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Crystal symmetry for incommensurate helical and cycloidal modulations

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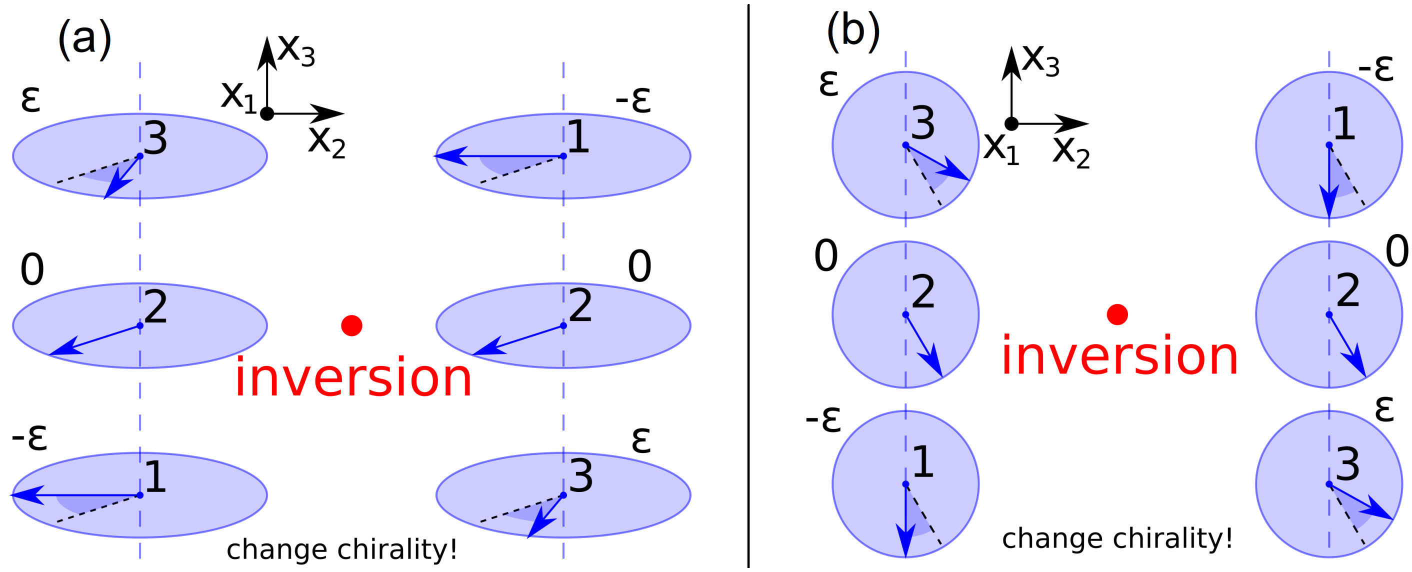

Fig. S1. Action of inversion on a helical (a) and cycloidal (b) magnetic modulation described both with the modulation vector $\mathbf{k} = [0, 0, k_i]$. The positions of magnetic ions from left column (numbered as $1, 2, 3$ located at $x_3 = -1, 0, 1$) and their magnetic moments are transformed by inversion giving magnetic ions shown in the right column: $(1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3)$. The angle between the magnetic moment direction and a reference direction in the spin rotation plane are shown (see text).

1 Examples of inversion, 2-fold axis and mirror

In this subsection the action of three representative symmetry operations: inversion, 2-fold rotation and mirror on a helical and cycloidal modulated ordering in the general elliptical case are explained in detail as given below in Figs. S1, S2 and S3, respectively. Each visualisation is followed by a calculation using eq. (6) from the main text. Let us consider a vertical column of magnetic ions located in adjacent unit cells along the modulation vector $\mathbf{k} = [0, 0, k_i]$, i.e. with coordinates $x_3 = -1, 0, 1 \ldots$. For the helical ordering, drawn in Fig. S1a, the magnetic ions can be described as $[M_{0x} \cos(2\pi k_i x_3), M_{0y} \sin(2\pi k_i x_3), 0]$, with the normal vector $\mathbf{n} = [0, 0, 1]$. We assume in general that $M_{0x} \neq M_{0y}$. For the cycliodal ordering, shown in Fig. S1b one has $[0, M_{0y} \cos(2\pi k_i x_3), M_{0z} \sin(2\pi k_i x_3)]$ with the normal vector $\mathbf{n} = [1, 0, 0]$. We assume in general that $M_{0y} \neq M_{0z}$.

The left column of magnetic ions transforms to the right column of magnetic ions but the ions in both columns belong to the same Wyckoff position. Such transformations between ions located in different 'columns' are not covered by the local symmetry analysis in [1, 2, 3].

