In[1]:= `SetDirectory["/Users/salvino/Desktop/WORK_IN_PRGS/PLATONIC_SOLD_CF_APPRXMTN"];`
`Directory[]`

Out[2]= /Users/salvino/Desktop/WORK_IN_PRGS/PLATONIC_SOLD_CF_APPRXMTN

Merano April/June 2020

We evaluate the approximae CF of a tetrahedron by the formulae reported in ANALYTIC_APPROXMTN_CF_CLD_BB.nb

The octahedron CF and its first two derivatives have been copied from the file contained in the folder "/STOCHASTICS/PLATONIC_CF"
stored in the memory of 1 Tb.

TETRAHEDRON CF

COPYED FROM TETRAHEDRON/Tetrahedron_CF _FNL with the renaming CFAA[r] -> TetrCFAA[r] etc

$$\text{TetrDist} = \left\{0, \frac{1}{\sqrt{2}}, \sqrt{\frac{2}{3}}, \frac{\sqrt{3}}{2}, 1\right\};$$

$$\alpha\text{Tetr} = \text{ArcCos}[1/3];\ \text{STetr} = \sqrt{3};\ \text{VTetr} = 1\Big/\left(6*\sqrt{2}\right);$$

`Theta[x_] := If[x > 0, 1, 0];`

$$(*\ \text{IF}\ 0 < r < \frac{1}{\sqrt{2}}\ *)$$

$$\text{TetrCFAA}[r\_] := 1 - 3\sqrt{\frac{3}{2}}\, r - \frac{\left(6 + 5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2 \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$(*\ \text{TetrDCFAA}[r\_]:=\text{Simplify}\left[D[\text{TetrCFAA}[r],r],\text{Assumptions}\to\left\{0<r<\frac{1}{\sqrt{2}}\right\}\right];$$

$$\text{TetrDDCFAA}[r\_]:=\text{Simplify}\left[D[\text{TetrDCFAA}[r],r],\text{Assumptions}\to\left\{0<r<\frac{1}{\sqrt{2}}\right\}\right];\ *)$$

$$\text{TetrDCFAA}[r\_] := -3\sqrt{\frac{3}{2}} - \frac{3\left(6 + 5\sqrt{3}\,\pi\right) r^2}{4\sqrt{2}\,\pi} + \frac{6 r \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TetrDDCFAA}[r\_] := -\frac{3\left(6\sqrt{2}\, r + \pi\left(-8 + 5\sqrt{6}\, r\right) + 8\left(-2\sqrt{2} + \text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

$$(*\ \text{IF}\ \frac{1}{\sqrt{2}} < r < \sqrt{\frac{2}{3}}\ *)\text{TetrCFBB}[r\_] :=$$

$$-2+\frac{3}{4\sqrt{2}\ r}-\frac{3\left(-3+\sqrt{3}\right)r}{\sqrt{2}}-\frac{\left(6-12\pi+5\sqrt{3}\ \pi\right)r^{3}}{4\sqrt{2}\ \pi}+\frac{3r^{2}\left(2\sqrt{2}-\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

```
(* TetrDCFBB[r_]:=Simplify[D[TetrCFBB[r],r],Assumptions→{1/√2<r<√(2/3)}];

TetrDDCFBB[r_]:=Simplify[D[TetrDCFBB[r],r],Assumptions→{1/√2<r<√(2/3)}]; *)
TetrDCFBB[r_] :=
```

$$\frac{3}{8}\left(-4\sqrt{2}\left(-3+\sqrt{3}\right)-\frac{\sqrt{2}}{r^{2}}-\frac{\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r^{2}}{\pi}-\frac{16r\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\text{TetrDDCFBB[r\_]} := \frac{3}{8}\left(\frac{2\sqrt{2}}{r^{3}}-\frac{2\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r}{\pi}-\frac{16\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

```
(* IF √(2/3) < r < √3/2 *)TetrCFCC[r_] :=
```

$$-6+\frac{9+8\sqrt{3}}{12\sqrt{2}\ r}+\frac{3\left(3+\sqrt{3}\right)r}{\sqrt{2}}+\frac{\left(-6+12\pi+\sqrt{3}\ \pi\right)r^{3}}{4\sqrt{2}\ \pi}+\frac{3r^{2}\left(2\sqrt{2}-3\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

```
(* TetrDCFCC[r_]:=Simplify[D[TetrCFCC[r],r],Assumptions→{√(2/3) < r < √3/2}];

TetrDDCFCC[r_]:=Simplify[D[TetrDCFCC[r],r],Assumptions→{√(2/3) < r < √3/2}]; *)
TetrDCFCC[r_] :=
```

$$\frac{3\left(3+\sqrt{3}\right)}{\sqrt{2}}-\frac{9+8\sqrt{3}}{12\sqrt{2}\ r^{2}}+\frac{3\left(-6+\left(12+\sqrt{3}\right)\pi\right)r^{2}}{4\sqrt{2}\ \pi}-\frac{6r\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

```
TetrDDCFCC[r_] :=
```

$$\frac{1}{12}\left(\frac{\sqrt{2}\left(9+8\sqrt{3}\right)}{r^{3}}+\frac{9\sqrt{2}\left(-6+\left(12+\sqrt{3}\right)\pi\right)r}{\pi}-\frac{72\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

```
(* IF √3/2 < r < 1 *)
```

TetrCFDD[r_] :=

$$3+\frac{3}{8\sqrt{2}\,r}+\frac{1}{\sqrt{6}\,r}+\frac{21}{2}\sqrt{\frac{3}{2}}\,r+\frac{9\,r}{\sqrt{2}}-3\,r^2+\frac{6\sqrt{2}\,r^2}{\pi}-\frac{1}{2}\sqrt{\frac{3}{2}}\,r^3+$$

$$3\sqrt{2}\,r^3-\frac{3\,r^3}{2\sqrt{2}\,\pi}-\frac{21\,r\sqrt{-6+8\,r^2}}{4\,\pi}-\frac{3\,r^2\,\mathrm{ArcCos}\left[\frac{1}{3}\right]}{\pi}-\frac{9\sqrt{6}\,r\,\mathrm{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4\,r^2}{3}}}\right]}{\pi}+$$

$$\frac{3\sqrt{\frac{3}{2}}\,r^3\,\mathrm{ArcTan}\left[\sqrt{-1+\frac{4\,r^2}{3}}\right]}{2\,\pi}-\frac{9\,\mathrm{ArcTan}\left[\frac{r}{\sqrt{-\frac{3}{2}+2\,r^2}}\right]}{\pi}+\frac{3\sqrt{\frac{3}{2}}\,r\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4\,r^2}}\right]}{2\,\pi}+$$

$$\frac{3\,\mathrm{ArcTan}\left[\frac{3+2\sqrt{2}\,r}{\sqrt{-3+4\,r^2}}\right]}{\pi}+\frac{3\sqrt{\frac{3}{2}}\,r\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4\,r^2}}\right]}{2\,\pi}-\frac{15\sqrt{2}\,r\,\mathrm{ArcTan}\left[\sqrt{-3+4\,r^2}\right]}{\pi}+$$

$$\frac{3\,\mathrm{ArcTan}\left[\frac{\sqrt{-3+4\,r^2}}{2-2\,r^2}\right]}{2\,\pi}-\frac{3\,r\,\mathrm{ArcTan}\left[\frac{\sqrt{-3+4\,r^2}}{2-2\,r^2}\right]}{\sqrt{2}\,\pi}-\frac{12\,\mathrm{ArcTan}\left[\frac{r}{\sqrt{-6+8\,r^2}}\right]}{\pi}+\frac{6\,r^2\,\mathrm{ArcTan}\left[\frac{r\,(-5+6\,r^2)}{\sqrt{-6+8\,r^2}}\right]}{\pi}-$$

$$\frac{\sqrt{\frac{2}{3}}\,\mathrm{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\sqrt{-9+12\,r^2}}\right]}{\pi\,r}-\frac{3\sqrt{\frac{3}{2}}\,r^3\,\mathrm{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\sqrt{-9+12\,r^2}}\right]}{2\,\pi}-$$

$$\frac{15\sqrt{\frac{3}{2}}\,r\,\mathrm{ArcTan}\left[\sqrt{-9+12\,r^2}\right]}{\pi}-\frac{3\sqrt{\frac{3}{2}}\,r^3\,\mathrm{ArcTan}\left[\frac{\sqrt{-9+12\,r^2}}{-3+2\,r^2}\right]}{\pi}+$$

$$\frac{3\,\mathrm{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\,(-1+r^2)\sqrt{-3+4\,r^2}}\right]}{4\sqrt{2}\,\pi\,r}+\frac{6\,r^2\,\mathrm{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\,(-1+r^2)\sqrt{-6+8\,r^2}}\right]}{\pi}+\frac{6\sqrt{2}\,r^3\,\mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\right]}{\pi};$$

(* TetrDCFDD[r_]:=Simplify[D[TetrCFDD[r],r],Assumptions→{ $\frac{\sqrt{3}}{2}$ < r < 1}];

TetrDDCFDD[r_]:=Simplify[D[TetrDCFDD[r],r],Assumptions→{ $\frac{\sqrt{3}}{2}$ < r < 1}]; *)

TetrDCFDD[r_] :=

$$-\Bigg(\left(2-3\,r^2\right)^2\Bigg(-1404\sqrt{2}\,r^2-5616\,r^3-936\sqrt{2}\,r^4+7488\,r^5+3744\sqrt{2}\,r^6+8\sqrt{6}\,\pi\sqrt{-3+4\,r^2}+$$

$$36\,\pi\,r\sqrt{-3+4\,r^2}-236\sqrt{6}\,\pi\,r^2\sqrt{-3+4\,r^2}-576\,\pi\,r^3\sqrt{-3+4\,r^2}-2304\,r^4\sqrt{-3+4\,r^2}-$$

$$468\sqrt{6}\,\pi r^4\sqrt{-3+4r^2}+432r^5\sqrt{-3+4r^2}-1152\pi r^5\sqrt{-3+4r^2}+$$
$$72\sqrt{6}\,\pi r^6\sqrt{-3+4r^2}+9\pi\sqrt{-6+8r^2}-198\pi r^2\sqrt{-6+8r^2}-576r^3\sqrt{-6+8r^2}+$$
$$108r^4\sqrt{-6+8r^2}-288\pi r^4\sqrt{-6+8r^2}-1152r^5\sqrt{-6+8r^2}+216r^6\sqrt{-6+8r^2}-$$
$$864\pi r^6\sqrt{-6+8r^2}+32\pi r\sqrt{-9+12r^2}-1008\pi r^3\sqrt{-9+12r^2}+144\pi r^5\sqrt{-9+12r^2}+$$
$$288r^3\sqrt{-3+4r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]+576r^5\sqrt{-3+4r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]+576r^4\sqrt{-6+8r^2}$$
$$\mathrm{ArcCos}\left[\frac{1}{3}\right]+432r^2\sqrt{-9+12r^2}\left(\sqrt{2}+4r+2\sqrt{2}\,r^2\right)\mathrm{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4r^2}{3}}}\right]-$$
$$108r^4\sqrt{-9+12r^2}\left(\sqrt{2}+4r+2\sqrt{2}\,r^2\right)\mathrm{ArcTan}\left[\sqrt{-1+\frac{4r^2}{3}}\right]-36\sqrt{6}\,r^2$$
$$\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]-72\sqrt{6}\,r^4\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]-$$
$$144r^3\sqrt{-9+12r^2}\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]-36\sqrt{6}\,r^2\sqrt{-3+4r^2}$$
$$\mathrm{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]-72\sqrt{6}\,r^4\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]-$$
$$144r^3\sqrt{-9+12r^2}\,\mathrm{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]+2880r^3\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\sqrt{-3+4r^2}\right]+$$
$$720r^2\sqrt{-6+8r^2}\,\mathrm{ArcTan}\left[\sqrt{-3+4r^2}\right]+1440r^4\sqrt{-6+8r^2}\,\mathrm{ArcTan}\left[\sqrt{-3+4r^2}\right]+$$
$$288r^3\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]+72r^2\sqrt{-6+8r^2}\,\mathrm{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]+$$
$$144r^4\sqrt{-6+8r^2}\,\mathrm{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]-576r^3\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{r\left(-5+6r^2\right)}{\sqrt{-6+8r^2}}\right]-$$
$$1152r^5\sqrt{-3+4r^2}\,\mathrm{ArcTan}\left[\frac{r\left(-5+6r^2\right)}{\sqrt{-6+8r^2}}\right]-1152r^4\sqrt{-6+8r^2}\,\mathrm{ArcTan}\left[\frac{r\left(-5+6r^2\right)}{\sqrt{-6+8r^2}}\right]-$$

$$16\sqrt{6}\,\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]-$$

$$32\sqrt{6}\,r^2\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]+$$

$$108\sqrt{6}\,r^4\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]+$$

$$216\sqrt{6}\,r^6\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]-$$

$$64\,r\sqrt{-9+12\,r^2}\,\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]+432\,r^5\sqrt{-9+12\,r^2}$$

$$\text{ArcTan}\left[\frac{7-9\,r^2}{3\,(-1+r^2)\,\sqrt{-9+12\,r^2}}\right]+360\sqrt{6}\,r^2\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\sqrt{-9+12\,r^2}\right]+$$

$$720\sqrt{6}\,r^4\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\sqrt{-9+12\,r^2}\right]+1440\,r^3\sqrt{-9+12\,r^2}$$

$$\text{ArcTan}\left[\sqrt{-9+12\,r^2}\right]+216\sqrt{6}\,r^4\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{\sqrt{-9+12\,r^2}}{-3+2\,r^2}\right]+432\sqrt{6}\,r^6$$

$$\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{\sqrt{-9+12\,r^2}}{-3+2\,r^2}\right]+864\,r^5\sqrt{-9+12\,r^2}\,\text{ArcTan}\left[\frac{\sqrt{-9+12\,r^2}}{-3+2\,r^2}\right]+$$

$$72\,r\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\,(-1+r^2)\,\sqrt{-3+4\,r^2}}\right]+18\sqrt{-6+8\,r^2}$$

$$\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\,(-1+r^2)\,\sqrt{-3+4\,r^2}}\right]+36\,r^2\sqrt{-6+8\,r^2}\,\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\,(-1+r^2)\,\sqrt{-3+4\,r^2}}\right]-$$

$$576\,r^3\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\,(-1+r^2)\,\sqrt{-6+8\,r^2}}\right]-$$

$$1152\,r^5\sqrt{-3+4\,r^2}\,\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\,(-1+r^2)\,\sqrt{-6+8\,r^2}}\right]-$$

$$1152\,r^4\sqrt{-6+8\,r^2}\,\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\,(-1+r^2)\,\sqrt{-6+8\,r^2}}\right]-$$

$$3456\, r^5 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right] - 864\, r^4 \sqrt{-6+8 r^2}$$

$$\left.\left.\text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right] - 1728\, r^6 \sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\right)\right) \Big/$$

$$\left(48\,\pi\, r^2 \left(1+2\sqrt{2}\, r+2 r^2\right)\left(2-2\sqrt{6}\, r+3 r^2\right)\left(2+2\sqrt{6}\, r+3 r^2\right)\sqrt{-3+4 r^2}\right);$$

TetrDDCFDD[r_] :=

$$\Big(540\sqrt{2}\, r^2 + 4320\, r^3 + 5760\sqrt{2}\, r^4 + 2880\, r^5 - 6480\sqrt{2}\, r^6 - 11\,520\, r^7 - 2880\sqrt{2}\, r^8 +$$

$$8\sqrt{6}\,\pi\sqrt{-3+4 r^2} + 72\,\pi\, r\sqrt{-3+4 r^2} + 96\sqrt{6}\,\pi\, r^2\sqrt{-3+4 r^2} + 2304\, r^4\sqrt{-3+4 r^2} -$$

$$4\sqrt{6}\,\pi\, r^4\sqrt{-3+4 r^2} - 864\, r^5\sqrt{-3+4 r^2} + 1728\,\pi\, r^5\sqrt{-3+4 r^2} +$$

$$4608\, r^6\sqrt{-3+4 r^2} - 432\sqrt{6}\,\pi\, r^6\sqrt{-3+4 r^2} - 1728\, r^7\sqrt{-3+4 r^2} +$$

$$6336\,\pi\, r^7\sqrt{-3+4 r^2} - 144\sqrt{6}\,\pi\, r^8\sqrt{-3+4 r^2} + 9\,\pi\sqrt{-6+8 r^2} + 108\,\pi\, r^2\sqrt{-6+8 r^2} +$$

$$288\, r^3\sqrt{-6+8 r^2} - 108\, r^4\sqrt{-6+8 r^2} - 108\,\pi\, r^4\sqrt{-6+8 r^2} + 3456\, r^5\sqrt{-6+8 r^2} -$$

$$1296\, r^6\sqrt{-6+8 r^2} + 4032\,\pi\, r^6\sqrt{-6+8 r^2} + 1152\, r^7\sqrt{-6+8 r^2} - 432\, r^8\sqrt{-6+8 r^2} +$$

$$1728\,\pi\, r^8\sqrt{-6+8 r^2} + 64\,\pi\, r\sqrt{-9+12 r^2} + 128\,\pi\, r^3\sqrt{-9+12 r^2} -$$

$$288\,\pi\, r^5\sqrt{-9+12 r^2} - 576\,\pi\, r^7\sqrt{-9+12 r^2} - 144\, r^3\sqrt{-3+4 r^2}\ \text{ArcCos}\left[\frac{1}{3}\right] -$$

$$1728\, r^5\sqrt{-3+4 r^2}\ \text{ArcCos}\left[\frac{1}{3}\right] - 576\, r^7\sqrt{-3+4 r^2}\ \text{ArcCos}\left[\frac{1}{3}\right] -$$

$$576\, r^4\sqrt{-6+8 r^2}\ \text{ArcCos}\left[\frac{1}{3}\right] - 1152\, r^6\sqrt{-6+8 r^2}\ \text{ArcCos}\left[\frac{1}{3}\right] +$$

$$108\, r^4\sqrt{-9+12 r^2}\left(\sqrt{2} + 8 r + 12\sqrt{2}\, r^2 + 16 r^3 + 4\sqrt{2}\, r^4\right)\text{ArcTan}\left[\sqrt{-1+\frac{4 r^2}{3}}\right] +$$

$$288\, r^3\sqrt{-3+4 r^2}\left(1 + 4\sqrt{2}\, r + 12 r^2 + 8\sqrt{2}\, r^3 + 4 r^4\right)\text{ArcTan}\left[\frac{r\left(-5+6 r^2\right)}{\sqrt{-6+8 r^2}}\right] -$$

$$16\sqrt{6}\sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] -$$

$$192\sqrt{6}\ r^2\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$
$$172\sqrt{6}\ r^4\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$
$$1296\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$
$$432\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$
$$128\,r\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-256\,r^3\sqrt{-9+12r^2}$$
$$\text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-864\,r^5\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$
$$1728\,r^7\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-216\sqrt{6}\ r^4\sqrt{-3+4r^2}$$
$$\text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-2592\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$
$$864\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-1728\,r^5\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$
$$3456\,r^7\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]+144\,r\sqrt{-3+4r^2}$$
$$\text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+288\,r^3\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+$$
$$18\sqrt{-6+8r^2}\ \text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+216\,r^2\sqrt{-6+8r^2}$$
$$\text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+72\,r^4\sqrt{-6+8r^2}\ \text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+$$

$$288 r^3 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r \left(-1+r^2\right) \sqrt{-6+8 r^2}}\right]+3456 r^5 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r \left(-1+r^2\right) \sqrt{-6+8 r^2}}\right]+1152 r^7 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r \left(-1+r^2\right) \sqrt{-6+8 r^2}}\right]+$$

$$1152 r^4 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r \left(-1+r^2\right) \sqrt{-6+8 r^2}}\right]+2304 r^6 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r \left(-1+r^2\right) \sqrt{-6+8 r^2}}\right]+6912 r^5 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$13\,824 r^7 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+864 r^4 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+10\,368 r^6 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$3456 r^8 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\Bigg) \Big/ \left(24 \pi r^3 \left(1+2 \sqrt{2}\ r+2 r^2\right)^2 \sqrt{-3+4 r^2}\right);$$

