FOUNDATIONS
ADVANCES

Volume 77 (2021)

Supporting information for article:

On symmetry breaking of dual polyhedra of non-crystallographic group *H*3

Mariia Myronova

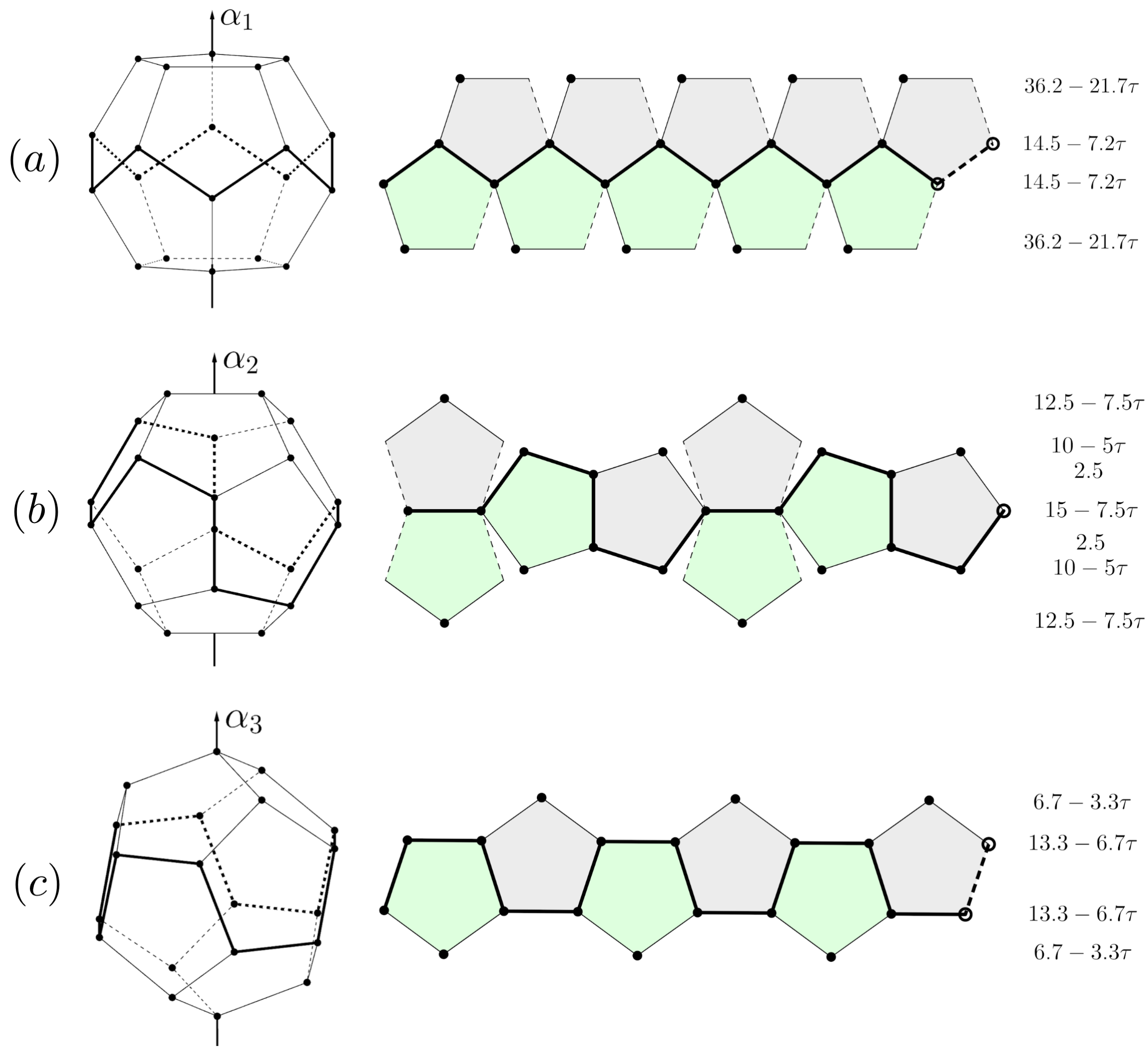

Figure 14. The polytope $\mathcal{V}_{H_3}(1,0,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(0,0,c')$ are depicted by black color.

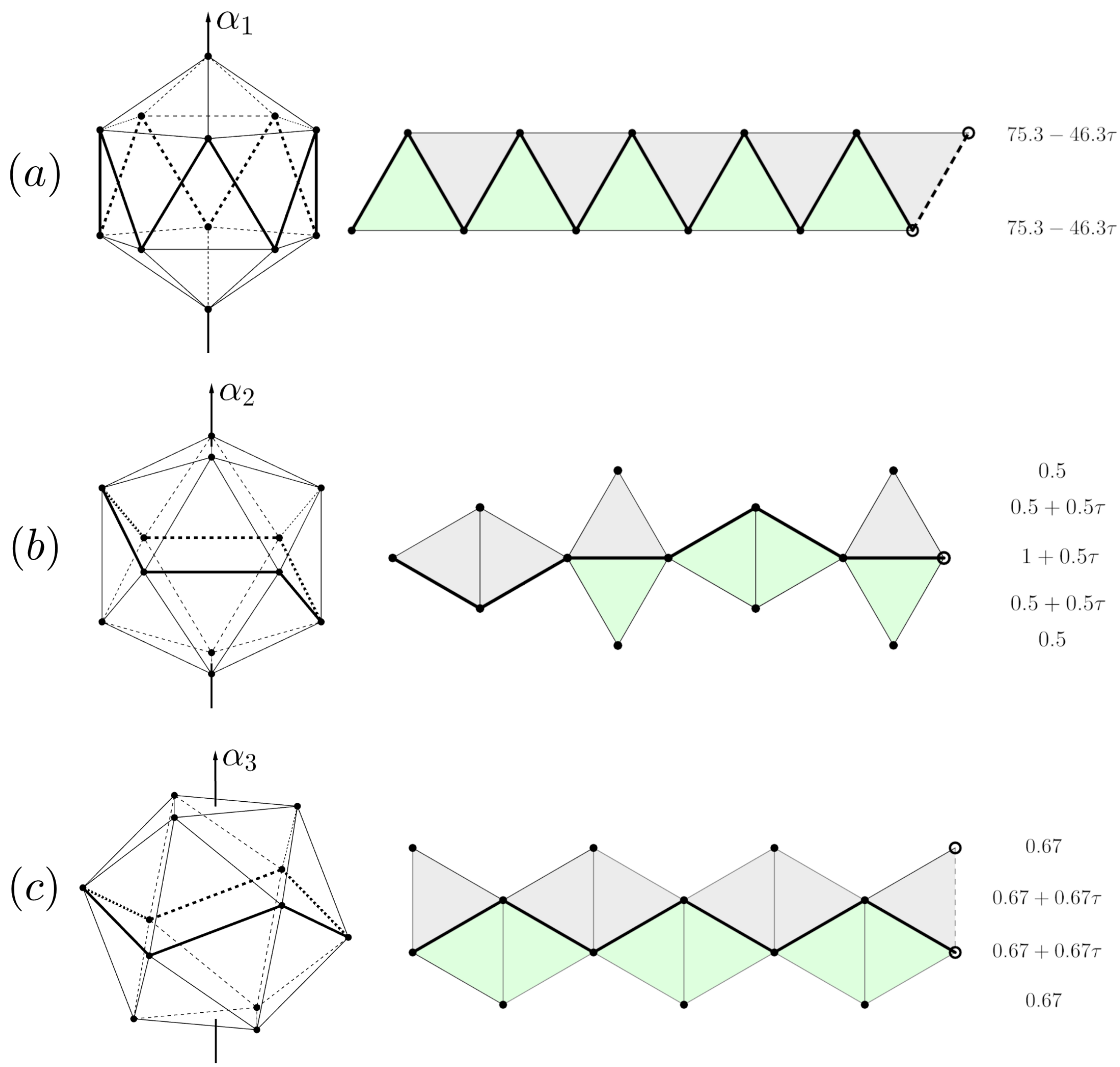

Figure 18. The polytope $\mathcal{V}_{H_3}(1,0,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(0,0,c')$ are depicted by black color.

