



FOUNDATIONS
ADVANCES

Volume 77 (2021)

Supporting information for article:

Magnetic modes compatible with the symmetry of crystals

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PART I

Tables of magetic modes without atomic coordinates (i.e. set-of-directions of the magnetic moments) for all crystallographic systems. These modes depend on the magnetic point group only.

Table S1a. Classification of all possible magnetic modes for triclinic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S1b.

| | WITH FERROMAGNETISM | WITHOUT FERROMAGNETISM |
|---|------------------------|--|
| Cat. No. → | 1 | 2 |
| Magnetic Class → | | |
| Magnetic site-symmetry point group ↓ | 1 $\bar{1}$ | $\bar{1}'$ $11'$ $\bar{1}\bar{1}'$ |
| 1 $\bar{1}$ | FFF | AAA |
| $\bar{1}'$ | | 000 |

Table S1b. Magnetic moment directions possible in triclinic symmetry.

| free param. | Mode | Directions |
|----------------|------|--|
| 3 | FFF | $[u, v, w]$ |
| 3 | AAA | $[u, v, w], [\bar{u}, \bar{v}, \bar{w}]$ |
| 0 | 000 | $[0, 0, 0]$ |

Table S2a. Classification of all possible magnetic modes for monoclinic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S2b. The unique axis along [010] is assumed. For all possible unique axis see Table S8.

| | WITH FERROMAGNETISM | | WITHOUT FERROMAGNETISM |
|--------------------------------------|---------------------|---------------|-------------------------------------|
| Cat. No. → | 4 | 3 | 5 |
| Magnetic Class → | | | 21' m1' 2'/m 2/m' 2/m1' |
| Magnetic site-symmetry point group ↓ | 2' m' 2'/m' | 2 m 2/m | |
| 1 1̄ | FAF | AFA | ABA |
| 2' m' 2'/m' | F0F | | A0A |
| 2 m 2/m | | 0F0 | 0A0 |
| 1' 2'/m 2/m' | | | 000 |

Table S2b. Magnetic moment directions possible in monoclinic symmetry. The unique axis along [010] is assumed.

| free param. | Mode | Directions |
|-------------|------|--|
| 3 | FAF | [u, v, w], [u, v̄, w] |
| 3 | AFA | [u, v, w], [ū, v, w̄] |
| 3 | ABA | [u, v, w], [ū, v, w̄], [ū, v̄, w̄], [u, v̄, w] |
| 2 | F0F | [u, 0, w] |
| 2 | A0A | [u, 0, w], [ū, 0, w̄] |
| 1 | 0F0 | [0, v, 0] |
| 1 | 0A0 | [0, v, 0], [0, v̄, 0] |
| 0 | 000 | [0, 0, 0] |

Table S3a. Classification of all possible magnetic modes for orthorhombic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S3b. For all possible permutations of directions see Table S9.

| | WITH FERROMAGNETISM | WITHOUT FERROMAGNETISM | |
|---|---|---------------------------------|---|
| Cat. No. → | 7 | 6 | 8 |
| Magnetic Class → | 2'2'2 2'm'm m'2'm m'm'2 m'm'm | 222 2mm m2m mm2 mmm | 2221' mm21' m'mm mm'm mmm' m'm'm' mmm1' |
| Magnetic site-symmetry point group ↓ | | | |
| 1 1̄ | ABF | ABC1 | ABC2 |
| 2'.. m'.. 2'/m'.. | 0AF | | 0AB |
| .2'. .m'. .2'/.m'. | A0F | | A0B |
| ..2' ..m' ..2'/m' | | | AB0 |
| 2.. m.. 2/m.. 22'2' 2m'm' m'2'm' mm'2' mm'm' | | A00 | A00 A00 |
| .2. .m. .2./m. 2'22' 2'mm' m'2m' m'm2' m'mm' | | 0A0 | 0A0 0A0 |
| ..2 ..m ..2/m 2'2'2 2'm'm m'2'm m'm'2 m'm'm | 00F 00F | 00A | 00A 00A |
| 1' 222 mmm mmm1' 2'/m.. .2'/m. ..2'/m 2/m'.. .2/m'. ..2/m' 2mm m2m mm2 m'mm mm'm mmm' | | 000 000 000 000 | 000 000 000 000 |

Table S3b. Magnetic moment directions possible in orthorhombic symmetry.

| free param. | Mode | Directions |
|-------------|------|--|
| 3 | ABF | [u, v, w], [ū, v̄, w], [u, v̄, w], [ū, v, w] |
| 3 | ABC1 | [u, v, w], [ū, v̄, w], [ū, v, w̄], [u, v̄, w̄] |
| 3 | ABC2 | [u, v, w], [ū, v̄, w], [ū, v, w̄], [u, v̄, w̄] [ū, v̄, w̄], [u, v, w̄], [u, v̄, w], [ū, v, w] |
| 2 | 0AF | [0, v, w], [0, v̄, w] |
| 2 | A0F | [u, 0, w], [ū, 0, w] |
| 2 | 0AB | [0, u, w], [0, u, w̄], [0, ū, w], [0, ū, w̄] |
| 2 | A0B | [u, 0, w], [u, 0, w̄], [ū, 0, w], [ū, 0, w̄] |
| 2 | AB0 | [u, v, 0], [u, v̄, 0], [ū, v̄, 0], [ū, v, 0] |
| 1 | 00F | [0, 0, w] |
| 1 | A00 | [u, 0, 0], [ū, 0, 0] |
| 1 | 0A0 | [0, v, 0], [0, v̄, 0] |
| 1 | 00A | [0, 0, w], [0, 0, w̄] |
| 0 | 000 | [0, 0, 0] |

Table S4a. Classification of all possible magnetic modes for tetragonal symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S4b.

| | | WITH FERROMAGNETISM | | | | | | WITHOUT FERROMAGNETISM | | | | | |
|--------------------------------------|-------|---------------------|-------|-------|-------|--------|--------|------------------------|--|----|--|--|--|
| Cat. No. → | | 9 | 13 | 10 | 11 | 12 | | 14 | | 15 | | | |
| Magnetic Class → | | | | | | | | | | | | | |
| Magnetic site-symmetry point group ↓ | | | | | | | | | | | | | |
| 1 $\bar{1}$ | Tetr1 | Tetr5 | Tetr2 | Tetr3 | Tetr4 | Tetr6a | Tetr6b | Tetr7 | | | | | |
| 2'.. m'.. 2'/m'.. | | | | | | | | | | | | | |
| .2'. .m'. 2'/m'. | | | | | | | | | | | | | |
| ..2' ..m' ..2'/m' | | | | | | | | | | | | | |
| 2.. m.. 2/m.. | | | | | | | | | | | | | |
| 2.2' 2.m' 2/m.. | | | | | | | | | | | | | |
| 22.2'. 2m'm' 2m' 2/m.. | | | | | | | | | | | | | |
| 4.. $\bar{4}$.. 4/m.. | | | | | | | | | | | | | |
| 42'2' 4m/m 42'm' 4/m.. | | | | | | | | | | | | | |
| .2.. .m.. 2/m.. | | | | | | | | | | | | | |
| .2.. ..m.. ..2/m.. | | | | | | | | | | | | | |
| other | | | | | | | | | | | | | |

Table S4b. Magnetic moment directions possible in tetragonal symmetry.

| free param. | Mode | Directions |
|-------------|---------|--|
| 3 | Tetr1 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{v}, u, w], [v, \bar{u}, w]$ |
| 3 | Tetr5 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{v}, u, w], [v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [\bar{u}, v, w], [\bar{v}, \bar{u}, w], [v, u, w]$ |
| 3 | Tetr2 | $[u, v, w], [\bar{u}, \bar{v}, w], [v, \bar{u}, \bar{w}], [\bar{v}, u, \bar{w}]$ |
| 3 | Tetr3 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{v}, u, w], [v, \bar{u}, w]$ $[\bar{u}, \bar{v}, \bar{w}], [u, v, \bar{w}], [v, \bar{u}, \bar{w}], [\bar{v}, u, \bar{w}]$ |
| 3 | Tetr4 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{v}, u, w], [v, \bar{u}, w]$ $[\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}], [v, u, \bar{w}], [\bar{v}, \bar{u}, \bar{w}]$ |
| 3 | Tetr6a | $[u, v, w], [\bar{u}, \bar{v}, w], [v, \bar{u}, \bar{w}], [\bar{v}, u, \bar{w}]$ $[\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}], [\bar{v}, \bar{u}, w], [v, u, w]$ |
| 3 | Tetr6b | $[u, v, w], [\bar{u}, \bar{v}, w], [v, \bar{u}, \bar{w}], [\bar{v}, u, \bar{w}]$ $[\bar{u}, \bar{v}, w], [\bar{u}, v, w], [v, u, \bar{w}], [\bar{v}, \bar{u}, \bar{w}]$ |
| 3 | Tetr7 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{v}, u, w], [v, \bar{u}, w]$ $[\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}], [v, u, \bar{w}], [\bar{v}, \bar{u}, \bar{w}]$ $[\bar{u}, \bar{v}, \bar{w}], [u, v, \bar{w}], [v, \bar{u}, \bar{w}], [\bar{v}, u, \bar{w}]$ $[\bar{u}, \bar{v}, w], [\bar{u}, v, w], [\bar{v}, \bar{u}, w], [v, u, w]$ |
| 2 | Tetr8a | $[u, 0, w], [\bar{u}, 0, w], [0, u, w], [0, \bar{u}, w]$ |
| 2 | Tetr8b | $[u, \bar{u}, w], [\bar{u}, u, w], [u, u, w], [\bar{u}, \bar{u}, w]$ |
| 2 | Tetr9 | $[u, v, 0], [\bar{u}, \bar{v}, 0], [v, \bar{u}, 0], [\bar{v}, u, 0]$ |
| 2 | Tetr10a | $[0, v, w], [0, \bar{v}, w], [v, 0, \bar{w}], [\bar{v}, 0, \bar{w}]$ |
| 2 | Tetr10b | $[u, \bar{u}, w], [\bar{u}, u, w], [\bar{u}, \bar{u}, \bar{w}], [u, u, \bar{w}]$ |
| 2 | Tetr11 | $[u, v, 0], [\bar{u}, v, 0], [v, u, 0], [v, \bar{u}, 0]$ $[\bar{u}, \bar{v}, 0], [u, \bar{v}, 0], [\bar{v}, \bar{u}, 0], [\bar{v}, u, 0]$ |
| 2 | Tetr12a | $[0, v, w], [0, \bar{v}, w], [v, 0, \bar{w}], [\bar{v}, 0, \bar{w}]$ $[0, \bar{v}, \bar{w}], [0, v, \bar{w}], [\bar{v}, 0, w], [v, 0, w]$ |
| 2 | Tetr12b | $[u, \bar{u}, w], [\bar{u}, u, w], [\bar{u}, \bar{u}, \bar{w}], [u, u, \bar{w}]$ $[\bar{u}, u, \bar{w}], [u, \bar{u}, \bar{w}], [u, u, w], [\bar{u}, \bar{u}, w]$ |
| 1 | 00F | $[0, 0, w]$ |
| 1 | 00A | $[0, 0, w], [0, 0, \bar{w}]$ |
| 1 | Tetr13a | $[u, 0, 0], [\bar{u}, 0, 0], [0, u, 0], [0, \bar{u}, 0]$ |
| 1 | Tetr13b | $[u, u, 0], [\bar{u}, \bar{u}, 0], [\bar{u}, u, 0], [u, \bar{u}, 0]$ |
| 0 | 000 | $[0, 0, 0]$ |

Table S5a. Classification of all possible magnetic modes for trigonal symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S5b.