1.1 Inversion, $\bar{1}$

The magnetic ions with their magnetic moments from the left column (numbered $1, 2, 3$) are transformed by inversion (eq. (6) from the main text) giving the magnetic ions in the right column $(1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3)$ as drawn in Fig. S1. Please note that $\det \bar{1} = -1$. The angles between the magnetic moment direction and an arbitrarily chosen reference 'zero' direction in the spin rotation plane are shown in Figs. S1a,b. For the left column these angles are: $-\epsilon, 0, \epsilon$ for ions no. $1, 2, 3$ while going up along $\mathbf{k}_i$, i.e. the magnetic modulation has positive chirality. The resulting angles for the transformed magnetic moments in the right column are: $\epsilon, 0, -\epsilon$, while going up along $\mathbf{k}_i$, i.e. negative chirality. The conclusion from Fig. S1 is that inversion changes the chirality of the magnetic modulation so a helical or cycloidal ordering with one chirality is not possible with centrosymmetric magnetic superspace groups.

With a rational part of the modulation vector i.e. $\mathbf{k}_r = (k_1, k_2, 0)$ the contribution to the scalar product is the same for all atoms in one column because the atomic coordinates x_1 and x_2 are the same in the whole column. It means that the rational part of the modulation vector introduces a phase shift of $2(k_1 x_1 + k_2 x_2)$ for inversion but it does not influence the change of chirality.

Instead of drawing the transformation by inversion one can also calculate the transformed magnetic moment components for atoms in the right column using eq. (6) from the main text. One gets $[M_x, M_y, M_z] \rightarrow [M_x, M_y, M_z]$ and the modulation vector $\mathbf{k}_i \rightarrow -\mathbf{k}_i$, so $x_4 \rightarrow -x_4$. When we start from $[\cos(2\pi x_4), \sin(2\pi x_4)]$, i.e. positive chirality, we can obtain the same chirality when the transformed modulation is either unchanged or with both signs changed i.e. $[-\cos(2\pi x_4), -\sin(2\pi x_4)]$. If only one component changes its sign: $[-\cos(2\pi x_4), \sin(2\pi x_4)]$ or $[\cos(2\pi x_4), -\sin(2\pi x_4)]$ then the transformed modulation has negative chirality. For the helical ordering shown in Fig. S1a one gets:

