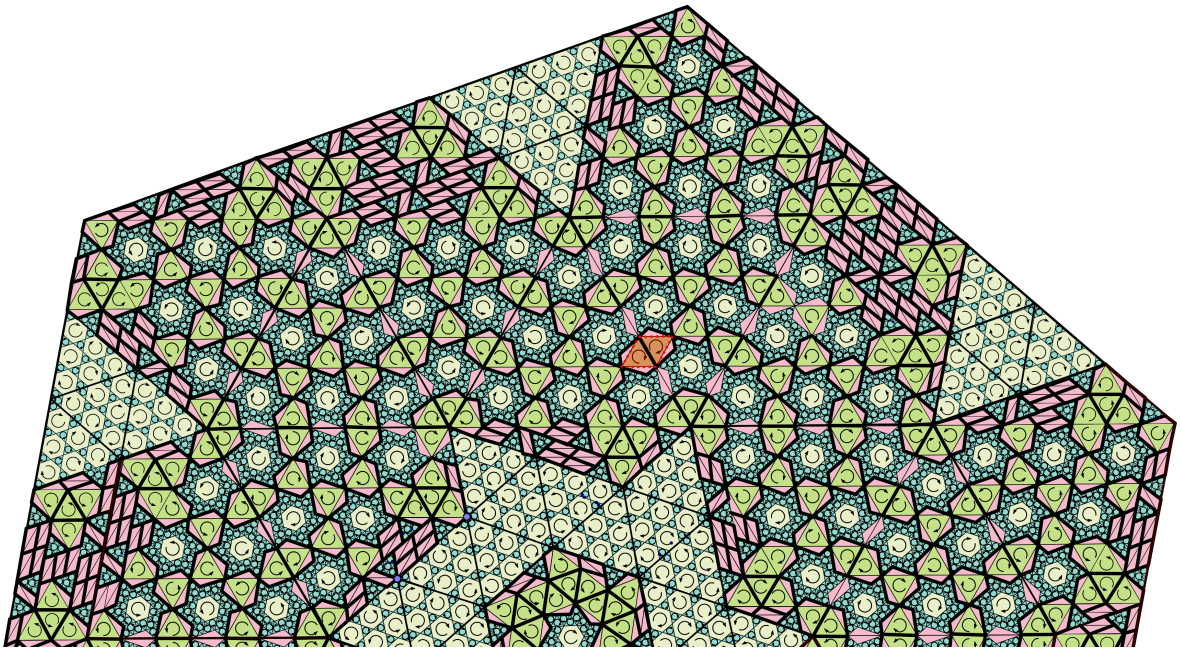
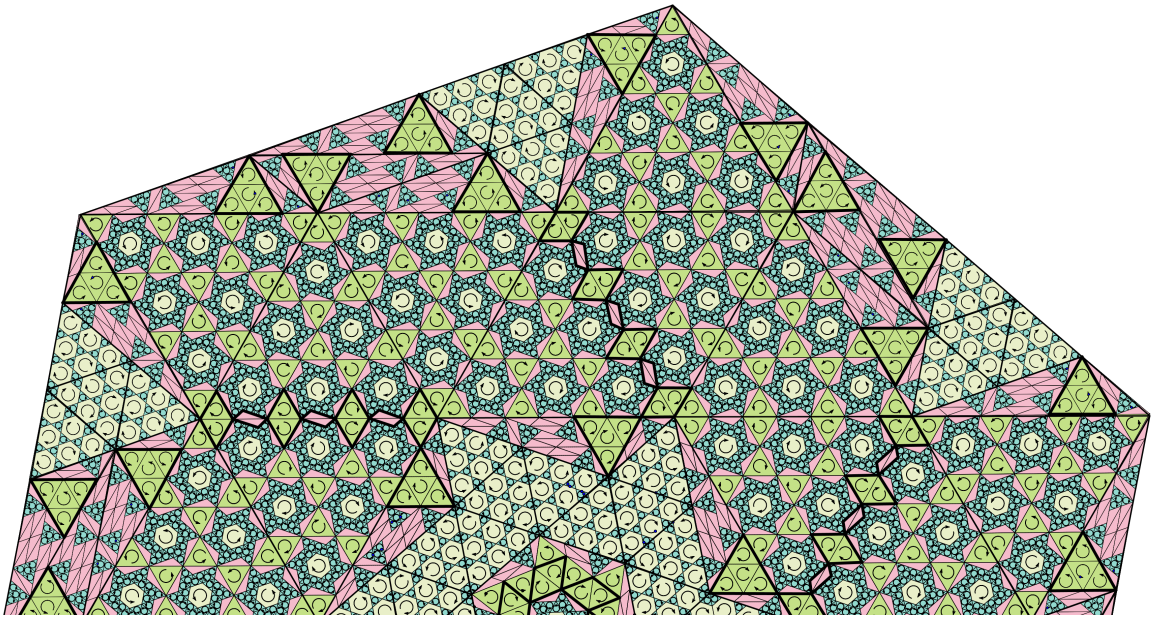


Figure 14 The substitution ω'.