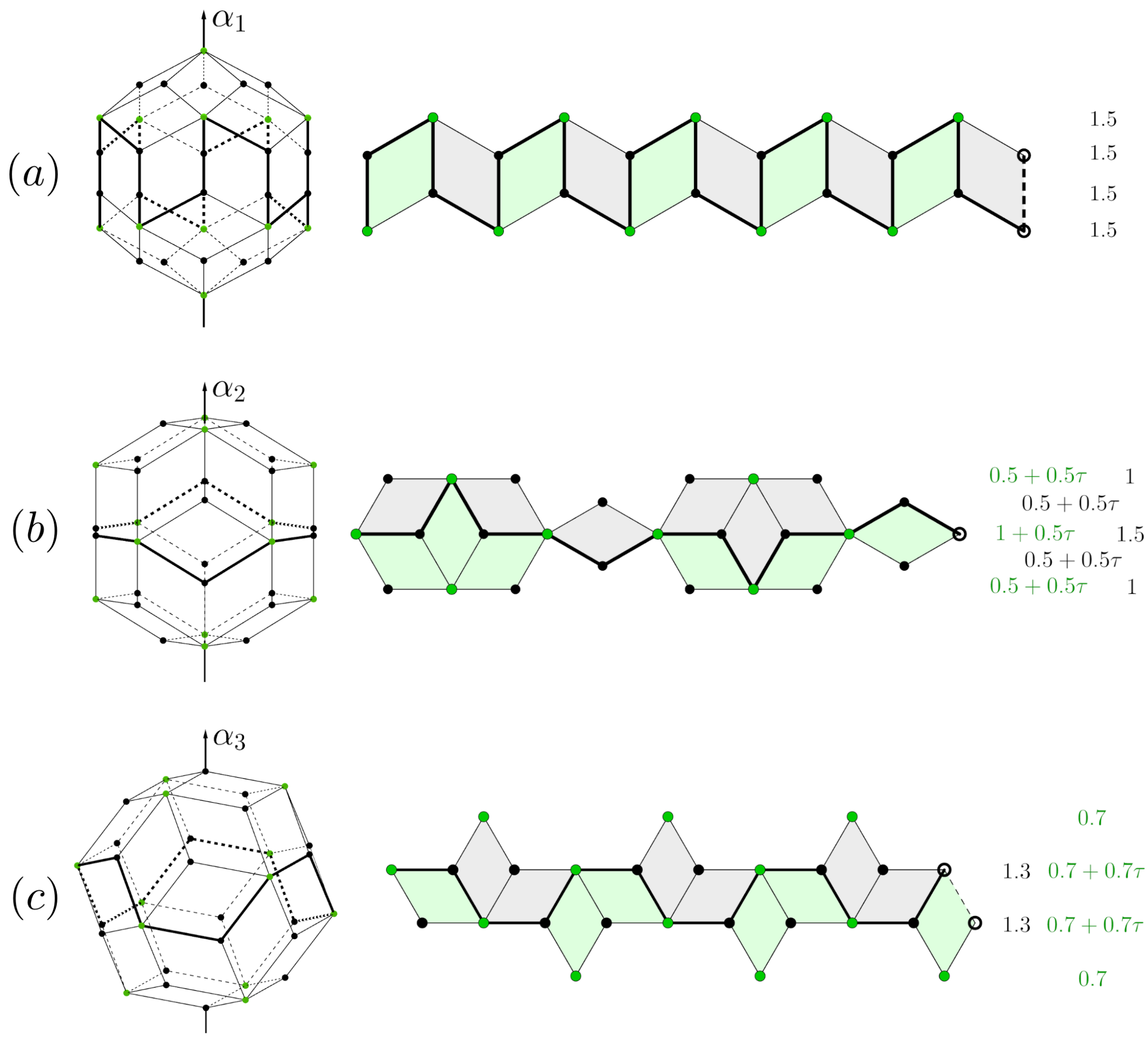

Figure 21. The polytope $\mathcal{V}_{H_3}(1,0,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(0,0,c')$ are depicted by black color.

