FOUNDATIONS ADVANCES

Volume 77 (2021)

Supporting information for article:

Algebraic approximations of a polyhedron correlation function stemming from its chord-length distribution

Salvino Ciccariello

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SetDirectory["/Users/salvino/Desktop/WORK_IN_PRGS/PLATONIC_SOLD_CF_APPRXMTN"];
Directory[]
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/Users/salvino/Desktop/WORK_IN_PRGS/PLATONIC_SOLD_CF_APPRXMTN
```

Merano, March - June 2020

Plots of the lowest order algebraic approximations of the CLD and CF of the cube, the octahedron and the tetrahedron.

These expressions have been copied from the files: CUBE_CF_APPROXMTN_FNL_BB.nb, TETRA_CF_APPRXMTN_FNL_BB.nb and OCTA_CF_APPRXMTN_FNL_BB.nb.

CUBE

```
Theta[x_] := If[x > 0, 1, 0];
```

$$\text{CBDst} = \left\{\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 1\right\};$$

$$\text{CBCFAA}[r\_] := 1 - \frac{3\sqrt{3}\, r}{2} + \frac{6 r^2}{\pi} - \frac{3\sqrt{3}\, r^3}{4\pi};$$

$$\text{CBDCFAA}[r\_] := -\frac{3\sqrt{3}}{2} + \frac{12 r}{\pi} - \frac{9\sqrt{3}\, r^2}{4\pi}; \; \text{CBDDCFAA}[r\_] := \frac{12}{\pi} - \frac{9\sqrt{3}\, r}{2\pi};$$

$$\text{CBCFBB}[r\_] := -2 + \frac{\sqrt{3}}{2 r} - \frac{1}{4\sqrt{3}\,\pi r} + 3\sqrt{3}\, r + \frac{3\sqrt{3}\, r}{2\pi} + \frac{3\sqrt{3}\, r^3}{2\pi} - \frac{2\left(\sqrt{3}\sqrt{-1+3r^2} + 6\sqrt{3}\, r^2\sqrt{-1+3r^2}\right)}{3\pi r} - \frac{6\sqrt{3}\, r\left(\frac{\pi}{2} - \text{ArcTan}\left[\sqrt{-1+3r^2}\right]\right)}{\pi};$$

$$\text{CBDCFBB}[r\_] := \frac{1}{4\pi r^2\sqrt{-3+9r^2}}\left(-8 + \sqrt{-1+3r^2} - 6\pi\sqrt{-1+3r^2} + 18 r^4\left(-16 + 3\sqrt{-1+3r^2}\right) + 6 r^2\left(20 + 3\sqrt{-1+3r^2}\right) + 72 r^2\sqrt{-1+3r^2}\,\text{ArcTan}\left[\sqrt{-1+3r^2}\right]\right);$$

$$\text{CBDDCFBB}[r\_] := \frac{8 + 24 r^2 - \sqrt{-1+3r^2} + 6\pi\sqrt{-1+3r^2} + 18 r^4\left(-8 + 3\sqrt{-1+3r^2}\right)}{2\pi r^3\sqrt{-3+9r^2}};$$

```
CBCFCC[r_] :=
```

$$-\frac{1}{12\pi r}\left(5\sqrt{3}-6\sqrt{3}\,\pi+24\pi r+18\sqrt{3}\,r^2-18\sqrt{3}\,\pi r^2+9\sqrt{3}\,r^4-8\sqrt{3}\,\sqrt{-2+3r^2}-24\sqrt{3}\,r^2\sqrt{-2+3r^2}+24\sqrt{3}\,\text{ArcTan}\left[\sqrt{-2+3r^2}\right]+36r\left(-2+\sqrt{3}\,r\right)\text{ArcTan}\left[\frac{\sqrt{-2+3r^2}}{2-\sqrt{3}\,r}\right]+72r\,\text{ArcTan}\left[\frac{\sqrt{-2+3r^2}}{2+\sqrt{3}\,r}\right]+36\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{\sqrt{-2+3r^2}}{2+\sqrt{3}\,r}\right]\right);$$

$$\text{CBDCFCC}[r\_]:=\frac{1}{4\sqrt{3}\,\pi r^2}\left(5-6\pi-18r^2+18\pi r^2-27r^4+\frac{16}{\sqrt{-2+3r^2}}-\frac{120r^2}{\sqrt{-2+3r^2}}+\frac{144r^4}{\sqrt{-2+3r^2}}+24\,\text{ArcTan}\left[\sqrt{-2+3r^2}\right]-36r^2\,\text{ArcTan}\left[\frac{\sqrt{-2+3r^2}}{2-\sqrt{3}\,r}\right]-36r^2\,\text{ArcTan}\left[\frac{\sqrt{-2+3r^2}}{2+\sqrt{3}\,r}\right]\right);$$

$$\text{CBDDCFCC}[r\_]:=-\frac{1}{2\pi r^3\sqrt{-6+9r^2}}\left(16+24r^2+5\sqrt{-2+3r^2}-6\pi\sqrt{-2+3r^2}+9r^4\left(-8+3\sqrt{-2+3r^2}\right)+24\sqrt{-2+3r^2}\,\text{ArcTan}\left[\sqrt{-2+3r^2}\right]\right);$$