TetrCFTot[r_] :=

$$\mathrm{Theta}\left[\frac{1}{\sqrt{2}}-r\right]*\mathrm{TetrCFAA}[r]+\mathrm{Theta}\left[\sqrt{\frac{2}{3}}-r\right]*\mathrm{Theta}\left[r-\frac{1}{\sqrt{2}}\right]*\mathrm{TetrCFBB}[r]+$$

$$\mathrm{Theta}\left[\frac{\sqrt{3}}{2}-r\right]*\mathrm{Theta}\left[r-\sqrt{\frac{2}{3}}\right]*\mathrm{TetrCFCC}[r]+$$

$$\mathrm{Theta}\left[r-\frac{\sqrt{3}}{2}\right]*\mathrm{Theta}[1-r]*\mathrm{TetrCFDD}[r];$$ **TetrDCFTot[r_] :=**

$$\mathrm{Theta}\left[\frac{1}{\sqrt{2}}-r\right]*\mathrm{TetrDCFAA}[r]+\mathrm{Theta}\left[\sqrt{\frac{2}{3}}-r\right]*\mathrm{Theta}\left[r-\frac{1}{\sqrt{2}}\right]*\mathrm{TetrDCFBB}[r]+$$

$$\mathrm{Theta}\left[\frac{\sqrt{3}}{2}-r\right]*\mathrm{Theta}\left[r-\sqrt{\frac{2}{3}}\right]*\mathrm{TetrDCFCC}[r]+$$

```
Theta[r - Sqrt[3]/2] * Theta[1 - r] * TetrDCFDD[r]; TetrDDCFTot[r_] :=
 Theta[1/Sqrt[2] - r] * TetrDDCFAA[r] + Theta[Sqrt[2/3] - r] * Theta[r - 1/Sqrt[2]] * TetrDDCFBB[r] +
  Theta[Sqrt[3]/2 - r] * Theta[r - Sqrt[2/3]] * TetrDDCFCC[r] +
  Theta[r - Sqrt[3]/2] * Theta[1 - r] * TetrDDCFDD[r];
```

TetrDist

$$\left\{0, \frac{1}{\sqrt{2}}, \sqrt{\frac{2}{3}}, \frac{\sqrt{3}}{2}, 1\right\}$$

```
TetrCFDD[r]

Simplify[TetrDCFDD[r], Assumptions → {TETRDst[[3]] < r < TETRDst[[4]]}]

FullSimplify[TetrDCFDD[r], Assumptions → {TETRDst[[3]] < r < TETRDst[[4]]}]
```

```
Theta[x_] := If[x > 0, 1, 0];
TETRDst = {1/Sqrt[2], Sqrt[2/3], Sqrt[3]/2, 1};
```

$$\text{TETRCFAA[r\_]} := 1 - 3\sqrt{\frac{3}{2}}\, r - \frac{\left(6 + 5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2 \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TETRDCFAA[r\_]} := -3\sqrt{\frac{3}{2}} - \frac{3\left(6 + 5\sqrt{3}\,\pi\right) r^2}{4\sqrt{2}\,\pi} + \frac{6 r \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TETRDDCFAA[r\_]} := -\frac{3\left(6\sqrt{2}\, r + \pi\left(-8 + 5\sqrt{6}\, r\right) + 8\left(-2\sqrt{2} + \text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

TETRCFBB[r_] :=

$$-2 + \frac{3}{4\sqrt{2}\, r} - \frac{3\left(-3 + \sqrt{3}\right) r}{\sqrt{2}} - \frac{\left(6 - 12\pi + 5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2 \left(2\sqrt{2} - \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TETRDCFBB}[r\_] := \frac{3}{8}\left(-4\sqrt{2}\left(-3+\sqrt{3}\right)-\frac{\sqrt{2}}{r^2}-\frac{\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r^2}{\pi}-\frac{16\,r\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\text{TETRDDCFBB}[r\_] := \frac{3}{8}\left(\frac{2\sqrt{2}}{r^3}-\frac{2\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r}{\pi}-\frac{16\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\text{TETRCFCC}[r\_] := -6+\frac{9+8\sqrt{3}}{12\sqrt{2}\,r}+\frac{3\left(3+\sqrt{3}\right)r}{\sqrt{2}}+\frac{\left(-6+12\pi+\sqrt{3}\,\pi\right)r^3}{4\sqrt{2}\,\pi}+\frac{3r^2\left(2\sqrt{2}-3\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TETRDCFCC}[r\_] := \frac{3\left(3+\sqrt{3}\right)}{\sqrt{2}}-\frac{9+8\sqrt{3}}{12\sqrt{2}\,r^2}+\frac{3\left(-6+\left(12+\sqrt{3}\right)\pi\right)r^2}{4\sqrt{2}\,\pi}-\frac{6\,r\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TETRDDCFCC}[r\_] := \frac{1}{12}\left(\frac{\sqrt{2}\left(9+8\sqrt{3}\right)}{r^3}+\frac{9\sqrt{2}\left(-6+\left(12+\sqrt{3}\right)\pi\right)r}{\pi}-\frac{72\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\text{TETRCFDD}[r\_] := 3+\frac{3}{8\sqrt{2}\,r}+\frac{1}{\sqrt{6}\,r}+\frac{21}{2}\sqrt{\frac{3}{2}}\,r+\frac{9r}{\sqrt{2}}-3r^2+\frac{6\sqrt{2}\,r^2}{\pi}-\frac{1}{2}\sqrt{\frac{3}{2}}\,r^3+3\sqrt{2}\,r^3-\frac{3r^3}{2\sqrt{2}\,\pi}-\frac{21\,r\sqrt{-6+8r^2}}{4\pi}-\frac{3r^2\,\text{ArcCos}\left[\frac{1}{3}\right]}{\pi}-\frac{9\sqrt{6}\,r\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4r^2}{3}}}\right]}{\pi}+\frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\sqrt{-1+\frac{4r^2}{3}}\right]}{2\pi}-\frac{9\,\text{ArcTan}\left[\frac{r}{\sqrt{-\frac{3}{2}+2r^2}}\right]}{\pi}+\frac{3\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{2\pi}+$$

$$\frac{3\,\text{ArcTan}\left[\frac{3+2\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{\pi}+\frac{3\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{2\pi}-\frac{15\sqrt{2}\,r\,\text{ArcTan}\left[\sqrt{-3+4r^2}\right]}{\pi}+$$

$$\frac{3\,\text{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]}{2\pi}-\frac{3\,r\,\text{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]}{\sqrt{2}\,\pi}-\frac{12\,\text{ArcTan}\left[\frac{r}{\sqrt{-6+8r^2}}\right]}{\pi}+\frac{6r^2\,\text{ArcTan}\left[\frac{r(-5+6r^2)}{\sqrt{-6+8r^2}}\right]}{\pi}-$$

$$\frac{\sqrt{\frac{2}{3}}\,\text{ArcTan}\left[\frac{7-9r^2}{3(-1+r^2)\sqrt{-9+12r^2}}\right]}{\pi r}-\frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\frac{7-9r^2}{3(-1+r^2)\sqrt{-9+12r^2}}\right]}{2\pi}-$$

$$\frac{15\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\sqrt{-9+12r^2}\right]}{\pi}-\frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]}{\pi}+$$

$$\frac{3\,\text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4(-1+r^2)\sqrt{-3+4r^2}}\right]}{4\sqrt{2}\,\pi r}+\frac{6r^2\,\text{ArcTan}\left[\frac{2-r^2-2r^4}{2r(-1+r^2)\sqrt{-6+8r^2}}\right]}{\pi}+\frac{6\sqrt{2}\,r^3\,\text{ArcTan}\left[\frac{1+\sqrt{-3+4r^2}}{-1+\sqrt{-3+4r^2}}\right]}{\pi};$$

$$\text{TETRDCFDD}[r\_]:=-\Bigg((2-3r^2)^2\Bigg(-1404\sqrt{2}\,r^2-5616r^3-936\sqrt{2}\,r^4+7488r^5+$$

$$3744\sqrt{2}\,r^6+8\sqrt{6}\,\pi\sqrt{-3+4r^2}+36\pi r\sqrt{-3+4r^2}-236\sqrt{6}\,\pi r^2\sqrt{-3+4r^2}-$$

$$576\pi r^3\sqrt{-3+4r^2}-2304r^4\sqrt{-3+4r^2}-468\sqrt{6}\,\pi r^4\sqrt{-3+4r^2}+$$

$$432r^5\sqrt{-3+4r^2}-1152\pi r^5\sqrt{-3+4r^2}+72\sqrt{6}\,\pi r^6\sqrt{-3+4r^2}+9\pi\sqrt{-6+8r^2}-$$

$$198\pi r^2\sqrt{-6+8r^2}-576r^3\sqrt{-6+8r^2}+108r^4\sqrt{-6+8r^2}-288\pi r^4\sqrt{-6+8r^2}-$$

$$1152r^5\sqrt{-6+8r^2}+216r^6\sqrt{-6+8r^2}-864\pi r^6\sqrt{-6+8r^2}+32\pi r\sqrt{-9+12r^2}-$$

$$1008\pi r^3\sqrt{-9+12r^2}+144\pi r^5\sqrt{-9+12r^2}+288r^3\sqrt{-3+4r^2}\,\text{ArcCos}\left[\frac{1}{3}\right]+$$

$$576r^5\sqrt{-3+4r^2}\,\text{ArcCos}\left[\frac{1}{3}\right]+576r^4\sqrt{-6+8r^2}\,\text{ArcCos}\left[\frac{1}{3}\right]+$$

$$432r^2\sqrt{-9+12r^2}\left(\sqrt{2}+4r+2\sqrt{2}\,r^2\right)\text{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4r^2}{3}}}\right]-$$

$$108\, r^4 \sqrt{-9+12 r^2} \left(\sqrt{2}+4 r+2 \sqrt{2} r^2\right) \text{ArcTan}\left[\sqrt{-1+\frac{4 r^2}{3}}\right] - 36 \sqrt{6} r^2 \sqrt{-3+4 r^2}$$

$$\text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] - 72\sqrt{6} r^4 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] -$$

$$144 r^3 \sqrt{-9+12 r^2} \text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] - 36\sqrt{6} r^2 \sqrt{-3+4 r^2}$$

$$\text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] - 72\sqrt{6} r^4 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] -$$

$$144 r^3 \sqrt{-9+12 r^2} \text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2} r}{\sqrt{-3+4 r^2}}\right] + 2880 r^3 \sqrt{-3+4 r^2} \text{ArcTan}\left[\sqrt{-3+4 r^2}\right] +$$

$$720 r^2 \sqrt{-6+8 r^2} \text{ArcTan}\left[\sqrt{-3+4 r^2}\right] + 1440 r^4 \sqrt{-6+8 r^2} \text{ArcTan}\left[\sqrt{-3+4 r^2}\right] +$$

$$288 r^3 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{\sqrt{-3+4 r^2}}{2-2 r^2}\right] + 72 r^2 \sqrt{-6+8 r^2} \text{ArcTan}\left[\frac{\sqrt{-3+4 r^2}}{2-2 r^2}\right] +$$

$$144 r^4 \sqrt{-6+8 r^2} \text{ArcTan}\left[\frac{\sqrt{-3+4 r^2}}{2-2 r^2}\right] - 576 r^3 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{r\left(-5+6 r^2\right)}{\sqrt{-6+8 r^2}}\right] -$$

$$1152 r^5 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{r\left(-5+6 r^2\right)}{\sqrt{-6+8 r^2}}\right] - 1152 r^4 \sqrt{-6+8 r^2} \text{ArcTan}\left[\frac{r\left(-5+6 r^2\right)}{\sqrt{-6+8 r^2}}\right] -$$

$$16\sqrt{6} \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] -$$

$$32\sqrt{6} r^2 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] +$$

$$108\sqrt{6} r^4 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] +$$

$$216\sqrt{6} r^6 \sqrt{-3+4 r^2} \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] -$$

$$64\, r \sqrt{-9+12 r^2}\ \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] + 432\, r^5 \sqrt{-9+12 r^2}$$

$$\text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right] + 360\sqrt{6}\ r^2 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\sqrt{-9+12 r^2}\right] +$$

$$720\sqrt{6}\ r^4 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\sqrt{-9+12 r^2}\right] + 1440\, r^3 \sqrt{-9+12 r^2}\ \text{ArcTan}\left[\sqrt{-9+12 r^2}\right] +$$

$$216\sqrt{6}\ r^4 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right] +$$

$$432\sqrt{6}\ r^6 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right] + 864\, r^5 \sqrt{-9+12 r^2}$$

$$\text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right] + 72\, r \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right] +$$

$$18\sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right] + 36\, r^2 \sqrt{-6+8 r^2}$$

$$\text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right] - 576\, r^3 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right] -$$

$$1152\, r^5 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right] -$$

$$1152\, r^4 \sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right] -$$

$$3456\, r^5 \sqrt{-3+4 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right] - 864\, r^4 \sqrt{-6+8 r^2}$$

$$\left.\left.\text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right] - 1728\, r^6 \sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\right)\right) \Big/$$

$$\left(48\,\pi\, r^2 \left(1+2\sqrt{2}\ r+2 r^2\right)\left(2-2\sqrt{6}\ r+3 r^2\right)\left(2+2\sqrt{6}\ r+3 r^2\right)\sqrt{-3+4 r^2}\right);$$

```
TETRDDCFDD[r_] :=
```

$$\Big(540\sqrt{2}\,r^2 + 4320\,r^3 +$$

$$5760\sqrt{2}\,r^4 + 2880\,r^5 - 6480\sqrt{2}\,r^6 -$$

$$11520\,r^7 - 2880\sqrt{2}\,r^8 + 8\sqrt{6}\,\pi\sqrt{-3+4\,r^2} +$$

$$72\,\pi\,r\sqrt{-3+4\,r^2} + 96\sqrt{6}\,\pi\,r^2\sqrt{-3+4\,r^2} +$$

$$2304\,r^4\sqrt{-3+4\,r^2} - 4\sqrt{6}\,\pi\,r^4\sqrt{-3+4\,r^2} -$$

$$864\,r^5\sqrt{-3+4\,r^2} + 1728\,\pi\,r^5\sqrt{-3+4\,r^2} +$$

$$4608\,r^6\sqrt{-3+4\,r^2} - 432\sqrt{6}\,\pi\,r^6\sqrt{-3+4\,r^2} -$$

$$1728\,r^7\sqrt{-3+4\,r^2} + 6336\,\pi\,r^7\sqrt{-3+4\,r^2} -$$

$$144\sqrt{6}\,\pi\,r^8\sqrt{-3+4\,r^2} + 9\,\pi\sqrt{-6+8\,r^2} +$$

$$108\,\pi\,r^2\sqrt{-6+8\,r^2} + 288\,r^3\sqrt{-6+8\,r^2} - 108\,r^4\sqrt{-6+8\,r^2} -$$

$$108\,\pi\,r^4\sqrt{-6+8\,r^2} + 3456\,r^5\sqrt{-6+8\,r^2} -$$

$$1296\,r^6\sqrt{-6+8\,r^2} + 4032\,\pi\,r^6\sqrt{-6+8\,r^2} +$$

$$1152\,r^7\sqrt{-6+8\,r^2} - 432\,r^8\sqrt{-6+8\,r^2} +$$

$$1728\,\pi\,r^8\sqrt{-6+8\,r^2} + 64\,\pi\,r\sqrt{-9+12\,r^2} +$$

$$128\,\pi\,r^3\sqrt{-9+12\,r^2} - 288\,\pi\,r^5\sqrt{-9+12\,r^2} -$$

$$576\,\pi\,r^7\sqrt{-9+12\,r^2} - 144\,r^3\sqrt{-3+4\,r^2}\,\text{ArcCos}\Big[\frac{1}{3}\Big] -$$

$$1728\,r^5\sqrt{-3+4\,r^2}\,\text{ArcCos}\Big[\frac{1}{3}\Big] - 576\,r^7\sqrt{-3+4\,r^2}\,\text{ArcCos}\Big[\frac{1}{3}\Big] -$$

$$576\,r^4\sqrt{-6+8\,r^2}\,\text{ArcCos}\Big[\frac{1}{3}\Big] - 1152\,r^6\sqrt{-6+8\,r^2}\,\text{ArcCos}\Big[\frac{1}{3}\Big] +$$

$$108\,r^4\sqrt{-9+12\,r^2}\,\left(\sqrt{2} + 8\,r + 12\sqrt{2}\,r^2 + 16\,r^3 + 4\sqrt{2}\,r^4\right)\text{ArcTan}\Big[\sqrt{-1+\frac{4\,r^2}{3}}\Big] +$$

$$288\,r^3\sqrt{-3+4\,r^2}\,\left(1 + 4\sqrt{2}\,r + 12\,r^2 + 8\sqrt{2}\,r^3 + 4\,r^4\right)\text{ArcTan}\Big[\frac{r\,(-5+6\,r^2)}{\sqrt{-6+8\,r^2}}\Big] -$$

$$16\sqrt{6}\sqrt{-3+4\,r^2}\,\text{ArcTan}\Big[\frac{7-9\,r^2}{3\,(-1+r^2)\sqrt{-9+12\,r^2}}\Big] -$$

$$192\sqrt{6}\ r^2\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$172\sqrt{6}\ r^4\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$1296\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$432\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$128\,r\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$256\,r^3\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$864\,r^5\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$1728\,r^7\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$216\sqrt{6}\ r^4\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$2592\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$864\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$1728\,r^5\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$3456\,r^7\sqrt{-9+12r^2}\ \mathrm{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]+$$

$$144\,r\sqrt{-3+4r^2}\ \mathrm{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+$$

$$288\,r^3\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\left(-1+r^2\right)\sqrt{-3+4\,r^2}}\right]+$$

$$18\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\left(-1+r^2\right)\sqrt{-3+4\,r^2}}\right]+$$

$$216\,r^2\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\left(-1+r^2\right)\sqrt{-3+4\,r^2}}\right]+$$

$$72\,r^4\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{-7+12\,r^2-4\,r^4}{4\left(-1+r^2\right)\sqrt{-3+4\,r^2}}\right]+$$

$$288\,r^3\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\left(-1+r^2\right)\sqrt{-6+8\,r^2}}\right]+$$

$$3456\,r^5\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\left(-1+r^2\right)\sqrt{-6+8\,r^2}}\right]+$$

$$1152\,r^7\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\left(-1+r^2\right)\sqrt{-6+8\,r^2}}\right]+$$

$$1152\,r^4\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\left(-1+r^2\right)\sqrt{-6+8\,r^2}}\right]+$$

$$2304\,r^6\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{2-r^2-2\,r^4}{2\,r\left(-1+r^2\right)\sqrt{-6+8\,r^2}}\right]+$$

$$6912\,r^5\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\right]+$$

$$13\,824\,r^7\sqrt{-3+4\,r^2}\;\text{ArcTan}\left[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\right]+$$

$$864\,r^4\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\right]+$$

$$10\,368\,r^6\sqrt{-6+8\,r^2}\;\text{ArcTan}\left[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\right]+$$

$$3456\, r^8 \sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\Bigg) \Big/$$

$$\left(24\,\pi\, r^3 \left(1+2\sqrt{2}\ r+2 r^2\right)^2 \sqrt{-3+4 r^2}\right);$$

```
TETRExctTotalCF[r_] := If[r < TETRDst[[1]], TETRCFAA[r],
  If[r < TETRDst[[2]], TETRCFBB[r], If[r < TETRDst[[3]], TETRCFCC[r]], TETRCFDD[r]]];
TETRExctTotalDCF[r_] := If[r < TETRDst[[1]], TETRDCFAA[r],
  If[r < TETRDst[[2]], TETRDCFBB[r], If[r < TETRDst[[3]], TETRDCFCC[r]], TETRDCFDD[r]]];
TETRExctTotalDDCF[r_] := If[r < TETRDst[[1]], TETRDDCFAA[r], If[r < TETRDst[[2]],
   TETRDDCFBB[r], If[r < TETRDst[[3]], TETRDDCFCC[r]], TETRDDCFDD[r]]];
```