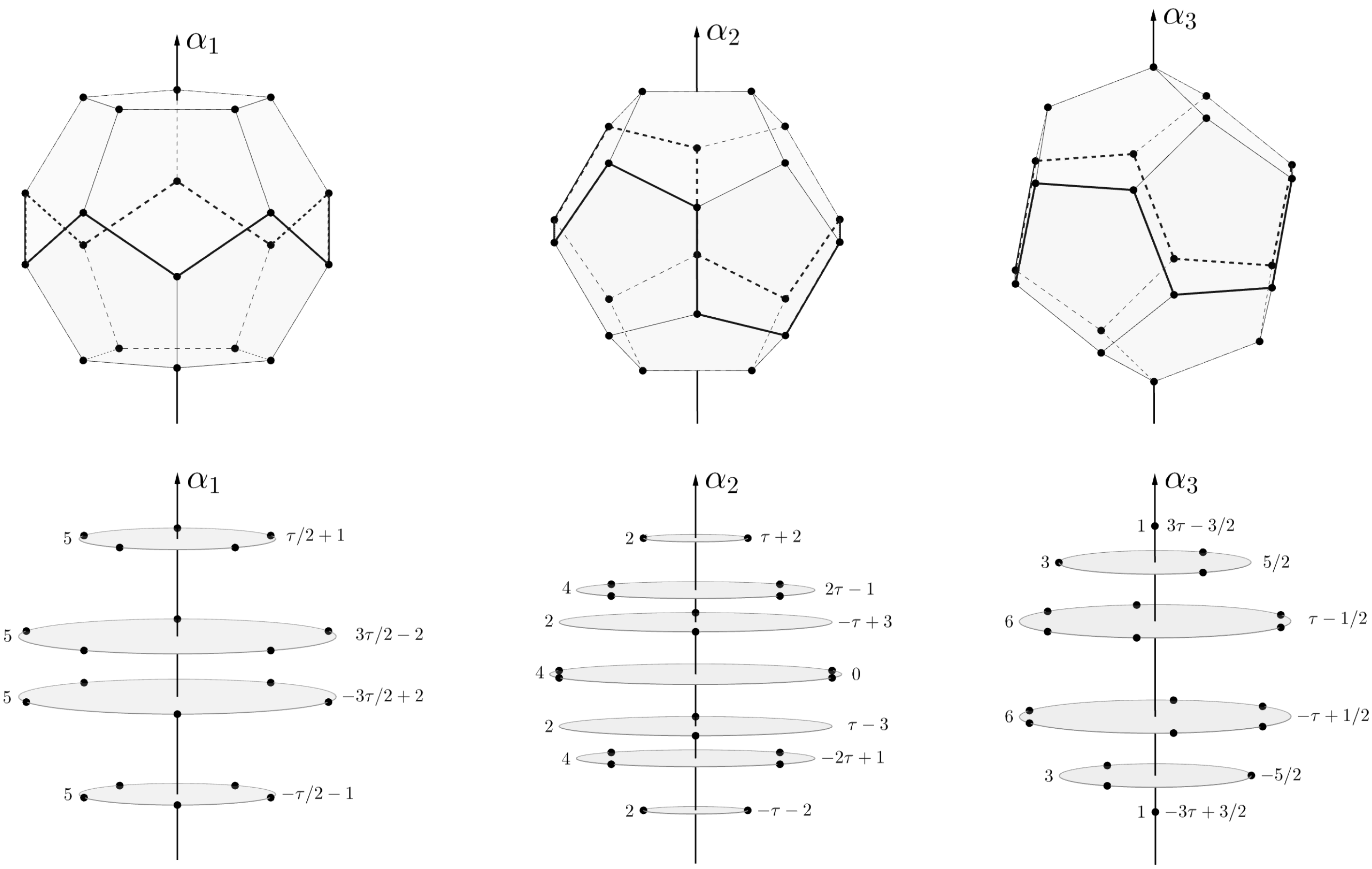

Figure 33. The polytope $\mathcal{V}_{H_3}(1,0,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(0,0,c')$ are depicted by black color.

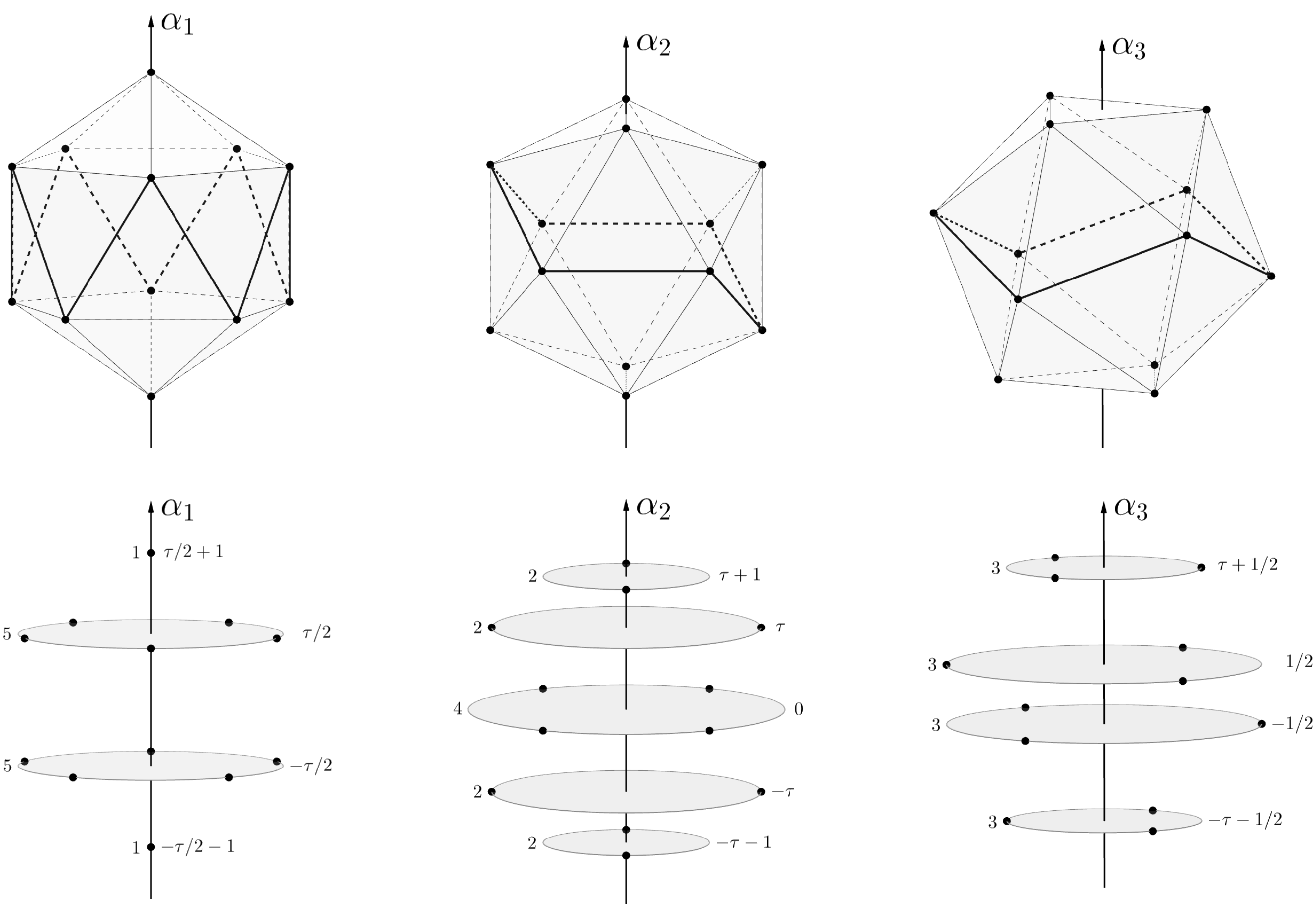

Figure 34. The polytope $\mathcal{V}_{H_3}(0,0,1)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$ are depicted by black color.