| | | WITH FERROMAGNETISM | | | WITHOUT FERROMAGNETISM | | | | |
|---|-------|---------------------|------------|-------|------------------------|--------|--------|------------|--------|
| Cat. No. → | | 16 | 19 | 19 | 17 | 18 | 18 | 20 | 20 |
| Magnetic Class → | | | | | | | | 3121' | 3211' |
| Magnetic site-symmetry point group ↓ | | 3 | 312' | 32'1 | 31' | 312 | 321 | 31m1' | 3m11' |
| | | 3 | 31m' | 3m'1 | 3' | 31m | 3m1 | 3'1m | 3'm1 |
| | | 3 | 31m' | 3m'1 | 31' | 31m | 3m1 | 3'1m' | 3'm'1 |
| 1 1 | Trig1 | Trig4a | Trig4b | Trig2 | Trig3a | Trig3b | Trig5a | Trig5b | |
| .2'. .m'. .2'/m'. | | | Trig6b | | | | | | Trig7b |
| ..2' ..m' ..2'/m' | | Trig6a | | | | | | Trig7a | |
| 3.. 3.. 32'. 3m'. 3m'. 3.2' 3.m' 3.m' | 00F | 00F | 00F 00F | 00A | 00A | 00A | 00A | 00A 00A | |
| .2. .m. .2/m. | | | | | | Trig8b | | | Trig9b |
| ..2 ..m ..2/m | | | | | Trig8a | | Trig9a | | |
| other | | | | 000 | 000 | 000 | 000 | 000 | 000 |

Table S5b. Magnetic moment directions possible in trigonal symmetry.

| free param. | Mode | Directions |
|-------------|--------|--|
| 3 | Trig1 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ |
| 3 | Trig4a | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[v, u, w], [u - v, \bar{v}, w], [\bar{u}, \bar{u} + v, w]$ |
| 3 | Trig4b | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [\bar{u} + v, v, w], [u, u - v, w]$ |
| 3 | Trig2 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, \bar{w}], [v, \bar{u} + v, \bar{w}], [u - v, u, \bar{w}]$ |
| 3 | Trig3a | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{v}, \bar{u}, \bar{w}], [\bar{u} + v, v, \bar{w}], [u, u - v, \bar{w}]$ |
| 3 | Trig3b | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[v, u, \bar{w}], [u - v, \bar{v}, \bar{w}], [\bar{u}, \bar{u} + v, \bar{w}]$ |
| 3 | Trig5a | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{v}, \bar{u}, \bar{w}], [\bar{u} + v, v, \bar{w}], [u, u - v, \bar{w}]$ $[\bar{u}, \bar{v}, \bar{w}], [v, \bar{u} + v, \bar{w}], [u - v, u, \bar{w}]$ $[v, u, w], [u - v, \bar{v}, w], [\bar{u}, \bar{u} + v, w]$ |
| 3 | Trig5b | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[v, u, \bar{w}], [u - v, \bar{v}, \bar{w}], [\bar{u}, \bar{u} + v, \bar{w}]$ $[\bar{u}, \bar{v}, \bar{w}], [v, \bar{u} + v, \bar{w}], [u - v, u, \bar{w}]$ $[\bar{v}, \bar{u}, w], [\bar{u} + v, v, w], [u, u - v, w]$ |
| 2 | Trig6a | $[u, u, w], [\bar{u}, 0, w], [0, \bar{u}, w]$ |
| 2 | Trig6b | $[u, 2u, w], [2\bar{u}, \bar{u}, w], [u, \bar{u}, w]$ |
| 2 | Trig7a | $[u, u, w], [\bar{u}, 0, w], [0, \bar{u}, w]$ $[\bar{u}, \bar{u}, \bar{w}], [u, 0, \bar{w}], [0, u, \bar{w}]$ |
| 2 | Trig7b | $[u, 2u, w], [2\bar{u}, \bar{u}, w], [u, \bar{u}, w]$ $[\bar{u}, 2\bar{u}, \bar{w}], [2u, u, \bar{w}], [\bar{u}, u, \bar{w}]$ |
| 1 | 00F | $[0, 0, w]$ |
| 1 | 00A | $[0, 0, w], [0, 0, \bar{w}]$ |
| 1 | Trig8a | $[u, \bar{u}, 0], [u, 2u, 0], [2\bar{u}, \bar{u}, 0]$ |
| 1 | Trig8b | $[u, 0, 0], [0, u, 0], [\bar{u}, \bar{u}, 0]$ |
| 1 | Trig9a | $[u, \bar{u}, 0], [u, 2u, 0], [2\bar{u}, \bar{u}, 0]$ $[\bar{u}, u, 0], [\bar{u}, 2\bar{u}, 0], [2u, u, 0]$ |
| 1 | Trig9b | $[u, 0, 0], [0, u, 0], [\bar{u}, \bar{u}, 0]$ $[\bar{u}, 0, 0], [0, \bar{u}, 0], [u, u, 0]$ |
| 0 | 000 | $[0, 0, 0]$ |

Table S6a. Classification of all possible magnetic modes for hexagonal symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S6b.

| Cat. No. → | WITH FERROMAGNETISM | | | | WITHOUT FERROMAGNETISM | | | |
|--|---------------------|---|----------------------------|---|--|---|---|------------------|
| | 21 | 25 | 22 | 23 | 24 | 26 | 27 | |
| Magnetic Class → | | | | | | | | |
| Magnetic site-symmetry point group ↓ | 6 6̄ 6/m | 62'2' 6m'm' 6̄m'2' 6̄2'm' 6/mm'm' | 6' 6̄' 6'/m' 6/m' | 61' 6̄1' 6/mm' 6̄m2 6̄2m 6/mmm | 6'22' 6'mm' 6̄'m2' 6̄'2m' 6'/m'mm' | 6'2'2' 6'm'm' 6̄'m'2' 6̄'2m' 6'/m'm'm | 6221' 6mm1' 6̄m21' 6̄2m1' 6'/mmm' 6'/mm'm 6/m'mm 6/m'm'm 6/m'mm1' | |
| 1 1 | Hex1 | Hex5 | Hex2 | Hex3 | Hex4 | Hex6a | Hex7 | |
| 2' .. m' .. 2'/m' .. | | | Hex9 | Hex10 | | Hex11a | Hex13 | |
| .2'. m'. .2'/m'. | | | Hex8a | | | | Hex12b | Hex14a |
| ..2' ..m' ..2'/m' | | Hex8b | | | | Hex12a | | Hex14b |
| 2.. m.. 2/m.. | 00F | 00F | 00A | 00A | | | | |
| 222' 2m'm' m2'm' mm'2' mm'm' | 00F | 00F | 00A | 00A | | | | |
| 3.. 3.. | 00F | 00F | 00A | 00A | | | | |
| 32'. 3m'. 3̄m'. | 00F | 00F | 00A | 00A | | | | |
| 3.2' 3.m' 3̄.m' | 00F | 00F | 00A | 00A | | | | |
| 6.. 6.. 6/m.. | 00F | 00F | 00A | 00A | | | | |
| 62'2' 6m'm' 6m'2' 6̄2'm' 6/mm'm' | 00F | | | | | | | |
| 2.. .m. .2/m. | | | | | | Hex15a | Hex16a | Hex15a Hex15a |
| 2'22' 2'mm' m'2m' m'm2' m'mm' | | | | | | Hex15b | Hex16b | Hex15b Hex15b |
| 2'2'2' 2'm'm' m'2'm' m'm'2' m'm'm' | | | | | | | | |
| other | | | 000 | 000 | 000 | 000 | 000 | |

Table S6b. Magnetic moment directions possible in hexagonal symmetry.

| free param. | Mode | Directions |
|-------------|-------|--|
| 3 | Hex1 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [v, \bar{u} + v, w], [u - v, u, w]$ |
| 3 | Hex5 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [v, \bar{u} + v, w], [u - v, u, w]$ $[\bar{v}, \bar{u}, w], [\bar{u} + v, v, w], [u, u - v, w]$ $[v, u, w], [u - v, \bar{v}, w], [\bar{u}, \bar{u} + v, w]$ |
| 3 | Hex2 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[u, v, \bar{w}], [\bar{v}, u - v, \bar{w}], [\bar{u} + v, \bar{u}, \bar{w}]$ |
| 3 | Hex3 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [v, \bar{u} + v, w], [u - v, u, w]$ $[\bar{u}, \bar{v}, \bar{w}], [v, \bar{u} + v, \bar{w}], [u - v, u, \bar{w}]$ $[u, v, \bar{w}], [\bar{v}, u - v, \bar{w}], [\bar{u} + v, \bar{u}, \bar{w}]$ |
| 3 | Hex4 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, w], [v, \bar{u} + v, w], [u - v, u, w]$ $[v, u, \bar{w}], [u - v, \bar{v}, \bar{w}], [\bar{u}, \bar{u} + v, \bar{w}]$ $[\bar{v}, \bar{u}, \bar{w}], [\bar{u} + v, v, \bar{w}], [u, u - v, \bar{w}]$ |
| 3 | Hex6a | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[u, v, \bar{w}], [\bar{v}, u - v, \bar{w}], [\bar{u} + v, \bar{u}, \bar{w}]$ $[v, u, \bar{w}], [u - v, \bar{v}, \bar{w}], [\bar{u}, \bar{u} + v, \bar{w}]$ $[v, u, w], [u - v, \bar{v}, w], [\bar{u}, \bar{u} + v, w]$ |
| 3 | Hex6b | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[u, v, \bar{w}], [\bar{v}, u - v, \bar{w}], [\bar{u} + v, \bar{u}, \bar{w}]$ $[\bar{v}, \bar{u}, w], [\bar{u} + v, v, w], [u, u - v, w]$ $[\bar{v}, \bar{u}, \bar{w}], [\bar{u} + v, v, \bar{w}], [u, u - v, \bar{w}]$ |
| 3 | Hex7 | $[u, v, w], [\bar{v}, u - v, w], [\bar{u} + v, \bar{u}, w]$ $[\bar{u}, \bar{v}, \bar{w}], [v, \bar{u} + v, \bar{w}], [u - v, u, \bar{w}]$ $[\bar{u}, \bar{v}, w], [v, \bar{u} + v, w], [u - v, u, w]$ $[u, v, \bar{w}], [\bar{v}, u - v, \bar{w}], [\bar{u} + v, \bar{u}, \bar{w}]$ $[v, u, \bar{w}], [u - v, \bar{v}, \bar{w}], [\bar{u}, \bar{u} + v, \bar{w}]$ $[\bar{v}, \bar{u}, w], [\bar{u} + v, v, w], [u, u - v, w]$ $[\bar{v}, \bar{u}, \bar{w}], [\bar{u} + v, v, \bar{w}], [u, u - v, \bar{w}]$ $[v, u, w], [u - v, \bar{v}, w], [\bar{u}, \bar{u} + v, w]$ |