```
CUBExctTotalCF[r_] :=
 If[r < CBDst[[1]], CBCFAA[r], If[r < CBDst[[2]], CBCFBB[r], CBCFCC[r]]];
CUBExctTotalDCF[r_] := If[r < CBDst[[1]], CBDCFAA[r],
  If[r < CBDst[[2]], CBDCFBB[r], CBDCFCC[r]]]; CUBExctTotalDDCF[r_] :=
 If[r < CBDst[[1]], CBDDCFAA[r], If[r < CBDst[[2]], CBDDCFBB[r], CBDDCFCC[r]]];
```

K=0 APPROXIMATIONS OF THE CUBE CF AND CLD

$$\text{CubeCFApprxFnl00AA[r\_] := } 1-\frac{3\sqrt{3}\,r}{2}+\frac{6r^2}{\pi}-\frac{3\sqrt{3}\,r^3}{4\pi};$$

$$\text{CubeCFApprxFnl00BB[r\_] := } -\frac{1}{280\left(-3+\sqrt{6}\right)\pi}9\left(\frac{1}{\sqrt{3}}-r\right)^3\left(\frac{1}{3}\left(577+408\sqrt{2}\right)\left(6550-4769\sqrt{2}+5772\sqrt{3}-3989\sqrt{6}+2\left(-6274+4459\sqrt{2}-4636\sqrt{3}+3263\sqrt{6}\right)\pi\right)\left(-2\sqrt{3}+5\sqrt{6}-9r\right)\left(\sqrt{3}-3r\right)-\left(248664+176007\sqrt{2}-123628\sqrt{3}-87342\sqrt{6}+\left(-81816-58122\sqrt{2}+40968\sqrt{3}+28844\sqrt{6}\right)\pi\right)\left(-2\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{3}}+r\right)\right)+$$
$$\frac{1}{1680\pi}\left(-3036+489\sqrt{2}+\left(2328-918\sqrt{2}\right)\pi+42\sqrt{3}\left(76+37\sqrt{2}+18\left(-4+\sqrt{2}\right)\pi\right)r+2\left(\frac{1}{\sqrt{2}}3(-17+6\pi)\left(70-70\sqrt{6}\,r+105r^2-4\sqrt{3}\left(1+\sqrt{2}\right)^{3/2}\left(-\frac{7}{\sqrt{3}}+2\sqrt{6}+r\right)\left(\sqrt{2}-\sqrt{3}\,r\right)^{5/2}\right)-6\left(1+\sqrt{2}\right)^{3/2}(5+6\pi)\left(14\sqrt{-1+\sqrt{2}}-22\sqrt{2\left(-1+\sqrt{2}\right)}-24\sqrt{-1+\sqrt{3}\,r}+28\sqrt{-2+2\sqrt{3}\,r}-12r^3\sqrt{-3+3\sqrt{3}\,r}+2\sqrt{3}\,r\left(-14\sqrt{-1+\sqrt{2}}+21\sqrt{2\left(-1+\sqrt{2}\right)}+2\left(11-14\sqrt{2}\right)\sqrt{-1+\sqrt{3}\,r}\right)-3r^2\left(-35\sqrt{-1+\sqrt{2}}+35\sqrt{2\left(-1+\sqrt{2}\right)}+4\left(4-7\sqrt{2}\right)\sqrt{-1+\sqrt{3}\,r}\right)\right)\right)\right);$$

$$\text{CubeCFApprxFnl00CC[r\_] := } -\frac{1}{560\left(-3+\sqrt{6}\right)\pi}3(-17+6\pi)\left(41\sqrt{2}-22\sqrt{3}-56\sqrt{2}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}+32\sqrt{3}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}+\left(70\sqrt{2}-84\sqrt{3}-88\sqrt{2}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}+112\sqrt{3}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}\right)r+\left(105\sqrt{2}-70\sqrt{3}-84\sqrt{2}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}+32\sqrt{3}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}\right)r^2+12\sqrt{2}\sqrt{\frac{\sqrt{6}-3r}{-3+\sqrt{6}}}\,r^3\right);$$

$$\mathrm{CubeCLDApprx00AA}[r\_] := \frac{24 - 9\sqrt{3}\, r}{2\pi};\ \mathrm{CubeCLDApprx00BB}[r\_] := -\frac{1}{16\pi} 3 \Bigg(-40 + 34\sqrt{2} - 17\left(-2+\sqrt{2}\right)\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{2}-\sqrt{3}\, r} + 6\pi\Bigg(-2\left(4+\sqrt{2}\right) + \sqrt{3\left(1+\sqrt{2}\right)}\, r \left(2\sqrt{\sqrt{2}-\sqrt{3}\, r} + \sqrt{2}\sqrt{\sqrt{2}-\sqrt{3}\, r} - 4\sqrt{-1+\sqrt{3}\, r} - 4\sqrt{-2+2\sqrt{3}\, r}\right) + \sqrt{1+\sqrt{2}} \left(-2\sqrt{\sqrt{2}-\sqrt{3}\, r} + \sqrt{2}\sqrt{\sqrt{2}-\sqrt{3}\, r} + 16\sqrt{-1+\sqrt{3}\, r} + 4\sqrt{-2+2\sqrt{3}\, r}\right)\Bigg) + \sqrt{1+\sqrt{2}}\left(20\left(4+\sqrt{2}\right)\sqrt{-1+\sqrt{3}\, r} - \sqrt{3}\, r\left(34\sqrt{\sqrt{2}-\sqrt{3}\, r} + 17\sqrt{2}\sqrt{\sqrt{2}-\sqrt{3}\, r} + 20\sqrt{-1+\sqrt{3}\, r} + 20\sqrt{-2+2\sqrt{3}\, r}\right)\right)\Bigg);\ \mathrm{CubeCLDApprx00CC}[r\_] :=$$

$$\left(3\,(-17+6\pi)\left(-4\sqrt{-\sqrt{2}+\sqrt{3}} + 2\sqrt{6\left(-\sqrt{2}+\sqrt{3}\right)} + 4\sqrt{-\sqrt{2}+\sqrt{3}\, r} - 3\sqrt{-6\sqrt{2}+6\sqrt{3}\, r} + r\sqrt{-6\sqrt{2}+6\sqrt{3}\, r}\right)\right) \Big/ \left(16\left(-\sqrt{2}+\sqrt{3}\right)^{3/2}\pi\right);$$

TETRAHEDRON

$$\mathrm{Theta}[x\_] := \mathrm{If}[x > 0, 1, 0];$$

$$\mathrm{TETRDst} = \left\{\frac{1}{\sqrt{2}}, \sqrt{\frac{2}{3}}, \frac{\sqrt{3}}{2}, 1\right\};$$

$$\mathrm{TETRCFAA}[r\_] := 1 - 3\sqrt{\frac{3}{2}}\, r - \frac{\left(6+5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2\left(2\sqrt{2}+\pi-\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\mathrm{TETRDCFAA}[r\_] := -3\sqrt{\frac{3}{2}} - \frac{3\left(6+5\sqrt{3}\,\pi\right) r^2}{4\sqrt{2}\,\pi} + \frac{6 r\left(2\sqrt{2}+\pi-\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\mathrm{TETRDDCFAA}[r\_] := -\frac{3\left(6\sqrt{2}\, r + \pi\left(-8+5\sqrt{6}\, r\right) + 8\left(-2\sqrt{2}+\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

$$\mathrm{TETRCFBB}[r\_] :=$$

$$-2 + \frac{3}{4\sqrt{2}\, r} - \frac{3\left(-3+\sqrt{3}\right) r}{\sqrt{2}} - \frac{\left(6-12\pi+5\sqrt{3}\,\pi\right) r^3}{4\sqrt{2}\,\pi} + \frac{3 r^2\left(2\sqrt{2}-\pi-\mathrm{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\texttt{TetrCFBB[r\_] :=} -2+\frac{3}{4\sqrt{2}\,r}-\frac{3\left(-3+\sqrt{3}\right)r}{\sqrt{2}}-\frac{\left(6-12\pi+5\sqrt{3}\,\pi\right)r^3}{4\sqrt{2}\,\pi}+\frac{3r^2\left(2\sqrt{2}-\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\texttt{TETRDCFBB[r\_] :=} \frac{3}{8}\left(-4\sqrt{2}\left(-3+\sqrt{3}\right)-\frac{\sqrt{2}}{r^2}-\frac{\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r^2}{\pi}-\frac{16\,r\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\texttt{TETRDDCFBB[r\_] :=} \frac{3}{8}\left(\frac{2\sqrt{2}}{r^3}-\frac{2\sqrt{2}\left(6+\left(-12+5\sqrt{3}\right)\pi\right)r}{\pi}-\frac{16\left(-2\sqrt{2}+\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\texttt{TETRCFCC[r\_] :=} -6+\frac{9+8\sqrt{3}}{12\sqrt{2}\,r}+\frac{3\left(3+\sqrt{3}\right)r}{\sqrt{2}}+\frac{\left(-6+12\pi+\sqrt{3}\,\pi\right)r^3}{4\sqrt{2}\,\pi}+\frac{3r^2\left(2\sqrt{2}-3\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\texttt{TETRDCFCC[r\_] :=} \frac{3\left(3+\sqrt{3}\right)}{\sqrt{2}}-\frac{9+8\sqrt{3}}{12\sqrt{2}\,r^2}+\frac{3\left(-6+\left(12+\sqrt{3}\right)\pi\right)r^2}{4\sqrt{2}\,\pi}-\frac{6\,r\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\texttt{TETRDDCFCC[r\_] :=} \frac{1}{12}\left(\frac{\sqrt{2}\left(9+8\sqrt{3}\right)}{r^3}+\frac{9\sqrt{2}\left(-6+\left(12+\sqrt{3}\right)\pi\right)r}{\pi}-\frac{72\left(-2\sqrt{2}+3\pi+\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi}\right);$$

$$\texttt{TETRCFDD[r\_] :=} 3+\frac{3}{8\sqrt{2}\,r}+\frac{1}{\sqrt{6}\,r}+\frac{21}{2}\sqrt{\frac{3}{2}}\,r+\frac{9r}{\sqrt{2}}-3r^2+\frac{6\sqrt{2}\,r^2}{\pi}-\frac{1}{2}\sqrt{\frac{3}{2}}\,r^3+3\sqrt{2}\,r^3-\frac{3r^3}{2\sqrt{2}\,\pi}-\frac{21\,r\sqrt{-6+8r^2}}{4\pi}-\frac{3r^2\,\text{ArcCos}\left[\frac{1}{3}\right]}{\pi}-\frac{9\sqrt{6}\,r\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4r^2}{3}}}\right]}{\pi}+$$

$$\frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\sqrt{-1+\frac{4r^2}{3}}\right]}{2\pi} - \frac{9\,\text{ArcTan}\left[\frac{r}{\sqrt{-\frac{3}{2}+2r^2}}\right]}{\pi} + \frac{3\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{2\pi} +$$

$$\frac{3\,\text{ArcTan}\left[\frac{3+2\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{\pi} + \frac{3\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\,r}{\sqrt{-3+4r^2}}\right]}{2\pi} - \frac{15\sqrt{2}\,r\,\text{ArcTan}\left[\sqrt{-3+4r^2}\right]}{\pi} +$$

$$\frac{3\,\text{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]}{2\pi} - \frac{3\,r\,\text{ArcTan}\left[\frac{\sqrt{-3+4r^2}}{2-2r^2}\right]}{\sqrt{2}\,\pi} - \frac{12\,\text{ArcTan}\left[\frac{r}{\sqrt{-6+8r^2}}\right]}{\pi} + \frac{6\,r^2\,\text{ArcTan}\left[\frac{r\left(-5+6r^2\right)}{\sqrt{-6+8r^2}}\right]}{\pi} -$$

$$\frac{\sqrt{\frac{2}{3}}\,\text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]}{\pi\,r} - \frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]}{2\pi} -$$

$$\frac{15\sqrt{\frac{3}{2}}\,r\,\text{ArcTan}\left[\sqrt{-9+12r^2}\right]}{\pi} - \frac{3\sqrt{\frac{3}{2}}\,r^3\,\text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]}{\pi} +$$

$$\frac{3\,\text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]}{4\sqrt{2}\,\pi\,r} + \frac{6\,r^2\,\text{ArcTan}\left[\frac{2-r^2-2r^4}{2r\left(-1+r^2\right)\sqrt{-6+8r^2}}\right]}{\pi} + \frac{6\sqrt{2}\,r^3\,\text{ArcTan}\left[\frac{1+\sqrt{-3+4r^2}}{-1+\sqrt{-3+4r^2}}\right]}{\pi};$$

$$\text{TETRDCFDD}[r\_] := -\Bigg(\left(2-3r^2\right)^2\Bigg(-1404\sqrt{2}\,r^2 - 5616\,r^3 - 936\sqrt{2}\,r^4 + 7488\,r^5 +$$

$$3744\sqrt{2}\,r^6 + 8\sqrt{6}\,\pi\sqrt{-3+4r^2} + 36\,\pi\,r\sqrt{-3+4r^2} - 236\sqrt{6}\,\pi\,r^2\sqrt{-3+4r^2} -$$

$$576\,\pi\,r^3\sqrt{-3+4r^2} - 2304\,r^4\sqrt{-3+4r^2} - 468\sqrt{6}\,\pi\,r^4\sqrt{-3+4r^2} +$$

$$432\,r^5\sqrt{-3+4r^2} - 1152\,\pi\,r^5\sqrt{-3+4r^2} + 72\sqrt{6}\,\pi\,r^6\sqrt{-3+4r^2} + 9\,\pi\sqrt{-6+8r^2} -$$

$$198\,\pi\,r^2\sqrt{-6+8r^2} - 576\,r^3\sqrt{-6+8r^2} + 108\,r^4\sqrt{-6+8r^2} - 288\,\pi\,r^4\sqrt{-6+8r^2} -$$

$$1152\,r^5\sqrt{-6+8r^2} + 216\,r^6\sqrt{-6+8r^2} - 864\,\pi\,r^6\sqrt{-6+8r^2} + 32\,\pi\,r\sqrt{-9+12r^2} -$$

$$1008\,\pi\,r^3\sqrt{-9+12r^2} + 144\,\pi\,r^5\sqrt{-9+12r^2} + 288\,r^3\sqrt{-3+4r^2}\,\text{ArcCos}\left[\frac{1}{3}\right] +$$

$$576\,r^5\sqrt{-3+4r^2}\,\text{ArcCos}\left[\frac{1}{3}\right] + 576\,r^4\sqrt{-6+8r^2}\,\text{ArcCos}\left[\frac{1}{3}\right] +$$

$$432\, r^2 \sqrt{-9+12\, r^2}\, \left(\sqrt{2}+4\, r+2\sqrt{2}\, r^2\right) \text{ArcTan}\left[\frac{1}{\sqrt{-1+\frac{4\, r^2}{3}}}\right] -$$

$$108\, r^4 \sqrt{-9+12\, r^2}\, \left(\sqrt{2}+4\, r+2\sqrt{2}\, r^2\right) \text{ArcTan}\left[\sqrt{-1+\frac{4\, r^2}{3}}\right] - 36\sqrt{6}\, r^2 \sqrt{-3+4\, r^2}$$

$$\text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] - 72\sqrt{6}\, r^4 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] -$$

$$144\, r^3 \sqrt{-9+12\, r^2}\, \text{ArcTan}\left[\frac{3\sqrt{3}-4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] - 36\sqrt{6}\, r^2 \sqrt{-3+4\, r^2}$$

$$\text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] - 72\sqrt{6}\, r^4 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] -$$

$$144\, r^3 \sqrt{-9+12\, r^2}\, \text{ArcTan}\left[\frac{3\sqrt{3}+4\sqrt{2}\, r}{\sqrt{-3+4\, r^2}}\right] + 2880\, r^3 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\sqrt{-3+4\, r^2}\right] +$$

$$720\, r^2 \sqrt{-6+8\, r^2}\, \text{ArcTan}\left[\sqrt{-3+4\, r^2}\right] + 1440\, r^4 \sqrt{-6+8\, r^2}\, \text{ArcTan}\left[\sqrt{-3+4\, r^2}\right] +$$

$$288\, r^3 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{\sqrt{-3+4\, r^2}}{2-2\, r^2}\right] + 72\, r^2 \sqrt{-6+8\, r^2}\, \text{ArcTan}\left[\frac{\sqrt{-3+4\, r^2}}{2-2\, r^2}\right] +$$

$$144\, r^4 \sqrt{-6+8\, r^2}\, \text{ArcTan}\left[\frac{\sqrt{-3+4\, r^2}}{2-2\, r^2}\right] - 576\, r^3 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{r\left(-5+6\, r^2\right)}{\sqrt{-6+8\, r^2}}\right] -$$

$$1152\, r^5 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{r\left(-5+6\, r^2\right)}{\sqrt{-6+8\, r^2}}\right] - 1152\, r^4 \sqrt{-6+8\, r^2}\, \text{ArcTan}\left[\frac{r\left(-5+6\, r^2\right)}{\sqrt{-6+8\, r^2}}\right] -$$

$$16\sqrt{6}\, \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{7-9\, r^2}{3\left(-1+r^2\right)\sqrt{-9+12\, r^2}}\right] -$$

$$32\sqrt{6}\, r^2 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{7-9\, r^2}{3\left(-1+r^2\right)\sqrt{-9+12\, r^2}}\right] +$$

$$108\sqrt{6}\, r^4 \sqrt{-3+4\, r^2}\, \text{ArcTan}\left[\frac{7-9\, r^2}{3\left(-1+r^2\right)\sqrt{-9+12\, r^2}}\right] +$$

$$216\sqrt{6}\, r^6 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right]-$$

$$64 r \sqrt{-9+12 r^2}\, \text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right]+432 r^5 \sqrt{-9+12 r^2}$$

$$\text{ArcTan}\left[\frac{7-9 r^2}{3\left(-1+r^2\right)\sqrt{-9+12 r^2}}\right]+360\sqrt{6}\, r^2 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\sqrt{-9+12 r^2}\right]+$$

$$720\sqrt{6}\, r^4 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\sqrt{-9+12 r^2}\right]+1440 r^3 \sqrt{-9+12 r^2}\, \text{ArcTan}\left[\sqrt{-9+12 r^2}\right]+$$

$$216\sqrt{6}\, r^4 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right]+$$

$$432\sqrt{6}\, r^6 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right]+864 r^5 \sqrt{-9+12 r^2}$$

$$\text{ArcTan}\left[\frac{\sqrt{-9+12 r^2}}{-3+2 r^2}\right]+72 r \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+$$

$$18\sqrt{-6+8 r^2}\, \text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+36 r^2 \sqrt{-6+8 r^2}$$

$$\text{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]-576 r^3 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]-$$

$$1152 r^5 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]-$$

$$1152 r^4 \sqrt{-6+8 r^2}\, \text{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]-$$

$$3456 r^5 \sqrt{-3+4 r^2}\, \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]-864 r^4 \sqrt{-6+8 r^2}$$

$$\left.\left.\text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]-1728 r^6 \sqrt{-6+8 r^2}\, \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\right)\right)\Big/$$

$$\left(48\,\pi\,r^2\left(1+2\sqrt{2}\,r+2\,r^2\right)\left(2-2\sqrt{6}\,r+3\,r^2\right)\left(2+2\sqrt{6}\,r+3\,r^2\right)\sqrt{-3+4\,r^2}\right);$$

TETRDDCFDD[r_] :=

$$\Big(540\sqrt{2}\,r^2+4320\,r^3+$$
$$5760\sqrt{2}\,r^4+2880\,r^5-6480\sqrt{2}\,r^6-$$
$$11520\,r^7-2880\sqrt{2}\,r^8+8\sqrt{6}\,\pi\sqrt{-3+4\,r^2}+$$
$$72\,\pi\,r\sqrt{-3+4\,r^2}+96\sqrt{6}\,\pi\,r^2\sqrt{-3+4\,r^2}+$$
$$2304\,r^4\sqrt{-3+4\,r^2}-4\sqrt{6}\,\pi\,r^4\sqrt{-3+4\,r^2}-$$
$$864\,r^5\sqrt{-3+4\,r^2}+1728\,\pi\,r^5\sqrt{-3+4\,r^2}+$$
$$4608\,r^6\sqrt{-3+4\,r^2}-432\sqrt{6}\,\pi\,r^6\sqrt{-3+4\,r^2}-$$
$$1728\,r^7\sqrt{-3+4\,r^2}+6336\,\pi\,r^7\sqrt{-3+4\,r^2}-$$
$$144\sqrt{6}\,\pi\,r^8\sqrt{-3+4\,r^2}+9\,\pi\sqrt{-6+8\,r^2}+$$
$$108\,\pi\,r^2\sqrt{-6+8\,r^2}+288\,r^3\sqrt{-6+8\,r^2}-108\,r^4\sqrt{-6+8\,r^2}-$$
$$108\,\pi\,r^4\sqrt{-6+8\,r^2}+3456\,r^5\sqrt{-6+8\,r^2}-$$
$$1296\,r^6\sqrt{-6+8\,r^2}+4032\,\pi\,r^6\sqrt{-6+8\,r^2}+$$
$$1152\,r^7\sqrt{-6+8\,r^2}-432\,r^8\sqrt{-6+8\,r^2}+$$
$$1728\,\pi\,r^8\sqrt{-6+8\,r^2}+64\,\pi\,r\sqrt{-9+12\,r^2}+$$
$$128\,\pi\,r^3\sqrt{-9+12\,r^2}-288\,\pi\,r^5\sqrt{-9+12\,r^2}-$$
$$576\,\pi\,r^7\sqrt{-9+12\,r^2}-144\,r^3\sqrt{-3+4\,r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]-$$
$$1728\,r^5\sqrt{-3+4\,r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]-576\,r^7\sqrt{-3+4\,r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]-$$
$$576\,r^4\sqrt{-6+8\,r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]-1152\,r^6\sqrt{-6+8\,r^2}\,\mathrm{ArcCos}\left[\frac{1}{3}\right]+$$
$$108\,r^4\sqrt{-9+12\,r^2}\left(\sqrt{2}+8\,r+12\sqrt{2}\,r^2+16\,r^3+4\sqrt{2}\,r^4\right)\mathrm{ArcTan}\left[\sqrt{-1+\frac{4\,r^2}{3}}\right]+$$
$$288\,r^3\sqrt{-3+4\,r^2}\left(1+4\sqrt{2}\,r+12\,r^2+8\sqrt{2}\,r^3+4\,r^4\right)\mathrm{ArcTan}\left[\frac{r\left(-5+6\,r^2\right)}{\sqrt{-6+8\,r^2}}\right]-$$
$$16\sqrt{6}\sqrt{-3+4\,r^2}\,\mathrm{ArcTan}\left[\frac{7-9\,r^2}{3\left(-1+r^2\right)\sqrt{-9+12\,r^2}}\right]-$$

$$192\sqrt{6}\ r^2\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$172\sqrt{6}\ r^4\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$1296\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$432\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$128\,r\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$256\,r^3\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$864\,r^5\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$1728\,r^7\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{7-9r^2}{3\left(-1+r^2\right)\sqrt{-9+12r^2}}\right]-$$

$$216\sqrt{6}\ r^4\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$2592\sqrt{6}\ r^6\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$864\sqrt{6}\ r^8\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$1728\,r^5\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]-$$

$$3456\,r^7\sqrt{-9+12r^2}\ \text{ArcTan}\left[\frac{\sqrt{-9+12r^2}}{-3+2r^2}\right]+$$

$$144\,r\sqrt{-3+4r^2}\ \text{ArcTan}\left[\frac{-7+12r^2-4r^4}{4\left(-1+r^2\right)\sqrt{-3+4r^2}}\right]+$$

$$288\, r^3 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+$$

$$18 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+$$

$$216\, r^2 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+$$

$$72\, r^4 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{-7+12 r^2-4 r^4}{4\left(-1+r^2\right)\sqrt{-3+4 r^2}}\right]+$$

$$288\, r^3 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]+$$

$$3456\, r^5 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]+$$

$$1152\, r^7 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]+$$

$$1152\, r^4 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]+$$

$$2304\, r^6 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{2-r^2-2 r^4}{2 r\left(-1+r^2\right)\sqrt{-6+8 r^2}}\right]+$$

$$6912\, r^5 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$13\,824\, r^7 \sqrt{-3+4 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$864\, r^4 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$10\,368\, r^6 \sqrt{-6+8 r^2}\ \mathrm{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]+$$

$$3456\, r^8 \sqrt{-6+8 r^2}\ \text{ArcTan}\left[\frac{1+\sqrt{-3+4 r^2}}{-1+\sqrt{-3+4 r^2}}\right]\Bigg) \Big/$$

$$\left(24\, \pi\, r^3 \left(1+2 \sqrt{2}\ r+2 r^2\right)^2 \sqrt{-3+4 r^2}\right);$$