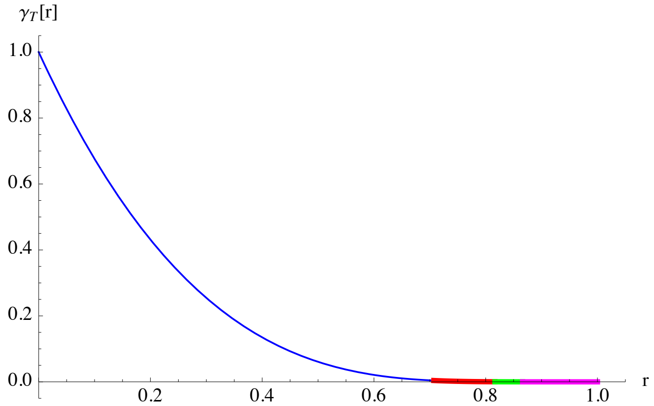
```
Plot[{TetrCFDD[r], TETRCFDD[r]}, {r, TETRDst[[3]], TETRDst[[4]]}]
```

```
TETRDst[[4]]
```

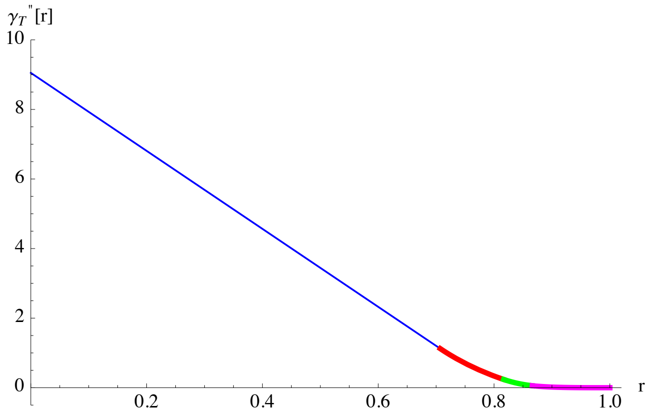
1

```
Plot[{TETRExctTotalCF[r]}, {r, 0, TETRDst[[4]]}, PlotRange → {{0, 1}, {0, 1}},
 AxesLabel → {"r", "γT[r]"}, PlotRange → {{0, 1.02}, {-0.01, 1.05}}]
```

```
Plot[{If[r < TETRDst[[1]], TETRCFAA[r]],
  If[r > TETRDst[[1]], If[r < TETRDst[[2]], TETRCFBB[r]]],
  If[r > TETRDst[[2]], If[r < TETRDst[[3]], TETRCFCC[r]]],
  If[r > TETRDst[[3]], If[r < TETRDst[[4]], TETRCFDD[r]]]}, {r, 0, TETRDst[[4]]},
 PlotRange → {{0, 1.05}, {-0.05, 1.05}}, AxesLabel → {"r", "γT[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]],
   Directive[Red, Thickness[0.009]], Directive[Green, Thickness[0.009]],
   Directive[Magenta, Thickness[0.009]]}]
```



```
Plot[{If[r < TETRDst[[1]], TETRDDCFAA[r]],
  If[r > TETRDst[[1]], If[r < TETRDst[[2]], TETRDDCFBB[r]]],
  If[r > TETRDst[[2]], If[r < TETRDst[[3]], TETRDDCFCC[r]]],
  If[r > TETRDst[[3]], If[r < TETRDst[[4]], TETRDDCFDD[r]]]}, {r, 0, TETRDst[[4]]},
 PlotRange → {{0, 1.02}, {-1 / 2, 10}}, AxesLabel → {"r", "γT''[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]],
   Directive[Red, Thickness[0.009]], Directive[Green, Thickness[0.009]],
   Directive[Magenta, Thickness[0.009]]}]
```




```
Plot[{If[r < TETRDst[[1]], TetrDDCFAA[r]],
  If[r > TETRDst[[1]], If[r < TETRDst[[2]], TetrDDCFBB[r]]],
  If[r > TETRDst[[2]], If[r < TETRDst[[3]], TetrDDCFCC[r]]],
  If[r > TETRDst[[3]], If[r < TETRDst[[4]], TetrDDCFDD[r]]]}, {r, 0, TETRDst[[4]]},
 PlotRange → {{0, 1.02}, {-1 / 2, 10}}, AxesLabel → {"r", "γT''[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]],
   Directive[Red, Thickness[0.009]], Directive[Green, Thickness[0.009]],
   Directive[Magenta, Thickness[0.009]]}]

Clear[TETRDDDCFAA]; Clear[TETRDDDCFBB]; Clear[TETRDDDCFCC]; Clear[TETRDDDCFDD];

TETRDDDCFAA[r_] := D[TETRDDCFAA[r], r];
TETRDDDCFBB[r_] := D[TETRDDCFBB[r], r];
TETRDDDCFCC[r_] := D[TETRDDCFCC[r], r]; TETRDDDCFDD[r_] :=
 Simplify[D[TETRDDCFDD[r], r], Assumptions → {TETRDst[[3]] < r < TETRDst[[4]]}];

TETRDDDCFAA[r]
TETRDDDCFBB[r]
TETRDDDCFCC[r]
TETRDDDCFDD[r]
```

$$\textbf{Plot}\Big[\Big\{\textbf{If}\Big[\textbf{r} < \textbf{TETRDst}[[1]],\ -\frac{3\left(6\sqrt{2}+5\sqrt{6}\,\pi\right)}{4\,\pi}\Big],$$

$$\textbf{If}\Big[\textbf{r} > \textbf{TETRDst}[[1]],\ \textbf{If}\Big[\textbf{r} < \textbf{TETRDst}[[2]],\ \frac{3}{8}\left(-\frac{2\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)}{\pi}-\frac{6\sqrt{2}}{r^4}\right)\Big]\Big],$$

$$\textbf{If}\Big[\textbf{r} > \textbf{TETRDst}[[2]],$$

$$\textbf{If}\Big[\textbf{r} < \textbf{TETRDst}[[3]],\ \frac{1}{12}\left(\frac{9\sqrt{2}\left(-6+\left(12+\sqrt{3}\right)\pi\right)}{\pi}-\frac{3\sqrt{2}\left(9+8\sqrt{3}\right)}{r^4}\right)\Big]\Big],$$

$$\textbf{If}\Big[\textbf{r} > \textbf{TETRDst}[[3]],\ \textbf{If}\Big[\textbf{r} < \textbf{TETRDst}[[4]],$$

$$-\Bigg(\left(\sqrt{2}+12\,r+30\sqrt{2}\,r^2+80\,r^3+60\sqrt{2}\,r^4+48\,r^5+8\sqrt{2}\,r^6\right)$$

$$\Bigg(108\,r^2-144\,r^4+9\,\pi\sqrt{-3+4\,r^2}+36\,r^4\sqrt{-3+4\,r^2}-144\,\pi\,r^4\sqrt{-3+4\,r^2}+$$

$$8\,\pi\sqrt{-9+12\,r^2}+12\,\pi\,r^4\sqrt{-9+12\,r^2}-36\,r^4\sqrt{-9+12\,r^2}\ \textbf{ArcTan}\Big[\sqrt{-1+\frac{4\,r^2}{3}}\Big]+$$

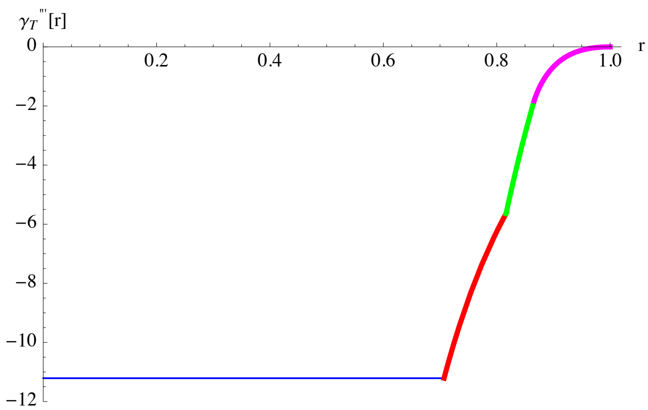
$$4\sqrt{-9+12\,r^2}\left(-4+9\,r^4\right)\textbf{ArcTan}\Big[\frac{7-9\,r^2}{3\left(-1+r^2\right)\sqrt{-9+12\,r^2}}\Big]+$$

$$72\,r^4\sqrt{-9+12\,r^2}\ \textbf{ArcTan}\Big[\frac{\sqrt{-9+12\,r^2}}{-3+2\,r^2}\Big]+18\sqrt{-3+4\,r^2}$$

$$\textbf{ArcTan}\Big[\frac{-7+12\,r^2-4\,r^4}{4\left(-1+r^2\right)\sqrt{-3+4\,r^2}}\Big]-288\,r^4\sqrt{-3+4\,r^2}\ \textbf{ArcTan}\Big[\frac{1+\sqrt{-3+4\,r^2}}{-1+\sqrt{-3+4\,r^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\left(8\,\pi\,r^4\left(1+2\sqrt{2}\,r+2\,r^2\right)^3\sqrt{-3+4\,r^2}\right)\Big]\Big]\Big\},\ \{r,0,1\},$$

```
AxesLabel → {"r", "γT''' [r]"}, PlotStyle →
 {Directive[
   Blue, Thickness[0.003]],
  Directive[Red, Thickness[0.009]], Directive[Green, Thickness[0.009]],
  Directive[Magenta, Thickness[0.009]]}, PlotRange → {{0, 1.02}, {-12, 0.1}}]
```



EVALUATION OF THE BEHAVIOUR AROUND D1$^+$, D2$^-$, D2$^+$, D3$^-$, D3$^+$, D4$^-$

```
Simplify[Series[TETRDDCFAA[x], {x, TETRDst[[1]], 3}]]
```

$$-\frac{3\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\,\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{4\pi}-\frac{3\left(6+5\sqrt{3}\,\pi\right)\left(x-\frac{1}{\sqrt{2}}\right)}{2\left(\sqrt{2}\,\pi\right)}+O\left[x-\frac{1}{\sqrt{2}}\right]^4$$

```
Simplify[Series[TETRDDCFBB[x], {x, TETRDst[[1]], 3}]]
```

$$-\frac{3\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\,\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{4\pi}-\frac{3\left(6+5\sqrt{3}\,\pi\right)\left(x-\frac{1}{\sqrt{2}}\right)}{2\left(\sqrt{2}\,\pi\right)}+36\left(x-\frac{1}{\sqrt{2}}\right)^2-60\sqrt{2}\left(x-\frac{1}{\sqrt{2}}\right)^3+O\left[x-\frac{1}{\sqrt{2}}\right]^4$$

```
Simplify[Series[TETRDDCFBB[x], {x, TETRDst[[2]], 3}]]
```

$$\frac{3\left(32\sqrt{2}-8\sqrt{3}-36\pi+19\sqrt{3}\,\pi-16\,\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{8\pi}-\frac{3\left(24+\left(-21+20\sqrt{3}\right)\pi\right)\left(x-\sqrt{\frac{2}{3}}\right)}{8\left(\sqrt{2}\,\pi\right)}+\frac{81}{8}\sqrt{3}\left(x-\sqrt{\frac{2}{3}}\right)^2-\frac{405\left(x-\sqrt{\frac{2}{3}}\right)^3}{8\sqrt{2}}+O\left[x-\sqrt{\frac{2}{3}}\right]^4$$

```
Simplify[Series[TETRDDCFCC[x], {x, TETRDst[[2]], 3}]]
```

$$\frac{3\left(32\sqrt{2}-8\sqrt{3}-36\pi+19\sqrt{3}\,\pi-16\,\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{8\pi}-\frac{3\left(24+\left(-21+20\sqrt{3}\right)\pi\right)\left(x-\sqrt{\frac{2}{3}}\right)}{8\left(\sqrt{2}\,\pi\right)}+\left(27+\frac{81\sqrt{3}}{8}\right)\left(x-\sqrt{\frac{2}{3}}\right)^2-\frac{45\left(9+8\sqrt{3}\right)\left(x-\sqrt{\frac{2}{3}}\right)^3}{8\sqrt{2}}+O\left[x-\sqrt{\frac{2}{3}}\right]^4$$

```
Simplify[Series[TETRDDCFCC[x], {x, TETRDst[[3]], 3}]]
```

$$\frac{\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{72\pi}+\left(5\sqrt{2}-\frac{101}{6\sqrt{6}}-\frac{9}{\sqrt{2}\,\pi}\right)\left(x-\frac{\sqrt{3}}{2}\right)+\frac{16}{9}\sqrt{2}\left(8+3\sqrt{3}\right)\left(x-\frac{\sqrt{3}}{2}\right)^2-\frac{160}{81}\left(\sqrt{2}\left(9+8\sqrt{3}\right)\right)\left(x-\frac{\sqrt{3}}{2}\right)^3+O\left[x-\frac{\sqrt{3}}{2}\right]^4$$

```
FullSimplify[Series[TETRDDCFDD[x], {x, TETRDst[[3]], 4}], Assumptions → {x > √3/2}]
```

$$\frac{1}{72}\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right)+\frac{1}{36\left(5+2\sqrt{6}\right)^{3}\pi}$$

$$\left(-162\left(485\sqrt{2}+396\sqrt{3}\right)+\left(-119\,988+87\,300\sqrt{2}+71\,280\sqrt{3}-48\,985\sqrt{6}\right)\pi\right)\left(x-\frac{\sqrt{3}}{2}\right)+$$

$$\frac{16}{9}\sqrt{2}\left(8+3\sqrt{3}\right)\left(x-\frac{\sqrt{3}}{2}\right)^{2}-\frac{1024\sqrt{2}\left(x-\frac{\sqrt{3}}{2}\right)^{5/2}}{5\left(3^{1/4}\pi\right)}-$$

$$\frac{160}{27}\sqrt{\frac{182}{3}+32\sqrt{3}}\left(x-\frac{\sqrt{3}}{2}\right)^{3}+\frac{57\,856\sqrt{2}\left(x-\frac{\sqrt{3}}{2}\right)^{7/2}}{35\times 3^{3/4}\pi}+O\left[x-\frac{\sqrt{3}}{2}\right]^{4}$$

```
FullSimplify[Series[TETRDDCFDD[x], {x, TETRDst[[4]], 4}], Assumptions → {x < 1}]
```

$$-\frac{24\sqrt{2}\,(x-1)^{3}}{\pi}+\frac{108\sqrt{2}\,(x-1)^{4}}{\pi}+O[x-1]^{5}$$

```
cfD1Minus =
 FullSimplify[(CoefficientList[Series[TETRDDCFAA[TETRDst[[1]] - x^2], {x, 0, 6}], x]) /.
   {ArcSec[-3] → π - ArcSec[3]}]
```

$$\text{cfD1Minus} = \left\{-\frac{3\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\,\text{ArcSec}[3]\right)}{4\pi},\ 0,\ \frac{3\left(6+5\sqrt{3}\,\pi\right)}{2\sqrt{2}\,\pi},\ 0,\ 0,\ 0,\ 0\right\};$$

```
cfD1Minus[[7]]
```

0

The 0s in red have been added to make the vector 7-dimensional

```
cfD1Plus =
 FullSimplify[(CoefficientList[Series[TETRDDCFBB[TETRDst[[1]] + x^2], {x, 0, 6}], x]),
  Assumptions → {x > 0}]
```

$$\text{cfD1Plus} = \left\{-\frac{3\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\,\text{ArcSec}[3]\right)}{4\pi},\ 0,\ -\frac{3\left(6+5\sqrt{3}\,\pi\right)}{2\sqrt{2}\,\pi},\ 0,\ 36,\ 0,\ -60\sqrt{2}\right\};\ \text{cfD1Plus}[[7]];$$

```
cfD2Minus =
 FullSimplify[(CoefficientList[Series[TETRDDCFBB[TETRDst[[2]] - x^2], {x, 0, 6}], x]),
  Assumptions → {x > 0}]
```

$$\text{cfD2Minus} = \left\{\frac{3}{8}\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right),\ 0,\ \frac{3\left(24+\left(-21+20\sqrt{3}\right)\pi\right)}{8\sqrt{2}\,\pi},\ 0,\ \frac{81\sqrt{3}}{8},\ 0,\ \frac{405}{8\sqrt{2}}\right\};\ \text{cfD2Minus}[[7]];$$

```
cfD2Plus =
 FullSimplify[(CoefficientList[Series[TETRDDCFCC[TETRDst[[2]] + x^2], {x, 0, 6}], x]),
  Assumptions → {x > 0}]
```

$$\text{cfD2Plus} = \left\{\frac{3}{8}\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right),\ 0,\ -\frac{3\left(24+\left(-21+20\sqrt{3}\right)\pi\right)}{8\sqrt{2}\,\pi},\ 0,\ 27+\frac{81\sqrt{3}}{8},\ 0,\ -\frac{45\left(9+8\sqrt{3}\right)}{8\sqrt{2}}\right\};\ \text{cfD2Plus}[[7]];$$

```
cfD3Minus =
 FullSimplify[(CoefficientList[Series[TETRDDCFCC[TETRDst[[3]] - x^2], {x, 0, 6}], x]) /.
   {ArcSec[-3] → π - ArcSec[3]}]
```

$$\text{cfD3Minus} = \left\{\frac{1}{72}\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right),\ 0,\ -5\sqrt{2}+\frac{101}{6\sqrt{6}}+\frac{9}{\sqrt{2}\,\pi},\ 0,\ \frac{16}{9}\sqrt{2}\left(8+3\sqrt{3}\right),\ 0,\ \frac{160}{81}\sqrt{2}\left(9+8\sqrt{3}\right)\right\};\ \text{cfD3Minus}[[7]];$$

```
CoefficientList[
 FullSimplify[Series[TETRDDCFDD[TETRDst[[3]] + x^2], {x, 0, 7}], Assumptions → {x > 0}], x]
```

$$\text{cfD3Plus} = \left\{\frac{1}{72}\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right),\ 0,\ \frac{1}{36\left(5+2\sqrt{6}\right)^{3}\pi}\left(-162\left(485\sqrt{2}+396\sqrt{3}\right)+\left(-119\,988+87\,300\sqrt{2}+71\,280\sqrt{3}-48\,985\sqrt{6}\right)\pi\right),\ 0,\ \frac{16}{9}\sqrt{2}\left(8+3\sqrt{3}\right),\ -\frac{1024\sqrt{2}}{5\times 3^{1/4}\pi},\ -\frac{160}{27}\sqrt{\frac{182}{3}+32\sqrt{3}}\right\};\ \text{cfD3Plus}[[7]];$$

```
CoefficientList[
 FullSimplify[Series[TETRDDCFDD[TETRDst[[4]] - x^2], {x, 0, 6}], Assumptions → {x > 0}], x]
```

$$\text{cfD4Minus} = \left\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{24\sqrt{2}}{\pi}\right\};\ \text{cfD4Minus}[[7]];$$