Table S6b. (continued)

| free param. | Mode | Directions |
|-------------|--------|--|
| 2 | Hex8a | $[u, 2u, w], [2\bar{u}, \bar{u}, w], [u, \bar{u}, w]$ $[\bar{u}, 2\bar{u}, w], [2u, u, w], [\bar{u}, u, w]$ |
| 2 | Hex8b | $[u, u, w], [\bar{u}, 0, w], [0, \bar{u}, w]$ $[\bar{u}, \bar{u}, w], [u, 0, w], [0, u, w]$ |
| 2 | Hex9 | $[u, v, 0], [\bar{v}, u - v, 0], [\bar{u} + v, \bar{u}, 0]$ |
| 2 | Hex10 | $[u, v, 0], [\bar{v}, u - v, 0], [\bar{u} + v, \bar{u}, 0]$ $[\bar{u}, \bar{v}, 0], [v, \bar{u} + v, 0], [u - v, u, 0]$ |
| 2 | Hex11a | $[u, v, 0], [\bar{v}, u - v, 0], [\bar{u} + v, \bar{u}, 0]$ $[v, u, 0], [u - v, \bar{v}, 0], [\bar{u}, \bar{u} + v, 0]$ |
| 2 | Hex11b | $[u, v, 0], [\bar{v}, u - v, 0], [\bar{u} + v, \bar{u}, 0]$ $[\bar{v}, \bar{u}, 0], [\bar{u} + v, v, 0], [u, u - v, 0]$ |
| 2 | Hex12a | $[u, u, w], [\bar{u}, 0, w], [0, \bar{u}, w]$ $[u, u, \bar{w}], [\bar{u}, 0, \bar{w}], [0, \bar{u}, \bar{w}]$ |
| 2 | Hex12b | $[u, 2u, w], [2\bar{u}, \bar{u}, w], [u, \bar{u}, w]$ $[u, 2u, \bar{w}], [2\bar{u}, \bar{u}, \bar{w}], [u, \bar{u}, \bar{w}]$ |
| 2 | Hex13 | $[u, v, 0], [\bar{v}, u - v, 0], [\bar{u} + v, \bar{u}, 0]$ $[v, u, 0], [u - v, \bar{v}, 0], [\bar{u}, \bar{u} + v, 0]$ $[\bar{u}, \bar{v}, 0], [v, \bar{u} + v, 0], [u - v, u, 0]$ $[\bar{v}, \bar{u}, 0], [\bar{u} + v, v, 0], [u, u - v, 0]$ |
| 2 | Hex14a | $[u, 2u, w], [2\bar{u}, \bar{u}, w], [u, \bar{u}, w]$ $[\bar{u}, 2\bar{u}, w], [2u, u, w], [\bar{u}, u, w]$ $[\bar{u}, 2\bar{u}, \bar{w}], [2u, u, \bar{w}], [\bar{u}, u, \bar{w}]$ $[u, 2u, \bar{w}], [2\bar{u}, \bar{u}, \bar{w}], [u, \bar{u}, \bar{w}]$ |
| 2 | Hex14b | $[u, u, w], [\bar{u}, 0, w], [0, \bar{u}, w]$ $[\bar{u}, \bar{u}, w], [u, 0, w], [0, u, w]$ $[\bar{u}, \bar{u}, \bar{w}], [u, 0, \bar{w}], [0, u, \bar{w}]$ $[u, u, \bar{w}], [\bar{u}, 0, \bar{w}], [0, \bar{u}, \bar{w}]$ |
| 1 | 00F | $[0, 0, w]$ |
| 1 | 00A | $[0, 0, w], [0, 0, \bar{w}]$ |
| 1 | Hex15a | $[u, 0, 0], [0, u, 0], [\bar{u}, \bar{u}, 0]$ $[\bar{u}, 0, 0], [0, \bar{u}, 0], [u, u, 0]$ |
| 1 | Hex15b | $[u, \bar{u}, 0], [u, 2u, 0], [2\bar{u}, \bar{u}, 0]$ $[\bar{u}, u, 0], [\bar{u}, 2\bar{u}, 0], [2u, u, 0]$ |
| 1 | Hex16a | $[u, 0, 0], [0, u, 0], [\bar{u}, \bar{u}, 0]$ |
| 1 | Hex16b | $[u, \bar{u}, 0], [u, 2u, 0], [2\bar{u}, \bar{u}, 0]$ |
| 0 | 000 | $[0, 0, 0]$ |

Table S7a. Classification of all possible magnetic modes for cubic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). The magnetic modes are given below in Table S7b.

| | | WITHOUT FERROMAGNETISM | | | | |
|--|-------------------------|-------------------------|----------------------|-------------------|----------------------------------|--------------------------|
| Cat. No. → | | 28 | 29 | 30 | 31 | 32 |
| Magnetic Class → | | | | | | 4321' 43m1' |
| Magnetic site-symmetry point group ↓ | | 23 m3 m31' | 231' m'3' m31' | 432 43m m3m | 4'32' 4'3m' m3m' | m'3'm m'3'm' m3m1' |
| 1 1 | Cub1 | Cub2 | Cub3 | Cub4 | Cub5 | |
| 2'.. m'.. 2'/m'.. | | Cub6 | | | | Cub8 |
| ..2' ..m' ..2'/m' | | | | | Cub7 | Cub9 |
| 2.. m.. 2/m'.. 22'2'.. 2m'm'.. m2'm'.. mm'2'.. mm'm'.. 4.. 4.. 4/m.. 42'.2' 4m'.m' 42'.m' 4m'.2' 4/mm'.m' | Cub10 Cub10 Cub10 | Cub10 Cub10 Cub10 | Cub10 | Cub10 | Cub10 Cub10 Cub10 Cub10 | Cub10 |
| .3. .3. .32' .3m' .3m' | Cub11 | Cub12 | Cub12 | Cub12 | Cub12 Cub12 | Cub12 |
| ..2 ..m ..2/m | | | Cub13 | Cub13 | Cub13 | |
| other | 000 | 000 | 000 | 000 | 000 | |

Table S7b. Magnetic moment directions possible in cubic symmetry.

| free param. | Mode | Directions |
|-------------|------|--|
| 3 | Cub1 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}]$ $[w, u, v], [w, \bar{u}, \bar{v}], [\bar{w}, \bar{u}, v], [\bar{w}, u, \bar{v}]$ $[v, w, u], [\bar{v}, w, \bar{u}], [v, \bar{w}, \bar{u}], [\bar{v}, \bar{w}, u]$ |
| 3 | Cub2 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}]$ $[\bar{u}, \bar{v}, \bar{w}], [u, v, \bar{w}], [u, \bar{v}, w], [\bar{u}, v, w]$ $[w, u, v], [w, \bar{u}, \bar{v}], [\bar{w}, \bar{u}, v], [\bar{w}, u, \bar{v}]$ $[\bar{w}, \bar{u}, \bar{v}], [\bar{w}, u, v], [w, u, \bar{v}], [w, \bar{u}, v]$ $[v, w, u], [\bar{v}, w, \bar{u}], [v, \bar{w}, \bar{u}], [\bar{v}, \bar{w}, u]$ $[\bar{v}, \bar{w}, \bar{u}], [v, \bar{w}, u], [\bar{v}, w, u], [v, w, \bar{u}]$ |
| 3 | Cub3 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}]$ $[w, u, v], [w, \bar{u}, \bar{v}], [\bar{w}, \bar{u}, v], [\bar{w}, u, \bar{v}]$ $[v, w, u], [\bar{v}, w, \bar{u}], [v, \bar{w}, \bar{u}], [\bar{v}, \bar{w}, u]$ $[v, u, \bar{w}], [\bar{v}, \bar{u}, \bar{w}], [v, \bar{u}, w], [\bar{v}, u, w]$ $[u, w, \bar{v}], [\bar{u}, w, v], [\bar{u}, \bar{w}, \bar{v}], [u, \bar{w}, v]$ $[w, v, \bar{u}], [w, \bar{v}, u], [\bar{w}, v, u], [\bar{w}, \bar{v}, \bar{u}]$ |
| 3 | Cub4 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}]$ $[w, u, v], [w, \bar{u}, \bar{v}], [\bar{w}, \bar{u}, v], [\bar{w}, u, \bar{v}]$ $[v, w, u], [\bar{v}, w, \bar{u}], [v, \bar{w}, \bar{u}], [\bar{v}, \bar{w}, u]$ $[\bar{v}, \bar{u}, w], [v, u, w], [\bar{v}, u, \bar{w}], [v, \bar{u}, \bar{w}]$ $[\bar{u}, \bar{w}, v], [u, \bar{w}, \bar{v}], [u, w, v], [\bar{u}, w, \bar{v}]$ $[\bar{w}, v, u], [w, v, \bar{u}], [w, \bar{v}, \bar{u}], [w, v, u]$ |
| 3 | Cub5 | $[u, v, w], [\bar{u}, \bar{v}, w], [\bar{u}, v, \bar{w}], [u, \bar{v}, \bar{w}]$ $[\bar{u}, \bar{v}, \bar{w}], [u, v, \bar{w}], [u, \bar{v}, w], [\bar{u}, v, w]$ $[w, u, v], [w, \bar{u}, \bar{v}], [\bar{w}, \bar{u}, v], [\bar{w}, u, \bar{v}]$ $[\bar{w}, \bar{u}, \bar{v}], [\bar{w}, u, v], [w, u, \bar{v}], [w, \bar{u}, v]$ $[v, w, u], [\bar{v}, w, \bar{u}], [v, \bar{w}, \bar{u}], [\bar{v}, \bar{w}, u]$ $[\bar{v}, \bar{w}, \bar{u}], [v, \bar{w}, u], [\bar{v}, w, u], [v, w, \bar{u}]$ $[\bar{u}, \bar{w}, v], [\bar{v}, \bar{w}, \bar{v}], [u, w, v], [\bar{u}, w, \bar{v}]$ $[\bar{u}, \bar{w}, \bar{v}], [u, \bar{w}, v], [\bar{u}, \bar{w}, \bar{v}], [u, \bar{w}, v]$ $[w, v, \bar{u}], [w, \bar{v}, u], [\bar{w}, v, u], [\bar{w}, \bar{v}, \bar{u}]$ $[\bar{w}, \bar{v}, u], [\bar{w}, v, \bar{u}], [w, \bar{v}, \bar{u}], [w, v, u]$ |

Table S7b. (continued)

| free param. | Mode | Directions |
|-------------|-------|--|
| 2 | Cub6 | $[0, v, w], [0, v, \bar{w}], [w, 0, v]$ $[0, \bar{v}, \bar{w}], [0, \bar{v}, w], [\bar{w}, 0, \bar{v}]$ $[\bar{w}, 0, v], [v, w, 0], [v, \bar{w}, 0]$ $[w, 0, \bar{v}], [\bar{v}, \bar{w}, 0], [\bar{v}, w, 0]$ |
| 2 | Cub7 | $[u, v, \bar{v}], [\bar{u}, \bar{v}, \bar{v}], [\bar{u}, v, v], [u, \bar{v}, v]$ $[\bar{v}, u, v], [\bar{v}, \bar{u}, \bar{v}], [v, \bar{u}, v], [v, u, \bar{v}]$ $[v, \bar{v}, u], [\bar{v}, \bar{v}, \bar{u}], [v, v, \bar{u}], [\bar{v}, v, u]$ |
| 2 | Cub8 | $[0, v, w], [0, v, \bar{w}], [w, 0, v], [\bar{w}, 0, v]$ $[0, \bar{v}, \bar{w}], [0, \bar{v}, w], [\bar{w}, 0, \bar{v}], [w, 0, \bar{v}]$ $[v, w, 0], [v, \bar{w}, 0], [v, 0, \bar{w}], [v, 0, w]$ $[\bar{v}, \bar{w}, 0], [\bar{v}, w, 0], [\bar{v}, 0, w], [\bar{v}, 0, \bar{w}]$ $[0, \bar{w}, v], [0, w, v], [w, v, 0], [\bar{w}, v, 0]$ $[0, w, \bar{v}], [0, \bar{w}, \bar{v}], [\bar{w}, \bar{v}, 0], [w, \bar{v}, 0]$ |
| 2 | Cub9 | $[u, v, \bar{v}], [\bar{u}, \bar{v}, \bar{v}], [\bar{u}, v, v], [u, \bar{v}, v]$ $[\bar{u}, \bar{v}, v], [u, v, v], [u, \bar{v}, \bar{v}], [\bar{u}, v, \bar{v}]$ $[\bar{v}, u, v], [\bar{v}, \bar{u}, \bar{v}], [v, \bar{u}, v], [v, u, \bar{v}]$ $[\bar{v}, \bar{u}, \bar{v}], [v, u, v], [\bar{v}, u, \bar{v}], [\bar{v}, \bar{u}, v]$ $[v, \bar{v}, u], [\bar{v}, \bar{v}, \bar{u}], [v, v, \bar{u}], [\bar{v}, v, u]$ $[\bar{v}, v, \bar{u}], [v, v, u], [\bar{v}, \bar{v}, u], [v, \bar{v}, \bar{u}]$ |
| 1 | Cub10 | $[u, 0, 0], [0, u, 0], [0, 0, u]$ $[\bar{u}, 0, 0], [0, \bar{u}, 0], [0, 0, \bar{u}]$ |
| 1 | Cub11 | $[u, u, u], [\bar{u}, \bar{u}, u], [\bar{u}, u, \bar{u}], [u, \bar{u}, \bar{u}]$ |
| 1 | Cub12 | $[u, u, u], [\bar{u}, \bar{u}, u], [\bar{u}, u, \bar{u}], [u, \bar{u}, \bar{u}]$ $[\bar{u}, \bar{u}, \bar{u}], [u, u, \bar{u}], [u, \bar{u}, u], [\bar{u}, u, u]$ |
| 1 | Cub13 | $[0, v, v], [v, 0, v], [v, v, 0]$ $[0, \bar{v}, \bar{v}], [\bar{v}, 0, \bar{v}], [\bar{v}, \bar{v}, 0]$ $[0, v, \bar{v}], [v, 0, \bar{v}], [v, \bar{v}, 0]$ $[0, \bar{v}, v], [\bar{v}, 0, v], [\bar{v}, v, 0]$ |
| 0 | 000 | $[0, 0, 0]$ |