```
TETRExctTotalCF[r_] := If[r < TETRDst[[1]], TETRCFAA[r],
  If[r < TETRDst[[2]], TETRCFBB[r], If[r < TETRDst[[3]], TETRCFCC[r]], TETRCFDD[r]]];
TETRExctTotalDCF[r_] := If[r < TETRDst[[1]], TETRDCFAA[r],
  If[r < TETRDst[[2]], TETRDCFBB[r], If[r < TETRDst[[3]], TETRDCFCC[r]], TETRDCFDD[r]]];
TETRExctTotalDDCF[r_] := If[r < TETRDst[[1]], TETRDDCFAA[r], If[r < TETRDst[[2]],
   TETRDDCFBB[r], If[r < TETRDst[[3]], TETRDDCFCC[r]], TETRDDCFDD[r]]];
```

$$\text{TetrCFApprxFnl00AA}[r\_] := 1-3\sqrt{\frac{3}{2}}\ r-\frac{\left(6+5\sqrt{3}\ \pi\right) r^3}{4\sqrt{2}\ \pi}+\frac{3 r^2 \left(2\sqrt{2}+\pi-\text{ArcCos}\left[\frac{1}{3}\right]\right)}{\pi};$$

$$\text{TetrCFApprxFnl00BB}[r\_] := -\frac{1}{40\pi}\ r\ \Bigg(-288+51\sqrt{2}-128\sqrt{3}+$$

$$\sqrt{2}\left(-177+131\sqrt{3}\right)\pi+120\sqrt{2}\ \text{ArcCos}\left[\frac{1}{3}\right]-48\sqrt{2}\ \text{ArcSec}[3]+32\sqrt{6}\ \text{ArcSec}[3]\Bigg)+$$

$$\frac{1}{1120\pi}\left(\left(2428-1007\sqrt{3}\right)\pi+4\left(51-952\sqrt{2}+118\sqrt{3}+420\,\text{ArcCos}\left[\frac{1}{3}\right]+56\,\text{ArcSec}[3]\right)\right)+$$

$$\frac{3}{280}\Bigg(-\Bigg(\Bigg(26\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-20\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+$$

$$84\sqrt{-\sqrt{2}+2 r}-12\sqrt{6}\ r^3\sqrt{-\sqrt{2}+2 r}-36\sqrt{-3\sqrt{2}+6 r}+r^2$$

$$\left(105\times 2^{1/4}\sqrt{-3+2\sqrt{3}}-70\times 2^{1/4}\sqrt{-9+6\sqrt{3}}+24\left(7-2\sqrt{3}\right)\sqrt{-\sqrt{2}+2 r}\right)+2 r$$

$$\left(-28\ 2^{3/4}\sqrt{-3+2\sqrt{3}}+21\times 2^{3/4}\sqrt{-9+6\sqrt{3}}+3\left(-28+11\sqrt{3}\right)\sqrt{-2\sqrt{2}+4 r}\right)\Bigg)$$

$$\left(-6+16\sqrt{2}+\left(8-5\sqrt{3}\right)\pi-8\,\text{ArcSec}[3]\right)\Bigg)\Big/\left(2^{1/4}\left(-3+2\sqrt{3}\right)^{3/2}\pi\right)+$$

$$35\left(\frac{1}{3}-\sqrt{\frac{2}{3}}\ r+\frac{r^2}{2}-\frac{2\times 2^{3/4}\left(\sqrt{6}-3 r\right)^{5/2}\left(-\frac{7}{\sqrt{2}}+2\sqrt{6}+r\right)}{105\left(-3+2\sqrt{3}\right)^{3/2}}\right)$$

$$\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right)\Bigg)-$$

$$\frac{1}{1680\,\pi}\left(\frac{1}{\sqrt{2}}-r\right)^3\left(2\left(\frac{1}{\sqrt{2}}-r\right)\left(-5\sqrt{\frac{2}{3}}+\sqrt{2}+3\,r\right)\right.$$
$$\left(\left(-441\,887\,094+284\,889\,498\sqrt{2}-255\,123\,640\sqrt{3}+164\,480\,973\sqrt{6}\right)\pi-\right.$$
$$324\left(-170\,664-391\,769\sqrt{2}-98\,532\sqrt{3}-226\,188\sqrt{6}+560\sqrt{2}\left(362+209\sqrt{3}\right)\right.$$
$$\left.\left.\text{ArcCos}\left[\frac{1}{3}\right]-4\left(-70\,588+50\,815\sqrt{2}-40\,754\sqrt{3}+29\,338\sqrt{6}\right)\text{ArcSec}[3]\right)\right)-$$
$$\left(-2\sqrt{\frac{2}{3}}+\frac{1}{\sqrt{2}}+r\right)\left(\left(-221\,874\,996+172\,383\,744\sqrt{2}-128\,100\,927\sqrt{3}+99\,525\,812\sqrt{6}\right)\pi+\right.$$
$$648\left(-153\,635-35\,520\sqrt{2}-88\,701\sqrt{3}-20\,511\sqrt{6}+140\left(575+332\sqrt{3}\right)\text{ArcCos}\left[\frac{1}{3}\right]+\right.$$
$$\left.\left.\left.\left(-79\,450+55\,384\sqrt{2}-45\,872\sqrt{3}+31\,976\sqrt{6}\right)\text{ArcSec}[3]\right)\right)\right);$$

$$\text{TetrCFApprxFn100CC}[r\_]:=\frac{1}{20\,160}\left(-\frac{1}{\left(-2+\sqrt{3}\right)\pi}28\left(-7+4\sqrt{3}\right)r\right.$$
$$\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)\right)+$$
$$\frac{1}{\left(-2+\sqrt{3}\right)\pi}\left(\left(-116\,640-40\,338\sqrt{2}+68\,688\sqrt{3}+22\,403\sqrt{6}\right)\pi-\right.$$
$$\left.54\left(\sqrt{2}\left(-1917+1118\sqrt{3}\right)-8\left(-90+53\sqrt{3}\right)\text{ArcSec}[3]\right)\right)+$$
$$4\left(-\frac{1}{4\pi}3\times3^{3/4}\left(3+2\sqrt{2}\right)\left(-465\,3^{1/4}+294\sqrt{2}\,3^{1/4}-768\sqrt{\frac{\sqrt{6}-3\,r}{-3+2\sqrt{2}}}+\right.\right.$$
$$672\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3\,r}-96\sqrt{6\left(3+2\sqrt{2}\right)}\,r^3\sqrt{-\sqrt{6}+3\,r}+$$
$$12\,r^2\left(-105\,3^{1/4}+70\sqrt{2}\,3^{1/4}-64\sqrt{\frac{\sqrt{6}-3\,r}{-3+2\sqrt{2}}}+84\sqrt{6+4\sqrt{2}}\sqrt{-\sqrt{6}+3\,r}\right)+$$

$$4\,r\left(7\times 3^{3/4}\left(-23+18\sqrt{2}\right)+176\sqrt{6\left(3+2\sqrt{2}\right)}\sqrt{-\sqrt{6}+3\,r}-336\sqrt{9+6\sqrt{2}}\sqrt{-\sqrt{6}+3\,r}\right)\right)\left(32\sqrt{2}-8\sqrt{3}-36\,\pi+19\sqrt{3}\,\pi-16\,\mathrm{ArcSec}[3]\right)+$$

$$70\left(\frac{3}{8}-\frac{\sqrt{3}\,r}{2}+\frac{r^2}{2}-\frac{3^{3/4}\left(\sqrt{3}-2\,r\right)^{5/2}\left(\frac{9-7\sqrt{2}}{\sqrt{3}}+r\right)}{35\left(3-2\sqrt{2}\right)^{3/2}}\right)$$

$$\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)}{\pi}\right)\right)\right);$$

$$\mathrm{TetrCFApprxFnl00DD}[r\_] := -\frac{1}{5040\left(-2+\sqrt{3}\right)\pi}$$

$$\left(30-13\sqrt{3}-42\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}+18\sqrt{3}\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}+\left(56-42\sqrt{3}-66\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}+56\sqrt{3}\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}\right)r+\left(70-35\sqrt{3}-56\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}+16\sqrt{3}\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}\right)r^2+8\sqrt{\frac{\sqrt{3}-2\,r}{-2+\sqrt{3}}}\,r^3\right)$$

$$\left(\left(-1296+209\sqrt{2}+372\sqrt{6}\right)\pi-54\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)\right);$$

$$\text{TetrCLDApprx00AA}[r\_] := -\frac{3\left(6\sqrt{2}\ r+\pi\left(-8+5\sqrt{6}\ r\right)+8\left(-2\sqrt{2}+\text{ArcCos}\left[\frac{1}{3}\right]\right)\right)}{4\pi};$$

$$\text{TetrCLDApprx00BB}[r\_] :=$$

$$\frac{3}{8}\left(-\frac{1}{\pi}2\left(1-\frac{3\left(-\sqrt{2}+\sqrt{6}-r\right)\sqrt{-3\sqrt{2}+6r}}{2^{3/4}\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(6-16\sqrt{2}+\left(-8+5\sqrt{3}\right)\pi+8\,\text{ArcSec}[3]\right)+\right.$$
$$\left.\left(1+\frac{\sqrt{\sqrt{6}-3r}\left(9-4\sqrt{3}-3\sqrt{2}\ r\right)}{2^{3/4}\left(-3+2\sqrt{3}\right)^{3/2}}\right)\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00CC}[r\_] := \frac{1}{72}\left(27\left(1+\frac{3^{1/4}\left(2\sqrt{\frac{2}{3}}-\frac{3\sqrt{3}}{2}+r\right)\sqrt{-2\sqrt{6}+6r}}{\left(3-2\sqrt{2}\right)^{3/2}}\right)\right.$$
$$\left(-36+19\sqrt{3}-\frac{8\left(-4\sqrt{2}+\sqrt{3}+2\,\text{ArcSec}[3]\right)}{\pi}\right)+\left(1-\frac{3^{3/4}\sqrt{\sqrt{3}-2r}\left(\sqrt{3}-\sqrt{6}+r\right)}{\left(3-2\sqrt{2}\right)^{3/2}}\right)$$
$$\left.\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{TetrCLDApprx00DD}[r\_] := \frac{1}{72}\left(1+\frac{\left(-3+\sqrt{3}+r\right)\sqrt{-\sqrt{3}+2r}}{\left(2-\sqrt{3}\right)^{3/2}}\right)$$
$$\left(-1296+209\sqrt{2}+372\sqrt{6}-\frac{54\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right);$$

OCTAHEDRON