The discontinuities at the points joining two different r-subintervals are

```
ΔCfD1 = Simplify[cfD1Plus - cfD1Minus]
```

$$\left\{0,\,0,\,-\frac{3\left(6+5\sqrt{3}\,\pi\right)}{\sqrt{2}\,\pi},\,0,\,36,\,0,\,-60\sqrt{2}\right\}$$

```
ΔCfD2 = Simplify[cfD2Plus - cfD2Minus]
```

$$\left\{0,\,0,\,-\frac{3\left(24+\left(-21+20\sqrt{3}\right)\pi\right)}{4\sqrt{2}\,\pi},\,0,\,27,\,0,\,-\frac{45\left(9+4\sqrt{3}\right)}{4\sqrt{2}}\right\}$$

```
ΔCfD3 = Simplify[cfD3Plus - cfD3Minus]
```

$$\left\{0,\,0,\,\frac{1}{18\left(5+2\sqrt{6}\right)^{3}\pi}\left(-162\left(485\sqrt{2}+396\sqrt{3}\right)+\left(-119\,988+87\,300\sqrt{2}+71\,280\sqrt{3}-48\,985\sqrt{6}\right)\pi\right),\right.$$
$$\left.0,\,0,\,-\frac{1024\sqrt{2}}{5\times 3^{1/4}\,\pi},\,-\frac{160}{81}\sqrt{2}\left(9+8\sqrt{3}+\sqrt{3\left(91+48\sqrt{3}\right)}\right)\right\}$$

Here we have a fractional behaviour $|x-\mathrm{D3}]^{5/2}$

```
ΔCfD4 = Simplify[-cfD4Minus]
```

$$\left\{0,\,0,\,0,\,0,\,0,\,0,\,-\frac{24\sqrt{2}}{\pi}\right\}$$

We construct now the polynomial approximations of the CLD and the CF, in terms of variables $\sqrt{r-\mathrm{D1}}$ and $\sqrt{\mathrm{D2}-r}$,
using the following formulae worked out in the file
" ANALYTIC_APPROXMTN_CF_CLD_BB.nb "

```
Clear[K]; Clear[cflft]; Clear[cfrgt];
Clear[a0]; Clear[a1]; Clear[a2]; Clear[a3]; Clear[a4]; Clear[a5];
Clear[a6]; Clear[a7]; Clear[a8]; Clear[a9]; Clear[a10];
Clear[b0]; Clear[b1]; Clear[b2]; Clear[b3]; Clear[b4]; Clear[b5];
Clear[b6]; Clear[b7]; Clear[b8]; Clear[b9]; Clear[b10];
(* Coefficients of the expansion of the 2nd derivaive around D1 and D2 *)
cflft = {a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10};
cfrgt = {b0, b1, b2, b3, b4, b5, b6, b7, b8, b9, b10}; GLftNw[ξ_, K_] :=
 If[K < 4, 4 * ξ^4 * Sum[(cflft[[i]] / ((3 + i) (1 + i))) * (ξ)^(i-1), {i, 1, 2 K + 1}]];
GRgtNw[η_, K_] :=
  If[K < 4, 4 * η^4 * Sum[(cfrgt[[i]] / ((3 + i) (1 + i))) * (η)^(i-1), {i, 1, 2 K + 1}]];
```

K = 0

```
LeftCLD00[r_, D1_, D2_] := a0 (1 + (Sqrt[-D1 + r] (2 D1 - 3 D2 + r))/(2 (-D1 + D2)^(3/2)));

RgtCLD00[r_, D1_, D2_] := b0 (1 - (Sqrt[D2 - r] (-3 D1 + 2 D2 + r))/(2 (-D1 + D2)^(3/2)));

CLDApprx00[r_, D1_, D2_] := LeftCLD00[r, D1, D2] + RgtCLD00[r, D1, D2 ] ;
```

$$\text{LEFTCFApprx00[r\_, D1\_, D2\_] := b0}\left(\frac{D2^2}{2} - D2\,r + \frac{r^2}{2} - \frac{2\,(D2-r)^{5/2}\,(-7\,D1+6\,D2+r)}{35\,(-D1+D2)^{3/2}}\right) + \frac{1}{70\,(D1-D2)}\,a0\left(-24\,D1^3\sqrt{\frac{D1-r}{D1-D2}} + 28\,D1^2\,D2\sqrt{\frac{D1-r}{D1-D2}} + 44\,D1^2\sqrt{\frac{D1-r}{D1-D2}}\,r - 56\,D1\,D2\sqrt{\frac{D1-r}{D1-D2}}\,r - 16\,D1\sqrt{\frac{D1-r}{D1-D2}}\,r^2 + 28\,D2\sqrt{\frac{D1-r}{D1-D2}}\,r^2 - 4\sqrt{\frac{D1-r}{D1-D2}}\,r^3 + (D1-D2)\left(24\,D1^2 + 8\,D1\,D2 + 3\,D2^2 - 14\,(4\,D1+D2)\,r + 35\,r^2\right)\right);$$

$$\text{RGHTCFApprx00[r\_, D1\_, D2\_] := a0}\left(\frac{D1^2}{2} - D1\,r + \frac{r^2}{2} - \frac{2}{35}\,(D1-r)\left(\frac{D1-r}{D1-D2}\right)^{3/2}(6\,D1-7\,D2+r)\right) - \frac{1}{70\,(-D1+D2)^{3/2}}\,b0\left(3\,D1^3\sqrt{-D1+D2} - 24\,D2^3\left(\sqrt{-D1+D2} - \sqrt{D2-r}\right) + D1^2\sqrt{-D1+D2}\,(5\,D2-14\,r) + D2^2\left(56\sqrt{-D1+D2} - 44\sqrt{D2-r}\right)r + D2\left(-35\sqrt{-D1+D2} + 16\sqrt{D2-r}\right)r^2 + 4\sqrt{D2-r}\,r^3 + D1\left(4\,D2^2\left(4\sqrt{-D1+D2} - 7\sqrt{D2-r}\right) + D2\left(-42\sqrt{-D1+D2} + 56\sqrt{D2-r}\right)r + 7\left(5\sqrt{-D1+D2} - 4\sqrt{D2-r}\right)r^2\right)\right);$$

$$\text{EXTRCNTRBLft00[r\_, D1\_, D2\_, a\_, b\_] :=}\ \frac{1}{60}\,(D2-r)^3\left(5\,a\,(-2\,D1+D2+r) + b\left(10\,D1^2 + 3\,D2^2 + 4\,D2\,r + 3\,r^2 - 10\,D1\,(D2+r)\right)\right);$$

$$\text{EXTRCNTRBRgt00[r\_, D1\_, D2\_, a\_, b\_] :=}\ -\frac{1}{60}\,(D1-r)^3\,(-5\,a\,(D1-2\,D2+r) + b\,(D1-r)\,(2\,D1-5\,D2+3\,r));$$

THE CASE $K = 0$

THE FINAL FORMULAE ARE:

```
Clear[K]; Clear[cflft]; Clear[cfrgt];
Clear[a0]; Clear[a1]; Clear[a2]; Clear[a3]; Clear[a4]; Clear[a5];
Clear[a6]; Clear[a7]; Clear[a8]; Clear[a9]; Clear[a10];
Clear[b0]; Clear[b1]; Clear[b2]; Clear[b3]; Clear[b4]; Clear[b5];
Clear[b6]; Clear[b7]; Clear[b8]; Clear[b9]; Clear[b10];
(* Coefficients of the expansion of the 2nd derivaive around D1 and D2 *)
cflft = {a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10};
cfrgt = {b0, b1, b2, b3, b4, b5, b6, b7, b8, b9, b10}; GLftNw[ξ_, K_] :=
 If[K < 4, 4 * ξ^4 * Sum[(cflft[[i]] / ((3 + i) (1 + i))) * (ξ)^(i-1), {i, 1, 2 K + 1}]];
GRgtNw[η_, K_] :=
  If[K < 4, 4 * η^4 * Sum[(cfrgt[[i]] / ((3 + i) (1 + i))) * (η)^(i-1), {i, 1, 2 K + 1}]];
```

K = 0

$$\text{LeftCLD00[r\_, D1\_, D2\_] := a0}\left(1+\frac{\sqrt{-D1+r}\,(2\,D1-3\,D2+r)}{2\,(-D1+D2)^{3/2}}\right);$$

$$\text{RgtCLD00[r\_, D1\_, D2\_] := b0}\left(1-\frac{\sqrt{D2-r}\,(-3\,D1+2\,D2+r)}{2\,(-D1+D2)^{3/2}}\right);$$

```
CLDApprx00[r_, D1_, D2_] := LeftCLD00[r, D1, D2] + RgtCLD00[r, D1, D2 ] ;
```

$$\text{LEFTCFApprx00[r\_, D1\_, D2\_] := (* } \int_r^{D2} F[x]\,dx \text{ *)}$$

$$b0\left(\frac{D2^2}{2}-D2\,r+\frac{r^2}{2}-\frac{2\,(D2-r)^{5/2}\,(-7\,D1+6\,D2+r)}{35\,(-D1+D2)^{3/2}}\right)+$$
$$\frac{1}{70\,(D1-D2)}\,a0\left(-24\,D1^3\sqrt{\frac{D1-r}{D1-D2}}+28\,D1^2\,D2\sqrt{\frac{D1-r}{D1-D2}}+44\,D1^2\sqrt{\frac{D1-r}{D1-D2}}\,r-\right.$$
$$56\,D1\,D2\sqrt{\frac{D1-r}{D1-D2}}\,r-16\,D1\sqrt{\frac{D1-r}{D1-D2}}\,r^2+28\,D2\sqrt{\frac{D1-r}{D1-D2}}\,r^2-$$
$$\left.4\sqrt{\frac{D1-r}{D1-D2}}\,r^3+(D1-D2)\left(24\,D1^2+8\,D1\,D2+3\,D2^2-14\,(4\,D1+D2)\,r+35\,r^2\right)\right);$$

$$\text{RGHTCFApprx00[r\_, D1\_, D2\_] := (* } \int_{D1}^{x} F[x]\,dx \text{ *)}$$

$$a0\left(\frac{D1^2}{2}-D1\,r+\frac{r^2}{2}-\frac{2}{35}\,(D1-r)\left(\frac{D1-r}{D1-D2}\right)^{3/2}(6\,D1-7\,D2+r)\right)-$$
$$\frac{1}{70\,(-D1+D2)^{3/2}}\,b0\left(3\,D1^3\sqrt{-D1+D2}-24\,D2^3\left(\sqrt{-D1+D2}-\sqrt{D2-r}\right)+\right.$$
$$D1^2\sqrt{-D1+D2}\,(5\,D2-14\,r)+D2^2\left(56\sqrt{-D1+D2}-44\sqrt{D2-r}\right)r+$$
$$D2\left(-35\sqrt{-D1+D2}+16\sqrt{D2-r}\right)r^2+4\sqrt{D2-r}\,r^3+D1\left(4\,D2^2\left(4\sqrt{-D1+D2}-7\sqrt{D2-r}\right)+\right.$$
$$\left.\left.D2\left(-42\sqrt{-D1+D2}+56\sqrt{D2-r}\right)r+7\left(5\sqrt{-D1+D2}-4\sqrt{D2-r}\right)r^2\right)\right);$$

$$\text{EXTRCNTRBLft00[r\_, D1\_, D2\_, a\_, b\_] := }\frac{1}{60}\,(D2-r)^3$$
$$\left(5\,a\,(-2\,D1+D2+r)+b\left(10\,D1^2+3\,D2^2+4\,D2\,r+3\,r^2-10\,D1\,(D2+r)\right)\right);$$

$$\text{EXTRCNTRBRgt00[r\_, D1\_, D2\_, a\_, b\_] := }$$
$$-\frac{1}{60}\,(D1-r)^3\,(-5\,a\,(D1-2\,D2+r)+b\,(D1-r)\,(2\,D1-5\,D2+3\,r));$$

```
Limit[LEFTCFApprx00[r, D1, D2], r → D2, Direction → 1]
```

0

```
Simplify[Series[EXTRCNTRBLft00[r, D1, D2, a, b], {r, D1, 4}]]
Simplify[Series[EXTRCNTRBLft00[r, D1, D2, a, b], {r, D2, 4}]]
Simplify[Series[EXTRCNTRBRgt00[r, D1, D2, a, b], {r, D1, 4}]]
Simplify[Series[EXTRCNTRBRgt00[r, D1, D2, a, b], {r, D2, 4}]]
```

The approximation is determined in this way:
1A) we first determine TetrCFApprxPrtlyMatchd00AA[r] equating it to TETRACFAA[r];
2A) we determine TetrCFApprxPrtlyMatchd00BB[r] using first LEFTCFApprxnn[..] within the subinterval [D1,D2] and then matching this expression to TETRCFAA[r] at r=D1. The error [TetrCFApprxFnl00BB[r] -TETRCFBB[r]] at r=D2 is ~0.005.
3A) we determine TetrCFApprxFnl00DD[r] by RGHTCFApprxnn[..]. The error (TetrCFApprxFnl00DD[r] -TETRCFDD[r]) is ~0.002. It cannot be cured and propagates to the left subinterval;
4A) we determine TetrCFApprxFnl00CC[r] by the above formula. It automatically matches TetrCFApprxFnl00DD[r] at r=D3 and is equal to 0 at r=D2.
5A) finally, TetrCFApprx00BB[r] is determined matching TetrCFApprxPrtlyMatchd00BB[r] to TetrCFApprxFnl00CC[r] at r=D2 adding the extracontribution EXTRCNTRB00[r].
The resulting inconsistencies are:
a small negativity on the left of D2; the vanishing at r=D2, r=D3 and r=D4.

Approximation within the interval [D0, D1]

```
TETRCFAA[r]
Simplify[D[D[TETRCFAA[r], r], r]]
```

$$\text{TetrCFApprxFnl00AA[r\_] := } 1 - 3\sqrt{\frac{3}{2}}\, r - \frac{\left(6 + 5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2 \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TetrCLDApprx00AA[r\_] := } -\frac{3\left(6\sqrt{2}\, r + \pi\left(-8 + 5\sqrt{6}\, r\right) + 8\left(-2\sqrt{2} + \text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

```
Plot[{TETRCFAA[r], TetrCFApprxFnl00AA[r]}, {r, 0, TETRDst[[1]]}]
```

Approximation within the interval [D1, D2]

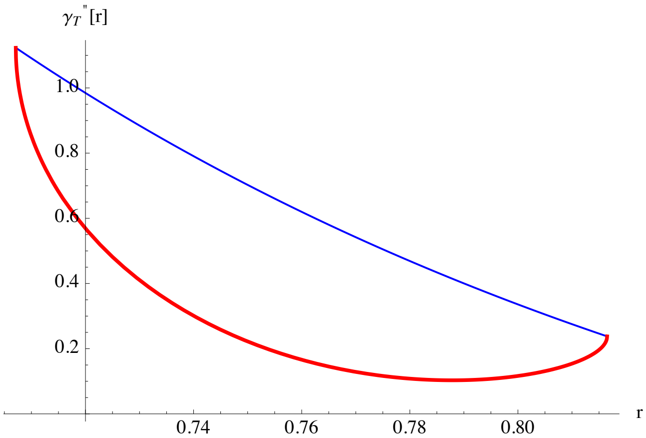
```
Clear[a0]; Clear[a1]; Clear[a2]; Clear[a3]; Clear[a4]; Clear[a5]; Clear[a6];
Clear[b0]; Clear[b1]; Clear[b2]; Clear[b3]; Clear[b4]; Clear[b5]; Clear[b6];

cfD1Plus
cfD2Minus

FullSimplify[((CLDApprx00[r, D1, D2]) /.
    {a0 → cfD1Plus[[1]], b0 → cfD2Minus[[1]], D1 → TETRDst[[1]], D2 → TETRDst[[2]]}) -
  TetrCLDApprx00BB[r], Assumptions → {TETRDst[[1]] < r < TETRDst[[2]]}]
```

0

approximation of the CLD

$$\text{TetrCLDApprx00BB[r\_] :=}$$
$$\frac{3}{8}\left(-\frac{1}{\pi}2\left(1 - \frac{3\left(-\sqrt{2} + \sqrt{6} - r\right)\sqrt{-3\sqrt{2} + 6 r}}{2^{3/4}\left(-3 + 2\sqrt{3}\right)^{3/2}}\right)\left(6 - 16\sqrt{2} + \left(-8 + 5\sqrt{3}\right)\pi + 8\,\text{ArcSec}[3]\right) + \left(1 + \frac{\sqrt{\sqrt{6} - 3 r}\left(9 - 4\sqrt{3} - 3\sqrt{2}\, r\right)}{2^{3/4}\left(-3 + 2\sqrt{3}\right)^{3/2}}\right)\left(-36 + 19\sqrt{3} - \frac{8\left(-4\sqrt{2} + \sqrt{3} + 2\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

```
Plot[{TETRDDCFBB[r], TetrCLDApprx00BB[r]}, {r, TETRDst[[1]], TETRDst[[2]]},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.006]]},
 AxesLabel → {"r", "γT''[r]"}]
```



approximation (not yet matched) of the CF

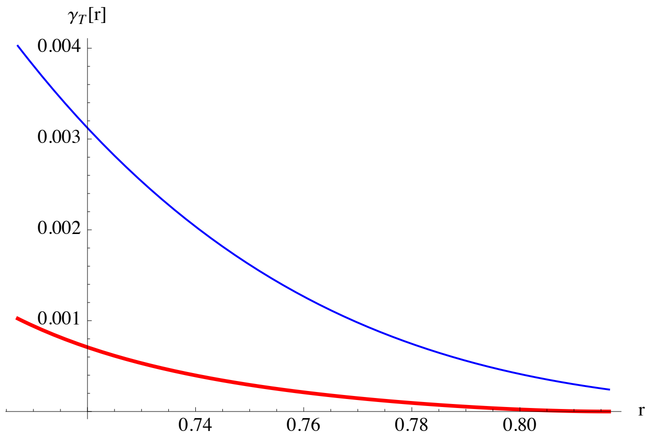
```
Simplify[((LEFTCFApprx00[r, D1, D2]) /.
    {a0 → cfD1Plus[[1]], b0 → cfD2Minus[[1]], D1 → TETRDst[[1]], D2 → TETRDst[[2]]}) -
  ApprxTetrCF00NotMatchBB[r], Assumptions → {TETRDst[[1]] < r < TETRDst[[2]]}]
```

0

```
ApprxTetrCF00NotMatchBB[r_] :=
```

$$\frac{3}{280}\left(-\left(\left(26\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-20\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+84\sqrt{-\sqrt{2}+2\,r}-12\sqrt{6}\,r^3\sqrt{-\sqrt{2}+2\,r}-36\sqrt{-3\sqrt{2}+6\,r}+r^2\left(105\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-70\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+24\left(7-2\sqrt{3}\right)\sqrt{-\sqrt{2}+2\,r}\right)+2\,r\left(-28\,2^{3/4}\sqrt{-3+2\sqrt{3}}+21\times 2^{3/4}\sqrt{-9+6\sqrt{3}}+3\left(-28+11\sqrt{3}\right)\sqrt{-2\sqrt{2}+4\,r}\right)\right)\left(-6+16\sqrt{2}+\left(8-5\sqrt{3}\right)\pi-8\,\mathrm{ArcSec}[3]\right)\right)\Big/\left(2^{1/4}\left(-3+2\sqrt{3}\right)^{3/2}\pi\right)+35\left(\frac{1}{3}-\sqrt{\frac{2}{3}}\,r+\frac{r^2}{2}-\frac{2\times 2^{3/4}\left(\sqrt{6}-3\,r\right)^{5/2}\left(-\frac{7}{\sqrt{2}}+2\sqrt{6}+r\right)}{105\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\mathrm{ArcSec}[3]\right)}{\pi}\right)\right);$$

```
Plot[{TETRCFBB[r], ApprxTetrCF00NotMatchBB[r]}, {r, TETRDst[[1]], TETRDst[[2]]},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.006]]},
 AxesLabel → {"r", "γT[r]"}]
```