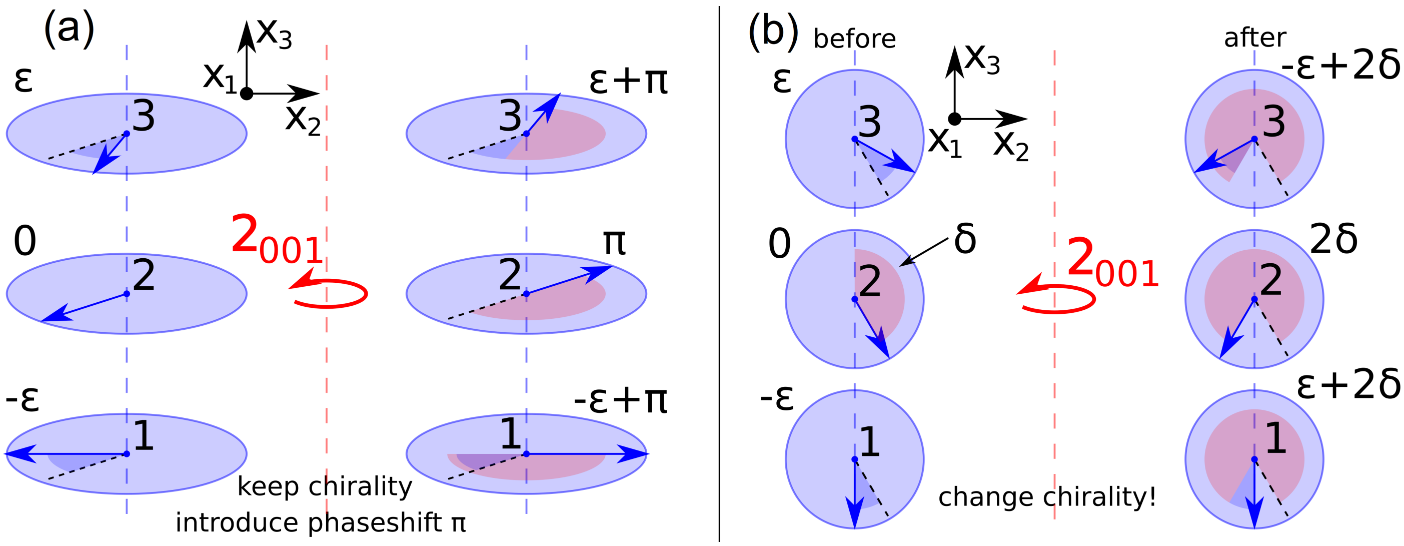

Fig. S2. Action of 2-fold rotation around [001] axis on a helical (a) and cycloidal (b) magnetic modulation described both with the modulation vector $\mathbf{k} = [0, 0, k_i]$. The positions of magnetic ions from left column (numbered as $1, 2, 3$ located at $x_3 = -1, 0, 1$) and their magnetic moments are transformed by 2_{001} giving magnetic ions shown in the right column: $(1 \to 1, 2 \to 2, 3 \to 3)$. The angle between the magnetic moment direction and a reference direction in the spin rotation plane, i.e. $-\epsilon, 0, \epsilon$ are shown (see text). The angle between the magnetic moment of the "spin0" ion (no. 2 left column in both panels) and the rotation axis direction is denoted as δ.

$$\begin{bmatrix} M_{0x} \cos(2\pi x_4) \\ M_{0y} \sin(2\pi x_4) \\ 0 \end{bmatrix} \to \begin{bmatrix} M_{0x} \cos(-2\pi x_4) \\ M_{0y} \sin(-2\pi x_4) \\ 0 \end{bmatrix} = \begin{bmatrix} M_{0x} \cos(2\pi x_4) \\ -M_{0y} \sin(2\pi x_4) \\ 0 \end{bmatrix}, \tag{1}$$

while for the cycloidal ordering shown in Fig. S1b:

$$\begin{bmatrix} 0 \\ M_{0y} \cos(2\pi x_4) \\ M_{0z} \sin(2\pi x_4) \end{bmatrix} \to \begin{bmatrix} 0 \\ M_{0y} \cos(-2\pi x_4) \\ M_{0z} \sin(-2\pi x_4) \end{bmatrix} = \begin{bmatrix} 0 \\ M_{0y} \cos(2\pi x_4) \\ -M_{0z} \sin(2\pi x_4) \end{bmatrix}, \tag{2}$$

i.e. both orderings change chirality due to inversion.

1.2 2-fold rotation, 2_{001}

Next one can consider the action of 2-fold rotation along the axis parallel to the irrational component of the modulation vector $\mathbf{k}_i$ as shown in Figs. S2a,b for helical and cycloidal modulations respectively. The magnetic ions (numbered $1, 2, 3$ are located at $z_3 = -1, 0, 1$) shown in the left column have the same magnetic moments arrangement as in the previous example, see Figs. S1. The transformation by 2-fold rotation around [001], denoted as 2_{001} (eq. (6) from the main text) gives the magnetic ions in the right column, $(1 \to 1, 2 \to 2, 3 \to 3)$, see Fig. S2. The angle between the magnetic moment of the atom no. 2 (left column in panel b) and the rotation axis is denoted as δ. For the helical ordering the resulting angles for the transformed magnetic moments in the right column are: $-\epsilon + \pi, \pi, \epsilon + \pi$, while going up along $\mathbf{k}_i$, i.e. positive chirality (same as in the left column) with a phase shift by π. For the cycloidal ordering the resulting angles for the transformed magnetic moments in the right column are: $\epsilon + 2\delta, 2\delta, -\epsilon + 2\delta$, while going up along $\mathbf{k}_i$, so there is a negative chirality.

For the 2-fold rotation 2_{001} from eq. (6) from the main text we get $[M_x, M_y, M_z] \to [-M_x, -M_y, M_z]$ and the modulation vector $\mathbf{k}_i \to \mathbf{k}_i$, so $x_4 \to x_4$. For the helical ordering shown in Fig. S2a by 2_{001} one gets:

$$\begin{bmatrix} M_{0x} \cos(2\pi x_4) \\ M_{0y} \sin(2\pi x_4) \\ 0 \end{bmatrix} \to \begin{bmatrix} M_{0x}(-1) \cos(2\pi x_4) \\ M_{0y}(-1) \sin(2\pi x_4) \\ 0 \end{bmatrix}, \tag{3}$$

corresponding to the positive chirality. For the cycloidal ordering shown in Fig. S2b by 2_{001}:

$$\begin{bmatrix} 0 \\ M_{0y} \cos(2\pi x_4) \\ M_{0z} \sin(2\pi x_4) \end{bmatrix} \to \begin{bmatrix} 0 \\ M_{0y}(-1) \cos(2\pi x_4) \\ M_{0z} \sin(2\pi x_4) \end{bmatrix}, \tag{4}$$