Table S8. Classification of all possible magnetic modes without atomic coordinates for monoclinic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). Variants for all choices of the unique axis along [100], [010] and [001] are written explicitly.

| Cat. No. → | WITH FERROMAGNETISM | | | | | | WITHOUT FERROMAGNETISM | | |
|---|---------------------|----------|-------------------|-------|--------|-------|------------------------|-------------------|-------------------|
| | 4 | | 3 | | | 5 | | | |
| Magnetic Class → | | | | | | | 21'.. | .21'. | ..21' |
| Magnetic site-symmetry point group ↓ | 2'.. | .2'. | ..2' | 2.. | .2. | ..2 | m1'.. | .m1'. | ..m1' |
| | m'.. | .m'. | ..m' | m.. | .m.. | ..m | 2'/m.. | .2'/m.. | ..2'/m.. |
| | 2'/'m'.. | .2'/'m'. | ..2'/'m' | 2/m.. | .2/m.. | ..2/m | 2/m1'.. | .2/m1'. | ..2/m1' |
| 1 1 | AFF | FAF | FFA | FAA | AFA | AAF | BAA | ABA | AAB |
| 2'.. m'.. 2'/'m'.. .2'. .m'. .2'/'m'. .2' ..m' ..2'/'m' | 0FF F0F FF0 | | | | | | 0AA A0A AA0 | | |
| 2.. m.. 2/m.. .2. .m. .2/m.. .2 ..m ..2/m | | | F00 OF0 OOF | | | | A00 0A0 00A | | |
| 1' 2'/'m.. .2'/'m.. ..2'/'m.. 2/m'.. .2/m'. ..2/m' | | | | | | | 000 000 000 | 000 000 000 | 000 000 000 |

Table S9 Classification of all possible magnetic modes without atomic coordinates for orthorhombic symmetry. Columns denote categories of magnetic space groups (i.e. magnetic classes) while rows denote magnetic site-symmetry groups (i.e. all Wyckoff positions). Variants for all assignments of the time reversal operation e.g. m'mm, mm'm and mmm' are written explicitly.

| Cat. No. → | WITH FERROMAGNETISM | | | WITHOUT FERROMAGNETISM | |
|--|--------------------------|-------------------|--------------------------|------------------------|--------------------------|
| | 7 | | 6 | 8 | |
| Magnetic Class → | 22'2' | 2'22' | 2'2'2 | 222 | 2221' mm21' |
| Magnetic site-symmetry point group ↓ | 2m'm' | 2'mm' | 2'm'm | 2mm | m'mm mm'm |
| | m2'm' | m'2m' | m'2'm | m2m | mmm' |
| | mm'2' | m'm2' | m'm'2 | mm2 | m'm'm' |
| | mm'm' | m'mm' | m'm'm | mmm | mmmm1' |
| 1 1 | FAB | AFB | ABF | ABC1 | ABC2 |
| 2'.. m'.. 2'/'m'.. .2'. .m'. .2'/'m'. .2' ..m' ..2'/'m' | 0FA F0A FA0 | 0AF A0F AF0 | | | 0AB A0B AB0 |
| 2.. m.. 2/m.. 22'2' 2m'm' m2'm' mm'2' mm'm' .2. .m. .2/m.. 2'22' 2'mm' m'2m' m'm2' m'mm' | F00 F00 0F0 0F0 | | A00 A00 0A0 0A0 | | A00 A00 0A0 0A0 |
| ..2 ..m ..2/m 2'2'2 2'm'm m'2'm m'm'2 m'm'm | | 00F 00F | 00A 00A | | 00A 00A |
| 1' 222 mmm mmm1' 2'/'m.. .2'/'m.. ..2'/'m.. 2/m'.. .2/m'.. ..2/m' 2mm m2m mm2 m'mm mm'm mmm' | | | 000 000 000 | | 000 000 000 |

PART II

Tables of magnetic space groups with all Wyckoff positions assigned to magnetic modes (set-of-directions of the magnetic moments)

Table S10. List of all Wyckoff positions of triclinic magnetic space groups assigned to magnetic modes. Group numbers, group symbols and Wyckoff positions symbols are according to the OG setting [Opechowski, W. & Guccione, R. (1965). *Magnetism*, ed. G.T. Rado and H. Suhl, Vol. 2A ch. 3, p. 105, New York,. Academic Press]. The symbol (****) mean FM ordering with three components. Please note that FFF and AAA modes are impossible in other magnetic space groups than listed in this table (see text).

| Number | Symbol | FFF | AAA | 000 |
|--------|-----------------|------------|-------------|---------|
| 1.1.1 | $P\bar{1}$ *** | (1a) | — | — |
| 1.3.3 | $P_{2s}\bar{1}$ | — | (2a) | — |
| 2.1.4 | $P\bar{1}$ *** | (1a-h, 2i) | — | — |
| 2.3.6 | $P\bar{1}'$ | — | (2i) | (1a-h) |
| 2.4.7 | $P_{2s}\bar{1}$ | — | (2acde, 4i) | (2bfgh) |

Table S11. List of all Wyckoff positions of monoclinic magnetic space groups assigned to magnetic modes. Group numbers, group symbols and Wyckoff positions symbols are according to the OG setting [Opechowski, W. & Guccione, R. (1965). *Magnetism*, ed. G.T. Rado and H. Suhl, Vol. 2A ch. 3, p. 105, New York, Academic Press]. The symbols (*) and (**) mean FM ordering with one or two components, respectively. Please note that FAF, FOF, AFA, A0A and A0A modes are impossible in other magnetic space groups than listed in this table.

| Number | Symbol | FAF | FOF | AFA | OF0 | ABA | A0A | A0A | 000 |
|--------|-----------------|------|---------|------|-------|------|---------|---------|-----|
| 3.1.8 | $P2^*$ | — | — | (2e) | — | — | — | — | — |
| 3.3.10 | $P2'^{**}$ | (2e) | (1abed) | — | — | (4e) | (2cd) | — | — |
| 3.4.11 | $P_{2a}2$ | — | — | — | — | (4e) | — | (2ab) | — |
| 3.5.12 | $P_{2b}2$ | — | — | — | — | (4e) | — | (2abcd) | — |
| 3.6.13 | P_C2 | — | — | — | — | (4e) | (2cd) | (2ab) | — |
| 3.7.14 | $P_{2b}2'$ | — | — | — | — | (4e) | (2abcd) | — | — |
| 4.1.15 | $P_{2_1}^*$ | — | — | (2a) | — | — | — | — | — |
| 4.3.17 | $P_{2_1'}^{**}$ | (2a) | — | — | — | (4a) | — | — | — |
| 4.4.18 | $P_{2a}2_1$ | — | — | — | — | (4a) | — | — | — |
| 5.1.19 | $C2^*$ | — | — | (4c) | (2ab) | — | — | — | — |
| 5.3.21 | $C2'^{**}$ | (4c) | (2ab) | — | — | — | — | — | — |
| 5.4.22 | $C_{2c}2$ | — | — | — | — | (8c) | (4b) | (4a) | — |
| 5.5.23 | C_P2 | — | — | — | — | (4c) | — | (2ab) | — |
| 5.6.24 | C_{P2}' | — | — | — | — | (4c) | (2ab) | — | — |
| 6.1.25 | Pm^* | — | — | (2c) | (1ab) | — | — | — | — |
| 6.3.27 | Pm'^{**} | (2c) | (1ab) | — | — | — | — | — | — |
| 6.4.28 | $P_{2a}m$ | — | — | — | — | (4c) | — | (2ab) | — |
| 6.5.29 | $P_{2b}m$ | — | — | — | — | (4c) | (2b) | (2a) | — |
| 6.6.30 | P_Cm | — | — | — | — | (4c) | (2b) | (2a) | — |
| 6.7.31 | $P_{2c}m'$ | — | — | — | — | (4c) | (2ab) | — | — |
| 7.1.32 | P_c^* | — | — | (2a) | — | — | — | — | — |
| 7.3.34 | $P_{c'}^{**}$ | (2a) | — | — | — | (4a) | — | (2ab) | — |
| 7.4.35 | $P_{2a}c$ | — | — | — | — | (4a) | — | (2a) | — |
| 7.5.36 | $P_{2b}c$ | — | — | — | — | (4a) | — | (2a) | — |
| 7.6.37 | P_{Cc} | — | — | — | — | (4a) | — | — | — |
| 8.1.38 | Cm^* | — | (4b) | (2a) | — | — | — | — | — |
| 8.3.40 | Cm'^{**} | (4b) | (2a) | — | — | (4a) | — | — | — |
| 8.4.41 | $C_{2c}m$ | — | — | — | — | (4a) | — | — | — |
| 8.5.42 | C_Pm | — | — | — | — | (8b) | — | (4a) | — |
| 8.6.43 | $C_{2c}m'$ | — | — | — | — | (4b) | — | (2a) | — |
| 8.7.44 | C_Pm' | — | — | — | — | (8b) | (4a) | — | — |
| 9.1.45 | Cc^* | — | — | (4a) | — | — | — | — | — |
| 9.3.47 | Cc'^{**} | (4a) | — | — | — | (4a) | — | — | — |
| 9.4.48 | C_Pc | — | — | — | — | (4a) | — | — | — |

Table S11. (continued)