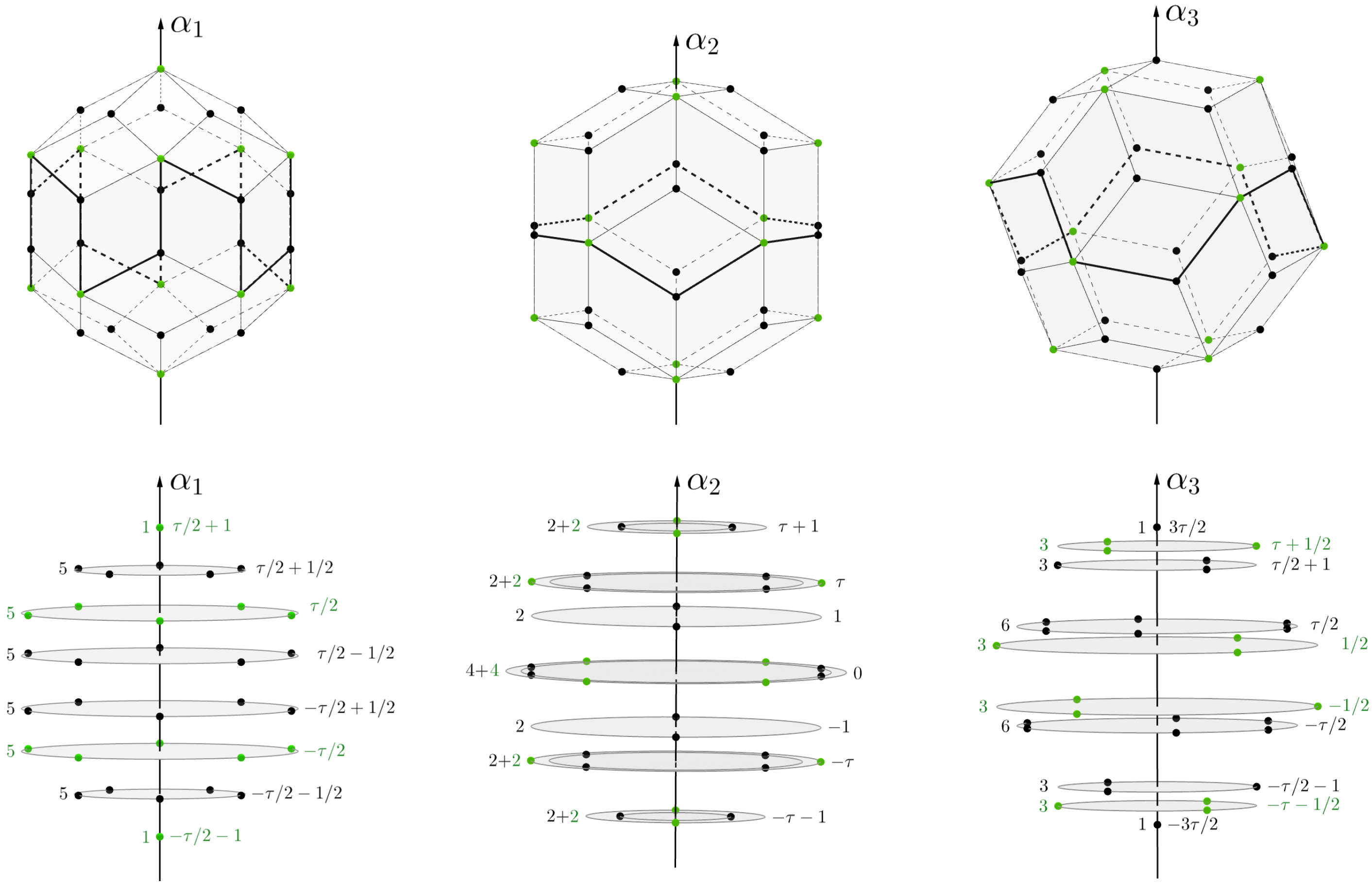

Figure 35. The polytope $\mathcal{V}_{H_3}(0,1,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$ and $O_{H_3}(0,0,c')$ are depicted by green and black colors, respectively.

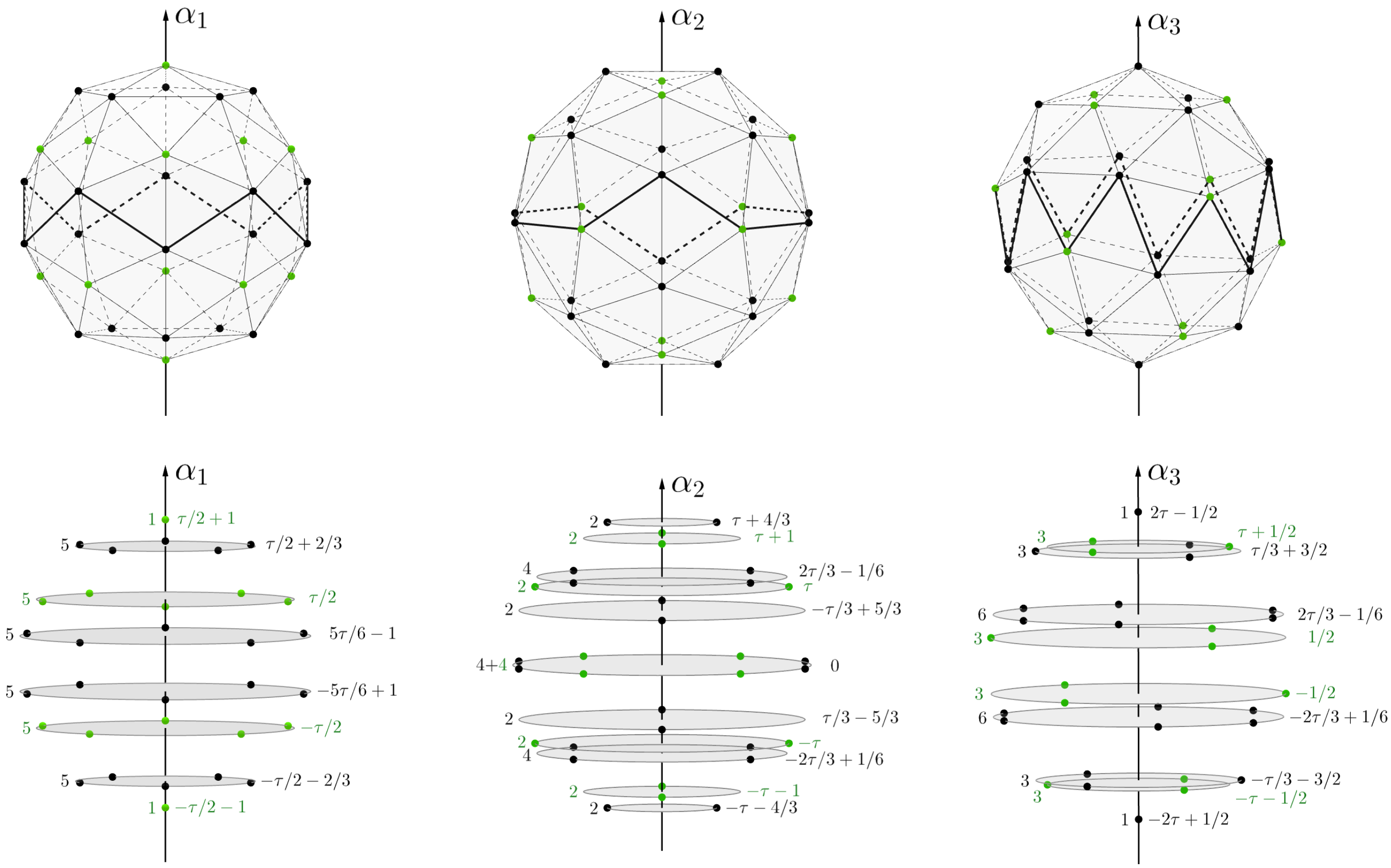

Figure 36. The polytope $\mathcal{V}_{H_3}(1,1,0)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$ and $O_{H_3}(0,0,c')$ are depicted by green and black colors, respectively.