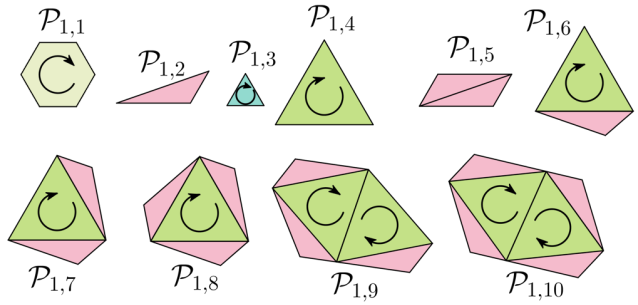


(a)

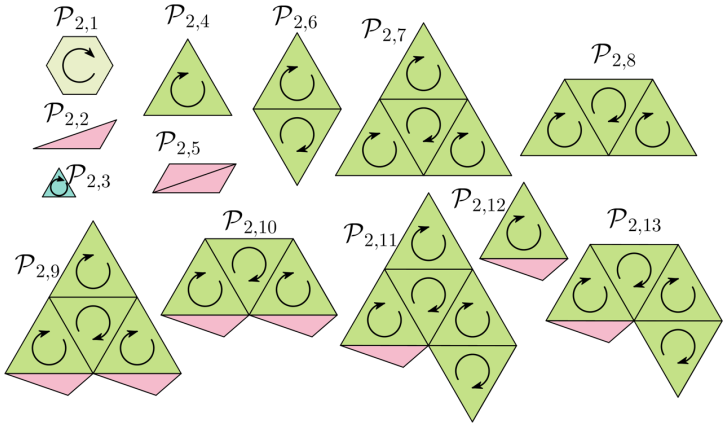


(b)

Figure 15 Portions of the partitions (a) $\mathcal{H}_{2,1}$ and (b) $\mathcal{H}_{2,2}$ of $\mathcal{T}_\omega$. A patch (in $\mathcal{H}_{2,1}$ or $\mathcal{H}_{2,2}$) consisting of more than one tile is enclosed by thick edges.



(a)



(b)

Figure 16 The complete lists of non-equivalent patches of (a) $\mathcal{H}_{2,1}$ and (b) $\mathcal{H}_{2,2}$.