```
Theta[x_] := If[x > 0, 1, 0];
```

$$\text{OCTDst} = \left\{\frac{1}{\sqrt{3}}, \sqrt{\frac{3}{8}}, \frac{1}{\sqrt{2}}, 1\right\};$$

$$\Delta\Delta = \left\{\frac{1}{\sqrt{3}}, \frac{3-2\sqrt{2}}{2\sqrt{6}}, \frac{2-\sqrt{3}}{2\sqrt{2}}, \frac{\sqrt{2}-1}{\sqrt{2}}\right\}; \text{VOcth} = \text{VOct} * \left(1 \middle/ \sqrt{2}\right)\hat{}\,3;$$

$$\text{OCTCFAA}[r\_] := 1 - \frac{3\sqrt{3}\,r}{2} + \frac{6\sqrt{2}\,r^2}{\pi} - \frac{\left(3-3\pi+\sqrt{3}\,\pi\right)r^3}{2\pi} - \frac{3\,r^2\,\text{ArcSec}[3]}{\pi};$$

$$\text{OCTDCFAA}[r\_] := -\frac{3\sqrt{3}}{2} + \frac{12\sqrt{2}\,r}{\pi} - \frac{3\left(3-3\pi+\sqrt{3}\,\pi\right)r^2}{2\pi} - \frac{6\,r\,\text{ArcSec}[3]}{\pi};$$

$$\text{OCTDDCFAA}[r\_] := \frac{6\left(2\sqrt{2}-\text{ArcSec}[3]\right)}{\pi} - \frac{3\left(3-3\pi+\sqrt{3}\,\pi\right)r}{\pi};$$

$$\text{OCTCFBB}[r\_] := -3 + \frac{2}{\sqrt{3}\,r} - \frac{\sqrt{3}\,r}{2} - \frac{\left(3+\left(-3+7\sqrt{3}\right)\pi\right)r^3}{2\pi} + \frac{3\,r^2\left(2\sqrt{2}+2\pi-\text{ArcSec}[3]\right)}{\pi};$$

$$\text{OCTDCFBB}[r\_] := -\frac{\sqrt{3}}{2} - \frac{2}{\sqrt{3}\,r^2} - \frac{3\left(3+\left(-3+7\sqrt{3}\right)\pi\right)r^2}{2\pi} + \frac{6\,r\left(2\sqrt{2}+2\pi-\text{ArcSec}[3]\right)}{\pi};$$

$$\text{OCTDDCFBB}[r\_] := \frac{4}{\sqrt{3}\,r^3} - \frac{3\left(3+\left(-3+7\sqrt{3}\right)\pi\right)r}{\pi} + \frac{6\left(2\sqrt{2}+2\pi-\text{ArcSec}[3]\right)}{\pi};$$

$$\text{OCTCFCC}[r\_] := -3 + \frac{2}{\sqrt{3}\,r} + 4\sqrt{3}\,r + 3\,r^2 + \frac{6\sqrt{2}\,r^2}{\pi} + \frac{3\,r^3}{2} - \frac{23\,r^3}{8\sqrt{3}} - \frac{3\,r^3}{2\pi} -$$
$$\frac{3\sqrt{-3+8\,r^2}}{4\pi r} - \frac{17\,r\sqrt{-3+8\,r^2}}{2\pi} - \frac{3\,r^2\,\text{ArcCos}\left[\frac{1}{3}\right]}{\pi} - \frac{15\sqrt{3}\,r^3\,\text{ArcTan}\left[\frac{1}{\sqrt{3}\sqrt{-3+8\,r^2}}\right]}{2\pi} -$$
$$\frac{9\sqrt{3}\,r\,\text{ArcTan}\left[\frac{\sqrt{3}}{\sqrt{-3+8\,r^2}}\right]}{\pi} + \frac{3\sqrt{3}\,r^3\,\text{ArcTan}\left[\frac{\sqrt{3}}{\sqrt{-3+8\,r^2}}\right]}{2\pi} + \frac{6\,r^2\,\text{ArcTan}\left[\frac{r}{\sqrt{-3+8\,r^2}}\right]}{\pi} +$$
$$\frac{2\,r^3\,\text{ArcTan}\left[\frac{-3+4\,r^2}{\sqrt{3}\sqrt{-3+8\,r^2}}\right]}{\sqrt{3}\,\pi} + \frac{\sqrt{3}\,r\,\text{ArcTan}\left[\sqrt{3}\sqrt{-3+8\,r^2}\right]}{\pi} - \frac{2\sqrt{3}\,r^3\,\text{ArcTan}\left[\frac{-5+12\,r^2}{\sqrt{3}\sqrt{-3+8\,r^2}}\right]}{\pi} +$$

$$\frac{r^3\,\text{ArcTan}\left[\frac{-9+24r^2-8r^4}{\sqrt{3}\,(-3+4r^2)\sqrt{-3+8r^2}}\right]}{4\sqrt{3}\,\pi}+\frac{\sqrt{3}\,r^3\,\text{ArcTan}\left[\frac{\sqrt{3}\,(27-180r^2+384r^4-272r^6+32r^8)}{\sqrt{-3+8r^2}\,(-27+144r^2-216r^4+80r^6)}\right]}{2\pi};$$

$$\text{OCTDCFCC}[r\_]:=4\sqrt{3}-\frac{2}{\sqrt{3}\,r^2}+6r+\frac{12\sqrt{2}\,r}{\pi}+\frac{9r^2}{2}-\frac{23\sqrt{3}\,r^2}{8}-\frac{9r^2}{2\pi}+$$

$$\frac{105}{2\pi\sqrt{-3+8r^2}}-\frac{9}{4\pi r^2\sqrt{-3+8r^2}}-\frac{124r^2}{\pi\sqrt{-3+8r^2}}-\frac{6r\,\text{ArcCos}\left[\frac{1}{3}\right]}{\pi}+$$

$$\frac{9\sqrt{3}\,(-2+r^2)\,\text{ArcTan}\left[\frac{\sqrt{3}}{\sqrt{-3+8r^2}}\right]}{2\pi}+\frac{12r\,\text{ArcTan}\left[\frac{r}{\sqrt{-3+8r^2}}\right]}{\pi}-\frac{45\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{1}{\sqrt{-9+24r^2}}\right]}{2\pi}+$$

$$\frac{2\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{-3+4r^2}{\sqrt{-9+24r^2}}\right]}{\pi}-\frac{6\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{-5+12r^2}{\sqrt{-9+24r^2}}\right]}{\pi}+\frac{\sqrt{3}\,\text{ArcTan}\left[\sqrt{-9+24r^2}\right]}{\pi}+$$

$$\frac{\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{-9+24r^2-8r^4}{(-3+4r^2)\sqrt{-9+24r^2}}\right]}{4\pi}+\frac{3\sqrt{3}\,r^2\,\text{ArcTan}\left[\frac{\sqrt{3}\,(27-180r^2+384r^4-272r^6+32r^8)}{\sqrt{-3+8r^2}\,(-27+144r^2-216r^4+80r^6)}\right]}{2\pi};$$

$$\text{OCTDDCFCC}[r\_]:=6+\frac{12\sqrt{2}}{\pi}+\frac{4}{\sqrt{3}\,r^3}+9r-\frac{23\sqrt{3}\,r}{4}-\frac{9r}{\pi}+\frac{9}{2\pi r^3\sqrt{-3+8r^2}}+\frac{21}{\pi r\sqrt{-3+8r^2}}-$$

$$\frac{88r}{\pi\sqrt{-3+8r^2}}-\frac{6\,\text{ArcCos}\left[\frac{1}{3}\right]}{\pi}+\frac{9\sqrt{3}\,r\,\text{ArcTan}\left[\frac{\sqrt{3}}{\sqrt{-3+8r^2}}\right]}{\pi}+\frac{12\,\text{ArcTan}\left[\frac{r}{\sqrt{-3+8r^2}}\right]}{\pi}-$$

$$\frac{45\sqrt{3}\,r\,\text{ArcTan}\left[\frac{1}{\sqrt{-9+24r^2}}\right]}{\pi}+\frac{4\sqrt{3}\,r\,\text{ArcTan}\left[\frac{-3+4r^2}{\sqrt{-9+24r^2}}\right]}{\pi}-\frac{12\sqrt{3}\,r\,\text{ArcTan}\left[\frac{-5+12r^2}{\sqrt{-9+24r^2}}\right]}{\pi}+$$

$$\frac{\sqrt{3}\,r\,\text{ArcTan}\left[\frac{-9+24r^2-8r^4}{(-3+4r^2)\sqrt{-9+24r^2}}\right]}{2\pi}+\frac{3\sqrt{3}\,r\,\text{ArcTan}\left[\frac{\sqrt{3}\,(27-180r^2+384r^4-272r^6+32r^8)}{\sqrt{-3+8r^2}\,(-27+144r^2-216r^4+80r^6)}\right]}{\pi};$$

$$\text{OCTCFDD}[r\_]:=-\frac{1}{12\pi r}\Bigg(6+4\sqrt{3}\,\pi-36\pi r+36r^2-16\sqrt{3}\,\pi r^2-18\pi r^3+18r^4-9\pi r^4+$$

$$12\sqrt{3}\,\pi r^4-12\sqrt{-1+2r^2}-48r^2\sqrt{-1+2r^2}+48\sqrt{3}\,r^2\,\text{ArcSin}\left[\sqrt{\frac{3-6r^2}{2-6r^2}}\right]+$$

$$24\sqrt{3}\,r^4\,\text{ArcSin}\left[\frac{5-9r^2}{\sqrt{2}\,\left(-1+3r^2\right)^{3/2}}\right]+144\,r\,\text{ArcSin}\left[\frac{r}{\sqrt{-1+3r^2}}\right]-24\sqrt{3}$$

$$\left.\text{ArcSin}\left[\frac{1}{\sqrt{-2+6r^2}}\right]-36\,r^3\,\text{ArcSin}\left[\frac{1+2r^2-7r^4}{\left(1-3r^2\right)^2}\right]+36\,r^4\,\text{ArcSin}\left[\frac{-1+r^2+\sqrt{-1+2r^2}}{\sqrt{2}\,r^2}\right]\right);$$

$$\text{OCTDCFDD}[r\_] := \frac{4}{\sqrt{3}}-\frac{3}{\pi}+\frac{1}{\sqrt{3}\,r^2}+\frac{1}{2\pi r^2}+3r+\frac{9r^2}{4}-3\sqrt{3}\,r^2-\frac{9r^2}{2\pi}-\frac{10}{\pi\sqrt{-1+2r^2}}+$$

$$\frac{1}{\pi r^2\sqrt{-1+2r^2}}+\frac{16r^2}{\pi\sqrt{-1+2r^2}}-\frac{4\sqrt{3}\,\text{ArcSin}\left[\sqrt{\frac{3-6r^2}{2-6r^2}}\right]}{\pi}-\frac{6\sqrt{3}\,r^2\,\text{ArcSin}\left[\frac{5-9r^2}{\sqrt{2}\,\left(-1+3r^2\right)^{3/2}}\right]}{\pi}-$$

$$\frac{2\sqrt{3}\,\text{ArcSin}\left[\frac{1}{\sqrt{-2+6r^2}}\right]}{\pi r^2}+\frac{6\,r\,\text{ArcSin}\left[\frac{1+2r^2-7r^4}{\left(1-3r^2\right)^2}\right]}{\pi}-\frac{9r^2\,\text{ArcSin}\left[\frac{-1+r^2+\sqrt{-1+2r^2}}{\sqrt{2}\,r^2}\right]}{\pi};$$

$$\text{OCTDDCFDD}[r\_] := 3-\frac{2}{\sqrt{3}\,r^3}-\frac{1}{\pi r^3}+\frac{9r}{2}-6\sqrt{3}\,r-\frac{9r}{\pi}+\frac{2\sqrt{-1+2r^2}\,\left(1+4r^2\right)}{\pi r^3}-$$

$$\frac{1}{\pi r^3}\,2\left(6\sqrt{3}\,r^4\,\text{ArcSin}\left[\frac{5-9r^2}{\sqrt{2}\,\left(-1+3r^2\right)^{3/2}}\right]-2\sqrt{3}\,\text{ArcSin}\left[\frac{1}{\sqrt{-2+6r^2}}\right]-\right.$$

$$\left.3r^3\,\text{ArcSin}\left[\frac{1+2r^2-7r^4}{\left(1-3r^2\right)^2}\right]+9r^4\,\text{ArcSin}\left[\frac{-1+r^2+\sqrt{-1+2r^2}}{\sqrt{2}\,r^2}\right]\right);$$

