Merano October/November, 2019

In[11]:=
```
SetDirectory[
  "/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH"];
Directory[]
```

Out[12]=
```
/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH
```

The following functions have been copied from the file:
"/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH/Integrand_Formulae.nb"

FILE CONTENT:
- the prmitives of CF3ATot[t] and CF3BTot[t] are evaluated,written in the most compact form (I have succeeded to do) as well as numerically checked comparing them to the numerival values of the integrals

Expressions of the final t-integrands once we set

$$\phi = \text{fff} + \text{ArcSin}\left[\frac{\text{AAA}+\text{BBB t}}{\sqrt{1-\text{t}^2}}\right]$$

```
CF1Aa[t_] := fff * (-a t^2 Cos[β] - b r t^3 Cos[β] Cot[β] - 1/2 b r t (1 - t^2) Sin[β]);

CF1Ab[t_] := (-a t^2 Cos[β] - b r t^3 Cos[β] Cot[β] - 1/2 b r t (1 - t^2) Sin[β]);

CF1ATot[t_] := CF1Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF1Ab[t];
```

```
CF1Ba[t_] := 1/2 t Sin[fff] (4 b r t (AAA + BBB t) Cos[β] +
     (2 a (AAA + BBB t) - b r (-1 + 2 AAA^2 + 4 AAA BBB t + t^2 + 2 BBB^2 t^2) Cos[fff]) Sin[β]);
CF1Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] * (1/2 t (b r (AAA + BBB t) Cos[fff]^2 Sin[β] -
       b r (AAA + BBB t) Sin[fff]^2 Sin[β] - 2 Cos[fff] (2 b r t Cos[β] + a Sin[β]))));
CF1BTot[t_] := CF1Ba[t] + CF1Bb[t]; CF1BTot[t];
```

```
CF3Aa[t_] := fff * ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3Ab[t_] := ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3ATot[t_] := CF3Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF3Ab[t]; CF3ATot[t];
CF3Ba[t_] := -1/2 t (-2 r t (AAA + BBB t) Cos[fff] Cos[β] + r (AAA + BBB t)^2 Cos[fff]^2 Sin[β] +
     Sin[fff] (-2 bp r t (AAA + BBB t) + (-2 ap (AAA + BBB t) + r (1 - t^2 - (AAA + BBB t)^2) Sin[fff])
         Sin[β])); CF3Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] *
   (-t (-r t Cos[β] Sin[fff] + Cos[fff] (bp r t + (ap + r (AAA + BBB t) Sin[fff]) Sin[β])));
CF3BTot[t_] := CF3Ba[t] + CF3Bb[t]; CF3BTot[t];

CF2Aa[t_] := ((CF1Aa[t]) /. {a → A, b → A}); CF2Ab[t_] := ((CF1Ab[t]) /. {a → A, b → A});
CF2ATot[t_] := CF2Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF2Ab[t];
Simplify[CF2ATot[t] - ((CF1ATot[t]) /. {a → A, b → A})]
CF2Ba[t_] := ((CF1Ba[t]) /. {a → A, b → B}); CF2Bb[t_] := ((CF1Bb[t]) /. {a → A, b → B});
CF2BTot[t_] := CF2Ba[t] + CF2Bb[t];
CF2BTot[t]; Simplify[(CF2BTot[t] - ((CF1BTot[t]) /. {a → A, b → B}))]

 CF4Aa[t_] := ((CF3Aa[t]) /. {ap → Ap, bp → Ap});
CF4Ab[t_] := ((CF3Ab[t]) /. {ap → Ap, bp → Ap});
CF4ATot[t_] := CF4Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF4Ab[t];
Simplify[CF4ATot[t] - ((CF3ATot[t]) /. {ap → Ap, bp → Ap})]
CF4Ba[t_] := ((CF3Ba[t]) /. {ap → Ap, bp → Bp});
CF4Bb[t_] := ((CF3Bb[t]) /. {ap → Ap, bp → Bp});
CF4BTot[t_] := CF4Ba[t] + CF4Bb[t];
CF4BTot[t]; Simplify[(CF4BTot[t] - ((CF3BTot[t]) /. {ap → Ap, bp → Bp}))]
```