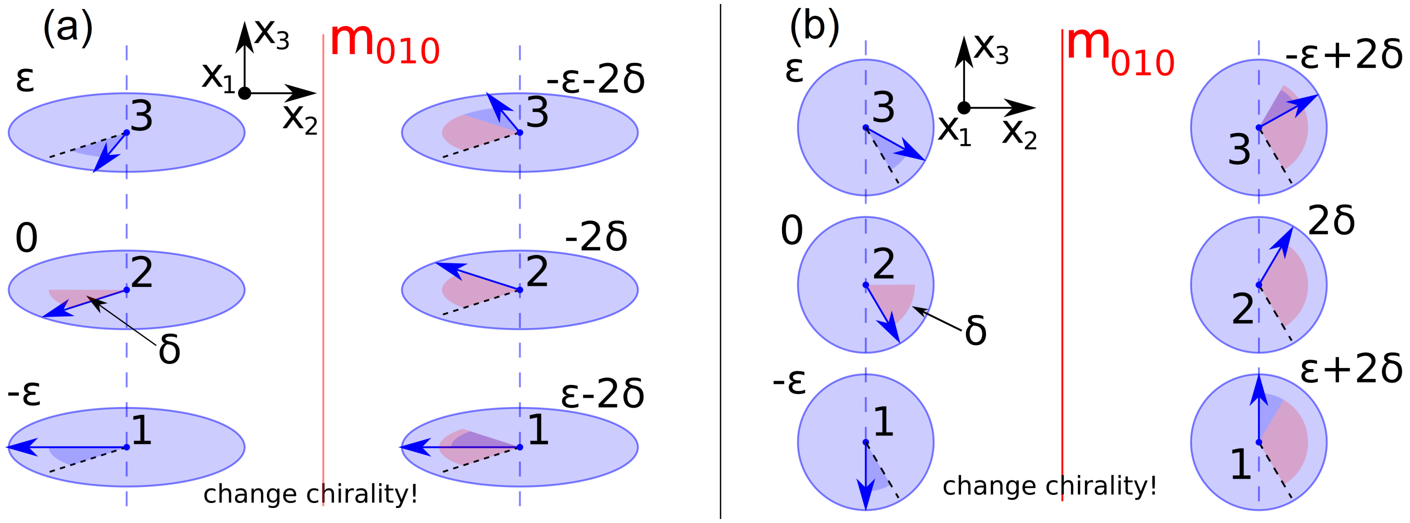

Fig. S3. Action of a mirror in the (010) plane on a helical (a) and cycloidal (b) magnetic modulation described both with the modulation vector $\mathbf{k} = [0, 0, k_i]$. The positions of magnetic ions from left column (numbered as $1, 2, 3$) and their magnetic moments are transformed by m_{010} giving magnetic ions shown in the right column: $(1 \to 1, 2 \to 2, 3 \to 3)$. The angle between the magnetic moment direction and a reference direction in the spin rotation plane are shown (see text). The angle between the magnetic moment of the "spin0" ion, no. 2 in left column in panel (b), and the axis perpendicular to the mirror plane: [010] is denoted as δ.

one gets the negative chirality.

1.3 Mirror, m_{010}

In the last example one can consider the action of a (010) mirror plane as shown in Figs. S3a,b for helical and cycloidal modulations respectively. The magnetic ions (numbered $1, 2, 3$ are located at $z_3 = -1, 0, 1$) shown in the left column have the same magnetic moments arrangement as in the previous example, see Figs. S1. The transformation by m_{010} (eq. (8) from the main text) gives the magnetic ions in the right column, (see Fig. S3: $1 \to 1, 2 \to 2, 3 \to 3$). For the helical ordering the resulting angles for the transformed magnetic moments in the right column are: $\epsilon - 2\delta, -2\delta, -\epsilon - 2\delta$ while going up along $\mathbf{k}_i$, i.e. negative chirality. For the cycloidal ordering the resulting angles for the transformed magnetic moments in the right column are: $\epsilon + 2\delta, 2\delta, -\epsilon + 2\delta$, i.e. also negative chirality.

For the mirror m_{010} from eq. (6) from the main text we get $[M_x, M_y, M_z] \to [-M_x, M_y, -M_z]$ (because mirrors have determinant -1) and the modulation vector $\mathbf{k}_i \to \mathbf{k}_i$, so $x_4 \to x_4$. For the helical ordering shown in Fig. S3a one gets:

$$\begin{bmatrix} M_{0x} \cos(2\pi x_4) \\ M_{0y} \sin(2\pi x_4) \\ 0 \end{bmatrix} \to \begin{bmatrix} M_{0x}(-1) \cos(2\pi x_4) \\ M_{0y}(+1) \sin(2\pi x_4) \\ 0 \end{bmatrix}, \tag{5}$$

while for the cycloidal ordering shown in Fig. S3b:

$$\begin{bmatrix} 0 \\ M_{0y} \cos(2\pi x_4) \\ M_{0z} \sin(2\pi x_4) \end{bmatrix} \to \begin{bmatrix} 0 \\ M_{0y}(+1) \cos(2\pi x_4) \\ M_{0z}(-1) \sin(2\pi x_4) \end{bmatrix}, \tag{6}$$

i.e. both orderings change chirality due to m_{010}.