| Number | Symbol | FAF | F0F | AFA | 0F0 | ABA | A0A | 0A0 | 000 |
|----------|-------------------|-------------|--------------|---------------|---------------|-----------------|------------------|------------------|------------------|
| 10.1.49 | $P2/m^*$ | — | — | (4o) | (1a-h, 2i-n) | — | — | — | — |
| 10.3.51 | $P2'/m$ | — | — | — | (4o) | (4o) | (2ijkl) (2mn) | (2mn) (2ijkl) | (1a-h) (1a-h) |
| 10.4.52 | $P2/m'$ | — | — | — | — | — | — | — | — |
| 10.5.53 | $P2'/m'**$ | (4o) | (1a-h, 2i-n) | — | (8o) | (4jl) | (2abcf, 4ikmn) | (2degh) | — |
| 10.6.54 | $P_{2a}2/m$ | — | — | — | (8o) | (4n) | (2acd, 4i-m) | (2befh) | — |
| 10.7.55 | $P_{2b}2/m$ | — | — | — | (8o) | (2eh, 4jln) | (2ac, 4ikm) | (2bdfg) | — |
| 10.8.56 | P_C2/m | — | — | — | (8o) | (2befh, 4ijkln) | (4m) | (2acd) | — |
| 10.9.57 | $P_{2b}2'/m$ | — | — | — | (8o) | (2cfcgh, 4klnm) | (4ij) | (2abde) | — |
| 10.10.58 | $P_{2c}2/m'$ | — | — | (2abcd, 4f) | (2e) | — | — | — | — |
| 11.1.59 | P_{21}/m^* | — | — | — | — | (4f) | (2e) | (2e) | — |
| 11.3.61 | P_{21}'/m | — | — | — | — | (4f) | — | — | (2abcd) |
| 11.4.62 | P_{21}'/m' | — | — | (2e) | — | — | — | — | (2abcd) |
| 11.5.63 | $P_{21}'/m'**$ | (2abcd, 4f) | — | — | — | — | — | — | — |
| 11.6.64 | $P_{2a}2_{1l}/m$ | — | — | — | — | (4ac, 8f) | (4e) | (4e) | (4bd) |
| 11.7.65 | $P_{2c}2_{1l}/m'$ | — | — | — | — | (4cd, 8f) | — | — | (4ab) |
| 12.1.66 | $C2/m^*$ | — | — | (4ef, 8j) | (2abcd, 4ghi) | — | — | — | — |
| 12.3.68 | $C2'/m$ | — | — | — | — | (4gh) | (4i) | (4i) | (2abcd, 4ef) |
| 12.4.69 | $C2'/m'$ | — | — | (2abcd, 4ghi) | — | (4gh) | (4gh) | (4gh) | (2abcd, 4ef) |
| 12.5.70 | $C2'/m'**$ | (4ef, 8j) | — | — | — | (8i) | — | — | — |
| 12.6.71 | $C_{2c}2/m$ | — | — | — | — | (8e, 16j) | (8h) | (8h) | (4cd, 8f) |
| 12.7.72 | C_P2/m | — | — | — | — | (8i) | (8i) | (8i) | (4ef, 8gi) |
| 12.8.73 | $C_{2c}2/m'$ | — | — | — | — | (8f, 16j) | (4cd, 8hi) | (4cd, 8gi) | (4ab, 8e) |
| 12.9.74 | C_P2'/m | — | — | — | — | (4ef, 8j) | (4gh) | (8g) | (2abcd) |
| 12.10.75 | C_P2'/m' | — | — | — | — | (4ef, 8j) | (4i) | (4i) | (2abcd) |
| 12.11.76 | C_P2'/m' | — | — | — | — | (8j) | (4gh) | (4gh) | (4ef) |
| 13.1.77 | $P2/c^*$ | — | — | (2abcd, 4g) | (2ef) | — | — | — | — |
| 13.3.79 | $P2'/c$ | — | — | — | — | (4g) | (2ef) | (2ef) | (2abcd) |
| 13.4.80 | $P2/c'$ | — | — | — | — | (4g) | — | — | (2abcd) |
| 13.5.81 | $P2'/c'**$ | (2abcd, 4g) | (2ef) | — | — | — | — | — | — |
| 13.6.82 | $P_{2a}2/c$ | — | — | — | — | (4ac, 8g) | (4f) | (4e) | (4bd) |
| 13.7.83 | $P_{2b}2/c$ | — | — | — | — | (4ad, 8g) | — | — | (4bc) |
| 13.8.84 | P_C2/c | — | — | — | — | (4ab, 8g) | (4f) | (4ef) | (4cd) |
| 13.9.85 | $P_{2b}2'/c$ | — | — | — | — | (4bc, 8g) | (4ef) | (4e) | (4ad) |
| 14.1.86 | P_{21}/c^* | — | — | (2abcd, 4e) | — | — | — | — | — |
| 14.3.88 | P_{21}'/c | — | — | — | — | (4e) | — | — | (2abcd) |
| 14.4.89 | $P_{21}'/c'**$ | (2abcd, 4e) | — | — | — | (4e) | — | — | (2abcd) |
| 14.5.90 | $P_{21}'/c''**$ | — | — | — | — | (4ac, 8e) | — | — | — |
| 14.6.91 | $P_{2a}2_{1l}/c$ | — | — | — | — | (4abcd, 8f) | (4e) | (4e) | (4bd) |
| 15.1.92 | $C2/c^*$ | — | — | — | — | (8f) | (4e) | (4e) | — |
| 15.3.94 | $C2'/c$ | — | — | — | — | (8f) | (4e) | (4e) | (4abcd) |
| 15.4.95 | $C2/c'$ | — | — | — | — | (8f) | — | — | (4abcd) |
| 15.5.96 | $C2/c'**$ | (4abcd, 8f) | (4e) | — | — | (4ab, 8f) | (4e) | (4e) | — |
| 15.6.97 | C_P2/c | — | — | — | — | (4cd, 8f) | (4e) | (4e) | (4ab) |
| 15.7.98 | C_P2/c' | — | — | — | — | — | — | — | — |

Table S12. List of all Wyckoff positions of orthorhombic magnetic space groups assigned to magnetic modes. Only groups that allow antiferromagnetism with (weak) ferromagnetic moment are listed. Group numbers, group symbols and Wyckoff positions symbols are according to the OG setting [Opechowski, W. & Guccione, R. (1965). *Magnetism*, ed. G.T. Rado and H. Suhl, Vol. 2A ch. 3, p. 105, New York,. Academic Press]. Please note that FA0 and FAB modes are impossible in other magnetic space groups than listed in this table.

| Number | Symbol | F00 | FA0 | FAB |
|----------|----------------|---------------|---------|-------|
| 16.3.101 | $P2'2'2$ | (1a-h, 2qrst) | (2i-p) | (4u) |
| 17.3.108 | $P2'2'2_1$ | — | (2abcd) | (4e) |
| 17.4.109 | $P22'2'_1$ | (2ab) | (2cd) | (4e) |
| 18.3.115 | $P2'_12'_12$ | (2ab) | — | (4c) |
| 18.4.116 | $P2_12'_12'$ | — | (2ab) | (4c) |
| 19.3.121 | $P2'_12'_12_1$ | — | — | (4a) |
| 20.3.124 | $C2'2'2_1$ | — | (4ab) | (8c) |
| 20.4.125 | $C22'2'_1$ | (4a) | (4b) | (8c) |
| 21.3.131 | $C2'2'2$ | (2abcd, 4ijk) | (4efgh) | (8l) |
| 21.4.132 | $C22'2'$ | (2abcd, 4ef) | (4g-k) | (8l) |
| 22.3.142 | $F2'2'2$ | (4abcd, 8gh) | (8efij) | (16k) |
| 23.3.147 | $I2'2'2$ | (2abcd, 4ij) | (4efgh) | (8k) |
| 24.3.152 | $I2'_12'_12_1$ | (4c) | (4ab) | (8d) |
| 25.3.157 | $Pm'm2'$ | (1abcd, 2ef) | (2gh) | (4i) |
| 25.4.158 | $Pm'm'2$ | (1abcd) | (2efgh) | (4i) |
| 26.3.170 | $Pm'c2'_1$ | — | (2ab) | (4c) |
| 26.4.171 | $Pmc'2'_1$ | (2ab) | — | (4c) |
| 26.5.172 | $Pm'c'2_1$ | — | (2ab) | (4c) |
| 27.3.180 | $Pc'c2'$ | — | (2abcd) | (4e) |
| 27.4.181 | $Pc'c'2$ | (2abcd) | — | (4e) |
| 28.3.187 | $Pm'a2'$ | — | (2abc) | (4d) |
| 28.4.188 | $Pma'2'$ | (2c) | (2ab) | (4d) |
| 28.5.189 | $Pm'a'2$ | (2ab) | (2c) | (4d) |
| 29.3.200 | $Pc'a2'_1$ | — | — | (4a) |
| 29.4.201 | $Pca'2'_1$ | — | — | (4a) |
| 29.5.202 | $Pc'a'2_1$ | — | — | (4a) |
| 30.3.207 | $Pn'c2'$ | — | (2ab) | (4c) |
| 30.4.208 | $Pnc'2'$ | — | (2ab) | (4c) |
| 30.5.209 | $Pn'c'2$ | (2ab) | — | (4c) |
| 31.3.214 | $Pm'n2'_1$ | — | (2a) | (4b) |
| 31.4.215 | $Pmn'2'_1$ | (2a) | — | (4b) |
| 31.5.216 | $Pm'n'2_1$ | — | (2a) | (4b) |
| 32.3.221 | $Pb'a2'$ | — | (2ab) | (4c) |
| 32.4.222 | $Pb'a'2$ | (2ab) | — | (4c) |
| 33.3.228 | $Pn'a2'_1$ | — | — | (4a) |
| 33.4.229 | $Pna'2'_1$ | — | — | (4a) |
| 33.5.230 | $Pn'a'2_1$ | — | — | (4a) |
| 34.3.233 | $Pn'n2'$ | — | (2ab) | (4c) |
| 34.4.234 | $Pn'n'2$ | (2ab) | — | (4c) |
| 35.3.238 | $Cm'm2'$ | (2ab, 4d) | (4ce) | (8f) |
| 35.4.239 | $Cm'm'2$ | (2ab, 4c) | (4de) | (8f) |
| 36.3.251 | $Cm'c2'_1$ | — | (4a) | (8b) |
| 36.4.252 | $Cmc'2'_1$ | (4a) | — | (8b) |
| 36.5.253 | $Cm'c'2_1$ | — | (4a) | (8b) |

Table S12. (continued)

| Number | Symbol | F00 | FA0 | FAB |
|----------|----------|-------------------|--------------------|-----------|
| 37.3.260 | $Cc'c2'$ | — | (4abc) | (8d) |
| 37.4.261 | $Cc'c'2$ | (4abc) | — | (8d) |
| 38.3.267 | $Am'm2'$ | (2ab, 4c) | (4de) | (8f) |
| 38.4.268 | $Amm'2'$ | (2ab, 4de) | (4c) | (8f) |
| 38.5.269 | $Am'm'2$ | (2ab) | (4cde) | (8f) |
| 39.3.280 | $Ab'm2'$ | (4c) | (4ab) | (8d) |
| 39.4.281 | $Abm'2'$ | — | (4abc) | (8d) |
| 39.5.282 | $Ab'm'2$ | (4ab) | (4c) | (8d) |
| 40.3.293 | $Am'a2'$ | — | (4ab) | (8c) |
| 40.4.294 | $Ama'2'$ | (4b) | (4a) | (8c) |
| 40.5.295 | $Am'a'2$ | (4a) | (4b) | (8c) |
| 41.3.302 | $Ab'a2'$ | — | (4a) | (8b) |
| 41.4.303 | $Aba'2'$ | — | (4a) | (8b) |
| 41.5.304 | $Ab'a'2$ | (4a) | — | (8b) |
| 42.3.311 | $Fm'm2'$ | (4a, 8d) | (8bc) | (16e) |
| 42.4.312 | $Fm'm'2$ | (4a, 8b) | (8cd) | (16e) |
| 43.3.322 | $Fd'd2'$ | — | (8a) | (16b) |
| 43.4.323 | $Fd'd'2$ | (8a) | — | (16b) |
| 44.3.326 | $Im'm2'$ | (2ab, 4c) | (4d) | (8e) |
| 44.4.327 | $Im'm'2$ | (2ab) | (4cd) | (8e) |
| 45.3.333 | $Ib'a2'$ | — | (4ab) | (8c) |
| 45.4.334 | $Ib'a'2$ | (4ab) | — | (8c) |
| 46.3.340 | $Im'a2'$ | — | (4ab) | (8c) |
| 46.4.341 | $Ima'2'$ | (4b) | (4a) | (8c) |
| 46.5.342 | $Im'a'2$ | (4a) | (4b) | (8c) |
| 47.4.350 | $Pm'm'm$ | (1a-h, 2i-t, 4yz) | (4uvwxyz) | (8α) |
| 48.4.361 | $Pn'n'n$ | (2abcd, 4kl) | (4ghij) | (4ef, 8m) |
| 49.5.368 | $Pc'c'm$ | (2a-h, 4m-q) | (4ijkl) | (8r) |
| 49.6.369 | $Pc'cm'$ | (2efgh, 4kl) | (2abcd, 4ij, 4m-q) | (8r) |
| 50.5.381 | $Pb'a'n$ | (2abcd, 4kl) | (4ghij) | (4ef, 8m) |
| 50.6.382 | $Pb'an'$ | (2abcd, 4ghij) | (4kl) | (4ef, 8m) |
| 51.6.392 | $Pm'm'a$ | (2ef) | (2abcd, 4g-k) | (8l) |
| 51.7.393 | $Pmm'a'$ | (2ef, 4k) | (2abcd, 4ghij) | (8l) |
| 51.8.394 | $Pm'ma'$ | (2a-f, 4ghij) | (4k) | (8l) |
| 52.6.411 | $Pn'n'a$ | (4c) | (4d) | (4ab, 8e) |
| 52.7.412 | $Pnn'a'$ | (4d) | (4c) | (4ab, 8e) |
| 52.8.413 | $Pn'na'$ | — | (4cd) | (8e) |
| 53.6.420 | $Pm'n'a$ | — | (2abcd, 4efgh) | (8i) |
| 53.7.421 | $Pmn'a'$ | (2abcd, 4efh) | (4g) | (8i) |
| 53.8.422 | $Pm'na'$ | (4g) | (2abcd, 4efh) | (8i) |