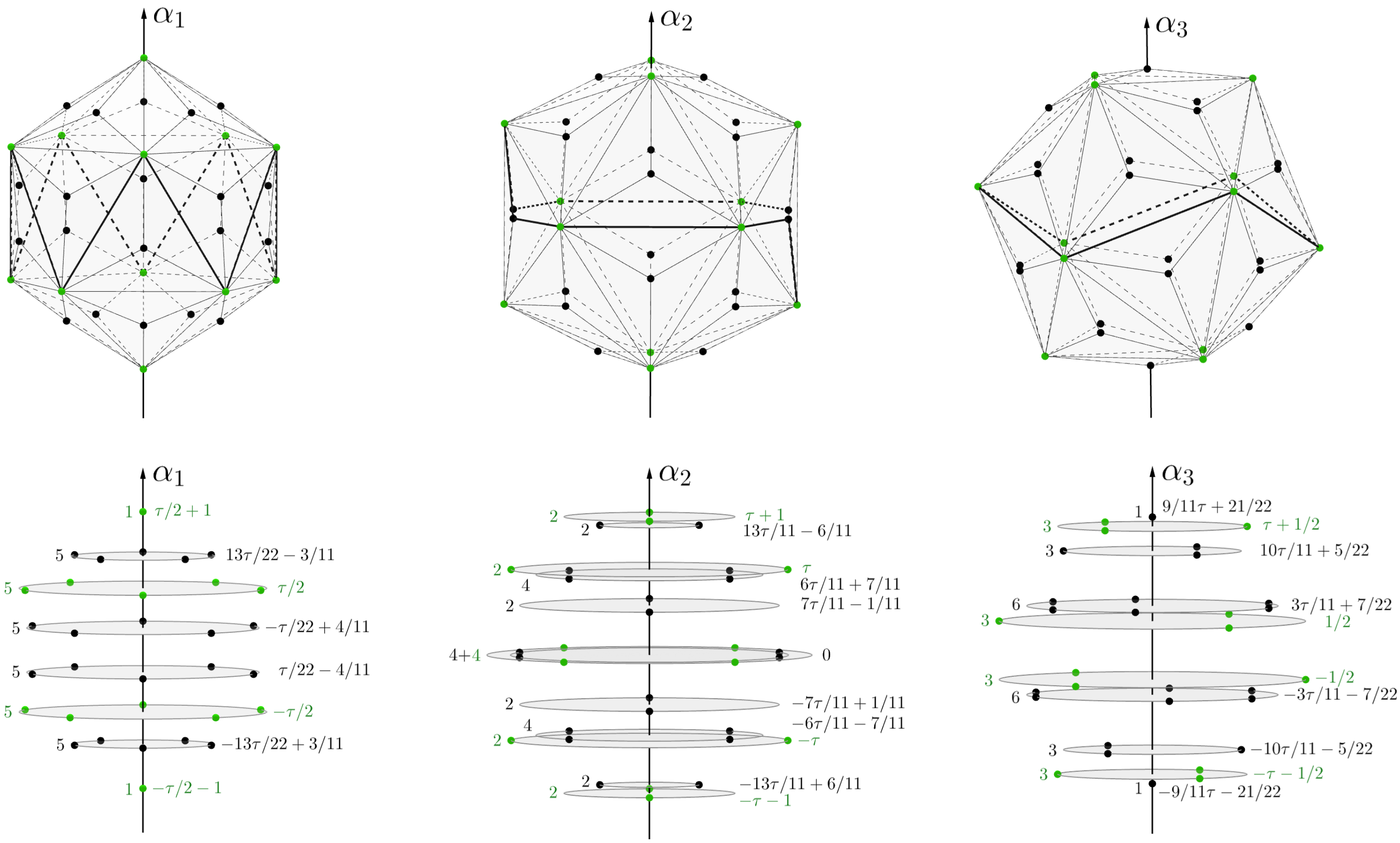

Figure 37. The polytope $\mathcal{V}_{H_3}(0,1,1)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$ and $O_{H_3}(0,0,c')$ are depicted by green and black colors, respectively.

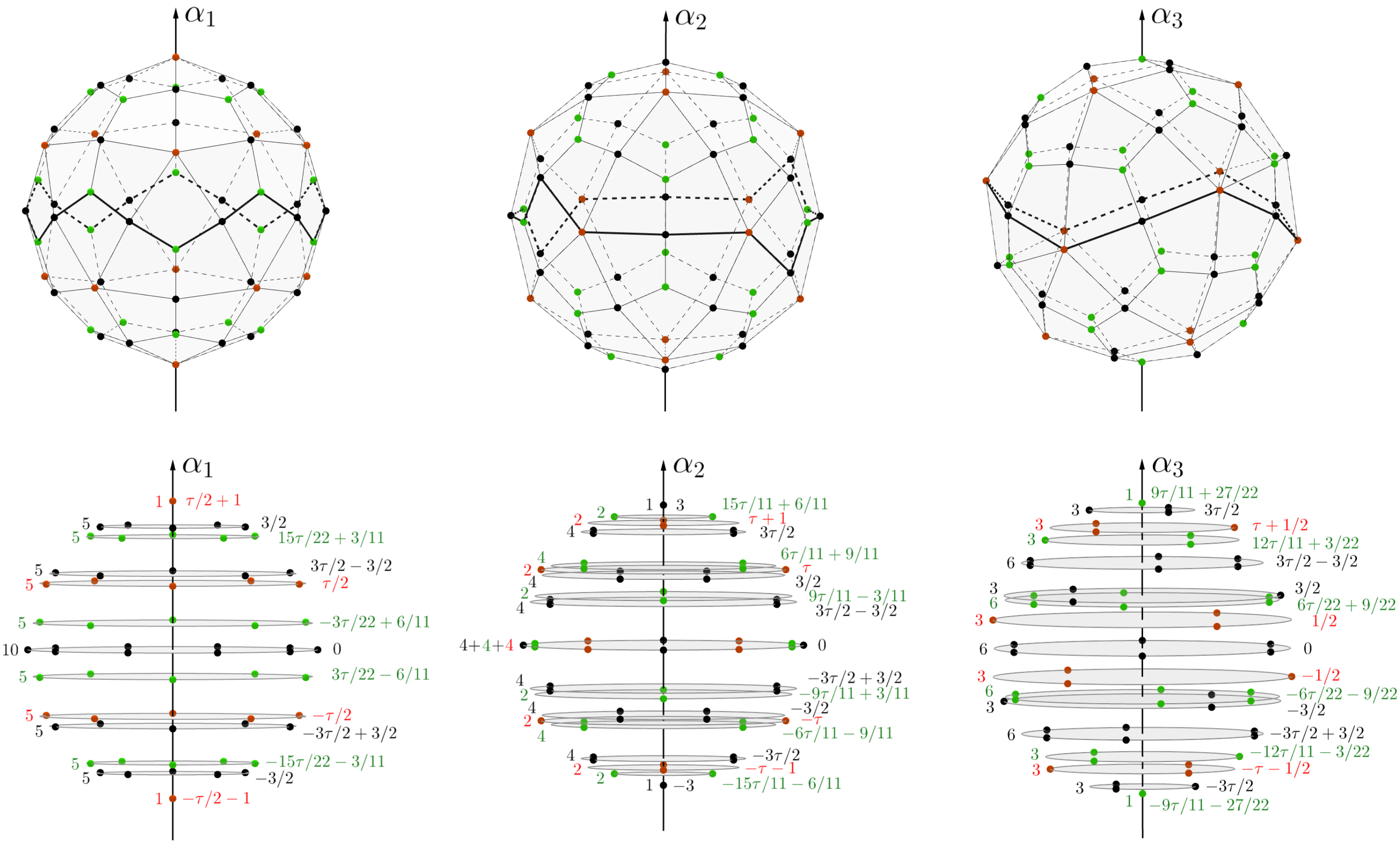

Figure 38. The polytope $\mathcal{V}_{H_3}(1,0,1)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$, $O_{H_3}(0,b',0)$ and $O_{H_3}(0,0,c')$ are depicted by orange, black and green colors, respectively.