2 Magnetic superspace groups which allow helical and cycloidal magnetic ordering

In this section one can find superspace groups which are compatible with the helical and cycloidal modulation. Tables S1 and S2 below contain all 'one-colour' (Type I, i.e. without antisymmetry ($'$)) magnetic superspace groups (in standard settings) which are build only from operators from Tables 2 and 3 from the main text. For practical magnetic structure search one should consider also all the magnetic superspace groups derived from the 'one-colour' group given below. This task can be done by using the routine MGENPOS from Bilbao Crystallographic Server [4] or using the ISOTROPY software suite [5, 6].

Columns "FM", "AFM" and "other" in Tables S1 and S2 indicate if given group is compatible with FM-type, AFM-type and other than FM and AFM modulation, respectively. We point out that mulitplication of one half of operation by $\{1'|0001/2\}$ changes the non-modulated part while keeping unchanged the magnetic modulation i.e. FM-type modulations remains FM-type, AFM-type remains AFM-type and 'other than FM and AFM'-type also does not change which we will denote as FM→FM, AFM→AFM, other→other. There are three ways to obtain magnetic superspace groups compatible with helical or cycloidal ordering from 'one-colour' (Type I) magnetic superspace groups:

- adding $\{1'|0001/2\}$ as new generator produce Type II groups which give FM→FM, AFM→AFM, other→other,
- mulitplication of one half of the group operations by $\{1'|0001/2\}$ produce Type III groups which give FM→FM, AFM→AFM, other→other,
- adding magnetic centering produce Type IV groups which give FM→AFM, AFM→AFM, other→other (type IV groups are not compatible with FM-type both modulated and non-modulated ordering).

If a helical FM ordering is possible in the general Wyckoff position than it is also possible in all Wyckoff positions of the magnetic superspace group. Modulations other than FM may not be possible in all Wyckoff positions because symmetry may restrict the magnetic moments to zero. Column 'Equiv.' shows equivalent sets of translations in internal coordinate as taken from [7]. Equivalent superspace groups are given in multiple rows. Their equivalence is based on a change of the modulation length by adding or subtracting a rational number, see e.g. [7]. Equivalent groups are duplicated in tables, because we assume fixed length of modulation vector.

Allowing for rational component of modulation vector, $\mathbf{k}_r$ will give magnetic superspace groups which keep chirality but classification to FM, AFM and other types are more difficult, because in that case relative phase between spin chains depend on position x_1, x_2, x_3 of magnetic moment in the unit cell.

Magnetic superspace groups which allow helical magnetic ordering

Table S1. HELIX - triclinic class 1 and monoclinic class 2 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$1(\alpha\beta\gamma)$	1.1	$P1(\alpha\beta\gamma)$		YES	no	no
$2(\alpha\beta0)0$	3.1 4.1 5.1	$P2(\alpha\beta0)0$ $P2_1(\alpha\beta0)0$ $B2(\alpha\beta0)0$		YES	YES	YES
$2(00\gamma)0$	3.3 4.2 5.2	$P2(00\gamma)0$ $P2_1(00\gamma)0$ $B2(00\gamma)0$	 0,s 	no	YES	no
$2(00\gamma)s$	3.4 4.2 5.3	$P2(00\gamma)s$ $P2_1(00\gamma)s$ $B2(00\gamma)s$	 0,s 	YES	no	no

Table S1. HELIX (continued) - orthorhombic class 222 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$222(00\gamma)000$	16.1	$P222(00\gamma)000$		no	YES	YES
	17.1	$P222_1(00\gamma)000$	000,00s			
	17.4	$P2_122(00\gamma)000$				
	18.1	$P2_12_12(00\gamma)000$				
	18.3	$P2_122_1(00\gamma)000$	000,00s			
	19.1	$P2_12_12_1(00\gamma)000$	000,00s			
	20.1	$C222_1(00\gamma)000$	000,00s			
	20.3	$A2_122(00\gamma)000$				
	21.1	$C222(00\gamma)000$				
	21.5	$A222(00\gamma)000$				
	22.1	$F222(00\gamma)000$				
	23.1	$I222(00\gamma)000$				
	24.1	$I2_12_12_1(00\gamma)000$				
$222(00\gamma)00s$	16.2	$P222(00\gamma)00s$		YES	YES	YES
	17.1	$P222_1(00\gamma)00s$	000,00s			
	17.5	$P2_122(00\gamma)00s$				
	18.2	$P2_12_12(00\gamma)00s$				
	18.3	$P2_122_1(00\gamma)00s$	000,00s			
	19.1	$P2_12_12_1(00\gamma)00s$	000,00s			
	20.1	$C222_1(00\gamma)00s$	000,00s			
	20.4	$A2_122(00\gamma)00s$				
	21.2	$C222(00\gamma)00s$				
	21.6	$A222(00\gamma)00s$				
	22.2	$F222(00\gamma)00s$				
	23.2	$I222(00\gamma)00s$				
	24.2	$I2_12_12_1(00\gamma)00s$				