Table S12. (continued)

| Number | Symbol | F00 | FA0 | FAB |
|----------|----------|---------------------|-------------------|-------------|
| 54.6.433 | $Pc'c'a$ | (4de) | (4c) | (4ab, 8f) |
| 54.7.434 | $Pcc'a'$ | — | (4cde) | (4ab, 8f) |
| 54.8.435 | $Pc'ca'$ | (4c) | (4de) | (4ab, 8f) |
| 55.5.445 | $Pb's'm$ | (2abcd, 4efgh) | — | (8i) |
| 55.6.446 | $Pb'am'$ | — | (2abcd, 4efgh) | (8i) |
| 56.5.455 | $Pc'c'n$ | (4cd) | — | (4ab, 8e) |
| 56.6.456 | $Pc'cn'$ | — | (4cd) | (4ab, 8e) |
| 57.6.463 | $Pb'c'm$ | (4d) | (4c) | (4ab, 8e) |
| 57.7.464 | $Pbc'm'$ | (4c) | (4d) | (4ab, 8e) |
| 57.8.465 | $Pb'cm'$ | — | (4cd) | (4ab, 8e) |
| 58.5.475 | $Pn'n'm$ | (2abcd, 4efg) | — | (8h) |
| 58.6.476 | $Pnn'm'$ | — | (2abcd, 4efg) | (8h) |
| 59.5.482 | $Pm'm'n$ | (2ab) | (4ef) | (4cd, 8g) |
| 59.6.483 | $Pmm'n'$ | (2ab, 4e) | (4f) | (4cd, 8g) |
| 60.6.493 | $Pb'c'n$ | — | (4c) | (4ab, 8d) |
| 60.7.494 | $Pbc'n'$ | — | (4c) | (4ab, 8d) |
| 60.8.495 | $Pb'cn'$ | (4c) | — | (4ab, 8d) |
| 61.4.500 | $Pb'c'a$ | — | — | (4ab, 8c) |
| 62.6.507 | $Pn'm'a$ | — | (4c) | (4ab, 8d) |
| 62.7.508 | $Pnm'a'$ | — | (4c) | (4ab, 8d) |
| 62.8.609 | $Pn'ma'$ | (4c) | — | (4ab, 8d) |
| 63.6.516 | $Cm'c'm$ | (4c, 8g) | (4ab, 8ef) | (8d, 16h) |
| 63.7.517 | $Cmc'm'$ | (4abc, 8ef) | (8g) | (8d, 16h) |
| 63.8.518 | $Cm'cm'$ | (4c) | (4ab, 8efg) | (8d, 16h) |
| 64.6.533 | $Cm'c'a$ | — | (4ab, 8def) | (8c, 16g) |
| 64.7.534 | $Cmc'a'$ | (4ab, 8df) | (8e) | (8c, 16g) |
| 64.8.535 | $Cm'ca'$ | (8e) | (4ab, 8df) | (8c, 16g) |
| 65.5.549 | $Cm'm'm$ | (2abcd, 4e-l, 8mpq) | (8no) | (16r) |
| 65.6.550 | $Cmm'm'$ | (2abcd, 4g-l, 8n) | (4ef, 8mopq) | (16r) |
| 66.5.568 | $Cc'c'm$ | (4a-f, 8ijkl) | (8gh) | (16m) |
| 66.6.569 | $Ccc'm'$ | (4ab, 8g) | (4cdef, 8h-1) | (16m) |
| 67.5.581 | $Cm'm'a$ | (4abg, 8l) | (4cdef, 8hijklmn) | (16o) |
| 67.6.582 | $Cmm'a'$ | (4abcdg, 8him) | (4ef, 8jklm) | (16o) |
| 68.5.598 | $Cc'c'a$ | (4ab, 8gh) | (8ef) | (8cd, 16i) |
| 68.6.599 | $Ccc'a'$ | (4ab, 8e) | (8fg) | (8cd, 16i) |
| 69.4.608 | $Fm'm'm$ | (4ab, 8e-i, 16jo) | (8cd, 16klmn) | (32p) |
| 70.4.619 | $Fd'd'd$ | (8ab, 16g) | (16ef) | (16cd, 32h) |
| 71.4.624 | $Im'm'm$ | (2abcd, 4e-j, 8n) | (8lm) | (8k, 16o) |
| 72.5.634 | $Ib'a'm$ | (4abcd, 8hij) | (8fg) | (8e, 16k) |
| 72.6.635 | $Iba'm'$ | (4ab, 8f) | (4cd, 8ghij) | (8e, 16k) |
| 73.4.646 | $Ib'c'a$ | (8e) | (8cd) | (8ab, 16f) |
| 74.5.654 | $Im'm'a$ | (4e) | (4abcd, 8fghi) | (16j) |
| 74.6.655 | $Imm'a'$ | (4abe, 8fh) | (4cd, 8gi) | (16j) |

PART III

Tables of magnetic space groups assigned to magnetic mode categories

Table S13 All triclinic, monoclinic and orthorhombic magentic spacegroup assigned to allowed modes and categories. Modes in specified Wyckoff positions of those groups are classified in Tables S1-S3, S8, S9 and Table 3 (main text).

| FFF modes (cat. 1) | | | | AAA modes (cat. 2) | | | |
|------------------------------------|--------------|----------|--------------|-------------------------|---------------|----------|---------------|
| 1.1.1 | $P1$ | | | 1.3.3 | P_{2s} | | |
| 2.1.4 | $P\bar{1}$ | | | 2.3.6 | $P\bar{1}'$ | | |
| AFA, 0F0 modes (cat. 3) | | | | FAF, F0F modes (cat. 4) | | | |
| 3.1.8 | $P2$ | 10.1.49 | $P2/m$ | 3.3.10 | $P2'$ | 10.5.53 | $P2'/m'$ |
| 4.1.15 | $P2_1$ | 11.1.59 | $P2_1/m$ | 4.3.17 | $P2'_1$ | 11.5.63 | $P2'_1/m'$ |
| 5.1.19 | $C2$ | 12.1.66 | $C2/m$ | 5.3.21 | $C2'$ | 12.5.70 | $C2'/m'$ |
| 6.1.25 | Pm | 13.1.77 | $P2/c$ | 6.3.27 | Pm' | 13.5.81 | $P2'/c'$ |
| 7.1.32 | Pc | 14.1.86 | $P2_1/c$ | 7.3.34 | Pc' | 14.5.90 | $P2'_1/c'$ |
| 8.1.38 | Cm | 15.1.92 | $C2/c$ | 8.3.40 | Cm' | 15.5.96 | $C2'/c'$ |
| 9.1.45 | Cc | | | 9.3.47 | Cc' | | |
| ABA, A0A, 0A0, 000 modes (cat. 5) | | | | | | | |
| 3.4.11 | $P_{2a}2$ | 7.5.36 | $P_{2b}c$ | 10.10.58 | $P_{2c}2/m'$ | 13.3.79 | $P2'/c$ |
| 3.5.12 | $P_{2b}2$ | 7.6.37 | P_Cc | 11.3.61 | $P2'_1/m$ | 13.4.80 | $P2/c'$ |
| 3.6.13 | P_C2 | 8.4.41 | C_{2cm} | 11.4.62 | $P2_1/m'$ | 13.6.82 | $P_{2a}2/c$ |
| 3.7.14 | $P_{2b}2'$ | 8.5.42 | C_Pm | 11.6.64 | $P_{2a}2_1/m$ | 13.7.83 | $P_{2b}2/c$ |
| 4.4.18 | $P_{2a}2_1$ | 8.6.43 | C_{2cm}' | 11.7.65 | $P_{2c}2_1/m$ | 13.8.84 | P_C2/c |
| 5.4.22 | $C_{2c}2$ | 8.7.44 | C_Pm' | 12.3.68 | $C2'/m$ | 13.9.85 | $P_{2b}2'/c$ |
| 5.5.23 | C_P2 | 9.4.48 | C_Pc | 12.4.69 | $C2/m'$ | 14.3.88 | $P2'_1/c$ |
| 5.6.24 | C_P2' | 10.3.51 | $P2'/m$ | 12.6.71 | $C_{2c}2/m$ | 14.3.89 | $P2_1/c'$ |
| 6.4.28 | $P_{2a}m$ | 10.4.52 | $P2/m'$ | 12.7.72 | C_P2/m | 14.6.91 | $P_{2a}2_1/c$ |
| 6.5.29 | $P_{2b}m$ | 10.6.54 | $P_{2a}2/m$ | 12.8.73 | $C_{2c}2/m'$ | 15.3.94 | $C2'/c$ |
| 6.6.30 | P_Cm | 10.7.55 | $P_{2b}2/m$ | 12.9.74 | C_P2'/m | 15.4.95 | $C2/c'$ |
| 6.7.31 | $P_{2c}m'$ | 10.8.56 | P_C2/m | 12.10.75 | C_P2/m' | 15.6.97 | C_P2/c |
| 7.4.35 | $P_{2a}c$ | 10.9.57 | $P_{2b}2'/m$ | 12.11.76 | C_P2'/m' | 15.7.98 | C_P2'/c |
| ABC1, AB0, 0A0, 000 modes (cat. 6) | | | | | | | |
| 16.1.99 | $P222$ | 31.1.212 | $Pmn2_1$ | 46.1.338 | $Ima2$ | 61.1.497 | $Pbca$ |
| 17.1.106 | $P222_1$ | 32.1.219 | $Pba2$ | 47.1.347 | $Pmmm$ | 62.1.502 | $Pnma$ |
| 18.1.113 | $P2_12_12$ | 33.1.226 | $Pna2_1$ | 48.1.358 | $Pnnm$ | 63.1.511 | $Cmcm$ |
| 19.1.119 | $P2_12_12_1$ | 34.1.231 | $Pnn2$ | 49.1.364 | $Pccm$ | 64.1.528 | $Cmca$ |
| 20.1.122 | $C222_1$ | 35.1.236 | $Cmm2$ | 50.1.377 | $Pban$ | 65.1.545 | $Cmmm$ |
| 21.1.129 | $C222$ | 36.1.249 | $Cmc2_1$ | 51.1.387 | $Pmma$ | 66.1.564 | $Cccm$ |
| 22.1.140 | $F222$ | 37.1.258 | $Ccc2$ | 52.1.406 | $Pnna$ | 67.1.577 | $Cmma$ |
| 23.1.145 | $I222$ | 38.1.265 | $Amm2$ | 53.1.415 | $Pmna$ | 68.1.594 | $Ccca$ |
| 24.1.150 | $I2_12_12_1$ | 39.1.278 | $Abm2$ | 54.1.428 | $Pcca$ | 69.1.605 | $Fmmm$ |
| 25.1.155 | $Pmm2$ | 40.1.291 | $Ama2$ | 55.1.441 | $Pbam$ | 70.1.616 | $Fddd$ |
| 26.1.168 | $Pmc2_1$ | 41.1.300 | $Aba2$ | 56.1.451 | $Pccn$ | 71.1.621 | $Immm$ |
| 27.1.178 | $Pcc2$ | 42.1.309 | $Fmm2$ | 57.1.458 | $Pbcm$ | 72.1.630 | $Ibam$ |
| 28.1.185 | $Pma2$ | 43.1.320 | $Fdd2$ | 58.1.471 | $Pnnm$ | 73.1.643 | $Ibca$ |
| 29.1.198 | $Pca2_1$ | 44.1.324 | $Imm2$ | 59.1.478 | $Pmmn$ | 74.1.650 | $Imma$ |
| 30.1.205 | $Pnc2$ | 45.1.331 | $Iba2$ | 60.1.488 | $Pbcn$ | | |