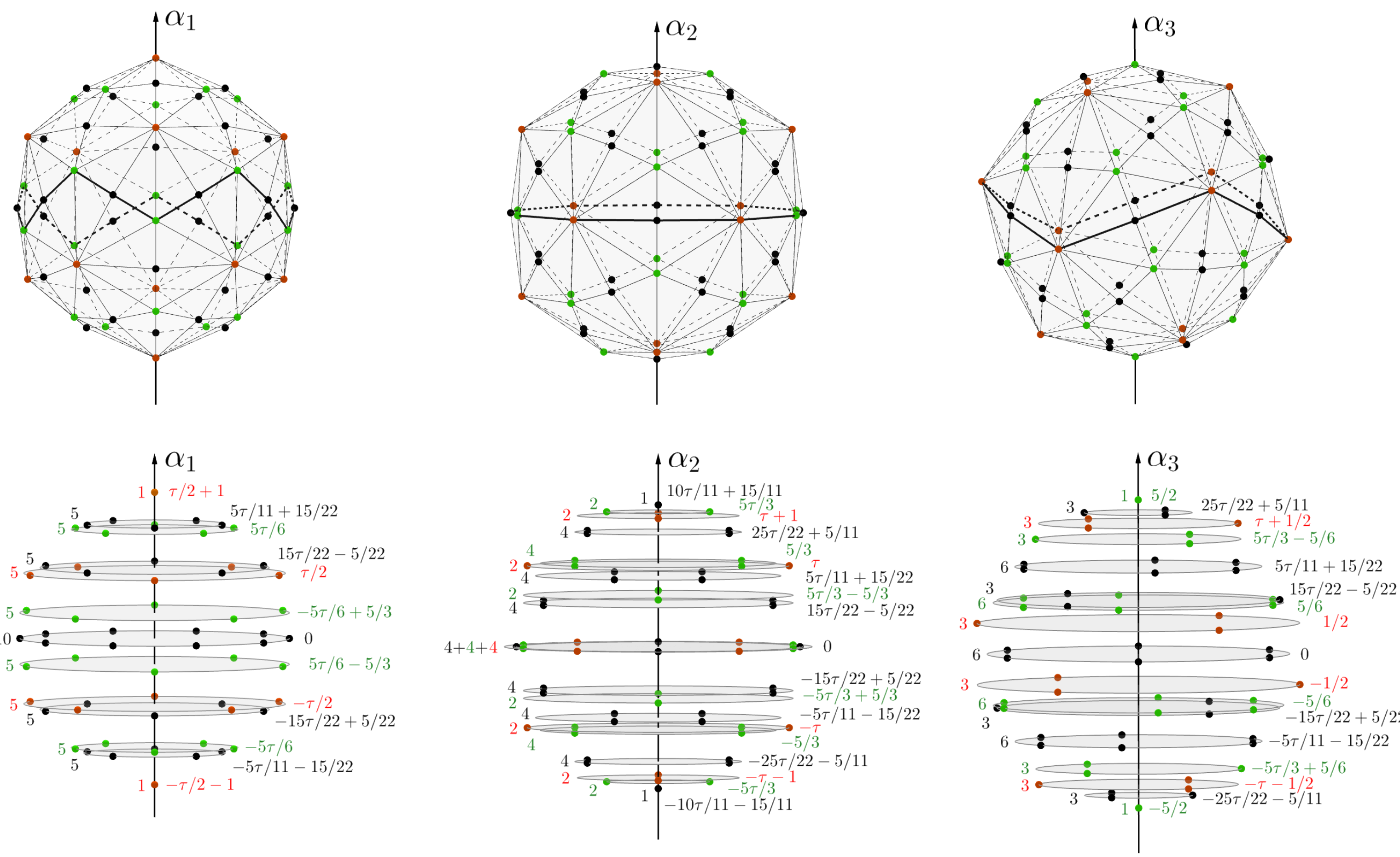

Figure 39. The polytope $\mathcal{V}_{H_3}(1,1,1)$ is oriented in the direction of the simple roots α_1, α_2 and α_3. The points of the orbits $O_{H_3}(a',0,0)$, $O_{H_3}(0,b',0)$ and $O_{H_3}(0,0,c')$ are depicted by orange, black and green colors, respectively.

	$\mathcal{V}_{H_3}(1,0,0)$			
	Orbit $c_3(0,0,1)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$(\frac{\tau}{2}+1,0,3\tau-4)$	$36.2-21.7\tau$	1.056	5
	$(\frac{3\tau}{2}-2,0,3-\tau)$	$14.5-7.2\tau$	2.764	5
	$(2-\frac{3\tau}{2},3-\tau,0)$	$14.5-7.2\tau$	2.764	5
	$(-\frac{\tau}{2}-1,3\tau-4,0)$	$36.2-21.7\tau$	1.056	5
$A_1\times A_1$	$(0,2+\tau,3\tau-4)$	$12.5-7.5\tau$	0.365	2
	$(3-\tau,2\tau-1,3-\tau)$	$10-5\tau$	1.910	4
	$(2\tau-1,3-\tau,0)$	2.5	2.5	2
	$(3\tau-4,0,2\tau-1)$	$15-7.5\tau$	2.865	4
	$(2\tau-1,\tau-3,0)$	2.5	2.5	2
	$(3-\tau,1-2\tau,3-\tau)$	$10-5\tau$	1.910	4
	$(0,-2-\tau,3\tau-4)$	$12.5-7.5\tau$	0.365	2
A_2	$(0,0,3\tau-\frac{3}{2})$	0	0	1
	$(0,3-\tau,\frac{5}{2})$	$6.7-3.3\tau$	1.273	3
	$(3-\tau,3\tau-4,\tau-\frac{1}{2})$	$13.3-6.7\tau$	2.546	6
	$(3\tau-4,3-\tau,\frac{1}{2}-\tau)$	$13.3-6.7\tau$	2.546	6
	$(3-\tau,0,-\frac{5}{2})$	$6.7-3.3\tau$	2.764	3
	$(0,0,\frac{3}{2}-3\tau)$	0	0	1

Table 11. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(1,0,0)$.

	$\mathcal{V}_{H_3}(0,0,1)$			
	Orbit $c_1(1,0,0)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$(1+\frac{\tau}{2},0,0)$	0	0	1
	$(\frac{\tau}{2},1,0)$	1.45	1.45	5
	$(-\frac{\tau}{2},0,1)$	1.45	1.45	5
	$(-1-\frac{\tau}{2},0,0)$	0	0	1
$A_1\times A_1$	$(1,\tau+1,0)$	0.5	0.5	2
	$(0,\tau,\tau)$	$0.5+0.5\tau$	1.309	2
	$(\tau,0,1)$	$1+0.5\tau$	1.809	4
	$(0,-\tau,\tau)$	$0.5+0.5\tau$	1.309	2
	$(1,-\tau-1,0)$	0.5	0.5	2
A_2	$(1,0,\tau+\frac{1}{2})$	0.667	0.667	3
	$(0,\tau,\frac{1}{2})$	$0.7+0.7\tau$	1.735	3
	$(\tau,0,-\frac{1}{2})$	$0.7+0.7\tau$	1.735	3
	$(0,1,-\tau-\frac{1}{2})$	0.667	0.667	3

Table 12. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(0,0,1)$.

	$\mathcal{V}_{H_3}(0,1,0)$			
	Orbit $c_3(0,0,1)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$(\frac{\tau}{2}+\frac{1}{2},0,\tau-1)$	$2.9-1.5\tau$	0.553	5
	$(\frac{\tau}{2}-\frac{1}{2},0,1)$	1.45	1.45	5
	$(\frac{\tau}{2}-\frac{1}{2},1,0)$	1.45	1.45	5
	$(-\frac{\tau}{2}-\frac{1}{2},\tau-1,0)$	$2.9-1.5\tau$	0.553	5
$A_1\times A_1$	$(0,1+\tau,\tau-1)$	$1-0.5\tau$	0.191	2
	$(1,\tau,1)$	1	1	4
	$(\tau,1,0)$	$0.5+0.5\tau$	1.309	2
	$(\tau-1,0,\tau)$	1.5	1.5	4
	$(\tau,-1,0)$	$0.5+0.5\tau$	1.309	2
	$(1,-\tau,1)$	1	1	4
	$(0,-1-\tau,\tau-1)$	$1-0.5\tau$	1.309	2
A_2	$(0,0,\frac{3}{2}\tau)$	0	0	3
	$(0,1,\tau+\frac{1}{2})$	0.667	0.667	3
	$(1,\tau-1,\frac{\tau}{2})$	1.333	1.333	6
	$(\tau-1,1,-\frac{\tau}{2})$	1.333	1.333	6
	$(1,0,-\frac{\tau}{2}-1)$	0.667	0.667	3
	$(0,0,-\frac{3}{2}\tau)$	0	0	3

Table 13. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(0,1,0)$.