Table S1. HELIX (continued) - tetragonal class 4 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$4(00\gamma)0$	75.1	$P4(00\gamma)0$		no	no	YES
	76.1	$P4_1(00\gamma)0$	0,s,q			
	77.1	$P4_2(00\gamma)0$	0,s			
	78.1	$P4_3(00\gamma)0$	0,s,q			
	79.1	$I4(00\gamma)0$				
	80.1	$I4_1(00\gamma)0$	0,s			
$4(00\gamma)s$	75.3	$P4(00\gamma)s$		no	no	YES
	76.1	$P4_1(00\gamma)s$	0,s,q			
	77.1	$P4_2(00\gamma)s$	0,s			
	78.1	$P4_3(00\gamma)s$	0,s,q			
	79.3	$I4(00\gamma)s$				
	80.1	$I4_1(00\gamma)s$	0,s			
$4(00\gamma)q$	75.2	$P4(00\gamma)q$		YES	YES	no
	76.1	$P4_1(00\gamma)q$	0,s,q			
	77.2	$P4_2(00\gamma)q$				
	78.1	$P4_3(00\gamma)q$	0,s,q			
	79.2	$I4(00\gamma)q$				
	80.2	$I4_1(00\gamma)q$				

Table S1. HELIX (continued) - tetragonal class 422 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
	89.1	P422(00γ)000				
	90.1	P42$_1$2(00γ)000				
	91.1	P4$_1$22(00γ)000	000,s00,q00			
	92.1	P4$_1$2$_1$2(00γ)000	000,s00,q00			
422(00γ)000	93.1	P4$_2$22(00γ)000	000,s00	no	no	YES
	94.1	P4$_2$2$_1$2(00γ)000	000,s00			
	95.1	P4$_3$22(00γ)000	000,s00,q00			
	96.1	P4$_3$2$_1$2(00γ)000	000,s00,q00			
	97.1	I422(00γ)000				
	98.1	I4$_1$22(00γ)000	000,s00			
	89.3	P422(00γ)s00				
	90.3	P42$_1$2(00γ)s00				
	91.1	P4$_1$22(00γ)s00	000,s00,q00			
	92.1	P4$_1$2$_1$2(00γ)s00	000,s00,q00			
422(00γ)s00	93.1	P4$_2$22(00γ)s00	000,s00	no	no	YES
	94.1	P4$_2$2$_1$2(00γ)s00	000,s00			
	95.1	P4$_3$22(00γ)s00	000,s00,q00			
	96.1	P4$_3$2$_1$2(00γ)s00	000,s00,q00			
	97.3	I422(00γ)s00				
	98.1	I4$_1$22(00γ)s00	000,s00			
	89.2	P422(00γ)q00				
	90.2	P42$_1$2(00γ)q00				
	91.1	P4$_1$22(00γ)q00	000,s00,q00			
	92.1	P4$_1$2$_1$2(00γ)q00	000,s00,q00			
422(00γ)q00	93.2	P4$_2$22(00γ)q00		YES	YES	YES
	94.2	P4$_2$2$_1$2(00γ)q00				
	95.1	P4$_3$22(00γ)q00	000,s00,q00			
	96.1	P4$_3$2$_1$2(00γ)q00	000,s00,q00			
	97.2	I422(00γ)q00				
	98.2	I4$_1$22(00γ)q00				

Table S1. HELIX (continued) - trigonal class 3 and 32 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
	143.2	P3(00γ)0				
3(00γ)0	144.2	P3$_1$(00γ)0	0,t	no	no	YES
	145.2	P3$_2$(00γ)0	0,t			
	146.1	R3(00γ)0				
	143.3	P3(00γ)t				
3(00γ)t	144.2	P3$_1$(00γ)t	0,t	YES	no	YES
	145.2	P3$_2$(00γ)t	0,t			
	146.2	R3(00γ)t				
	149.2	P312(00γ)000				
	150.1	P321(00γ)000				
	151.2	P3$_1$12(00γ)000	000,t00			
32(00γ)000	152.1	P3$_1$21(00γ)000	000,t00	no	no	YES
	153.2	P3$_2$12(00γ)000	000,t00			
	154.1	P3$_2$21(00γ)000	000,t00			
	155.1	R32(00γ)00				
	149.3	P312(00γ)t00				
	150.2	P321(00γ)t00				
	151.2	P3$_1$12(00γ)t00	000,t00			
32(00γ)t00	152.1	P3$_1$21(00γ)t00	000,t00	YES	YES	YES
	153.2	P3$_2$12(00γ)t00	000,t00			
	154.1	P3$_2$21(00γ)t00	000,t00			
	155.2	R32(00γ)t0				