The approximation is matched at r=D1

```
Solve[{Limit[ApprxTetrCF00NotMatchBB[r] + a + b r, r → TETRDst[[1]], Direction → -1] -
     Limit[TetrCFApprxFnl00AA[r], r → TETRDst[[1]], Direction → 1] == 0 &&
   Limit[D[ApprxTetrCF00NotMatchBB[r] + a + b r, r], r → TETRDst[[1]], Direction → -1] -
     Limit[D[TetrCFApprxFnl00AA[r], r], r → TETRDst[[1]], Direction → 1] == 0}, {a, b}]
```

```
Simplify[Simplify[(ApprxTetrCF00NotMatchBB[r] + a + b r) /. {a →
     -1/(1120 π) (-204 + 3808 √2 - 472 √3 - 2428 π + 1007 √3 π - 1680 ArcCos[1/3] - 224 ArcSec[3]),
     b → -1/(40 π) (-288 + 51 √2 - 128 √3 - 177 √2 π + 131 √6 π +
         120 √2 ArcCos[1/3] - 48 √2 ArcSec[3] + 32 √6 ArcSec[3])}, ,
   Assumptions → {TETRDst[[1]] < r < TETRDst[[2]]}] - ApprxTetrCF00PrtlyMatchBB[r]]
```

0

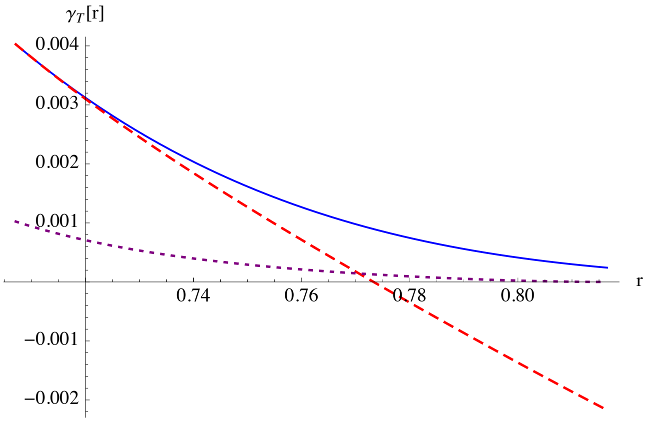
ApprxTetrCF00PrtlyMatchBB[r_] :=

$$-\frac{1}{40\pi}\, r \left(-288+51\sqrt{2}-128\sqrt{3}+\sqrt{2}\left(-177+131\sqrt{3}\right)\pi+120\sqrt{2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]-48\sqrt{2}\,\mathrm{ArcSec}[3]+32\sqrt{6}\,\mathrm{ArcSec}[3]\right)+$$

$$\frac{1}{1120\pi}\left(\left(2428-1007\sqrt{3}\right)\pi+4\left(51-952\sqrt{2}+118\sqrt{3}+420\,\mathrm{ArcCos}\left[\frac{1}{3}\right]+56\,\mathrm{ArcSec}[3]\right)\right)+$$

$$\frac{3}{280}\Bigg(-\Big(\Big(26\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-20\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+84\sqrt{-\sqrt{2}+2r}-12\sqrt{6}\,r^3\sqrt{-\sqrt{2}+2r}-36\sqrt{-3\sqrt{2}+6r}+r^2\left(105\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-70\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+24\left(7-2\sqrt{3}\right)\sqrt{-\sqrt{2}+2r}\right)+2r\left(-28\,2^{3/4}\sqrt{-3+2\sqrt{3}}+21\times 2^{3/4}\sqrt{-9+6\sqrt{3}}+3\left(-28+11\sqrt{3}\right)\sqrt{-2\sqrt{2}+4r}\right)\Big)\left(-6+16\sqrt{2}+\left(8-5\sqrt{3}\right)\pi-8\,\mathrm{ArcSec}[3]\right)\Big)\Big/\left(2^{1/4}\left(-3+2\sqrt{3}\right)^{3/2}\pi\right)+35\left(\frac{1}{3}-\sqrt{\frac{2}{3}}\,r+\frac{r^2}{2}-\frac{2\times 2^{3/4}\left(\sqrt{6}-3r\right)^{5/2}\left(-\frac{7}{\sqrt{2}}+2\sqrt{6}+r\right)}{105\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\mathrm{ArcSec}[3]\right)}{\pi}\right)\Bigg);$$

```
Plot[{TETRCFBB[r], ApprxTetrCF00PrtlyMatchBB[r], ApprxTetrCF00NotMatchBB[r]},
 {r, TETRDst[[1]], TETRDst[[2]]},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dashed],
   Directive[Purple, Thickness[0.004], Dotted]}, AxesLabel → {"r", "γT[r]"}]
```

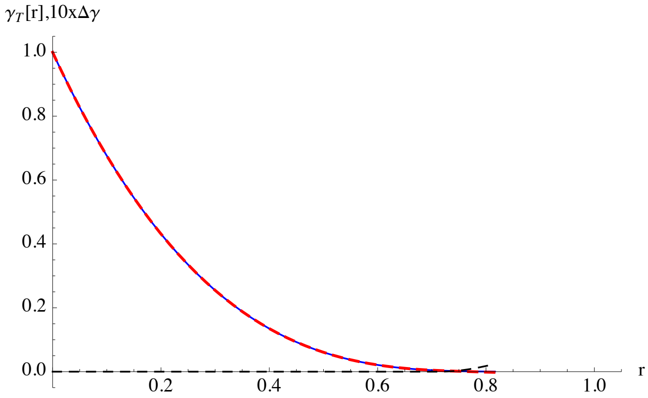


The error at r=D2 is ~0.00241293.

```
N[Limit[TETRCFBB[r] - ApprxTetrCF00PrtlyMatchBB[r], r → TETRDst[[2]], Direction → 1]]
```

```
0.00241293
```

```
ausfiga = Plot[{If[r < TETRDst[[1]], TETRCFAA[r], TETRCFBB[r] ],
   If[r < TETRDst[[1]], TetrCFApprxFnl00AA[r], ApprxTetrCF00PrtlyMatchBB[r]],
   10 (If[r < TETRDst[[1]], TETRCFAA[r], TETRCFBB[r] ] -
      If[r < TETRDst[[1]], TetrCFApprxFnl00AA[r], ApprxTetrCF00PrtlyMatchBB[r]])},
  {r, 0, TETRDst[[2]]}, PlotRange → {{0, 1.05}, {-0.05, 1.05}}, PlotStyle →
   {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.005], Dashed],
    Directive[Black, Thickness[0.003], Dashed]}, AxesLabel → {"r", "γT[r],10xΔγ"}]
```



We start now from D4 and proceed towards the left

Approximation within the interval [D3, D4]

```
Clear[a0]; Clear[a1]; Clear[a2]; Clear[a3]; Clear[a4]; Clear[a5]; Clear[a6];
Clear[b0]; Clear[b1]; Clear[b2]; Clear[b3]; Clear[b4]; Clear[b5]; Clear[b6];
```

```
cfD3Plus
cfD4Minus
```

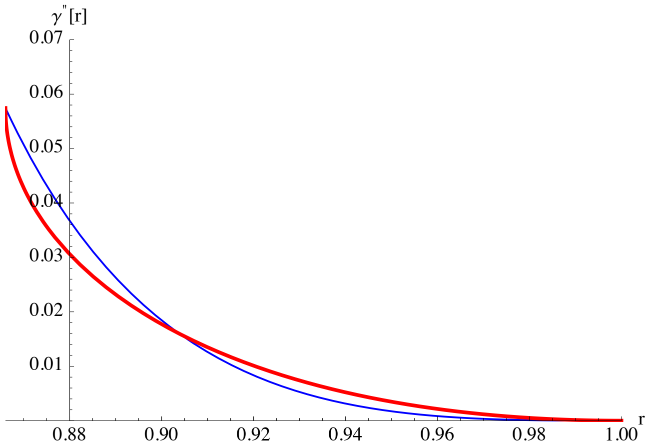
evaluation of the approximation of the CLD

```
Simplify[Simplify[(CLDApprx00[r, D1, D2]) /.
    {a0 → cfD3Plus[[1]], b0 → cfD4Minus[[1]], D1 → TETRDst[[3]], D2 → TETRDst[[4]]},
   Assumptions → {TETRDst[[3]] < r < TETRDst[[4]]}] - TetrCLDApprx00DD[r]]
```

0

TetrCLDApprx00DD[r_] :=
$$\frac{1}{72}\left(1+\frac{\left(-3+\sqrt{3}+r\right)\sqrt{-\sqrt{3}+2\,r}}{\left(2-\sqrt{3}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)}{\pi}\right);$$

```
Plot[{TETRDDCFDD[r], TetrCLDApprx00DD[r]},
{r, TETRDst[[3]], TETRDst[[4]]}, AxesLabel → {"r", "γ''[r]"},
PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.006]]},
PlotRange → {{TETRDst[[3]], TETRDst[[4]]}, {0, 0.07}}, AxesLabel → {"r", "γT''[r],10xΔγ"}]
```



evaluation of the approximation of the CF

```
Simplify[Simplify[(LEFTCFApprx00[r, D1, D2]) /.
    {a0 → cfD3Plus[[1]], b0 → cfD4Minus[[1]], D1 → TETRDst[[3]], D2 → TETRDst[[4]]},
  Assumptions → {TETRDst[[3]] < r < TETRDst[[4]]}] - TetrCFApprxFnl00DD[r]]
```

0

TetrCFApprxFnl00DD[r_] :=

$$-\frac{1}{5040\left(-2+\sqrt{3}\right)\pi}\left(30-13\sqrt{3}-42\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+18\sqrt{3}\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+\left(56-42\sqrt{3}-66\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+56\sqrt{3}\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}\right)r+\left(70-35\sqrt{3}-56\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+16\sqrt{3}\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}\right)r^2+8\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}\,r^3\right)\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)\right);$$

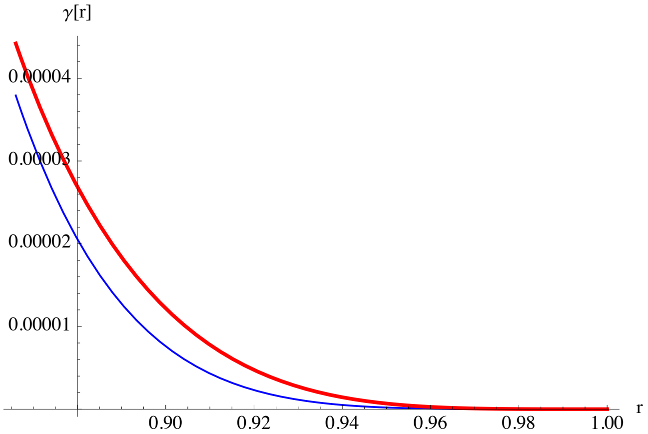
The error of the CF-approximation is -6.28505×10^{-6} at D_3, i.e. ~17%

```
N[Limit[TETRCFDD[r] - TetrCFApprxFnl00DD[r], r → TETRDst[[3]], Direction → -1]]
N[Limit[TETRCFDD[r] - TetrCFApprxFnl00DD[r], r → TETRDst[[3]], Direction → -1] /
  Limit[TETRCFDD[r], r → TETRDst[[3]], Direction → -1]]
```

-6.28505×10^{-6}

-0.165743

```
Plot[{TETRCFDD[r], TetrCFApprxFnl00DD[r]},
 {r, TETRDst[[3]], TETRDst[[4]]}, AxesLabel → {"r", "γ[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.006]]},
 AxesLabel → {"r", "γT[r],10xΔγ"}]
```



Approximation within the interval [D2, D3]

```
Clear[a0]; Clear[a1]; Clear[a2]; Clear[a3]; Clear[a4]; Clear[a5]; Clear[a6];
Clear[b0]; Clear[b1]; Clear[b2]; Clear[b3]; Clear[b4]; Clear[b5]; Clear[b6];
```

```
cfD2Plus
cfD3Minus
```

```
cfD2Plus[[1]]
cfD3Minus[[1]]
N[cfD2Plus[[1]]]
N[cfD3Minus[[1]]]
```

approximation of the CLD

```
Simplify[Simplify[(CLDApprx00[r, D1, D2]) /.
    {a0 → cfD2Plus[[1]], b0 → cfD3Minus[[1]], D1 → TETRDst[[2]], D2 → TETRDst[[3]]},
   Assumptions → {TETRDst[[2]] < r < TETRDst[[3]]}] - TetrCLDApprx00CC[r]]
```

0

$$\text{TetrCLDApprx00AA[r\_] := } -\frac{3\left(6\sqrt{2}\ r+\pi\left(-8+5\sqrt{6}\ r\right)+8\left(-2\sqrt{2}+\text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

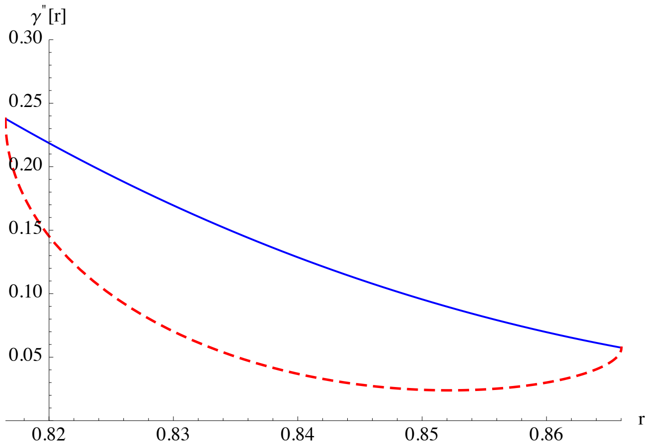
$$\text{TetrCLDApprx00BB[r\_] :=}$$
$$\frac{3}{8}\left(-\frac{1}{\pi}2\left(1-\frac{3\left(-\sqrt{2}+\sqrt{6}-r\right)\sqrt{-3\sqrt{2}+6\ r}}{2^{3/4}\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\ \text{ArcSec[3]}\right)+\left(1+\frac{\sqrt{\sqrt{6}-3\ r}\left(9-4\sqrt{3}-3\sqrt{2}\ r\right)}{2^{3/4}\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\ \text{ArcSec[3]}\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00CC[r\_] := }\frac{1}{72}\left(27\left(1+\frac{3^{1/4}\left(2\sqrt{\frac{2}{3}}-\frac{3\sqrt{3}}{2}+r\right)\sqrt{-2\sqrt{6}+6\ r}}{\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\ \text{ArcSec[3]}\right)}{\pi}\right)+\left(1-\frac{3^{3/4}\sqrt{\sqrt{3}-2\ r}\left(\sqrt{3}-\sqrt{6}+r\right)}{\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\ \text{ArcSec[3]}\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00DD[r\_] := }\frac{1}{72}\left(1+\frac{\left(-3+\sqrt{3}+r\right)\sqrt{-\sqrt{3}+2\ r}}{\left(2-\sqrt{3}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\ \text{ArcSec[3]}\right)}{\pi}\right);$$

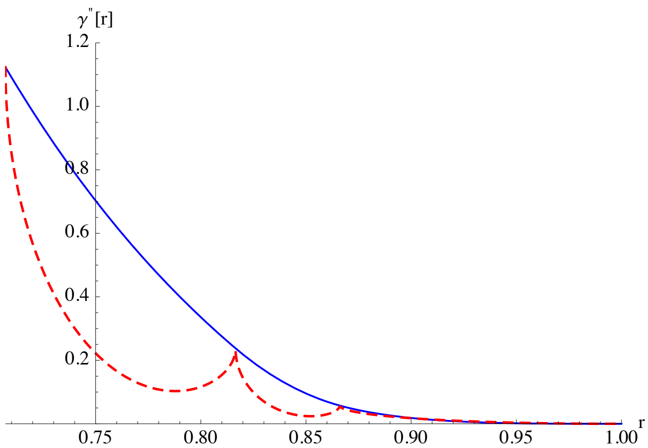
$$\text{TetrCLDApprx00CC[r\_] := }\frac{1}{72}\left(27\left(1+\frac{3^{1/4}\left(2\sqrt{\frac{2}{3}}-\frac{3\sqrt{3}}{2}+r\right)\sqrt{-2\sqrt{6}+6\ r}}{\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\ \text{ArcSec[3]}\right)}{\pi}\right)+\left(1-\frac{3^{3/4}\sqrt{\sqrt{3}-2\ r}\left(\sqrt{3}-\sqrt{6}+r\right)}{\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\ \text{ArcSec[3]}\right)}{\pi}\right)\right);$$

```
Plot[{TETRDDCFCC[r], TetrCLDApprx00CC[r]},
{r, TETRDst[[2]], TETRDst[[3]]}, AxesLabel → {"r", "γ''[r]"}, PlotStyle →
 {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dashed]},
PlotRange → {{TETRDst[[2]], TETRDst[[3]]}, {0, 0.3}}, AxesLabel → {"r", "γT''[r],10xΔγ"}]
```



The largest error is about 62% around 0.832

```
Plot[{TETRDDCFCC[r] - TetrCLDApprx00CC[r]}, {r, TETRDst[[2]], TETRDst[[3]]}]

N[(TETRDDCFCC[832 / 1000] - TetrCLDApprx00CC[832 / 1000]) / TETRDDCFCC[832 / 1000]]

0.617684

Plot[
 {If[r < TETRDst[[2]], TETRDDCFBB[r], If[r < TETRDst[[3]], TETRDDCFCC[r], TETRDDCFDD[r]]],
  If[r < TETRDst[[2]], TetrCLDApprx00BB[r],
   If[r < TETRDst[[3]], TetrCLDApprx00CC[r], TetrCLDApprx00DD[r]]]},
 {r, TETRDst[[1]], TETRDst[[4]]}, AxesLabel → {"r", "γ''[r]"}, PlotStyle →
  {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dashed]},
 PlotRange → {{TETRDst[[1]], TETRDst[[4]]}, {0, 1.2}}, AxesLabel → {"r", "γT''[r],10xΔγ"}]
```



Overall the CLD approximation does not very accurate. Nonetheless that of the CF will be.

approximation (not matched) of the CF

```
FullSimplify[((LEFTCFApprx00[r, D1, D2]) /.
    {a0 → cfD2Plus[[1]], b0 → cfD3Minus[[1]], D1 → TETRDst[[2]], D2 → TETRDst[[3]]}) -
  ApprxTetrCF00NotMatchCC[r], Assumptions → {TETRDst[[2]] < r < TETRDst[[3]]}]

0
```

ApprxTetrCF00NotMatchCC[r_] :=

$$\frac{1}{5040}\left(-\frac{1}{4\pi}3\times 3^{3/4}\left(3+2\sqrt{2}\right)\left(-465\,3^{1/4}+294\sqrt{2}\,3^{1/4}-768\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+672\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}-96\sqrt{6\left(3+2\sqrt{2}\right)}\,r^3\sqrt{-\sqrt{6}+3r}+12r^2\left(-105\,3^{1/4}+70\sqrt{2}\,3^{1/4}-64\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+84\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}\right)+4r\left(7\times 3^{3/4}\left(-23+18\sqrt{2}\right)+176\sqrt{6\left(3+2\sqrt{2}\right)}\sqrt{-\sqrt{6}+3r}-336\sqrt{9+6\sqrt{2}}\sqrt{-\sqrt{6}+3r}\right)\right)\left(32\sqrt{2}-8\sqrt{3}-36\pi+19\sqrt{3}\pi-16\,\text{ArcSec}[3]\right)+70\left(\frac{3}{8}-\frac{\sqrt{3}\,r}{2}+\frac{r^2}{2}-\frac{3^{3/4}\left(\sqrt{3}-2r\right)^{5/2}\left(\frac{9-7\sqrt{2}}{\sqrt{3}}+r\right)}{35\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

```
Plot[{TETRCFCC[r], ApprxTetrCF00NotMatchCC[r]},
  {r, TETRDst[[2]], TETRDst[[3]]}, AxesLabel → {"r", "γ[r]"},
  PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.006]]},
  AxesLabel → {"r", "γT[r],10xΔγ"}] ;
```