```
OCTAxctTotalCF[r_] := If[r < OCTDst[[1]], OCTCFAA[r],
  If[r < OCTDst[[2]], OCTCFBB[r], If[r < OCTDst[[3]], OCTCFCC[r]], OCTCFDD[r]]];
OCTAxctTotalDCF[r_] := If[r < OCTDst[[1]], OCTDCFAA[r],
  If[r < OCTDst[[2]], OCTDCFBB[r], If[r < OCTDst[[3]], OCTDCFCC[r]], OCTDCFDD[r]]];
OCTAxctTotalDDCF[r_] := If[r < OCTDst[[1]], OCTDDCFAA[r],
  If[r < OCTDst[[2]], OCTDDCFBB[r], If[r < OCTDst[[3]], OCTDDCFCC[r]], OCTDDCFDD[r]]];
```

$$\text{OcthCLDApprx00AA}[r\_] := \frac{12\sqrt{2}}{\pi} + 9\,r - 3\sqrt{3}\,r - \frac{9\,r}{\pi} - \frac{6\,\text{ArcSec}[3]}{\pi};\ \text{OcthCLDApprx00BB}[r\_] :=$$

$$\frac{1}{36}\left(\frac{1}{\pi}\,108\left(1 + \frac{\sqrt{-\sqrt{3}+3\,r}\,\left(8\sqrt{3} - 9\sqrt{6} + 12\,r\right)}{3^{3/4}\left(-4+3\sqrt{2}\right)^{3/2}}\right)\left(4\sqrt{2} - \sqrt{3} + \pi + \sqrt{3}\,\pi - 2\,\text{ArcSec}[3]\right) + \right.$$

$$\left(1 - \frac{3^{3/4}\sqrt{\sqrt{6}-4\,r}\,\left(-2\sqrt{3}+\sqrt{6}+2\,r\right)}{\left(-4+3\sqrt{2}\right)^{3/2}}\right)$$

$$\left.\left(432 - 311\sqrt{2} + 81\sqrt{6} - \frac{27\left(-16\sqrt{2}+3\sqrt{6}+8\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{OcthCLDApprx00CC}[r\_] := \frac{1}{36\sqrt{2-\sqrt{3}}\,\left(-2+\sqrt{3}\right)\pi}$$

$$\left(-639\sqrt{2\left(2-\sqrt{3}\right)} + 360\sqrt{6\left(2-\sqrt{3}\right)} - 1296\sqrt{2-\sqrt{3}}\,\pi + 181\sqrt{2\left(2-\sqrt{3}\right)}\,\pi + \right.$$

$$648\sqrt{3\left(2-\sqrt{3}\right)}\,\pi - 71\sqrt{6\left(2-\sqrt{3}\right)}\,\pi - 936\sqrt{1-\sqrt{2}\,r} + 702\sqrt{3}\sqrt{1-\sqrt{2}\,r} +$$

$$1728\,\pi\sqrt{1-\sqrt{2}\,r} + 432\sqrt{2}\,\pi\sqrt{1-\sqrt{2}\,r} - 966\sqrt{3}\,\pi\sqrt{1-\sqrt{2}\,r} - 324\sqrt{6}\,\pi\sqrt{1-\sqrt{2}\,r} -$$

$$468\sqrt{2}\,r\sqrt{1-\sqrt{2}\,r} + 432\,\pi\,r\sqrt{1-\sqrt{2}\,r} + 324\sqrt{2}\,\pi\,r\sqrt{1-\sqrt{2}\,r} -$$

$$240\sqrt{6}\,\pi\,r\sqrt{1-\sqrt{2}\,r} + 1539\sqrt{2}\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 675\sqrt{6}\sqrt{-\sqrt{3}+2\sqrt{2}\,r} +$$

$$1296\,\pi\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 1176\sqrt{2}\,\pi\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 432\sqrt{3}\,\pi\sqrt{-\sqrt{3}+2\sqrt{2}\,r} +$$

$$554\sqrt{6}\,\pi\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 864\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r} + 162\sqrt{3}\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r} +$$

$$622\,\pi\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 432\sqrt{2}\,\pi\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r} - 162\sqrt{3}\,\pi\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r} +$$

$$432\sqrt{2-\sqrt{3}}\,\text{ArcSec}[3] - 216\sqrt{3\left(2-\sqrt{3}\right)}\,\text{ArcSec}[3] - 648\sqrt{-\sqrt{3}+2\sqrt{2}\,r}\,\text{ArcSec}[3] +$$

$$\left.216\sqrt{3}\sqrt{-\sqrt{3}+2\sqrt{2}\,r}\,\text{ArcSec}[3] + 216\sqrt{2}\,r\sqrt{-\sqrt{3}+2\sqrt{2}\,r}\,\text{ArcSec}[3]\right);$$

$$\text{OcthCLDApprx00DD}[r\_] := \left(6 - 10\sqrt{\frac{2}{3}} + \frac{9}{\sqrt{2}} - \frac{13}{\sqrt{2}\,\pi}\right)\left(1 + \frac{\left(-3+\sqrt{2}+r\right)\sqrt{-\sqrt{2}+2\,r}}{\left(2-\sqrt{2}\right)^{3/2}}\right);$$

$$\text{OcthCFApprxFnl00AA}[r\_] := 1 - \frac{3\sqrt{3}\, r}{2} + \frac{6\sqrt{2}\, r^2}{\pi} + \frac{3 r^3}{2} - \frac{\sqrt{3}\, r^3}{2} - \frac{3 r^3}{2\pi} - \frac{3 r^2\, \text{ArcSec}[3]}{\pi};$$

$$\text{OcthCFApprxFnl00BB}[r\_] := \frac{1}{20160\,\pi}\left(\left(8324 + 12959\sqrt{2} - 1932\sqrt{3} - 3753\sqrt{6}\right)\pi + 3\left(-14112\sqrt{2} + 644\sqrt{3} + 1251\sqrt{6} + 7056\,\text{ArcSec}[3]\right)\right) + \frac{1}{1080\,\pi}\, r\left(\left(1701 - 4173\sqrt{3} + 1432\sqrt{6}\right)\pi - 81\left(21 - 32\sqrt{3} - 32\sqrt{6} + 8\sqrt{3}\left(2+\sqrt{2}\right)\text{ArcSec}[3]\right)\right) + \frac{1}{1260}\left(-\frac{1}{4\left(-4+3\sqrt{2}\right)^{3/2}\pi}\, 3\times 3^{3/4}\left(588\times 3^{1/4}\sqrt{-4+3\sqrt{2}} - 465\times 3^{1/4}\sqrt{-8+6\sqrt{2}} - 1536\sqrt{-\sqrt{3}+3r} + 1344\sqrt{-2\sqrt{3}+6r} - 768 r^3\sqrt{-3\sqrt{3}+9r} - 24 r^2\left(-140\, 3^{1/4}\sqrt{-4+3\sqrt{2}} + 105\times 3^{1/4}\sqrt{-8+6\sqrt{2}} + 8\left(16-21\sqrt{2}\right)\sqrt{-\sqrt{3}+3r}\right) + 8 r\left(-161\, 3^{3/4}\sqrt{-4+3\sqrt{2}} + 126\times 3^{3/4}\sqrt{-8+6\sqrt{2}} + 16\left(22-21\sqrt{2}\right)\sqrt{-3\sqrt{3}+9r}\right)\right)\left(4\sqrt{2} - \sqrt{3} + \pi + \sqrt{3}\,\pi - 2\,\text{ArcSec}[3]\right) + 35\left(\frac{3}{16} - \frac{1}{2}\sqrt{\frac{3}{2}}\, r + \frac{r^2}{2} - \frac{3^{3/4}\left(\sqrt{6}-4r\right)^{5/2}\left(3\sqrt{\frac{3}{2}} - \frac{7}{\sqrt{3}} + r\right)}{70\left(-4+3\sqrt{2}\right)^{3/2}}\right)\left(432 - 311\sqrt{2} + 81\sqrt{6} - \frac{27\left(-16\sqrt{2} + 3\sqrt{6} + 8\,\text{ArcSec}[3]\right)}{\pi}\right)\right);$$

$$\text{OcthCFApprxFnl00CC}[r\_] := \frac{1}{2520}\left(-\frac{6\left(-819 + 507\sqrt{2} + \left(99 + 27\sqrt{2} - 420\sqrt{3} + 260\sqrt{6}\right)\pi\right)}{\left(-2+\sqrt{2}\right)\pi} + \right.$$

$$\frac{84\left(39\left(4-3\sqrt{2}\right)+\left(9\sqrt{2}+80\sqrt{3}-60\sqrt{6}\right)\pi\right)r}{\left(-2+\sqrt{2}\right)\pi}+$$

$$2520\left(6-10\sqrt{\frac{2}{3}}+\frac{9}{\sqrt{2}}-\frac{13}{\sqrt{2}\,\pi}\right)\left(\frac{1}{4}-\frac{r}{\sqrt{2}}+\frac{r^2}{2}-\frac{2^{3/4}\left(\sqrt{2}-2\,r\right)^{5/2}\left(-\frac{7\sqrt{\frac{3}{2}}}{2}+3\sqrt{2}+r\right)}{35\left(2-\sqrt{3}\right)^{3/2}}\right)-$$

$$\frac{1}{2\left(2-\sqrt{3}\right)^{3/2}\pi}\Big(13\sqrt{6-3\sqrt{3}}-30\sqrt{2-\sqrt{3}}+42\sqrt{-\sqrt{3}+2\sqrt{2}\,r}-$$

$$16\,r^3\sqrt{-2\sqrt{3}+4\sqrt{2}\,r}-18\sqrt{-3\sqrt{3}+6\sqrt{2}\,r}+$$

$$2\sqrt{2}\,r\left(21\sqrt{6-3\sqrt{3}}-28\sqrt{2-\sqrt{3}}+\left(33-28\sqrt{3}\right)\sqrt{-\sqrt{3}+2\sqrt{2}\,r}\right)+$$

$$2\,r^2\left(35\sqrt{6-3\sqrt{3}}-70\sqrt{2-\sqrt{3}}+8\left(7-2\sqrt{3}\right)\sqrt{-\sqrt{3}+2\sqrt{2}\,r}\right)\Big)$$

$$\Big(\left(432-311\sqrt{2}+81\sqrt{6}\right)\pi-27\left(-16\sqrt{2}+3\sqrt{6}+8\,\mathrm{ArcSec}[3]\right)\Big)\Bigg)+$$

$$\frac{1}{945\,\pi}\,2\left(\frac{1}{\sqrt{2}}-r\right)^3\Bigg(8\left(\frac{21}{4}+2\sqrt{2}\,r+3\,r^2-5\sqrt{\frac{3}{2}}\left(\frac{1}{\sqrt{2}}+r\right)\right)$$

$$\Big(\Big(-37\,158\,831+16\,174\,533\sqrt{2}-21\,453\,639\sqrt{3}+9\,338\,357\sqrt{6}\Big)\pi-9\Big(232\,650-2\,173\,797$$

$$\sqrt{2}+134\,319\sqrt{3}-1\,255\,042\sqrt{6}+840\left(-2340+724\sqrt{2}-1351\sqrt{3}+418\sqrt{6}\right)$$

$$\mathrm{ArcSec}[-3]+24\left(-32\,760+23\,381\sqrt{2}-18\,914\sqrt{3}+13\,499\sqrt{6}\right)\mathrm{ArcSec}[3]\Big)\Big)+$$

$$3\left(-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}+r\right)\Big(\Big(-14\,556\,942+16\,833\,121\sqrt{2}-8\,404\,210\sqrt{3}+9\,718\,610\sqrt{6}\Big)\pi-$$

$$3\Big(5\,887\,008-268\,875\sqrt{2}+3\,398\,906\sqrt{3}-155\,202\sqrt{6}+$$

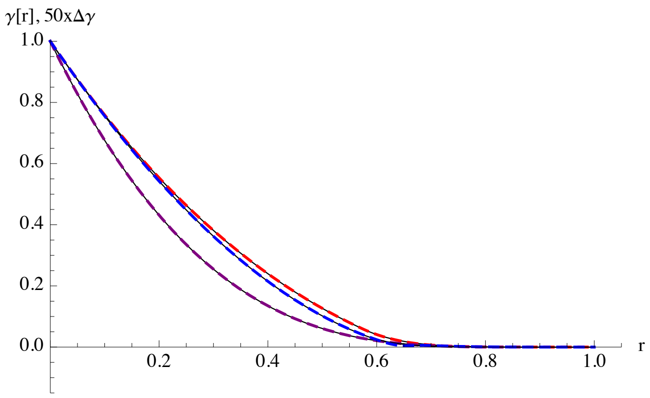
$$3360\left(-485+795\sqrt{2}-280\sqrt{3}+459\sqrt{6}\right)\mathrm{ArcSec}[-3]+$$

$$12\left(-127337+89040\sqrt{2}-73516\sqrt{3}+51408\sqrt{6}\right)\text{ArcSec}[3]\Big)\Big)\Bigg);$$

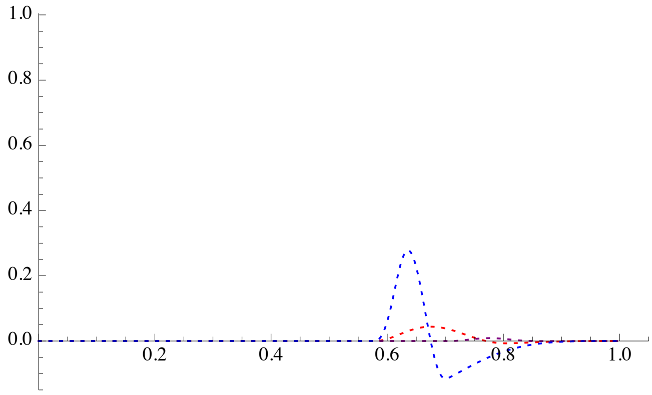
$$\text{OcthCFApprxFnl00DD}[r\_] := \Bigg(\left(39\sqrt{2}+\left(-36-27\sqrt{2}+20\sqrt{6}\right)\pi\right)$$
$$\Bigg(22\sqrt{-1+\sqrt{2}}-7\sqrt{2\left(-1+\sqrt{2}\right)}-28\sqrt{-1+\sqrt{2}\,r}+8r^3\sqrt{-1+\sqrt{2}\,r}+12\sqrt{-2+2\sqrt{2}\,r}+$$
$$r^2\left(70\sqrt{-1+\sqrt{2}}-35\sqrt{2\left(-1+\sqrt{2}\right)}+8\left(-7+2\sqrt{2}\right)\sqrt{-1+\sqrt{2}\,r}\right)+$$
$$r\left(28\sqrt{-1+\sqrt{2}}-42\sqrt{2\left(-1+\sqrt{2}\right)}+4\left(-11+14\sqrt{2}\right)\sqrt{-1+\sqrt{2}\,r}\right)\Bigg)\Bigg)\Big/$$
$$\left(420\left(-2+\sqrt{2}\right)\sqrt{-1+\sqrt{2}}\,\pi\right);$$

Figure of the CFs and CLDs

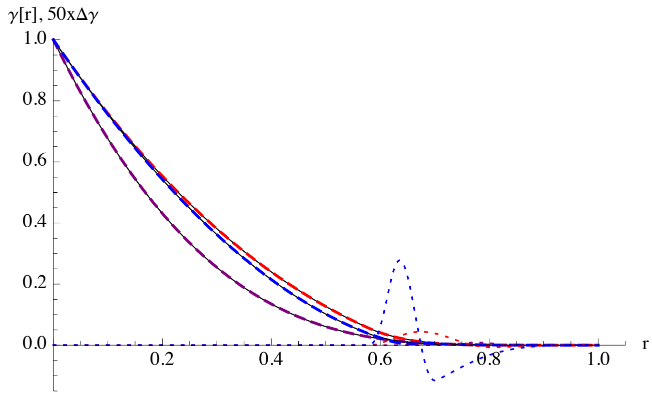
```
cfplt = Plot[{If[r < CBDst[[1]], CBCFAA[r], If[r < CBDst[[2]], CBCFBB[r], CBCFCC[r]]],
    If[r < CBDst[[1]], CubeCFApprxFnl00AA[r], If[r < CBDst[[2]], CubeCFApprxFnl00BB[r],
      CubeCFApprxFnl00CC[r]]], If[r < TETRDst[[1]], TETRCFAA[r],
     If[r < TETRDst[[2]], TETRCFBB[r], If[r < TETRDst[[3]], TETRCFCC[r], TETRCFDD[r]]]],
    If[r < TETRDst[[1]], TetrCFApprxFnl00AA[r], If[r < TETRDst[[2]], TetrCFApprxFnl00BB[r],
      If[r < TETRDst[[3]], TetrCFApprxFnl00CC[r], TetrCFApprxFnl00DD[r]]]],
    If[r < OCTDst[[1]], OCTCFAA[r], If[r < OCTDst[[2]], OCTCFBB[r],
      If[r < OCTDst[[3]], OCTCFCC[r], OCTCFDD[r]]]],
    If[r < OCTDst[[1]], OcthCFApprxFnl00AA[r], If[r < OCTDst[[2]], OcthCFApprxFnl00BB[r],
      If[r < OCTDst[[3]], OcthCFApprxFnl00CC[r], OcthCFApprxFnl00DD[r]]]]},
   {r, 0, 1}, PlotStyle → {Directive[Black, Thickness[0.002]],
     Directive[Red, Thickness[0.005], Dashed], Directive[Black, Thickness[0.002]],
     Directive[Purple, Thickness[0.005], Dashed],
     Directive[Black, Thickness[0.002]], Directive[Blue, Thickness[0.005], Dashed]},
   PlotRange → {{0, 1.05}, {-0.15, 1.005}}, AxesLabel → {"r", "γ[r], 50xΔγ"}]
```