Table S1. HELIX (continued) - hexagonal class 6 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$6(00\gamma)0$	168.1	$P6(00\gamma)0$		no	no	YES
	169.1	$P6_1(00\gamma)0$	0,h,t,s			
	170.1	$P6_5(00\gamma)0$	0,h,t,s			
	171.1	$P6_2(00\gamma)0$	0,t			
	172.1	$P6_4(00\gamma)0$	0,t			
	173.1	$P6_3(00\gamma)0$	0,s			
$6(00\gamma)$s	168.4	$P6(00\gamma)$s		no	no	YES
	169.1	$P6_1(00\gamma)$s	0,h,t,s			
	170.1	$P6_5(00\gamma)$s	0,h,t,s			
	171.2	$P6_2(00\gamma)$s	h,s			
	172.2	$P6_4(00\gamma)$s	h,s			
	173.1	$P6_3(00\gamma)$s	0,s			
$6(00\gamma)$h	168.2	$P6(00\gamma)$h		YES	no	YES
	169.1	$P6_1(00\gamma)$h	0,h,t,s			
	170.1	$P6_5(00\gamma)$h	0,h,t,s			
	171.2	$P6_2(00\gamma)$h	h,s			
	172.2	$P6_4(00\gamma)$h	h,s			
	173.2	$P6_3(00\gamma)$h	h,t			
$6(00\gamma)$t	168.3	$P6(00\gamma)$t		no	YES	YES
	169.1	$P6_1(00\gamma)$t	0,h,t,s			
	170.1	$P6_5(00\gamma)$t	0,h,t,s			
	171.1	$P6_2(00\gamma)$t	0,t			
	172.1	$P6_4(00\gamma)$t	0,t			
	173.2	$P6_3(00\gamma)$t	h,t			

Table S1. HELIX (continued) - hexagonal class 622 superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$622(00\gamma)000$	177.1	$P622(00\gamma)000$		no	no	YES
	178.1	$P6_122(00\gamma)000$	000,h00,t00,s00			
	179.1	$P6_522(00\gamma)000$	000,h00,t00,s00			
	180.1	$P6_222(00\gamma)000$	000,t00			
	181.1	$P6_422(00\gamma)000$	000,t00			
	182.1	$P6_322(00\gamma)000$	000,s00			
$622(00\gamma)$s00	177.4	$P622(00\gamma)$s00		no	no	YES
	178.1	$P6_122(00\gamma)$s00	000,h00,t00,s00			
	179.1	$P6_522(00\gamma)$s00	000,h00,t00,s00			
	180.2	$P6_222(00\gamma)$s00	h00,s00			
	181.2	$P6_422(00\gamma)$s00	h00,s00			
	182.1	$P6_322(00\gamma)$s00	000,s00			
$622(00\gamma)$h00	177.2	$P622(00\gamma)$h00		YES	YES	YES
	178.1	$P6_122(00\gamma)$h00	000,h00,t00,s00			
	179.1	$P6_522(00\gamma)$h00	000,h00,t00,s00			
	180.2	$P6_222(00\gamma)$h00	h00,s00			
	181.2	$P6_422(00\gamma)$h00	h00,s00			
	182.2	$P6_322(00\gamma)$h00	h00,t00			
$622(00\gamma)$t00	177.3	$P622(00\gamma)$t00		no	YES	YES
	178.1	$P6_122(00\gamma)$t00	000,h00,t00,s00			
	179.1	$P6_522(00\gamma)$t00	000,h00,t00,s00			
	180.1	$P6_222(00\gamma)$t00	000,t00			
	181.1	$P6_422(00\gamma)$t00	000,t00			
	182.2	$P6_322(00\gamma)$h00	h00,t00			

Magnetic superspace groups which allow cycloidal magnetic ordering

Table S2. CYCLOID - triclinic class 1 and monoclinic class 2 and class m superspace groups