The approximation is matched at r=D3

```
Simplify[Solve[
  {Simplify[Limit[ApprxTetrCF00NotMatchCC[r] + a + b r, r → TETRDst[[3]], Direction → 1] -
       Limit[TetrCFApprxFnl00DD[r], r → TETRDst[[3]], Direction → -1]] == 0 &&
    Simplify[Limit[D[ApprxTetrCF00NotMatchCC[r] + a + b r, r],
        r → TETRDst[[3]], Direction → 1] -
       Limit[D[TetrCFApprxFnl00DD[r], r], r → TETRDst[[3]], Direction → -1]] == 0}, {a, b}]]
```

Simplify[Simplify[(ApprxTetrCF00NotMatchCC[r] + a + b r) /.

$$\left\{a\to\left(\left(-116\,640-40\,338\sqrt{2}+68\,688\sqrt{3}+22\,403\sqrt{6}\right)\pi-54\left(\sqrt{2}\left(-1917+1118\sqrt{3}\right)-8\left(-90+53\sqrt{3}\right)\text{ArcSec}[3]\right)\right)\Big/\left(20\,160\left(-2+\sqrt{3}\right)\pi\right),\ b\to-\frac{1}{720\left(-2+\sqrt{3}\right)\pi}\left(-7+4\sqrt{3}\right)\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)\right)\right\},$$

Assumptions → {TETRDst[[2]] < r < TETRDst[[3]]}] - TetrCFApprxFnl00CC[r]]

0

$$\mathbf{TetrCFApprxFnl00CC[r\_]} := \frac{1}{20\,160}\left(-\frac{1}{\left(-2+\sqrt{3}\right)\pi}\right.$$

$$28\left(-7+4\sqrt{3}\right) r\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)\right)+$$

$$\frac{1}{\left(-2+\sqrt{3}\right)\pi}\left(\left(-116\,640-40\,338\sqrt{2}+68\,688\sqrt{3}+22\,403\sqrt{6}\right)\pi-\right.$$

$$\left.54\left(\sqrt{2}\left(-1917+1118\sqrt{3}\right)-8\left(-90+53\sqrt{3}\right)\mathrm{ArcSec}[3]\right)\right)+$$

$$4\left(-\frac{1}{4\pi}3\times 3^{3/4}\left(3+2\sqrt{2}\right)\left(-465\;3^{1/4}+294\sqrt{2}\;3^{1/4}-768\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+\right.\right.$$

$$672\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}-96\sqrt{6\left(3+2\sqrt{2}\right)}\,r^3\sqrt{-\sqrt{6}+3r}+$$

$$12r^2\left(-105\;3^{1/4}+70\sqrt{2}\;3^{1/4}-64\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+84\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}\right)+$$

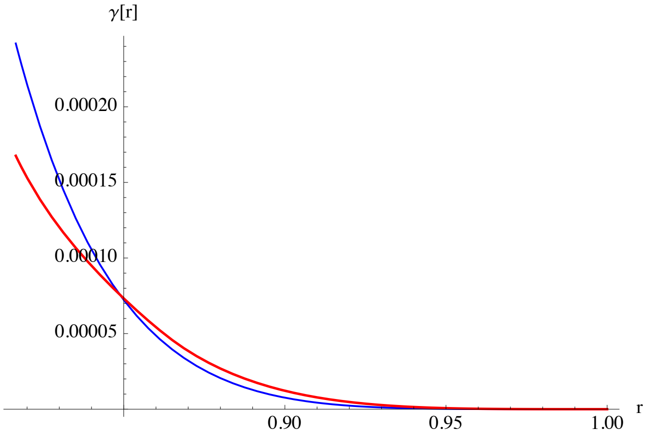
$$4r\left(7\times 3^{3/4}\left(-23+18\sqrt{2}\right)+176\sqrt{6\left(3+2\sqrt{2}\right)}\sqrt{-\sqrt{6}+3r}-336\sqrt{9+6\sqrt{2}}\right.$$

$$\left.\left.\sqrt{-\sqrt{6}+3r}\right)\right)\left(32\sqrt{2}-8\sqrt{3}-36\pi+19\sqrt{3}\,\pi-16\,\mathrm{ArcSec}[3]\right)+$$

$$70\left(\frac{3}{8}-\frac{\sqrt{3}\,r}{2}+\frac{r^2}{2}-\frac{3^{3/4}\left(\sqrt{3}-2r\right)^{5/2}\left(\frac{9-7\sqrt{2}}{\sqrt{3}}+r\right)}{35\left(3-2\sqrt{2}\right)^{3/2}}\right)\left(-1296+209\sqrt{2}+\right.$$

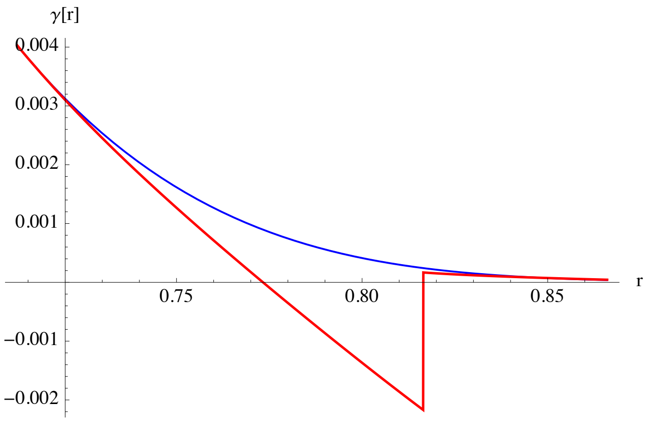
$$\left.\left.\left.372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)}{\pi}\right)\right)\right);$$

```
Plot[{If[r < TETRDst[[3]], TETRCFCC[r], TETRCFDD[r]],
  If[r < TETRDst[[3]], TetrCFApprxFnl00CC[r], TetrCFApprxFnl00DD[r]]},
 {r, TETRDst[[2]], TETRDst[[4]]}, AxesLabel → {"r", "γ[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004]]}]
```



The situation around D2 is shown in the figure.

```
Plot[{If[r < TETRDst[[2]], TETRCFBB[r], TETRCFCC[r]],
  If[r < TETRDst[[2]], ApprxTetrCF00PrtlyMatchBB[r], TetrCFApprxFnl00CC[r]]},
 {r, TETRDst[[1]], TETRDst[[3]]}, AxesLabel → {"r", "γ[r]"},
 PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004]]}]
```



The figure shows that end-value of TetrCFApprxFnl00CC[r] is closer to the exact one.
Thus the match at r=D2 is made adding to ApprxTetrCF00PrtlyMatchBB[r]
the contribution EXTRCNTRBRgt00[r, D1, D2, a, b] and matching the result
to TetrCFApprxFnl00CC[r].
The smaller lenght of [D2,D3] in comparison to [D1,D2]'s suggests that TetrCFApprxFnl00CC[r] is closer than ApprxTetrCF00PrtlyMatchBB[r] to the
exact CF. Thus, the choice can be done without knowing the exact CF.

We match ApprxTetrCF00PrtlyMatchBB[r] to TetrCFApprxFnl00CC[r] at r=D2 by adding to the first the contribution EXTRCNTRB00[r, D1, D2, a, b]

```
Simplify[(EXTRCNTRBRgt00[r, D1, D2, a, b]) /. {D1 → TETRDst[[1]], D2 → TETRDst[[2]]},
 Assumptions → {TETRDst[[1]] < r < TETRDst[[2]]}]
```

$$-\frac{1}{60}\left(\frac{1}{\sqrt{2}}-r\right)^{3}\left(-5\,a\left(-2\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{2}}+r\right)+b\left(\frac{1}{\sqrt{2}}-r\right)\left(-5\sqrt{\frac{2}{3}}+\sqrt{2}+3\,r\right)\right)$$

```
EXTRCNTRB00BB[r_, a_, b_] :=
```

$$-\frac{1}{60}\left(\frac{1}{\sqrt{2}}-r\right)^3\left(-5\,a\left(-2\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{2}}+r\right)+b\left(\frac{1}{\sqrt{2}}-r\right)\left(-5\sqrt{\frac{2}{3}}+\sqrt{2}+3\,r\right)\right)$$

```
Simplify[Solve[
  {Simplify[Limit[ApprxTetrCF00PrtlyMatchBB[r] + EXTRCNTRB00BB[r, a, b], r → TETRDst[[2]],
       Direction → 1] - Limit[TetrCFApprxFnl00CC[r],
       r → TETRDst[[2]], Direction → -1]] == 0 &&
    Simplify[Limit[D[ApprxTetrCF00PrtlyMatchBB[r] + EXTRCNTRB00BB[r, a, b], r],
       r → TETRDst[[2]], Direction → 1] -
      Limit[D[TetrCFApprxFnl00CC[r], r], r → TETRDst[[2]], Direction → -1]] == 0}, {a, b}]]
```

Simplify[Simplify[(ApprxTetrCF00PrtlyMatchBB[r] + EXTRCNTRB00BB[r, a, b]) /.

$$\Big\{a\to\frac{1}{140\,\pi}\Big(\Big(-221\,874\,996+172\,383\,744\sqrt{2}-128\,100\,927\sqrt{3}+99\,525\,812\sqrt{6}\Big)\pi+ 648\Big(-153\,635-35\,520\sqrt{2}-88\,701\sqrt{3}-20\,511\sqrt{6}+140\left(575+332\sqrt{3}\right)\mathrm{ArcCos}\left[\tfrac{1}{3}\right]+\left(-79\,450+55\,384\sqrt{2}-45\,872\sqrt{3}+31\,976\sqrt{6}\right)\mathrm{ArcSec}[3]\Big)\Big),$$

$$b\to\frac{1}{14\,\pi}\Big(\Big(-441\,887\,094+284\,889\,498\sqrt{2}-255\,123\,640\sqrt{3}+164\,480\,973\sqrt{6}\Big)\pi- 324\Big(-170\,664-391\,769\sqrt{2}-98\,532\sqrt{3}-226\,188\sqrt{6}+560\sqrt{2}\left(362+209\sqrt{3}\right)\mathrm{ArcCos}\left[\tfrac{1}{3}\right]-4\left(-70\,588+50\,815\sqrt{2}-40\,754\sqrt{3}+29\,338\sqrt{6}\right)\mathrm{ArcSec}[3]\Big)\Big)\Big\},$$

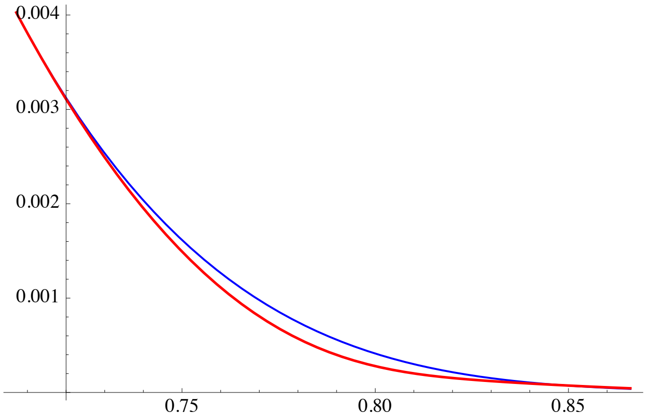
Assumptions → {TETRDst[[1]] < r < TETRDst[[2]]}] - TetrCFApprxFnl00BB[
r]]

0

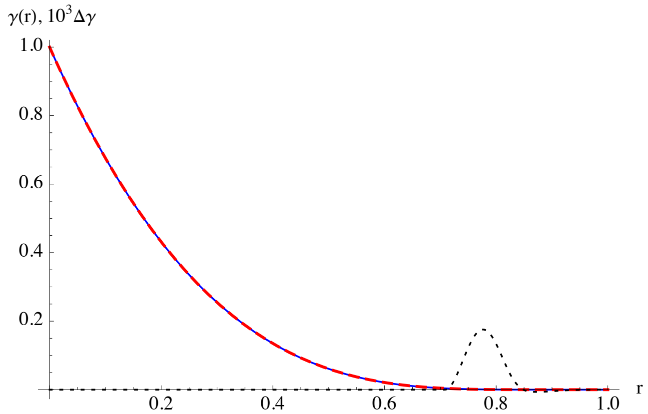
TetrCFApprxFnl00BB[r_] :=

$$
\begin{aligned}
&-\frac{1}{40\,\pi}\, r \Big(-288 + 51\sqrt{2} - 128\sqrt{3} + \sqrt{2}\left(-177 + 131\sqrt{3}\right)\pi + 120\sqrt{2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right] - \\
&\quad 48\sqrt{2}\,\mathrm{ArcSec}[3] + 32\sqrt{6}\,\mathrm{ArcSec}[3]\Big) + \\
&\frac{1}{1120\,\pi}\left(\left(2428 - 1007\sqrt{3}\right)\pi + 4\left(51 - 952\sqrt{2} + 118\sqrt{3} + 420\,\mathrm{ArcCos}\left[\frac{1}{3}\right] + 56\,\mathrm{ArcSec}[3]\right)\right) + \\
&\frac{3}{280}\Bigg(-\Bigg(\Bigg(\Bigg(26\times 2^{1/4}\sqrt{-3+2\sqrt{3}} - 20\times 2^{1/4}\sqrt{-9+6\sqrt{3}} + 84\sqrt{-\sqrt{2}+2\,r} - \\
&\qquad 12\sqrt{6}\,r^3\sqrt{-\sqrt{2}+2\,r} - 36\sqrt{-3\sqrt{2}+6\,r} + r^2\Big(105\times 2^{1/4}\sqrt{-3+2\sqrt{3}} - 70\times \\
&\qquad\quad 2^{1/4}\sqrt{-9+6\sqrt{3}} + 24\left(7-2\sqrt{3}\right)\sqrt{-\sqrt{2}+2\,r}\Big) + 2\,r\Big(-28\,2^{3/4} \\
&\qquad\quad \sqrt{-3+2\sqrt{3}} + 21\times 2^{3/4}\sqrt{-9+6\sqrt{3}} + 3\left(-28+11\sqrt{3}\right)\sqrt{-2\sqrt{2}+4\,r}\Big)\Bigg) \\
&\qquad \left(-6 + 16\sqrt{2} + \left(8-5\sqrt{3}\right)\pi - 8\,\mathrm{ArcSec}[3]\right)\Bigg) \Big/ \left(2^{1/4}\left(-3+2\sqrt{3}\right)^{3/2}\pi\right) + \\
&\quad 35\left(\frac{1}{3} - \sqrt{\frac{2}{3}}\,r + \frac{r^2}{2} - \frac{2\times 2^{3/4}\left(\sqrt{6}-3\,r\right)^{5/2}\left(-\frac{7}{\sqrt{2}} + 2\sqrt{6} + r\right)}{105\left(-3+2\sqrt{3}\right)^{3/2}}\right) \\
&\quad \left(-36 + 19\sqrt{3} - \frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\mathrm{ArcSec}[3]\right)}{\pi}\right)\Bigg) - \\
&\frac{1}{1680\,\pi}\left(\frac{1}{\sqrt{2}} - r\right)^3\Bigg(2\left(\frac{1}{\sqrt{2}} - r\right)\left(-5\sqrt{\frac{2}{3}} + \sqrt{2} + 3\,r\right) \\
&\quad \Big(\left(-441\,887\,094 + 284\,889\,498\sqrt{2} - 255\,123\,640\sqrt{3} + 164\,480\,973\sqrt{6}\right)\pi - \\
&\qquad 324\Big(-170\,664 - 391\,769\sqrt{2} - 98\,532\sqrt{3} - 226\,188\sqrt{6} + 560\sqrt{2}\left(362+209\sqrt{3}\right) \\
&\qquad\quad \mathrm{ArcCos}\left[\frac{1}{3}\right] - 4\left(-70\,588 + 50\,815\sqrt{2} - 40\,754\sqrt{3} + 29\,338\sqrt{6}\right)\mathrm{ArcSec}[3]\Big)\Big) - \\
&\quad \left(-2\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} + r\right)\Big(\left(-221\,874\,996 + 172\,383\,744\sqrt{2} - 128\,100\,927\sqrt{3} + 99\,525\,812\sqrt{6}\right)\pi + \\
&\qquad 648\Big(-153\,635 - 35\,520\sqrt{2} - 88\,701\sqrt{3} - 20\,511\sqrt{6} + 140\left(575+332\sqrt{3}\right)\mathrm{ArcCos}\left[\frac{1}{3}\right] + \\
&\qquad\quad \left(-79\,450 + 55\,384\sqrt{2} - 45\,872\sqrt{3} + 31\,976\sqrt{6}\right)\mathrm{ArcSec}[3]\Big)\Big)\Bigg);
\end{aligned}
$$

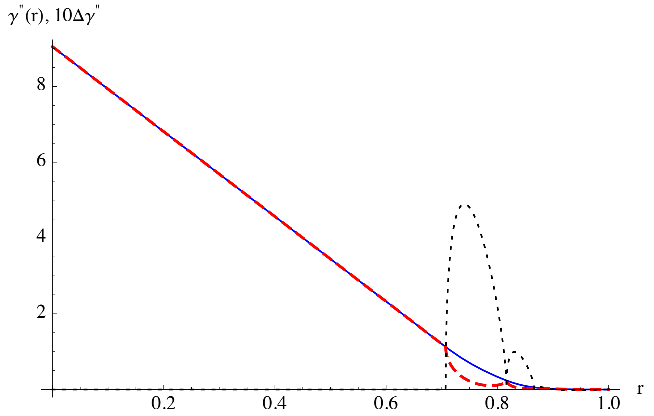
```
Plot[{If[r < TETRDst[[2]], TETRCFBB[r], TETRCFCC[r]],
   If[r < TETRDst[[2]], TetrCFApprxFnl00BB[r], TetrCFApprxFnl00CC[r]]},
  {r, TETRDst[[1]], TETRDst[[3]]},
  PlotStyle → {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004]]}]
```




```
Plot[{If[r < TETRDst[[1]], TETRCFAA[r],
    If[r < TETRDst[[2]], TETRCFBB[r], If[r < TETRDst[[3]], TETRCFCC[r], TETRCFDD[r]]]],
   If[r < TETRDst[[1]], TetrCFApprxFnl00AA[r], If[r < TETRDst[[2]], TetrCFApprxFnl00BB[r],
     If[r < TETRDst[[3]], TetrCFApprxFnl00CC[r], TetrCFApprxFnl00DD[r]]]],
   1000 * If[r < TETRDst[[1]], (TETRCFAA[r] - TetrCFApprxFnl00AA[r]),
     If[r < TETRDst[[2]], (TETRCFBB[r] - TetrCFApprxFnl00BB[r]), If[r < TETRDst[[3]],
       (TETRCFCC[r] - TetrCFApprxFnl00CC[r]), (TETRCFDD[r] - TetrCFApprxFnl00DD[r])]]]},
  {r, 0, 1}, PlotStyle → {Directive[Blue, Thickness[0.003]],
    Directive[Red, Thickness[0.005], Dashed], Directive[Black, Thickness[0.003], Dotted]},
  AxesLabel → {"r", " γ(r), 10^3Δγ"}]
```



```
Plot[{If[r < TETRDst[[1]], TETRDDCFAA[r], If[r < TETRDst[[2]],
    TETRDDCFBB[r], If[r < TETRDst[[3]], TETRDDCFCC[r], TETRDDCFDD[r]]]],
  If[r < TETRDst[[1]], TetrCLDApprx00AA[r], If[r < TETRDst[[2]], TetrCLDApprx00BB[r],
    If[r < TETRDst[[3]], TetrCLDApprx00CC[r], TetrCLDApprx00DD[r]]]],
  10 * If[r < TETRDst[[1]], (TETRDDCFAA[r] - TetrCLDApprx00AA[r]),
    If[r < TETRDst[[2]], (TETRDDCFBB[r] - TetrCLDApprx00BB[r]), If[r < TETRDst[[3]],
      (TETRDDCFCC[r] - TetrCLDApprx00CC[r]), (TETRDDCFDD[r] - TetrCLDApprx00DD[r])]]]},
 {r, -0, 1}, PlotStyle → {Directive[Blue, Thickness[0.003]],
   Directive[Red, Thickness[0.005], Dashed], Directive[Black, Thickness[0.003], Dotted]},
 AxesLabel → {"r", " γ'' (r), 10Δγ''"}]
```