```
errplt = Plot[{50 * If[r < CBDst[[1]], (CBCFAA[r] - CubeCFApprxFnl00AA[r]), If[r < CBDst[[2]],
        (CBCFBB[r] - CubeCFApprxFnl00BB[r]), (CBCFCC[r] - CubeCFApprxFnl00CC[r]) ]],
    50 * If[r < TETRDst[[1]], (TETRCFAA[r] - TetrCFApprxFnl00AA[r]),
      If[r < TETRDst[[2]], (TETRCFBB[r] - TetrCFApprxFnl00BB[r]), If[r < TETRDst[[3]],
        (TETRCFCC[r] - TetrCFApprxFnl00CC[r]), (TETRCFDD[r] - TetrCFApprxFnl00DD[r])]]],
    50 * If[r < OCTDst[[1]], (OCTCFAA[r] - OcthCFApprxFnl00AA[r]),
      If[r < OCTDst[[2]], (OCTCFBB[r] - OcthCFApprxFnl00BB[r]), If[r < OCTDst[[3]],
        (OCTCFCC[r] - OcthCFApprxFnl00CC[r]), (OCTCFDD[r] - OcthCFApprxFnl00DD[r])]]]},
   {r, 0, 1}, PlotStyle → {Directive[Red, Thickness[0.003], Dotted],
     Directive[Purple, Thickness[0.003], Dotted],
     Directive[Blue, Thickness[0.003], Dotted]}, PlotRange → {{0, 1.05}, {-0.15, 1.005}}]
```

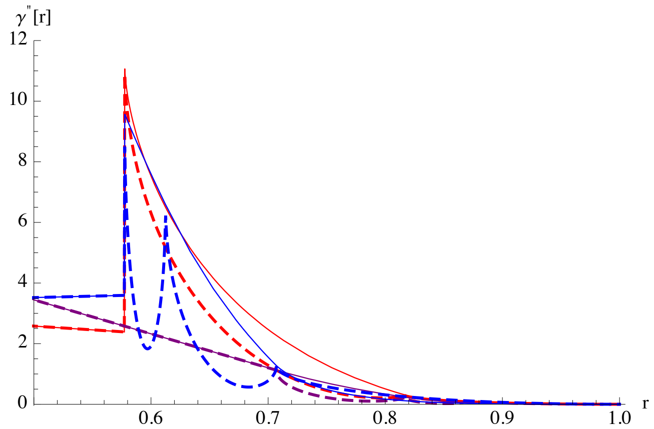