Table S11 (continued)

| AFB, AF0, 0F0 modes (cat. 7) | | | | | | | |
|------------------------------------|-------------------|-----------|------------------|-----------|-----------------|-----------|----------------|
| 16.3.101 | $P2'2'2$ | 32.3.221 | $Pb'a2'$ | 45.3.333 | $Ib'a2'$ | 59.5.482 | $Pm'm'n$ |
| 17.3.108 | $P2'2'2_1$ | 32.4.222 | $Pb'a'2$ | 45.4.334 | $Ib'a'2$ | 59.6.483 | $Pmm'n'$ |
| 17.4.109 | $P22'2'_1$ | 33.3.228 | $Pn'a2'_1$ | 46.3.340 | $Im'a2'$ | 60.6.493 | $Pb'c'n$ |
| 18.3.115 | $P2'_12'_12$ | 33.4.229 | $Pna'2'_1$ | 46.4.341 | $Ima'2'$ | 60.7.494 | $Pbc'n'$ |
| 18.4.116 | $P2_12'_12'$ | 33.5.230 | $Pn'a'2_1$ | 46.5.342 | $Im'a'2$ | 60.8.495 | $Pb'cn'$ |
| 19.3.121 | $P2'_12'_12_1$ | 34.3.233 | $Pn'n2'$ | 47.4.350 | $Pm'm'm$ | 61.4.500 | $Pb'c'a$ |
| 20.3.124 | $C2'2'2_1$ | 34.4.234 | $Pn'n'2$ | 48.4.361 | $Pn'n'n$ | 62.6.507 | $Pn'm'a$ |
| 20.4.125 | $C22'2'_1$ | 35.3.238 | $Cm'm2'$ | 49.5.368 | $Pc'c'm$ | 62.7.508 | $Pnm'a'$ |
| 21.3.131 | $C2'2'2$ | 35.4.239 | $Cm'm'2$ | 49.6.369 | $Pc'cm'$ | 62.8.509 | $Pn'ma'$ |
| 21.4.132 | $C22'2'$ | 36.3.251 | $Cm'c2'_1$ | 50.5.381 | $Pb'a'n$ | 63.6.516 | $Cm'c'm$ |
| 22.3.142 | $F2'2'2$ | 36.4.252 | $Cmc'2'_1$ | 50.6.382 | $Pb'an'$ | 63.7.517 | $Cmc'm'$ |
| 23.3.147 | $I2'2'2$ | 36.5.253 | $Cm'c'2_1$ | 51.6.392 | $Pm'm'a$ | 63.8.518 | $Cm'cm'$ |
| 24.3.152 | $I2'_12'_12_1$ | 37.3.260 | $Cc'c2'$ | 51.7.393 | $Pmm'a'$ | 64.6.533 | $Cm'c'a$ |
| 25.3.157 | $Pm'm2'$ | 37.4.261 | $Cc'c'2$ | 51.8.394 | $Pm'ma'$ | 64.7.534 | $Cmc'a'$ |
| 25.4.158 | $Pm'm'2$ | 38.3.267 | $Am'm2'$ | 52.6.411 | $Pn'n'a$ | 64.8.535 | $Cm'ca'$ |
| 26.3.170 | $Pm'c2'_1$ | 38.4.268 | $Amm'2'$ | 52.7.412 | $Pnn'a'$ | 65.5.549 | $Cm'm'm$ |
| 26.4.171 | $Pmc'2'_1$ | 38.5.269 | $Am'm'2$ | 52.8.413 | $Pn'na'$ | 65.6.550 | $Cmm'm'$ |
| 26.5.172 | $Pm'c'2'_1$ | 39.3.280 | $Ab'm2'$ | 53.6.420 | $Pm'n'a$ | 66.5.568 | $Cc'c'm$ |
| 27.3.180 | $Pc'c2'$ | 39.4.281 | $Abm'2'$ | 53.7.421 | $Pmn'a'$ | 66.6.569 | $Ccc'm'$ |
| 27.4.181 | $Pc'c'2$ | 39.5.282 | $Ab'm'2$ | 53.8.422 | $Pm'na'$ | 67.5.581 | $Cm'm'a$ |
| 28.3.187 | $Pm'a2'$ | 40.3.293 | $Am'a2'$ | 54.6.433 | $Pc'c'a$ | 67.6.582 | $Cmm'a'$ |
| 28.4.188 | $Pma'2'$ | 40.4.294 | $Ama'2'$ | 54.7.434 | $Pcc'a'$ | 68.5.598 | $Cc'c'a$ |
| 28.5.189 | $Pm'a'2$ | 40.5.295 | $Am'a'2$ | 54.8.435 | $Pc'ca'$ | 68.6.599 | $Ccc'a'$ |
| 29.3.200 | $Pc'a2'_1$ | 41.3.302 | $Ab'a2'$ | 55.5.445 | $Pb'a'm$ | 69.4.608 | $Fm'm'm$ |
| 29.4.201 | $Pca'2'_1$ | 41.4.303 | $Aba'2'$ | 55.6.446 | $Pb'am'$ | 70.4.619 | $Fd'd'd$ |
| 29.5.202 | $Pc'a'2_1$ | 41.5.304 | $Ab'a'2$ | 56.5.455 | $Pc'c'n$ | 71.4.624 | $Im'm'm$ |
| 30.3.207 | $Pn'c2'$ | 42.3.311 | $Fm'm2'$ | 56.6.456 | $Pc'cn'$ | 72.5.634 | $Ib'a'm$ |
| 30.4.208 | $Pnc'2'$ | 42.4.312 | $Fm'm'2$ | 57.6.463 | $Pb'c'm$ | 72.6.635 | $Iba'm'$ |
| 30.5.209 | $Pn'c'2$ | 43.3.322 | $Fd'd2'$ | 57.7.464 | $Pbc'm'$ | 73.4.646 | $Ib'c'a$ |
| 31.3.214 | $Pm'n2'_1$ | 43.4.323 | $Fd'd'2$ | 57.8.465 | $Pb'cm'$ | 74.5.654 | $Im'm'a$ |
| 31.4.215 | $Pmn'2'_1$ | 44.3.326 | $Im'm2'$ | 58.5.475 | $Pn'n'm$ | 74.6.655 | $Imm'a'$ |
| 31.5.216 | $Pm'n'2_1$ | 44.4.327 | $Im'm'2$ | 58.6.476 | $Pnn'm'$ | | |
| ABC2, AB0, 0A0, 000 modes (cat. 8) | | | | | | | |
| 16.4.102 | $P_{2a}222$ | 22.5.144 | $F_C22'2'$ | 27.6.183 | $P_{Ccc}2$ | 35.5.240 | $C_{2c}mm2$ |
| 16.5.103 | P_C222 | 23.4.148 | I_P222 | 27.7.184 | $P_{2b}c'c2'$ | 35.6.241 | $C_{Pmm}2$ |
| 16.6.104 | P_F222 | 23.5.149 | $I_P2'2'2$ | 28.6.190 | $P_{2b}ma2$ | 35.7.242 | $C_{Imm}2$ |
| 16.7.105 | $P_{2c}22'2'$ | 24.4.153 | $I_P2_12_12_1$ | 28.7.191 | $P_{2c}ma2$ | 35.8.243 | $C_{2c}m'm2'$ |
| 17.5.110 | $P_{2a}222_1$ | 24.5.154 | $I_P2'_12'_12_1$ | 28.8.192 | P_Ama2 | 35.9.244 | $C_{2c}m'm'2$ |
| 17.6.111 | P_C222_1 | 25.5.159 | $P_{2c}mm2$ | 28.9.193 | $P_{2b}m'a2'$ | 35.10.245 | $C_{Pm}'m2'$ |
| 17.7.112 | $P_{2a}2'2'2_1$ | 25.6.160 | $P_{2a}mm2$ | 28.10.194 | $P_{2c}m'a2'$ | 35.11.246 | $C_{Pm}'m'2$ |
| 18.5.117 | $P_{2c}2_12_12$ | 25.7.161 | $P_{Cmm}2$ | 28.11.195 | $P_{2c}ma'2'$ | 35.12.247 | $C_{Im}'m2'$ |
| 18.6.118 | $P_{2c}2_12'_12'$ | 25.8.162 | $P_{Amm}2$ | 28.12.196 | $P_{2c}m'a'2$ | 35.13.248 | $C_{Im}'m'2$ |
| 20.5.126 | C_P222_1 | 25.9.163 | $P_{Fmm}2$ | 28.13.197 | $P_{Am}'a'2$ | 36.6.254 | $C_{Pmc}2_1$ |
| 20.6.127 | $C_P2'2'2_1$ | 25.10.164 | $P_{2c}mm'2'$ | 29.6.203 | $P_{2b}ca2_1$ | 36.7.255 | $C_{Pm}'c2'_1$ |
| 20.7.128 | $C_P22'2'_1$ | 25.11.165 | $P_{2c}m'm'2$ | 29.7.204 | $P_{2b}c'a'2_1$ | 36.8.256 | $C_{Pmc}2'_1$ |
| 21.5.133 | $C_{2c}222$ | 25.12.166 | $P_{2a}m'm'2$ | 30.6.210 | $P_{2a}nc2$ | 36.9.257 | $C_{Pm}'c2_1$ |
| 21.6.134 | C_P222 | 25.13.167 | $P_{Am}'m'2$ | 30.7.211 | $P_{2a}nc'2'$ | 37.5.262 | $C_{Pcc}2$ |
| 21.7.135 | C_I222 | 26.6.173 | $P_{2a}mc2_1$ | 31.6.217 | $P_{2b}mn2_1$ | 37.6.263 | $C_{Pc}'c2'$ |
| 21.8.136 | $C_{2c}22'2'$ | 26.7.174 | $P_{2b}mc2_1$ | 31.7.218 | $P_{2b}m'n2'_1$ | 37.7.264 | $C_{Pc}'c'2$ |
| 21.9.137 | $C_P2'2'2$ | 26.8.175 | $P_{Cmc}2_1$ | 32.5.223 | $P_{2c}ba2$ | 38.6.270 | $A_{2a}mm2$ |
| 21.10.138 | $C_P22'2'$ | 26.9.176 | $P_{2a}mc'2'_1$ | 32.6.224 | $P_{2c}b'a2'$ | 38.7.271 | $A_{Pmm}2$ |
| 21.11.139 | $C_I2'2'2'$ | 26.10.177 | $P_{2b}m'c'2_1$ | 32.7.225 | $P_{2c}b'a'2$ | 38.8.272 | $A_{Imm}2$ |
| 22.4.143 | F_C222 | 27.5.182 | $P_{2a}cc2$ | 34.5.235 | $P_{Fnn}2$ | 38.9.273 | $A_{2a}mm'2'$ |

continued on the next page

Table S11 (continued)