The final formulae of the tetrahedron-CF and CLD approximation for the case K=0 and procedure B are

$$\text{TetrCFApprxFnl00AA[r\_]} := 1 - 3\sqrt{\frac{3}{2}}\, r - \frac{\left(6 + 5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2 \left(2\sqrt{2} + \pi - \text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TetrCFApprxFnl00BB[r\_]} := -\frac{1}{40\pi} r \Big(-288 + 51\sqrt{2} - 128\sqrt{3} + \sqrt{2}\left(-177 + 131\sqrt{3}\right)\pi + 120\sqrt{2}\,\text{ArcCos}\left[\frac{1}{3}\right] - 48\sqrt{2}\,\text{ArcSec}[3] + 32\sqrt{6}\,\text{ArcSec}[3]\Big) + \frac{1}{1120\pi}\left(\left(2428 - 1007\sqrt{3}\right)\pi + 4\left(51 - 952\sqrt{2} + 118\sqrt{3} + 420\,\text{ArcCos}\left[\frac{1}{3}\right] + 56\,\text{ArcSec}[3]\right)\right) + \frac{3}{280}\Bigg(-\Bigg(\Bigg(26 \times 2^{1/4}\sqrt{-3 + 2\sqrt{3}} - 20 \times 2^{1/4}\sqrt{-9 + 6\sqrt{3}} + 84\sqrt{-\sqrt{2} + 2 r} - 12\sqrt{6}\, r^3\sqrt{-\sqrt{2} + 2 r} - 36\sqrt{-3\sqrt{2} + 6 r} + r^2 \left(105 \times 2^{1/4}\sqrt{-3 + 2\sqrt{3}} - 70 \times 2^{1/4}\sqrt{-9 + 6\sqrt{3}} + 24\left(7 - 2\sqrt{3}\right)\sqrt{-\sqrt{2} + 2 r}\right) + 2 r$$

$$\left(-28\ 2^{3/4}\sqrt{-3+2\sqrt{3}}+21\times 2^{3/4}\sqrt{-9+6\sqrt{3}}+3\left(-28+11\sqrt{3}\right)\sqrt{-2\sqrt{2}+4\,r}\right)\Big)$$

$$\left(-6+16\sqrt{2}+\left(8-5\sqrt{3}\right)\pi-8\,\text{ArcSec}[3]\right)\Big)\Big/\left(2^{1/4}\left(-3+2\sqrt{3}\right)^{3/2}\pi\right)+$$

$$35\left(\frac{1}{3}-\sqrt{\frac{2}{3}}\,r+\frac{r^2}{2}-\frac{2\times 2^{3/4}\left(\sqrt{6}-3\,r\right)^{5/2}\left(-\frac{7}{\sqrt{2}}+2\sqrt{6}+r\right)}{105\left(-3+2\sqrt{3}\right)^{3/2}}\right)$$

$$\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right)\Bigg)-$$

$$\frac{1}{1680\,\pi}\left(\frac{1}{\sqrt{2}}-r\right)^3\left(2\left(\frac{1}{\sqrt{2}}-r\right)\left(-5\sqrt{\frac{2}{3}}+\sqrt{2}+3\,r\right)\right.$$

$$\Big(\left(-441\,887\,094+284\,889\,498\sqrt{2}-255\,123\,640\sqrt{3}+164\,480\,973\sqrt{6}\right)\pi-$$

$$324\left(-170\,664-391\,769\sqrt{2}-98\,532\sqrt{3}-226\,188\sqrt{6}+560\sqrt{2}\left(362+209\sqrt{3}\right)\right.$$

$$\left.\text{ArcCos}\left[\frac{1}{3}\right]-4\left(-70\,588+50\,815\sqrt{2}-40\,754\sqrt{3}+29\,338\sqrt{6}\right)\text{ArcSec}[3]\right)\Big)-$$

$$\left(-2\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{2}}+r\right)\Big(\left(-221\,874\,996+172\,383\,744\sqrt{2}-128\,100\,927\sqrt{3}+99\,525\,812\sqrt{6}\right)\pi+$$

$$648\left(-153\,635-35\,520\sqrt{2}-88\,701\sqrt{3}-20\,511\sqrt{6}+140\left(575+332\sqrt{3}\right)\text{ArcCos}\left[\frac{1}{3}\right]+\right.$$

$$\left.\left(-79\,450+55\,384\sqrt{2}-45\,872\sqrt{3}+31\,976\sqrt{6}\right)\text{ArcSec}[3]\right)\Big)\Big)\Big);$$

$$\text{TetrCFApprxFnl00CC}[r\_]:=\frac{1}{20\,160}\left(-\frac{1}{\left(-2+\sqrt{3}\right)\pi}28\left(-7+4\sqrt{3}\right)r\right.$$

$$\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)\right)+$$

$$\frac{1}{\left(-2+\sqrt{3}\right)\pi}\Big(\left(-116\,640-40\,338\sqrt{2}+68\,688\sqrt{3}+22\,403\sqrt{6}\right)\pi-$$

$$54\left(\sqrt{2}\left(-1917+1118\sqrt{3}\right)-8\left(-90+53\sqrt{3}\right)\text{ArcSec}[3]\right)\Big)+$$

$$4\left(-\frac{1}{4\pi}3\times3^{3/4}\left(3+2\sqrt{2}\right)\left(-465\ 3^{1/4}+294\sqrt{2}\ 3^{1/4}-768\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+\right.\right.$$

$$672\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}-96\sqrt{6\left(3+2\sqrt{2}\right)}\ r^3\sqrt{-\sqrt{6}+3r}+$$

$$12\,r^2\left(-105\ 3^{1/4}+70\sqrt{2}\ 3^{1/4}-64\sqrt{\frac{\sqrt{6}-3r}{-3+2\sqrt{2}}}+84\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3r}\right)+$$

$$4\,r\left(7\times3^{3/4}\left(-23+18\sqrt{2}\right)+176\sqrt{6\left(3+2\sqrt{2}\right)}\sqrt{-\sqrt{6}+3r}-\right.$$

$$\left.\left.336\sqrt{9+6\sqrt{2}}\sqrt{-\sqrt{6}+3r}\right)\right)\left(32\sqrt{2}-8\sqrt{3}-36\pi+19\sqrt{3}\pi-16\,\text{ArcSec}[3]\right)+$$

$$70\left(\frac{3}{8}-\frac{\sqrt{3}\,r}{2}+\frac{r^2}{2}-\frac{3^{3/4}\left(\sqrt{3}-2r\right)^{5/2}\left(\frac{9-7\sqrt{2}}{\sqrt{3}}+r\right)}{35\left(3-2\sqrt{2}\right)^{3/2}}\right)$$

$$\left.\left.\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right)\right)\right);$$

$$\text{TetrCFApprxFnl00DD}[r\_]:=-\frac{1}{5040\left(-2+\sqrt{3}\right)\pi}$$

$$\left(30-13\sqrt{3}-\right.$$

$$42\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+18\sqrt{3}\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+$$

$$\left(56-42\sqrt{3}-66\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}+56\sqrt{3}\sqrt{\frac{\sqrt{3}-2r}{-2+\sqrt{3}}}\right)r+$$

$$\left(70 - 35\sqrt{3} - 56\sqrt{\frac{\sqrt{3} - 2\,r}{-2 + \sqrt{3}}} + 16\sqrt{3}\sqrt{\frac{\sqrt{3} - 2\,r}{-2 + \sqrt{3}}}\right) r^2 +$$
$$\left. 8\sqrt{\frac{\sqrt{3} - 2\,r}{-2 + \sqrt{3}}}\, r^3 \right)$$
$$\left(\left(-1296 + 209\sqrt{2} + 372\sqrt{6}\right)\pi - 54\left(-16\sqrt{2} + 3\sqrt{6} + 8\,\text{ArcSec}[3]\right)\right);$$

$$\text{TetrCLDApprx00AA}[r\_] := -\frac{3\left(6\sqrt{2}\,r + \pi\left(-8 + 5\sqrt{6}\,r\right) + 8\left(-2\sqrt{2} + \text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

$$\text{TetrCLDApprx00BB}[r\_] :=$$
$$\frac{3}{8}\left(-\frac{1}{\pi}2\left(1 - \frac{3\left(-\sqrt{2} + \sqrt{6} - r\right)\sqrt{-3\sqrt{2} + 6\,r}}{2^{3/4}\left(-3 + 2\sqrt{3}\right)^{3/2}}\right)\left(6 - 16\sqrt{2} + \left(-8 + 5\sqrt{3}\right)\pi + 8\,\text{ArcSec}[3]\right) +\right.$$
$$\left.\left(1 + \frac{\sqrt{\sqrt{6} - 3\,r}\left(9 - 4\sqrt{3} - 3\sqrt{2}\,r\right)}{2^{3/4}\left(-3 + 2\sqrt{3}\right)^{3/2}}\right)\left(-36 + 19\sqrt{3} - \frac{8\left(-4\sqrt{2} + \sqrt{3} + 2\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00CC}[r\_] := \frac{1}{72}\left(27\left(1 + \frac{3^{1/4}\left(2\sqrt{\frac{2}{3}} - \frac{3\sqrt{3}}{2} + r\right)\sqrt{-2\sqrt{6} + 6\,r}}{\left(3 - 2\sqrt{2}\right)^{3/2}}\right)\right.$$
$$\left(-36 + 19\sqrt{3} - \frac{8\left(-4\sqrt{2} + \sqrt{3} + 2\,\text{ArcSec}[3]\right)}{\pi}\right) + \left(1 - \frac{3^{3/4}\sqrt{\sqrt{3} - 2\,r}\left(\sqrt{3} - \sqrt{6} + r\right)}{\left(3 - 2\sqrt{2}\right)^{3/2}}\right)$$
$$\left.\left(-1296 + 209\sqrt{2} + 372\sqrt{6} - \frac{54\left(-16\sqrt{2} + 3\sqrt{6} + 8\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00DD}[r\_] := \frac{1}{72}\left(1 + \frac{\left(-3 + \sqrt{3} + r\right)\sqrt{-\sqrt{3} + 2\,r}}{\left(2 - \sqrt{3}\right)^{3/2}}\right)$$
$$\left(-1296 + 209\sqrt{2} + 372\sqrt{6} - \frac{54\left(-16\sqrt{2} + 3\sqrt{6} + 8\,\text{ArcSec}[3]\right)}{\pi}\right);$$

Check of the sum rule $4\pi\int_0^\infty r^2\,\gamma(r)\,dr = V_p$

In our case $V_p =$ VTetr $= 1\Big/\left(6 * \sqrt{2}\right)$.

We find 0.11777 with an error ~-0.000092 (~ 0.07%) that will appear in the FT, i.e. ΔI(0) ~ 0.000092.

```
N[4 π NIntegrate[r^2 * TetrCFApprxFnl00AA[r],
    {r, 0, TETRDst[[1]]}, WorkingPrecision → 30, PrecisionGoal -> 15] +
  4 π NIntegrate[r^2 * TetrCFApprxFnl00BB[r], {r, TETRDst[[1]], TETRDst[[2]]},
    WorkingPrecision → 30, PrecisionGoal -> 15] +
  4 π NIntegrate[r^2 * TetrCFApprxFnl00CC[r], {r, TETRDst[[2]], TETRDst[[3]]},
    WorkingPrecision → 30, PrecisionGoal -> 15] +
  4 π NIntegrate[r^2 * TetrCFApprxFnl00DD[r], {r, TETRDst[[3]], TETRDst[[4]]},
    WorkingPrecision → 30, PrecisionGoal -> 15]]
```

```
0.117759
```

```
N[1 / (6 * √2) - 0.11775891860778369`]
N[(1 / (6 * √2) - 0.11775891860778369`) / (1 / (6 * √2))]
```

```
0.0000922116

0.000782441
```

We don't consider the cases K = 1, 2

because the agreement achieved in the case K=0 is satisfactory
both in direct and reciprocal spaces as shown in the following calculations.

EVALUATION OF THE FOURIER TRANSFORM

We have $I(q) = \frac{4\pi}{q}\int_0^\infty r\,\mathrm{Sin}[q\,r]\,\gamma[r]\,dr = \frac{2\pi S}{V\,q^4} - \frac{4\pi}{q^4}\int_0^\infty (2\,\mathrm{Cos}[q\,r] + r\,q\,\mathrm{Sin}[q\,r])\,\gamma''[r]\,dr$

The last expression is obtained integrating twice by part the first integral considering the successive primitives of r Sin[q r] [See Ciccariello J. Appl. Cryst. 38, 97, (2005)].
The use of the last expression is "dangerous" when q < 1 as we already found in the tetrahedron case. Consequently, we shall confine ourselves to the first formula.

```
Npnt = 100; Solve[Log[10, 10^(-5) f^20] == Log[10, 10^(-2)], f];
Solve[{Log[10, 10^(-2) f^30] == Log[10, 99 / 100]}, f];
Qstep = 8 / 10; Q0 = 1;
Qgrid = Table[If[i ≤ 21, 10^(-5) ((10^(3/20))^(i - 1)), If[i ≤ 51, ((3^(1/15) 11^(1/30))^(i - 21) / 100),
    Q0 + Qstep * (i - 51)]], {i, 1, Npnt}]; N[Qgrid]; N[Qgrid[[Npnt]]]
```

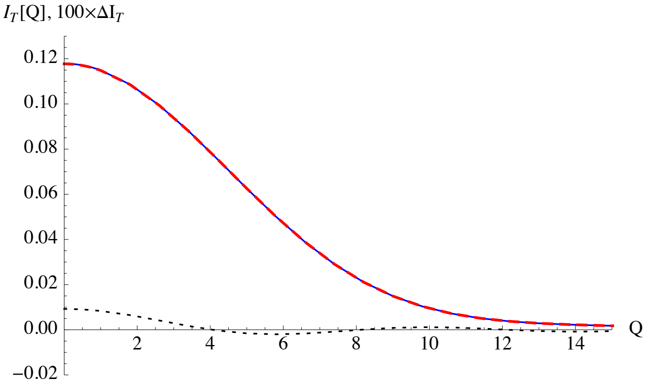
```
40.2
```

```
TetrExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
```

```
Do[qact = Qgrid[[i]]; TetrExctFormFact[[i, 2]] = (4 π / qact)
    (NIntegrate[TETRCFAA[r] * r * Sin[qact * r], {r, 0, TETRDst[[1]]}, WorkingPrecision → 50,
       PrecisionGoal → 15] + NIntegrate[TETRCFBB[r] * r * Sin[qact * r],
       {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFCC[r] * r * Sin[qact * r], {r, TETRDst[[2]], TETRDst[[3]]},
       WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFDD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];
```

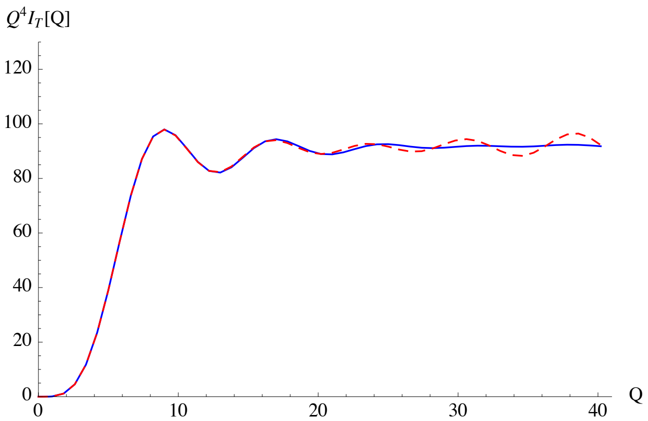
```
Do[qact = Qgrid[[i]]; TetrApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[TetrCFApprxFnl00AA[r] * r * Sin[qact * r],
       {r, 0, TETRDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
      NIntegrate[TetrCFApprxFnl00BB[r] * r * Sin[qact * r],
       {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
      NIntegrate[TetrCFApprxFnl00CC[r] * r * Sin[qact * r], {r, TETRDst[[2]],
        TETRDst[[3]]}, WorkingPrecision → 100, PrecisionGoal → 15] + NIntegrate[
       TetrCFApprxFnl00DD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
       WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];
```

```
TETR00FigFormFact = ListPlot[{N[TetrExctFormFact], N[ TetrApprxtd00FormFact],
   Table[{N[TetrExctFormFact[[i, 1]]],
     100 * N[TetrExctFormFact[[i, 2]] - TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
  Joined → True, PlotRange → {{0, 15}, {-0.02, 0.13}}, PlotStyle →
   {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.005], Dashed],
    Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "I_T[Q], 100×ΔI_T"} ]
```

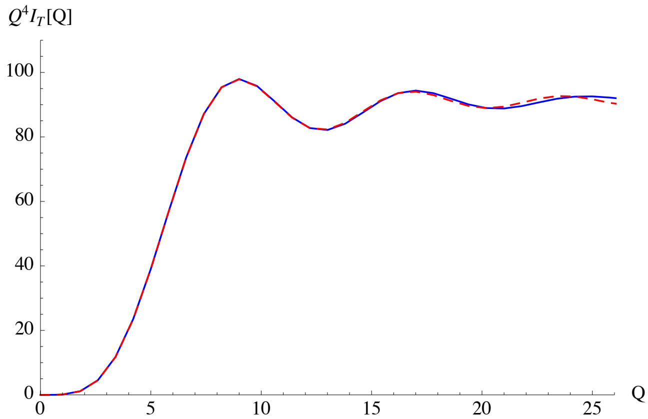



```
Export["TETR00FigFormFact_BB.eps", TETR00FigFormFact];
Export["TETR00FigFormFact_BB.PDF", TETR00FigFormFact];
```

```
ListPlot[{Table[{N[TetrExctFormFact[[i, 1]]],
     N[TetrExctFormFact[[i, 1]] ^4 * TetrExctFormFact[[i, 2]]]},
    {i, 1, Npnt}], Table[{N[TetrApprxtd00FormFact[[i, 1]]],
     N[TetrApprxtd00FormFact[[i, 1]] ^4 * TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
 Joined → True, PlotRange → {{0, 41}, {-0, 130}}, PlotStyle →
  {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.003], Dashed],
   Directive[Black, Thickness[0.003], Dashed],
   Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4 I_T[Q]"} ]
```



```
TETRFigPorPlt00 =
 ListPlot[{Table[{N[TetrExctFormFact[[i, 1]]], N[TetrExctFormFact[[i, 1]]^4 *
      TetrExctFormFact[[i, 2]]]}, {i, 1, Npnt}], Table[{N[TetrApprxtd00FormFact[[i, 1]]],
    N[TetrApprxtd00FormFact[[i, 1]]^4 * TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
  Joined → True, PlotRange → {{0, 26}, {0, 110}}, PlotStyle →
   {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.003], Dashed],
    Directive[Black, Thickness[0.003], Dashed],
    Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4 I_T[Q]"}]
```



Note that the approximated intensity goes out of phase as Q increases.
The following plot shows that, as Q increases, the discrepancies at first increase and then decrease once we have reached the true asymptotic region that lies in the range $Q>2\pi/\Delta_{\rm min}$, $\Delta_{\rm min}$ denoting the smallest of the widths of the small peaks present in the plot of the approximated γ''[r]. The last value is around 0.005 and the asymptotic region lies beyond Q=1000

```
Export["TETRFigPorPlt00_BB.eps", TETRFigPorPlt00];
Export["TETRFigPorPlt00_BB.PDF", TETRFigPorPlt00];
```

We consider a grid of larger Q-values in order to see where the onset of the exact Porod behavior occurs.

```
Npnt = 1000; Qstep = 8 / 10; Q0 = 250;
Qgrid = Table[Q0 + Qstep * i, {i, 1, Npnt}]; N[Qgrid[[Npnt]]]
 TetrExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd11FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
```

```
1050.
```