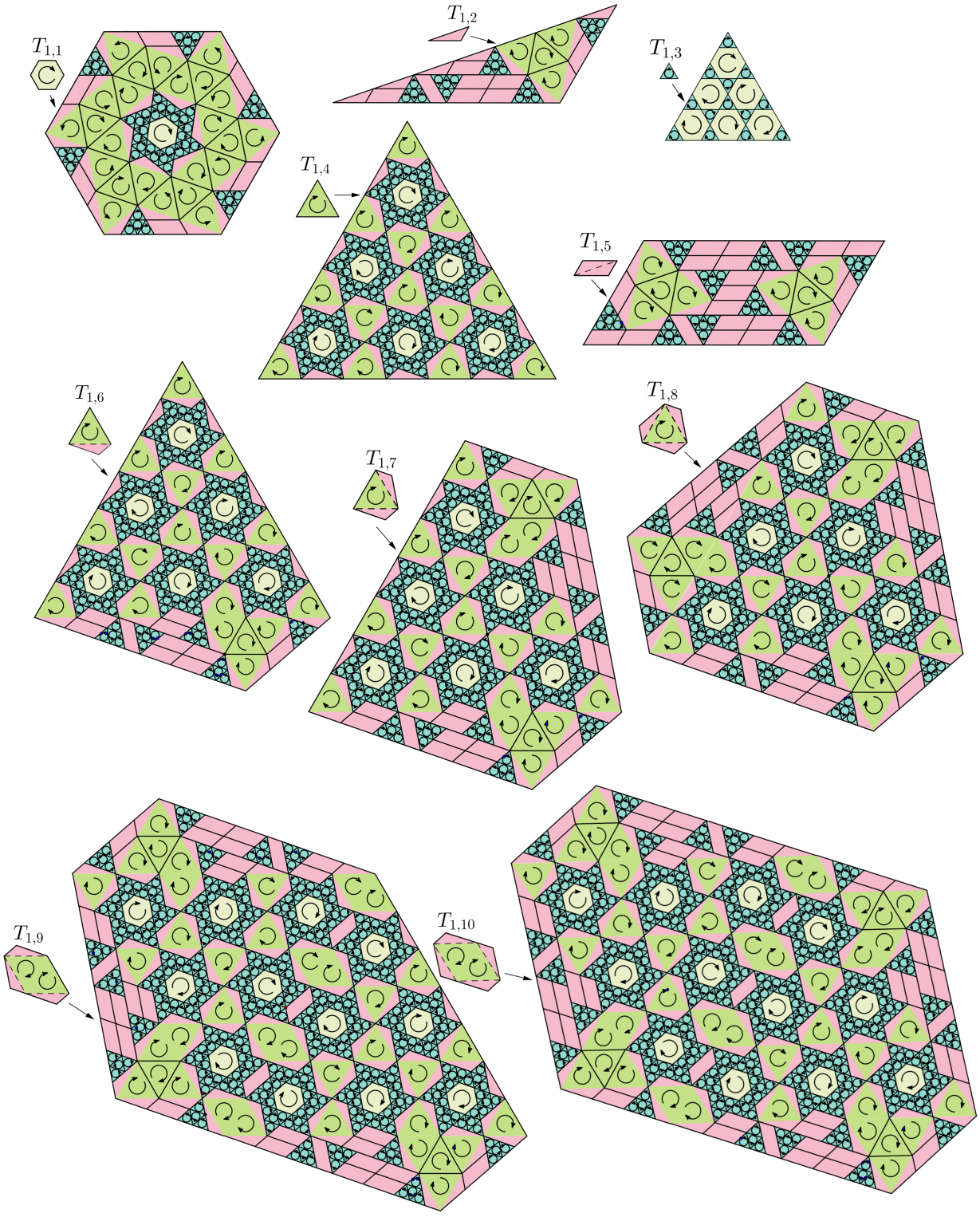


Figure 17 The substitution ω_1.

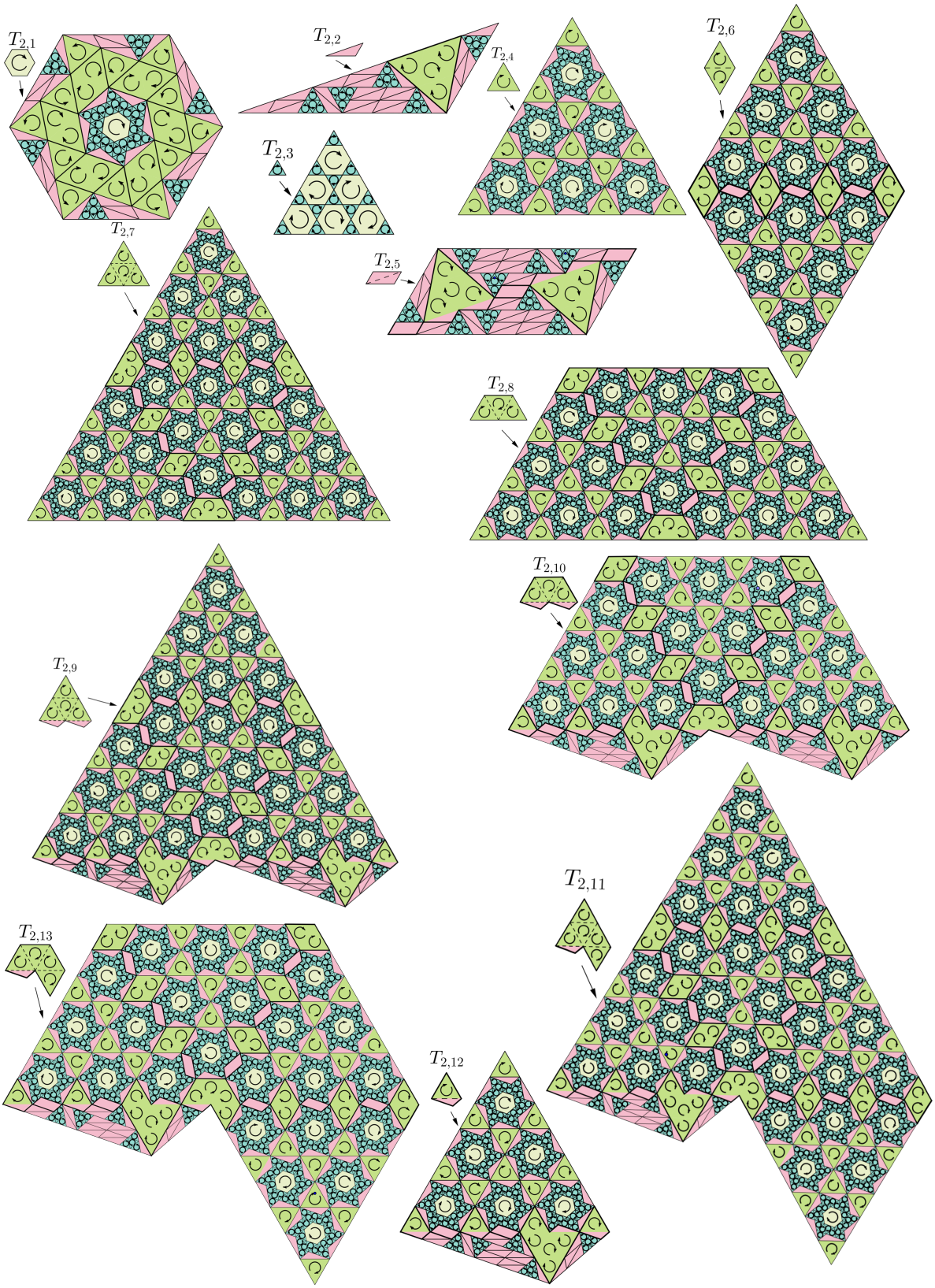


Figure 18 The substitution ω_2.