| ABC2, AB0, 0A0, 000 modes (cat. 8) continued | | | | | | | |
|--|----------------|-----------|-----------------|-----------|----------------|-----------|----------------|
| 38.10.274 | $A_{Pm'm'2'}$ | 50.8.384 | $P_{2c}ban$ | 59.7.484 | $Pm'm'n'$ | 66.11.574 | $C_{Pc'c'm}$ |
| 38.11.275 | $A_{Pmm'2'}$ | 50.9.385 | $P_{2c}b'an$ | 59.8.485 | $P_{2c}mmn$ | 66.12.575 | $C_{Pcc'm'}$ |
| 38.12.276 | $A_{Pm'm'2}$ | 50.10.386 | $P_{2c}b'a'n$ | 59.9.486 | $P_{2c}m'mn$ | 66.13.576 | $C_{Pc'c'm'}$ |
| 38.13.277 | $A_{Im'm'2}$ | 51.3.389 | $Pm'ma$ | 59.10.487 | $P_{2c}m'm'n$ | 67.3.579 | $Cm'ma$ |
| 39.6.283 | $A_{2a}bm2$ | 51.4.390 | $Pmm'a$ | 60.3.490 | $Pb'cn$ | 67.4.580 | $Cmma'$ |
| 39.7.284 | A_{Pbm2} | 51.5.391 | $Pmma'$ | 60.4.491 | $Pbc'n$ | 67.7.583 | $Cm'm'a'$ |
| 39.8.285 | A_Ibm2 | 51.9.395 | $Pm'm'a'$ | 60.5.492 | $Pbcn'$ | 67.8.584 | $C_{2c}mma$ |
| 39.9.286 | $A_{2a}b'm'2$ | 51.10.396 | $P_{2b}mma$ | 60.9.496 | $Pb'c'n'$ | 67.9.585 | C_{Pmma} |
| 39.10.287 | $A_{Pb'm'2'}$ | 51.11.397 | $P_{2c}mma$ | 61.3.499 | $Pb'ca$ | 67.10.586 | C_{Imma} |
| 39.11.288 | $A_{Pbm'2'}$ | 51.12.398 | P_{AMma} | 61.5.501 | $Pb'c'a'$ | 67.11.587 | $C_{2c}m'ma$ |
| 39.12.289 | $A_{Pb'm'2}$ | 51.13.399 | $P_{2b}b'm'ma$ | 62.3.504 | $Pn'ma$ | 67.12.588 | $C_{2c}m'm'a'$ |
| 39.13.290 | $A_Ib'm'2$ | 51.14.400 | $P_{2b}mma'$ | 62.4.505 | $Pnm'a$ | 67.13.589 | $C_{Pm'ma}$ |
| 40.6.296 | A_{Pma2} | 51.15.401 | $P_{2b}b'm'ma'$ | 62.5.506 | $Pnma'$ | 67.14.590 | $C_{Pmm'a}$ |
| 40.7.297 | $A_{Pm'a2'}$ | 51.16.402 | $P_{2c}m'ma$ | 62.9.510 | $Pn'm'a'$ | 67.15.591 | $C_{Pmma'}$ |
| 40.8.298 | $A_{Pma'2'}$ | 51.17.403 | $P_{2c}mm'a$ | 63.3.513 | $Cm'cm$ | 67.16.592 | $C_{Imm'a}$ |
| 40.9.299 | $A_{Pm'a'2}$ | 51.18.404 | $P_{2c}m'm'a$ | 63.4.514 | $Cmc'm$ | 67.17.593 | $C_{Im'm'a'}$ |
| 41.6.305 | A_{Pba2} | 51.19.405 | $P_{AM}m'ma$ | 63.5.515 | $Cmcm'$ | 68.3.596 | $Cc'ca$ |
| 41.7.306 | $A_{Pb'a'2'}$ | 52.3.408 | $Pn'na$ | 63.9.519 | $Cm'c'm'$ | 68.4.597 | $Ccca'$ |
| 41.8.307 | $A_{Pba'2'}$ | 52.4.409 | $Pnn'a$ | 63.10.520 | $Cpmcm$ | 68.7.600 | $Cc'c'a'$ |
| 41.9.308 | $A_{Pb'a'2}$ | 52.5.410 | $Pnna'$ | 63.11.521 | $Cpm'cm$ | 68.8.601 | C_{Pcca} |
| 42.5.313 | F_{Cmm2} | 52.9.414 | $Pn'n'a'$ | 63.12.522 | $Cpmc'm$ | 68.9.602 | $C_{Pc'ca}$ |
| 42.6.314 | F_{Amm2} | 53.3.417 | $Pm'na$ | 63.13.523 | $Cpmcm'$ | 68.10.603 | $C_{Pcca'}$ |
| 42.7.315 | $F_{Cmm'2'}$ | 53.4.418 | $Pmn'a$ | 63.14.524 | $Cpm'c'm$ | 68.11.604 | $C_{Pcc'a'}$ |
| 42.8.316 | $F_{Cm'm'2'}$ | 53.5.419 | $Pmna'$ | 63.15.525 | $Cpmc'm'$ | 69.3.607 | $Fm'mm$ |
| 42.9.317 | $F_{Am'm'2'}$ | 53.9.423 | $Pm'n'a'$ | 63.16.526 | $Cpm'cm'$ | 69.5.609 | $Fm'm'm'$ |
| 42.10.318 | $F_{Amm'2'}$ | 53.10.424 | $P_{2b}mna$ | 63.17.527 | $Cpm'c'm'$ | 69.6.610 | F_{Cmmm} |
| 42.11.319 | $F_{Am'm'2}$ | 53.11.425 | $P_{2b}b'm'na$ | 64.3.530 | $Cm'ca$ | 69.7.611 | $F_{Cm'mm}$ |
| 44.5.328 | I_{Pmm2} | 53.12.426 | $P_{2b}bmnna'$ | 64.4.531 | $Cmc'a$ | 69.8.612 | $F_{Cmmm'}$ |
| 44.6.329 | $I_{Pmm'2'}$ | 53.13.427 | $P_{2b}bm'na'$ | 64.5.532 | $Cmca'$ | 69.9.613 | $F_{Cm'm'm'}$ |
| 44.7.330 | $I_{Pm'm'2}$ | 54.3.430 | $Pc'ca$ | 64.9.536 | $Cm'c'a'$ | 69.10.614 | $F_{Cmm'm'}$ |
| 45.5.335 | I_{Pba2} | 54.4.431 | $Pcc'a$ | 64.10.537 | $Cpmca$ | 69.11.615 | $F_{Cm'm'm'}$ |
| 45.6.336 | $I_{Pba'2'}$ | 54.5.432 | $Pcca'$ | 64.11.538 | $Cpm'ca$ | 70.3.618 | $Fd'dd$ |
| 45.7.337 | $I_{Pb'a'2}$ | 54.9.436 | $Pc'c'a'$ | 64.12.539 | $Cpmc'a$ | 70.5.620 | $Fd'd'd'$ |
| 46.6.343 | I_{Pma2} | 54.10.437 | $P_{2b}cca$ | 64.13.540 | $Cpmca'$ | 71.3.623 | $Im'mm$ |
| 46.7.344 | $I_{Pm'a2'}$ | 54.11.438 | $P_{2b}b'ca$ | 64.14.541 | $Cpm'c'a$ | 71.5.625 | $Im'm'm'm'$ |
| 46.8.345 | $I_{Pma'2'}$ | 54.12.439 | $P_{2b}bca'$ | 64.15.542 | $Cpmc'a'$ | 71.6.626 | I_{Pmmm} |
| 46.9.346 | $I_{Pm'a'2}$ | 54.13.440 | $P_{2b}b'ca'$ | 64.16.543 | $Cpm'ca'$ | 71.7.627 | $I_{Pm'mm}$ |
| 47.3.349 | $Pm'mm$ | 55.3.443 | $Pb'am$ | 64.17.544 | $Cpm'c'a'$ | 71.8.628 | $I_{Pm'm'm'}$ |
| 47.5.351 | $Pm'm'm'$ | 55.4.444 | $Pbam'$ | 65.3.547 | $Cm'mm$ | 71.9.629 | $I_{Pm'm'm'}$ |
| 47.6.352 | $P_{2a}mmm$ | 55.7.447 | $Pb'a'm'$ | 65.4.548 | $Cmmm'$ | 72.3.632 | $Ib'am$ |
| 47.7.353 | P_{Cmmm} | 55.8.448 | $P_{2c}bam$ | 65.7.551 | $Cm'm'm'$ | 72.4.633 | $Ibam'$ |
| 47.8.354 | P_{Fmmm} | 55.9.449 | $P_{2c}b'am$ | 65.8.552 | $C_{2c}mmm$ | 72.7.636 | $Ib'a'm'$ |
| 47.9.355 | $P_{2a}mmm'$ | 55.10.450 | $P_{2c}b'a'm$ | 65.9.553 | $Cpmmm$ | 72.8.637 | I_{Pbam} |
| 47.10.356 | $P_{2c}m'm'm$ | 56.3.453 | $Pc'cn$ | 65.10.554 | C_{Immm} | 72.9.638 | $I_{Pb'am}$ |
| 47.11.357 | $P_{Cmmm'}$ | 56.4.454 | $Pccn'$ | 65.11.555 | $C_{2c}m'm'm'$ | 72.10.639 | $I_{Pbam'}$ |
| 48.3.360 | $Pn'nn$ | 56.7.457 | $Pc'c'n'$ | 65.12.556 | $C_{2c}mm'm'$ | 72.11.640 | $I_{Pb'a'm}$ |
| 48.5.362 | $Pn'n'n'$ | 57.3.460 | $Pb'cm$ | 65.13.557 | $Cpm'mm$ | 72.12.641 | $I_{Pb'am'}$ |
| 48.6.363 | $P_{Fn}nn$ | 57.4.461 | $Pbc'm$ | 65.14.558 | $Cpmmm'$ | 72.13.642 | $I_{Pb'a'm'}$ |
| 49.3.366 | $Pc'cm$ | 57.5.462 | $Pbcm'$ | 65.15.559 | $Cpm'm'm$ | 73.3.645 | $Ib'ca$ |
| 49.4.367 | $Pccm'$ | 57.9.466 | $Pb'c'm'$ | 65.16.560 | $Cpm'm'm'$ | 73.5.647 | $Ib'c'a'$ |
| 49.7.370 | $Pc'c'm'$ | 57.10.467 | $P_{2a}bcm$ | 65.17.561 | $Cpm'm'm'$ | 73.6.648 | I_{Pbca} |
| 49.8.371 | $P_{2a}ccm$ | 57.11.468 | $P_{2a}bc'm$ | 65.18.562 | $Cim'mm$ | 73.7.649 | $I_{Pb'ca}$ |
| 49.9.372 | P_{Cccm} | 57.12.469 | $P_{2a}bcm'$ | 65.19.563 | $Cim'm'm$ | 74.3.652 | $Im'ma$ |
| 49.10.373 | $P_{2a}ccm'$ | 57.13.470 | $P_{2a}bc'm'$ | 66.3.566 | $Cc'cm$ | 74.4.653 | $Imma'$ |
| 49.11.374 | $P_{2a}c'c'm$ | 58.3.473 | $Pn'nm$ | 66.4.567 | $Cccm'$ | 74.7.656 | $Im'm'a'$ |
| 49.12.375 | $P_{2a}c'c'm'$ | 58.4.474 | $Pnnm'$ | 66.7.570 | $Cc'c'm'$ | 74.8.657 | I_{Pmma} |
| 49.13.376 | $P_{Cccm'}$ | 58.7.477 | $Pn'n'm'$ | 66.8.571 | $Cpcem$ | 74.9.658 | $I_{Pm'm'a}$ |
| 50.3.379 | $Pb'an$ | 59.3.480 | $Pm'mn$ | 66.9.572 | $Cpc'm$ | 74.10.659 | $I_{Pmm'a'}$ |
| 50.4.380 | $Pban'$ | 59.4.481 | $Pmmn'$ | 66.10.573 | $Cpcem'$ | 74.11.660 | $I_{Pm'ma'}$ |
| 50.7.383 | $Pb'a'n'$ | | | | | | |

PART IV

New classification of magnetic point groups

Table 2 from the main paper collect together magnetic point groups which describe orderings with the same sets-of-directions of \mathbf{M} . This table was achieved by using a specific way of enumerating the groups which is different from the traditional way of derivation of magnetic point groups. It is obtained in the following steps.

Step 1a Take standard crystallographic point groups with proper rotations only. There are 11 such groups, called chiral point groups whose symbols have numbers without primes, i.e. 1, 2, 222, 4, 422, 3, 32, 6, 622, 23, 432, see Table 2 col. 1 (main paper).

Step 1b Add new groups by multiplying proper half of the operations of groups from step 1a by the time reversal operation $(1')$. One obtains 10 black-and-white magnetic point groups whose symbols have primes (but no $1'$ at the end), e.g. $2', 2'2'2$, see Table 2 col. 1 (main paper).

Step 1c Add new groups by multiplying all the operations of groups from step 1a by the time reversal operation $(1')$. One obtains 11 grey magnetic point groups, whose symbol have $1'$ at the end, see e.g. $21', 2221'$, see Table 2 col. 1 (main paper).

At the end of steps 1a-c one obtains 32 magnetic groups without improper rotations, shown all in 32 rows in Table 2 col. 1 (main paper).

Step 2 Add new groups by multiplying proper half of the operations of groups from steps 1a-c (i.e. all groups from col. 1) by inversion $\bar{1}$. One obtains 58 magnetic point groups with improper rotations, shown in Table 2 col. 2 (main paper). Please note that multiplying by inversion does not change the set-of-directions of \mathbf{M} .

Step 3 Add new groups by multiplying all the operations of groups from steps 1a-c (i.e. all groups from col. 1) by inversion $\bar{1}$. One obtains the remaining 32 centrosymmetric magnetic point groups, shown in Table 2 col. 3 (main paper). Please note that multiplying by inversion does not change the set-of-directions of \mathbf{M} .

At the end of these steps one obtains all the 122 magnetic point groups as shown in Table 2 (main paper).