```
FigAll00CF = Show[{cfplt, errplt}]
```




```
Export["FigAll00CF.eps", FigAll00CF];
```

```
FigAll00CLD =
 Plot[{If[r < CBDst[[1]], CBDDCFAA[r], If[r < CBDst[[2]], CBDDCFBB[r], CBDDCFCC[r]]],
   If[r < CBDst[[1]], CubeCLDApprx00AA[r],
    If[r < CBDst[[2]], CubeCLDApprx00BB[r], CubeCLDApprx00CC[r]]],
   If[r < TETRDst[[1]], TETRDDCFAA[r], If[r < TETRDst[[2]], TETRDDCFBB[r],
     If[r < TETRDst[[3]], TETRDDCFCC[r], TETRDDCFDD[r]]]],
   If[r < TETRDst[[1]], TetrCLDApprx00AA[r], If[r < TETRDst[[2]], TetrCLDApprx00BB[r],
     If[r < TETRDst[[3]], TetrCLDApprx00CC[r], TetrCLDApprx00DD[r]]]],
   If[r < OCTDst[[1]], OCTDDCFAA[r], If[r < OCTDst[[2]], OCTDDCFBB[r],
     If[r < OCTDst[[3]], OCTDDCFCC[r], OCTDDCFDD[r]]]],
   If[r < OCTDst[[1]], OcthCLDApprx00AA[r], If[r < OCTDst[[2]], OcthCLDApprx00BB[r],
     If[r < OCTDst[[3]], OcthCLDApprx00CC[r], OcthCLDApprx00DD[r]]]]}, {r, 0, 1},
  PlotStyle → {Directive[Red, Thickness[0.002]], Directive[Red, Thickness[0.005], Dashed],
    Directive[Purple, Thickness[0.002]], Directive[Purple, Thickness[0.005], Dashed],
    Directive[Blue, Thickness[0.002]], Directive[Blue, Thickness[0.005], Dashed]},
  PlotRange → {{0.5, 1.005}, {-0.05, 12}}, AxesLabel → {"r", "γ''[r]"}]
```