	$\mathcal{V}_{H_3}(1,1,0)$			
	Orbit $c_3(0,0,1)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$\frac{1}{6}(3\tau+4,0,10\tau-12)$	$9.8-5.6\tau$	0.702	5
	$\frac{1}{6}(5\tau-6,0,10-6\tau)$	$4.2-1.45\tau$	1.839	5
	$\frac{1}{6}(6-5\tau,10-6\tau,0)$	$4.2-1.45\tau$	1.839	5
	$\frac{1}{6}(-3\tau-4,10\tau-12,0)$	$9.8-5.6\tau$	0.702	5
$A_1\times A_1$	$\frac{1}{3}(0,3\tau+4,5\tau-6)$	$3.4-1.9\tau$	0.243	2
	$\frac{1}{3}(5-\tau,4\tau-3,5-\tau)$	$2.9-\tau$	1.271	4
	$\frac{1}{3}(4\tau-1,5-\tau,0)$	$0.9+0.4\tau$	1.664	2
	$\frac{1}{3}(5\tau-6,0,4\tau-1)$	$4.3-1.5\tau$	1.906	4
	$\frac{1}{3}(4\tau-1,\tau-5,0)$	$0.9+0.4\tau$	1.664	2
	$\frac{1}{3}(5-\tau,3-4\tau,5-\tau)$	$2.9-\tau$	1.271	4
	$\frac{1}{3}(0,-3\tau-4,5\tau-6)$	$3.4-1.9\tau$	0.243	2
A_2	$\frac{1}{6}(0,0,12\tau-3)$	0	0	3
	$\frac{1}{6}(0,10-2\tau,2\tau+9)$	$1.9-0.7\tau$	0.847	3
	$\frac{1}{6}(10-2\tau,10\tau-12,4\tau-1)$	$3.9-1.3\tau$	1.695	6
	$\frac{1}{6}(10\tau-12,10-2\tau,1-4\tau)$	$3.9-1.3\tau$	1.695	6
	$\frac{1}{6}(10-2\tau,0,-2\tau-9)$	$1.9-0.7\tau$	0.847	3
	$\frac{1}{6}(0,0,3-12\tau)$	0	0	3

Table 14. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(1,1,0)$.

	$\mathcal{V}_{H_3}(0,1,1)$			
	Orbit $c_3(0,0,1)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$\frac{1}{11}(\frac{13\tau}{2}+3,0,8-\tau)$	$0.8-0.2\tau$	0.487	5
	$\frac{1}{11}(4-\frac{\tau}{2},0,7\tau-1)$	$0.6+0.4\tau$	1.275	5
	$\frac{1}{11}(\frac{\tau}{2}-4,0,7\tau-1)$	$0.6+0.4\tau$	1.275	5
	$\frac{1}{11}(-\frac{13\tau}{2}-3,0,8-\tau)$	$0.8-0.2\tau$	0.487	5
$A_1\times A_1$	$\frac{1}{11}(13\tau+6,0,8-\tau)$	$0.3-0.1\tau$	0.168	4
	$\frac{1}{11}(6\tau+7,7\tau-1,7\tau-1)$	$0.4+0.3\tau$	0.881	4
	$\frac{1}{11}(7\tau+1,6\tau+7,0)$	$0.4+0.5\tau$	1.154	2
	$\frac{1}{11}(0,8-\tau,7+6\tau)$	$0.6+0.4\tau$	1.322	2
	$\frac{1}{11}(-7\tau-1,6\tau+7,0)$	$0.4+0.5\tau$	1.154	2
	$\frac{1}{11}(-6\tau-7,7\tau-1,7\tau-1)$	$0.4+0.3\tau$	0.881	4
	$(\frac{1}{11}(-13\tau-6,0,8-\tau))$	$0.3-0.1\tau$	0.168	4
A_2	$\frac{9}{22}(0,0,18\tau+21)$	0	0	1
	$\frac{1}{11}(0,7\tau-1,10+8\tau)$	$0.3+0.2\tau$	0.588	3
	$\frac{1}{11}(7\tau-1,8-\tau,12\tau+14)$	$0.6+0.4\tau$	1.175	6
	$\frac{1}{11}(7\tau-1,8-\tau,-12\tau-14)$	$0.6+0.4\tau$	1.175	6
	$\frac{1}{11}(0,7\tau-1,-10-8\tau)$	$0.3+0.2\tau$	0.588	3
	$\frac{9}{22}(0,0,-18\tau-21)$	0	0	1

Table 15. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(0,1,1)$.