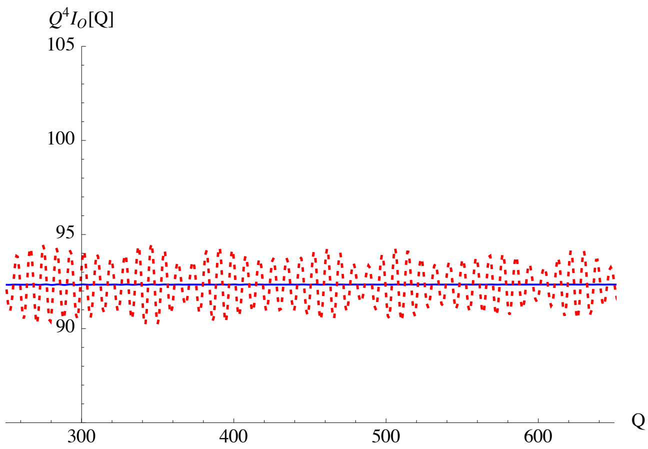
```
Do[qact = Qgrid[[i]]; TetrExctFormFact[[i, 2]] = (4 π / qact)
    (NIntegrate[TETRCFAA[r] * r * Sin[qact * r], {r, 0, TETRDst[[1]]}, WorkingPrecision → 50,
       PrecisionGoal → 15] + NIntegrate[TETRCFBB[r] * r * Sin[qact * r],
       {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
     NIntegrate[TETRCFCC[r] * r * Sin[qact * r], {r, TETRDst[[2]], TETRDst[[3]]},
       WorkingPrecision → 50, PrecisionGoal → 15] +
     NIntegrate[TETRCFDD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];
```

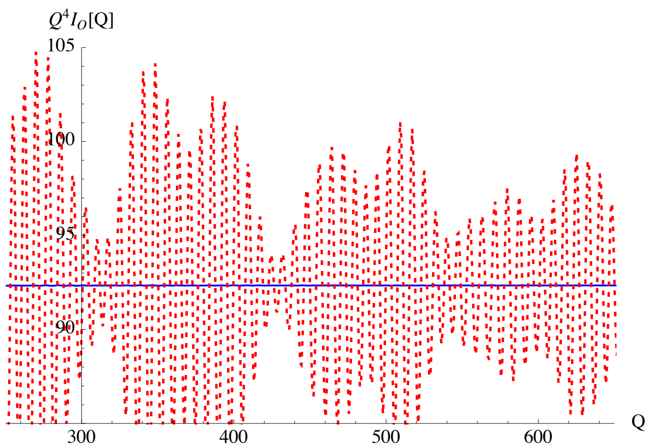
```
Do[qact = Qgrid[[i]]; TetrApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[TetrCFApprxFnl00AA[r] * r * Sin[qact * r],
        {r, 0, TETRDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00BB[r] * r * Sin[qact * r],
        {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00CC[r] * r * Sin[qact * r], {r, TETRDst[[2]],
          TETRDst[[3]]}, WorkingPrecision → 100, PrecisionGoal → 15] + NIntegrate[
        TetrCFApprxFnl00DD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
        WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];
```

The resulting intensity vectors are stored in new variables

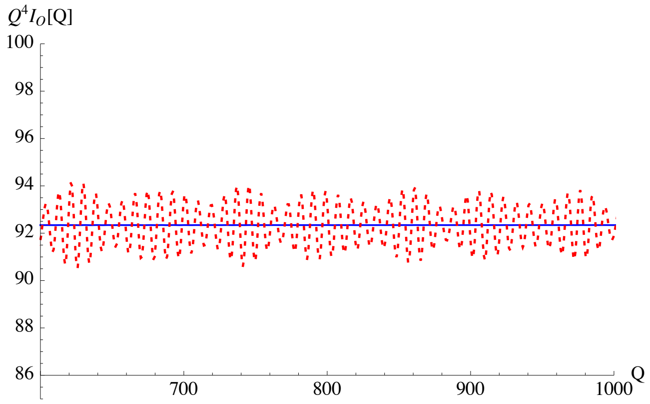
```
MQgrid = Qgrid; MTetrExctFormFact = TetrExctFormFact;
MTetrApprxtd00FormFact = TetrApprxtd00FormFact;
```

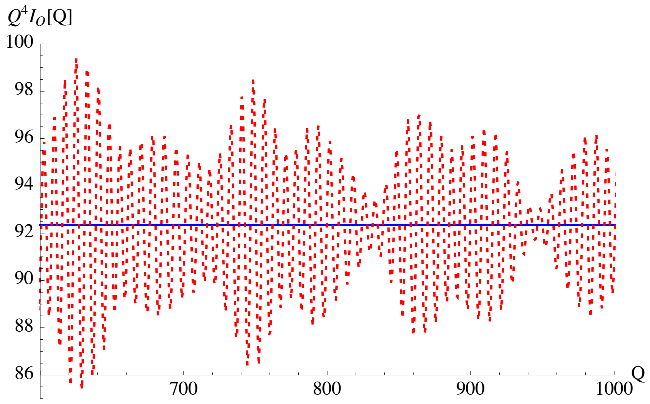
```
TetrAprx00LrgQPrdPltFigD =
 ListPlot[{Table[{N[TetrExctFormFact[[i, 1]]], N[TetrExctFormFact[[i, 1]]^4 *
        TetrExctFormFact[[i, 2]]]}, {i, 1, Npnt}], Table[{N[TetrApprxtd00FormFact[[i, 1]]],
      N[TetrApprxtd00FormFact[[i, 1]]^4 * TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{250, 650}, {85, 105}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4 I_O[Q]"}]
```



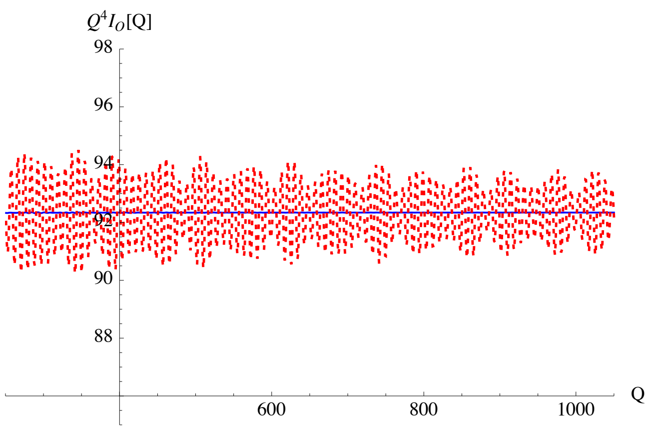


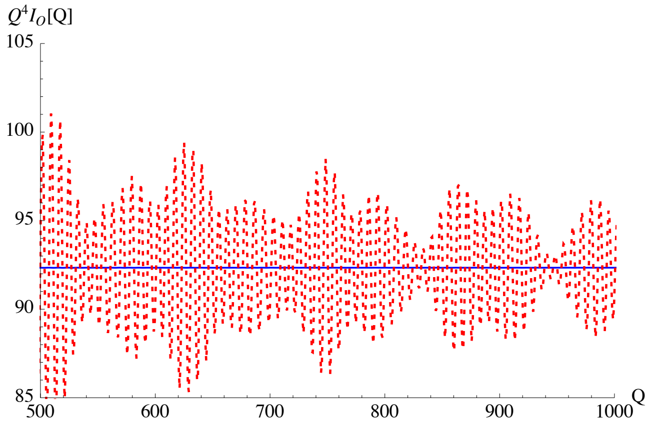
```
TetrAprx00LrgQPrdPltFigD =
 ListPlot[{Table[{N[TetrExctFormFact[[i, 1]]], N[TetrExctFormFact[[i, 1]]^4 *
       TetrExctFormFact[[i, 2]]]}, {i, 1, Npnt}], Table[{N[TetrApprxtd00FormFact[[i, 1]]],
     N[TetrApprxtd00FormFact[[i, 1]]^4 * TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
  Joined → True, PlotRange → {{600, 1000}, {85, 100}}, PlotStyle →
   {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
    Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4 I_O[Q]"}]
```






```
TetrAprx00LrgQPrdPltFigD =
 ListPlot[{Table[{N[TetrExctFormFact[[i, 1]]], N[TetrExctFormFact[[i, 1]]^4 *
       TetrExctFormFact[[i, 2]]]}, {i, 1, Npnt}], Table[{N[TetrApprxtd00FormFact[[i, 1]]],
     N[TetrApprxtd00FormFact[[i, 1]]^4 * TetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
  Joined → True, PlotRange → {{250, 1050}, {85, 98}}, PlotStyle →
   {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
    Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4 I_O[Q]"}]
```






```
Export["TetrAprx00LrgQPrdPltFigD.PDF", TetrAprx00LrgQPrdPltFigD]
Export["TetrAprx00LrgQPrdPltFigC.PDF", TetrAprx00LrgQPrdPltFigC]
Export["TetrAprx00LrgQPrdPltFigB.PDF", TetrAprx00LrgQPrdPltFigB]
Export["TetrAprx00LrgQPrdPltFigA.PDF", TetrAprx00LrgQPrdPltFigA]
Export["TetrAprx00LrgQPrdPltFigD.eps", TetrAprx00LrgQPrdPltFigD]
Export["TetrAprx00LrgQPrdPltFigC.eps", TetrAprx00LrgQPrdPltFigC]
Export["TetrAprx00LrgQPrdPltFigB.eps", TetrAprx00LrgQPrdPltFigB]
Export["TetrAprx00LrgQPrdPltFigA.eps", TetrAprx00LrgQPrdPltFigA]
```

```
N[Qgrid[[Npnt]]]
```

```
1050.
```

We consider a grid of even larger Q-values in order to see where the onset of the exact Porod behavior occurs.

```
Npnt = 5000; Qstep = 9 / 10; Q0 = 1050;
Qgrid = Table[Q0 + Qstep * i, {i, 1, Npnt}]; N[Qgrid[[Npnt]]]
 TetrExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd11FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
```

```
5550.
```

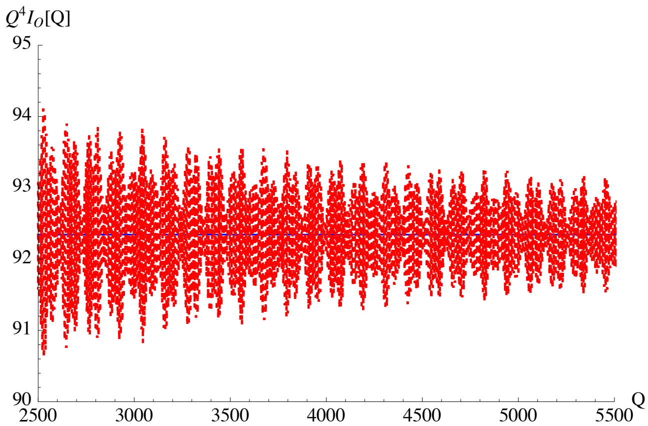
```
Do[qact = Qgrid[[i]]; TetrExctFormFact[[i, 2]] = (4 π / qact)
    (NIntegrate[TETRCFAA[r] * r * Sin[qact * r], {r, 0, TETRDst[[1]]}, WorkingPrecision → 50,
       PrecisionGoal → 15] + NIntegrate[TETRCFBB[r] * r * Sin[qact * r],
       {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFCC[r] * r * Sin[qact * r], {r, TETRDst[[2]], TETRDst[[3]]},
       WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFDD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];
```

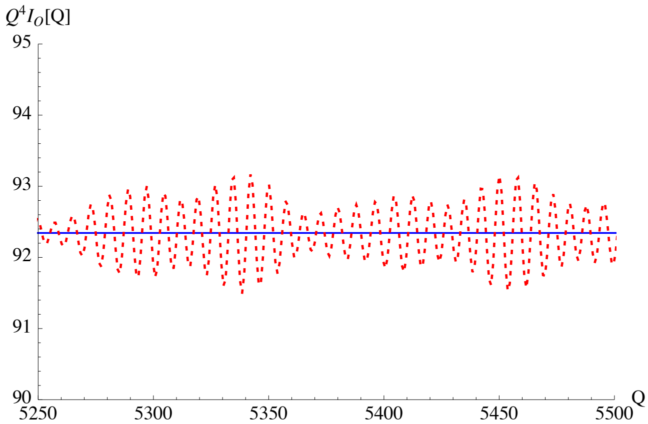
```
Do[qact = Qgrid[[i]]; TetrApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[TetrCFApprxFnl00AA[r] * r * Sin[qact * r],
        {r, 0, TETRDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00BB[r] * r * Sin[qact * r],
        {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00CC[r] * r * Sin[qact * r], {r, TETRDst[[2]],
         TETRDst[[3]]}, WorkingPrecision → 100, PrecisionGoal → 15] + NIntegrate[
        TetrCFApprxFnl00DD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
        WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];
```

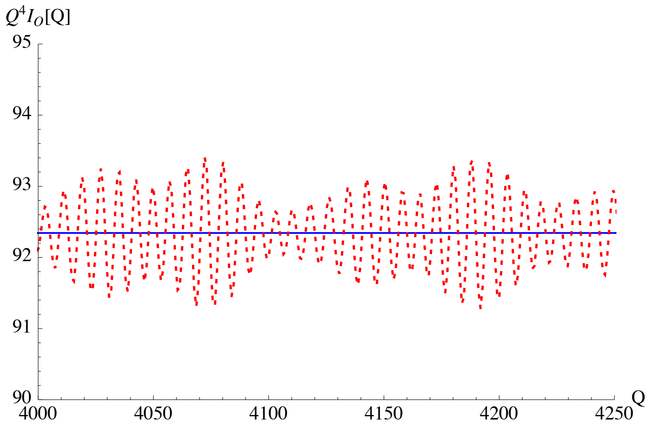
The resulting intensity vectors are stored in new variables

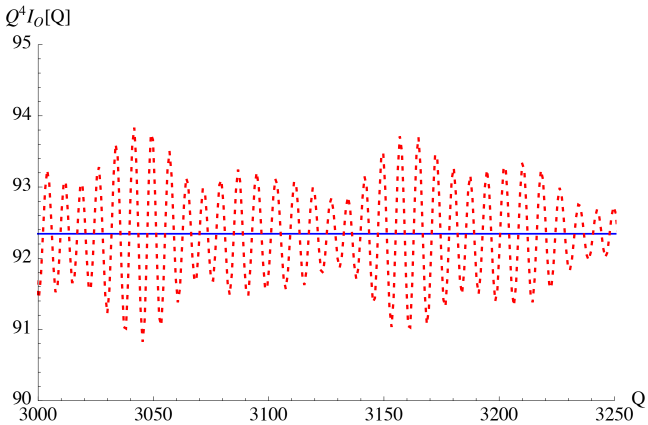
```
LQgrid = Qgrid; LTetrExctFormFact = TetrExctFormFact;
LTetrApprxtd00FormFact = TetrApprxtd00FormFact;
```

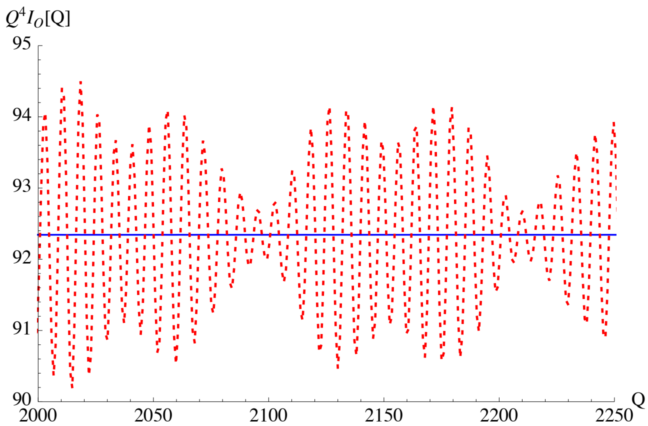
```
TetrAprx11LrgQPrdPltFigE = ListPlot[{Table[{N[LTetrExctFormFact[[i, 1]]],
      N[LTetrExctFormFact[[i, 1]]^4 * TetrExctFormFact[[i, 2]]]},
     {i, 1, Npnt}], Table[{N[LTetrApprxtd00FormFact[[i, 1]]],
      N[LTetrApprxtd00FormFact[[i, 1]]^4 * LTetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{2500, 5500}, {90, 95}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4I_O[Q]"}]
TetrAprx11LrgQPrdPltFigE = ListPlot[{Table[{N[LTetrExctFormFact[[i, 1]]],
      N[LTetrExctFormFact[[i, 1]]^4 * TetrExctFormFact[[i, 2]]]},
     {i, 1, Npnt}], Table[{N[LTetrApprxtd00FormFact[[i, 1]]],
      N[LTetrApprxtd00FormFact[[i, 1]]^4 * LTetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{5250, 5500}, {90, 95}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4I_O[Q]"}]
TetrAprx11LrgQPrdPltFigD = ListPlot[{Table[{N[LTetrExctFormFact[[i, 1]]],
      N[LTetrExctFormFact[[i, 1]]^4 * TetrExctFormFact[[i, 2]]]},
     {i, 1, Npnt}], Table[{N[LTetrApprxtd00FormFact[[i, 1]]],
      N[LTetrApprxtd00FormFact[[i, 1]]^4 * LTetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{4000, 4250}, {90, 95}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4I_O[Q]"}]
TetrAprx11LrgQPrdPltFigC = ListPlot[{Table[{N[LTetrExctFormFact[[i, 1]]],
      N[LTetrExctFormFact[[i, 1]]^4 * LTetrExctFormFact[[i, 2]]]},
     {i, 1, Npnt}], Table[{N[LTetrApprxtd00FormFact[[i, 1]]],
      N[LTetrApprxtd00FormFact[[i, 1]]^4 * LTetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{3000, 3250}, {90, 95}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4I_O[Q]"}]
TetrAprx11LrgQPrdPltFigB = ListPlot[{Table[{N[LTetrExctFormFact[[i, 1]]],
      N[LTetrExctFormFact[[i, 1]]^4 * LTetrExctFormFact[[i, 2]]]},
     {i, 1, Npnt}], Table[{N[LTetrApprxtd00FormFact[[i, 1]]],
      N[LTetrApprxtd00FormFact[[i, 1]]^4 * LTetrApprxtd00FormFact[[i, 2]]]}, {i, 1, Npnt}]},
   Joined → True, PlotRange → {{2000, 2250}, {90, 95}}, PlotStyle →
    {Directive[Blue, Thickness[0.003]], Directive[Red, Thickness[0.004], Dotted],
     Directive[Black, Thickness[0.003], Dotted]}, AxesLabel → {"Q", "Q^4I_O[Q]"}]
```




$Q^4 I_O$[Q]
95
94
93
92
91
90
5250
5300
5350
5400
5450
5500
Q


$Q^4 I_O$[Q]
95
94
93
92
91
90
4000
4050
4100
4150
4200
4250
Q


$Q^4 I_O$[Q]
95
94
93
92
91
90
3000
3050
3100
3150
3200
3250
Q


$Q^4 I_O$[Q]
95
94
93
92
91
90
2000
2050
2100
2150
2200
2250
Q

```
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Export["TetrAprx11LrgQPrdPltFigC.PDF", TetrAprx11LrgQPrdPltFigC]
Export["TetrAprx11LrgQPrdPltFigB.PDF", TetrAprx11LrgQPrdPltFigB]
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Export["TetrAprx11LrgQPrdPltFigB.eps", TetrAprx11LrgQPrdPltFigB]
Export["TetrAprx11LrgQPrdPltFigA.eps", TetrAprx11LrgQPrdPltFigA]
```

The above figures show that

- at very large Q the approximaed intensity is always positive and approaches the exact one.
- this happens at very large q's because the oscillatory features related to the polynomial must be fully resolved. Since these oscillations are for some r's very narrow, if Δr denotes the smallest oscillations witdth, the oscillation is fully resolved in reciprocal space at Qs such that $Q\,\Delta r > 2\pi$. In the reported figure cases we have Q~12000 so that $\Delta r \sim 10^{-4}$.