Class	No.	Symbol	Equiv.	FM	AFM	other
$1(\alpha\beta\gamma)$	1.1	$P1(\alpha\beta\gamma)$		YES	no	no
$2(\alpha\beta0)0$	3.1	$P2(\alpha\beta0)0$		YES	YES	YES
	4.1	$P2_1(\alpha\beta0)0$				
	5.1	$B2(\alpha\beta0)0$				
$m(\alpha\beta0)0$	6.1	$Pm(\alpha\beta0)0$		no	YES	no
	7.1	$Pb(\alpha\beta0)0$	0,s			
	8.1	$Bm(\alpha\beta0)0$				
	9.1	$Bb(\alpha\beta0)0$	0,s			
$m(\alpha\beta0)s$	6.2	$Pm(\alpha\beta0)s$		YES	no	no
	7.1	$Pb(\alpha\beta0)s$	0,s			
	8.2	$Bm(\alpha\beta0)s$				
	9.1	$Bb(\alpha\beta0)s$	0,s			
$m(00\gamma)0$	6.4	$Pm(00\gamma)0$		YES	YES	YES
	7.3	$Pb(00\gamma)0$				
	8.3	$Bm(00\gamma)0$				
	9.2	$Bb(00\gamma)0$				

Table S2. CYCLOID (continued) - orthorhombic class $mm2$ superspace groups, part 1/2

Class	No.	Symbol	Equiv.	FM	AFM	other
2mm(00γ)000 and m2m(00γ)000	25.9	P2mm(00γ)000		no	YES	YES
	26.7	P2$_1$am(00γ)000				
	26.9	P2$_1$ma(00γ)000				
	27.5	P2aa(00γ)000				
	28.10	P2cm(00γ)000	000,0s0			
	28.11	P2mb(00γ)000				
	29.4	P2$_1$ca(00γ)000	000,0s0			
	29.5	P2$_1$ab(00γ)000				
	30.4	P2na(00γ)000	000,0s0			
	30.5	P2an(00γ)000				
	31.5	P2$_1$nm(00γ)000	000,0s0			
	31.6	P2$_1$mn(00γ)000				
	32.6	P2cb(00γ)000	000,0s0			
	33.4	P2$_1$nb(00γ)000	000,0s0			
	33.5	P2$_1$cn(00γ)000	000,0s0			
	34.4	P2nn(00γ)000	000,0s0			
	35.7	A2mm(00γ)000				
	36.5	A2$_1$am(00γ)000				
	36.7	A2$_1$ma(00γ)000				
	37.5	A2aa(00γ)000				
	38.1	C2mm(00γ)000				
	38.11	Am2m(00γ)000				
	39.1	C2mb(00γ)000				
	39.11	Ac2m(00γ)000				
	40.1	C2cm(00γ)000	000,0s0			
	40.7	Am2a(00γ)000				
	41.1	C2cb(00γ)000	000,0s0			
	41.7	Ac2a(00γ)000				
	42.7	F2mm(00γ)000				
	43.3	F2dd(00γ)000	000,0s0			
	44.4	I2mm(00γ)000				
	45.4	I2cb(00γ)000				
	46.5	I2mb(00γ)000				
	46.7	I2cm(00γ)000				

Table S2. CYCLOID (continued) - orthorhombic class $mm2$ superspace groups, part 2/2

Class	No.	Symbol	Equiv.	FM	AFM	other
2mm(00γ)0s0 and m2m(00γ)s00	25.10	P2mm(00γ)0s0		YES	YES	YES
	26.8	P2$_1$am(00γ)0s0				
	26.10	P2$_1$ma(00γ)0s0				
	27.6	P2aa(00γ)0s0				
	28.10	P2cm(00γ)0s0	000,0s0			
	28.12	P2mb(00γ)0s0				
	29.4	P2$_1$ca(00γ)0s0	000,0s0			
	29.6	P2$_1$ab(00γ)0s0				
	30.4	P2na(00γ)0s0	000,0s0			
	30.6	P2an(00γ)0s0				
	31.5	P2$_1$nm(00γ)0s0	000,0s0			
	31.7	P21mn(00γ)0s0				
	32.6	P2cb(00γ)0s0	000,0s0			
	33.4	P2$_1$nb(00γ)0s0	000,0s0			
	33.5	P2$_1$cn(00γ)0s0	000,0s0			
	34.4	P2nn(00γ)0s0	000,0s0			
	35.8	A2mm(00γ)0s0				
	36.6	A2$_1$am(00γ)0s0				
	36.8	A2$_1$ma(00γ)0s0				
	37.6	A2aa(00γ)0s0				
	38.2	C2mm(00γ)0s0				
	38.12	Am2m(00γ)s00				
	39.2	C2mb(00γ)0s0				
	39.12	Ac2m(00γ)s00				
	40.1	C2cm(00γ)0s0	000,0s0			
	40.8	Am2a(00γ)s00				
	41.1	C2cb(00γ)0s0	000,0s0			
	41.8	Ac2a(00γ)s00				
	42.8	F2mm(00γ)0s0				
	43.3	F2dd(00γ)0s0	000,0s0			
	44.5	I2mm(00γ)0s0				
	45.5	I2cb(00γ)0s0				
	46.6	I2mb(00γ)0s0				
	46.8	I2cm(00γ)0s0				

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