	$\mathcal{V}_{H_3}(1,0,1)$			
	Orbit $c_2(0,1,0)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$\frac{3}{2}(1,2-\tau,0)$	$16.3-9.8\tau$	0.475	5
	$\frac{3}{2}(\tau-1,0,\tau-1)$	$6.5-3.3\tau$	1.244	5
	$\frac{3}{2}(0,2-\tau,2-\tau)$	$29-16.8\tau$	1.719	10
	$\frac{3}{2}(1-\tau,\tau-1,0)$	$6.5-3.3\tau$	1.244	5
	$\frac{3}{2}(-1,0,2-\tau)$	$16.3-9.8\tau$	0.475	5
$A_1\times A_1$	$\frac{3}{2}(0,2,0)$	0	0	1
	$\frac{3}{2}(2-\tau,\tau,\tau-1)$	$7.9-4.5\tau$	0.594	4
	$\frac{3}{2}(1,1,2-\tau)$	$6.8-3.4\tau$	1.289	4
	$\frac{3}{2}(\tau-1,\tau-1,1)$	$3.4-1.1\tau$	1.555	4
	$\frac{3}{2}(2\tau-2,0,0)$	$9-4.5\tau$	1.719	2
	$\frac{3}{2}(0,0,2\tau-2)$	$9-4.5\tau$	1.719	2
	$\frac{3}{2}(\tau-1,1-\tau,1)$	$3.4-1.1\tau$	1.555	4
	$\frac{3}{2}(1,-1,2-\tau)$	$6.8-3.4\tau$	1.289	4
	$\frac{3}{2}(2-\tau,-\tau,-2)$	$7.9-4.5\tau$	0.594	4
	$\frac{3}{2}(0,-2,0)$	0	0	2
A_2	$\frac{3}{2}(0,2-\tau,\tau)$	$7.5-4.5\tau$	0.219	3
	$\frac{3}{2}(2-\tau,\tau-1,1)$	$6-3\tau$	1.459	6
	$\frac{3}{2}(1,0,\tau-1)$	1.5	1.5	3
	$\frac{3}{2}(\tau-1,\tau-1,0)$	$9-4.5\tau$	1.719	6
	$\frac{3}{2}(0,1,1-\tau)$	1.5	1.5	3
	$\frac{3}{2}(\tau-1,2-\tau,-1)$	$6-3\tau$	1.459	6
	$\frac{3}{2}(2-\tau,0,-\tau)$	$7.5-4.5\tau$	0.219	3
	Orbit $c_3(0,0,1)$			
H_2	$\frac{3}{22}(2+5\tau,0,8-2\tau)$	$1.8-0.8\tau$	0.612	5
	$\frac{3}{22}(4-\tau,0,6\tau-2)$	$1.1-0.3\tau$	1.599	5
	$\frac{3}{22}(\tau-4,6\tau-2,0)$	$1.1-0.3\tau$	1.599	5
	$\frac{3}{22}(-2-5\tau,8-2\tau,0)$	$1.8-0.8\tau$	0.612	5
$A_1\times A_1$	$\frac{3}{11}(0,2+5\tau,4-\tau)$	$0.6-0.3\tau$	0.211	2
	$\frac{3}{11}(3\tau-1,3+2\tau,3\tau-1)$	$0.7+0.2\tau$	1.105	4
	$\frac{3}{11}(3+2\tau,3\tau-1,0)$	$0.5+0.6\tau$	1.446	2
	$\frac{3}{11}(4-\tau,0,3+2\tau)$	$1.1+0.3\tau$	1.657	4
	$\frac{3}{11}(3+2\tau,1-3\tau,0)$	$0.5+0.6\tau$	1.446	2
	$\frac{3}{11}(3\tau-1,-3-2\tau,3\tau-1)$	$0.7+0.2\tau$	1.105	4
	$\frac{3}{11}(0,-2-5\tau,4-\tau)$	$0.6-0.3\tau$	0.211	2
A_2	$\frac{3}{22}(0,0,9+6\tau)$	0	0	1
	$\frac{3}{22}(0,6\tau-2,1+8\tau)$	$0.5+0.1\tau$	0.737	3
	$\frac{3}{22}(6\tau-2,8-2\tau,3+2\tau)$	$1+0.3\tau$	1.473	6
	$\frac{3}{22}(8-2\tau,-2+6\tau,-3-2\tau)$	$1+0.3\tau$	1.473	6
	$\frac{3}{22}(6\tau-2,0,-1-8\tau)$	$0.5+0.1\tau$	0.737	3
	$\frac{3}{22}(0,0,-9-6\tau)$	0	0	1

Table 16. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(1,0,1)$.

	$\mathcal{V}_{H_3}(1,1,1)$			
	Orbit $c_2(0,1,0)$			
G'	μ	$R^2(\mu')$	$N[R^2(\mu')]$	N_p
H_2	$\frac{1}{22}(10\tau+15, 20-5\tau, 0)$	$1.3-0.5\tau$	0.424	5
	$\frac{1}{22}(15\tau-5, 0, 15\tau-5)$	$0.8+0.2\tau$	1.110	5
	$\frac{1}{22}(0, 20\tau-5, 20\tau-5)$	$2.5-0.6\tau$	1.535	10
	$\frac{1}{22}(5-15\tau, 15\tau-5, 0)$	$0.8+0.2\tau$	1.110	5
	$\frac{1}{22}(-10\tau-15, 0, 20-5\tau)$	$1.3-0.5\tau$	0.424	5
$A_1\times A_1$	$\frac{1}{22}(0, 30+20\tau, 0)$	0	0	1
	$\frac{1}{22}(20-5\tau, 25\tau+10, 15\tau-5)$	$0.7+0.1\tau$	0.530	4
	$\frac{1}{22}(10\tau+15, 10\tau+15, 20-5\tau)$	$0.8+0.2\tau$	1.151	4
	$\frac{1}{22}(15\tau-5, 15\tau-5, 10\tau+15)$	$0.6+0.5\tau$	1.388	4
	$\frac{1}{22}(30\tau-10, 0, 0)$	$1+0.3\tau$	1.535	2
	$\frac{1}{22}(0, 0, 30\tau-10)$	$1+0.3\tau$	1.535	2
	$\frac{1}{22}(15\tau-5, 5-15\tau, 10\tau+15)$	$0.6+0.5\tau$	1.388	4
	$\frac{1}{22}(10\tau+15, 10\tau-15, 20-5\tau)$	$0.8+0.2\tau$	1.151	4
	$\frac{1}{22}(20-5\tau, -10-25\tau, 15\tau-5)$	$0.7+0.1\tau$	0.530	4
	$\frac{1}{22}(0, -30-20\tau, 0)$	0	0	1
A_2	$\frac{1}{22}(0, 20-5\tau, 25\tau+10)$	$0.6-0.2\tau$	0.195	3
	$\frac{1}{22}(20-5\tau, 15\tau-5, 10\tau+15)$	$0.7+0.2\tau$	1.023	6
	$\frac{1}{22}(10\tau+15, 0, 15\tau-5)$	$0.4+0.6\tau$	1.339	3
	$\frac{1}{22}(15\tau-5, 15\tau-5, 0)$	$1+0.3\tau$	1.535	6
	$\frac{1}{22}(0, 10\tau+15, 5-15\tau)$	$0.4+0.6\tau$	1.339	3
	$\frac{1}{22}(15\tau-5, 20-5\tau, -10\tau-15)$	$0.7+0.2\tau$	1.023	6
	$\frac{1}{22}(20-5\tau, 0, -25\tau-10)$	$0.6-0.2\tau$	0.195	3
	Orbit $c_3(0,0,1)$			
H_2	$\frac{1}{6}(10-\tau, 0, 10\tau-10)$	$8-4\tau$	1.536	5
	$\frac{1}{6}(5\tau, 0, 20-10\tau)$	$20.1-12.1\tau$	0.587	5
	$\frac{1}{6}(5\tau-10, 10\tau-10, 0)$	$8-4\tau$	1.536	5
	$\frac{1}{6}(-5\tau, 20-10\tau, 0)$	$20.1-12.1\tau$	0.587	5
$A_1\times A_1$	$\frac{1}{3}(0, 5\tau, 10-5\tau)$	$6.9-4.2\tau$	0.203	2
	$\frac{1}{3}(5\tau-5, 5, 5\tau-5)$	$5.6-2.8\tau$	1.061	4
	$\frac{1}{3}(5, 5\tau-5, 0)$	1.389	1.389	2
	$\frac{1}{3}(10-5\tau, 0, 5)$	$8.3-4.2\tau$	1.592	4
	$\frac{1}{3}(5, 5-5\tau, 0)$	1.389	1.389	2
	$\frac{1}{3}(5\tau-5, -5, 5\tau-5)$	$5.6-2.8\tau$	1.061	4
	$\frac{1}{3}(0, -5\tau, 10-5\tau)$	$6.9-4.2\tau$	0.203	2
A_2	$\frac{1}{6}(0, 0, 15)$	0	0	1
	$\frac{1}{6}(0, 10\tau-10, 10\tau-5)$	$3.7-1.9\tau$	0.707	3
	$\frac{1}{6}(10\tau-10, 20\tau-10, 5)$	$7.4-3.7\tau$	1.415	6
	$\frac{1}{6}(20-10\tau, 10\tau-10, -5)$	$7.4-3.7\tau$	1.415	6
	$\frac{1}{6}(10\tau-10, 0, 5-10\tau)$	$3.7-1.9\tau$	0.707	3
	$\frac{1}{6}(0, 0, -15)$	0	0	1

Table 17. The orbit decomposition of the polytope $\mathcal{V}_{H_3}(1,1,1)$.