```
Export["FigAll00CLD.eps", FigAll00CLD];
```

FOURIER TRANSFORMS OF THE CFs

```
Npnt = 100; Solve[Log[10, 10^(-5) f^20] == Log[10, 10^(-2)], f];
Solve[{Log[10, 10^(-2) f^30] == Log[10, 99 / 100]}, f];
Qstep = 8 / 10; Q0 = 1;
Qgrid = Table[If[i ≤ 21, 10^(-5) ((10^(3/20))^(i - 1)), If[i ≤ 51, ((3^(1/15) 11^(1/30))^(i - 21) / 100),
     Q0 + Qstep * (i - 51)]], {i, 1, Npnt}]; N[Qgrid]; N[Qgrid[[Npnt]]]
```

```
40.2
```

```
CubExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
CubApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;

Do[qact = Qgrid[[i]]; CubExctFormFact[[i, 2]] =
   (4 π / qact) (NIntegrate[CBCFAA[r] * r * Sin[qact * r], {r, 0, CBDst[[1]]},
       WorkingPrecision → 50, PrecisionGoal → 15] + NIntegrate[CBCFBB[r] * r * Sin[qact * r],
       {r, CBDst[[1]], CBDst[[2]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[CBCFCC[r] * r * Sin[qact * r], {r, CBDst[[2]], CBDst[[3]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];

Do[qact = Qgrid[[i]]; CubApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[CubeCFApprxFnl00AA[r] * r * Sin[qact * r],
        {r, 0, CBDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[CubeCFApprxFnl00BB[r] * r * Sin[qact * r],
        {r, CBDst[[1]], CBDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[CubeCFApprxFnl00CC[r] * r * Sin[qact * r], {r, CBDst[[2]], CBDst[[3]]},
        WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];

TetrExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
TetrApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
```

```
Do[qact = Qgrid[[i]]; TetrExctFormFact[[i, 2]] = (4 π / qact)
    (NIntegrate[TETRCFAA[r] * r * Sin[qact * r], {r, 0, TETRDst[[1]]}, WorkingPrecision → 50,
       PrecisionGoal → 15] + NIntegrate[TETRCFBB[r] * r * Sin[qact * r],
       {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFCC[r] * r * Sin[qact * r], {r, TETRDst[[2]], TETRDst[[3]]},
       WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[TETRCFDD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];

Do[qact = Qgrid[[i]]; TetrApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[TetrCFApprxFnl00AA[r] * r * Sin[qact * r],
        {r, 0, TETRDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00BB[r] * r * Sin[qact * r],
        {r, TETRDst[[1]], TETRDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[TetrCFApprxFnl00CC[r] * r * Sin[qact * r], {r, TETRDst[[2]],
         TETRDst[[3]]}, WorkingPrecision → 100, PrecisionGoal → 15] + NIntegrate[
        TetrCFApprxFnl00DD[r] * r * Sin[qact * r], {r, TETRDst[[3]], TETRDst[[4]]},
        WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];

OcthExctFormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;
OcthApprxtd00FormFact = Table[{N[Qgrid[[i]], 15], 0}, {i, 1, Npnt}] ;

Do[qact = Qgrid[[i]];
  OcthExctFormFact[[i, 2]] = (4 π / qact) (NIntegrate[OCTCFAA[r] * r * Sin[qact * r],
       {r, 0, OCTDst[[1]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[OCTCFBB[r] * r * Sin[qact * r], {r, OCTDst[[1]], OCTDst[[2]]},
       WorkingPrecision → 50, PrecisionGoal → 15] + NIntegrate[OCTCFCC[r] * r * Sin[qact * r],
       {r, OCTDst[[2]], OCTDst[[3]]}, WorkingPrecision → 50, PrecisionGoal → 15] +
      NIntegrate[OCTCFDD[r] * r * Sin[qact * r], {r, OCTDst[[3]], OCTDst[[4]]},
       WorkingPrecision → 50, PrecisionGoal → 15]), {i, 1, Npnt, 1}];

Do[qact = Qgrid[[i]]; OcthApprxtd00FormFact[[i, 2]] =
   (4 π / qact) N[(NIntegrate[OcthCFApprxFnl00AA[r] * r * Sin[qact * r],
        {r, 0, OCTDst[[1]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[OcthCFApprxFnl00BB[r] * r * Sin[qact * r],
        {r, OCTDst[[1]], OCTDst[[2]]}, WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[OcthCFApprxFnl00CC[r] * r * Sin[qact * r], {r, OCTDst[[2]], OCTDst[[3]]},
        WorkingPrecision → 100, PrecisionGoal → 15] +
       NIntegrate[OcthCFApprxFnl00DD[r] * r * Sin[qact * r], {r, OCTDst[[3]], OCTDst[[4]]},
        WorkingPrecision → 100, PrecisionGoal → 15])];, {i, 1, Npnt, 1}];
```

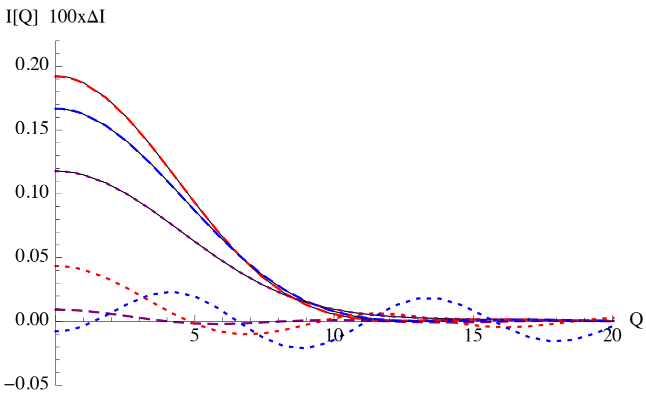
VECTORS CONTAINING THE FOURIER TRANSFORMS

```
DiffCubFormFact = Table[{CubExctFormFact[[i, 1]],
    100 * (CubExctFormFact[[i, 2]] - CubApprxtd00FormFact[[i, 2]])}, {i, 1, 100}];

DiffTetrFormFact = Table[{TetrExctFormFact[[i, 1]],
    100 * (TetrExctFormFact[[i, 2]] - TetrApprxtd00FormFact[[i, 2]])}, {i, 1, 100}];

DiffOcthFormFact = Table[{OcthExctFormFact[[i, 1]],
    100 * (OcthExctFormFact[[i, 2]] - OcthApprxtd00FormFact[[i, 2]])}, {i, 1, 100}];
```

```
FigFormFactAll = ListPlot[{CubExctFormFact, TetrExctFormFact,
    OcthExctFormFact, CubApprxtd00FormFact, DiffTetrFormFact, OcthApprxtd00FormFact,
    DiffCubFormFact, TetrApprxtd00FormFact, DiffOcthFormFact}, Joined → True,
  PlotRange → {{0, 20}, {-0.05, 0.22}}, PlotStyle → {Directive[Black, Thickness[0.002]],
     Directive[Black, Thickness[0.002]], Directive[Black, Thickness[0.002]],
     Directive[Red, Thickness[0.004], Dashed], Directive[Purple, Thickness[0.004], Dashed],
     Directive[Blue, Thickness[0.004], Dashed], Directive[Red, Thickness[0.004], Dotted],
     Directive[Purple, Thickness[0.004], Dotted],
     Directive[Blue, Thickness[0.004], Dotted]}, AxesLabel → {"Q", "I[Q]   100xΔI"}]
```




```
Export["FigFormFactAll.eps", FigFormFactAll];

CubExctFormFactPrdPlt = Table[{CubExctFormFact[[i, 1]],
     CubExctFormFact[[i, 1]]^4 CubExctFormFact[[i, 2]]}, {i, 1, 100}];
CubApprxtd00FormFactPrdPlt = Table[{CubApprxtd00FormFact[[i, 1]],
     CubApprxtd00FormFact[[i, 1]]^4 CubApprxtd00FormFact[[i, 2]]}, {i, 1, 100}];

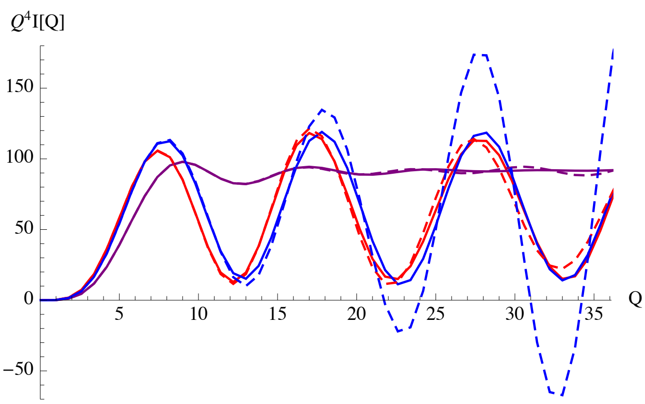
TetrExctFormFactPrdPlt = Table[{TetrExctFormFact[[i, 1]],
     TetrExctFormFact[[i, 1]]^4 TetrExctFormFact[[i, 2]]}, {i, 1, 100}];
TetrApprxtd00FormFactPrdPlt = Table[{TetrApprxtd00FormFact[[i, 1]],
     TetrApprxtd00FormFact[[i, 1]]^4 TetrApprxtd00FormFact[[i, 2]]}, {i, 1, 100}];

OcthExctFormFact[[100, 1]]

40.2000000000000

OcthExctFormFactPrdPlt = Table[{OcthExctFormFact[[i, 1]],
     OcthExctFormFact[[i, 1]]^4 OcthExctFormFact[[i, 2]]}, {i, 1, 100}];
OcthApprxtd00FormFactPrdPlt = Table[{OcthApprxtd00FormFact[[i, 1]],
     OcthApprxtd00FormFact[[i, 1]]^4 OcthApprxtd00FormFact[[i, 2]]}, {i, 1, 100}];

FigPorodPlotAll = ListPlot[{CubExctFormFactPrdPlt,
    TetrExctFormFactPrdPlt, OcthExctFormFactPrdPlt, CubApprxtd00FormFactPrdPlt,
    TetrApprxtd00FormFactPrdPlt, OcthApprxtd00FormFactPrdPlt}, Joined → True,
  PlotRange → {{0, 361 / 10}, {-70, 180}}, PlotStyle → {Directive[Red, Thickness[0.004]],
     Directive[Purple, Thickness[0.004]], Directive[Blue, Thickness[0.004]],
     Directive[Red, Thickness[0.004], Dashed], Directive[Purple, Thickness[0.004], Dashed],
     Directive[Blue, Thickness[0.004], Dashed]}, AxesLabel → {"Q", "Q^4 I[Q]  "}]
```



```
Export["FigPorodPlotAll.eps", FigPorodPlotAll];
```