0

0

0

0

```
CF3Aa[t_] := fff * ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3Ab[t_] := ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3ATot[t_] := CF3Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF3Ab[t]; CF3ATot[t];
CF3Ba[t_] := -1/2 t (-2 r t (AAA + BBB t) Cos[fff] Cos[β] + r (AAA + BBB t)^2 Cos[fff]^2 Sin[β] +
     Sin[fff] (-2 bp r t (AAA + BBB t) + (-2 ap (AAA + BBB t) + r (1 - t^2 - (AAA + BBB t)^2) Sin[fff])
         Sin[β])); CF3Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] *
   (-t (-r t Cos[β] Sin[fff] + Cos[fff] (bp r t + (ap + r (AAA + BBB t) Sin[fff]) Sin[β])));
CF3BTot[t_] := CF3Ba[t] + CF3Bb[t]; CF3BTot[t];
```

EVALUATION of the PRIMITIVE of CF3ATot[t]

```
CF3Aa[t_] := fff * ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3Ab[t_] := ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3ATot[t_] := CF3Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF3Ab[t]; CF3ATot[t];
```

PRIMITIVE OF CF3Aa[t]

```
Simplify[Integrate[CF3Aa[t], t]]
```

```
PrmitiveF3Aa[t_] := -1/12 fff t^3 Cot[β] (3 bp r t + 4 ap Sin[β]);
```

EVALUATION of the PRIMITIVE of $\mathrm{ArcSin}\left[\frac{AAA+BBB\,t}{\sqrt{1-t^2}}\right] * \mathrm{CF3Ab}[t]$

We proceed integrating by parts.
The first contribution is

```
Integrate[CF3Ab[t], t] ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]
```

```
PrmitiveF3AbIPP[t_] := ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] (-1/3 ap t^3 Cos[β] - 1/4 bp r t^4 Cot[β]);
```

```
Simplify[D[(PrmitiveF3AbIPP[t] / ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]), t] - CF3Ab[t]]
```

```
0
```

The remaining integrand is (including the minus sign)

```
rmnIntgrndF3Ab[t_] :=
  Simplify[Together[-Integrate[CF3Ab[t], t] D[ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]], t]],
   Assumptions → {-1 < t < 1}]; rmnIntgrndF3Ab[t]
```

```
NumRmnIntgrndF3Ab[t_] := (t^3 (BBB + AAA t) Cot[β] (3 bp r t + 4 ap Sin[β]))/12;
DenRmnIntgrndF3Ab[t_] := (1 - t^2);
```

```
Simplify[NumRmnIntgrndF3Ab[t] /
   (DenRmnIntgrndF3Ab[t] * Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]) - rmnIntgrndF3Ab[t]]
```

0

$$\text{rmnIntgrndF3Ab[t]} = \frac{\text{NumRmnIntgrndF3Ab}[t]}{\text{DenRmnIntgrndF3Ab}[t]\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}$$

```
rmnIntgrndF3Ab[t]
```

```
CfNum3A = Simplify[CoefficientList[NumRmnIntgrndF3Ab[t], t]] ;
CfNum3A[[6]];
```

```
Simplify[Sum[CfNum3A[[j]] * t^(j - 1), {j, 3, 6}] - NumRmnIntgrndF3Ab[t]]
```

0

I use Caccioppoli's substitution defined
by the formulae in the below magenta frame
[these identities have been derived in "Primtive_CF1.nb"]

$$\{(1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2) \to \Delta 1\}$$

$$\text{Reduce}\left[\{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2 > 0\ \&\&\ -1 < t < 1\},\ \{BBB, AAA, t\},\ \text{Reals}\right]$$

$$\left\{\left\{mua \to \frac{-AAA\,BBB - \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mua\ *)\right\},\ \left\{mub \to \frac{-AAA\,BBB + \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mub\ *)\right\}\right\}$$

$$\text{Simplify}\left[\text{Solve}\left[\{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2 == 0\},\ t\right]\right]$$

TABLE OF THE IDENTITIES

$$\left\{\left\{mua \to \frac{-AAA\,BBB - \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mua\ *)\right\},\ \left\{mub \to \frac{-AAA\,BBB + \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mub\ *)\right\}\right\}$$

$$\left\{\xi \to \sqrt{\frac{mub - t}{t - mua}},\quad t \to \frac{mub + mua\,\xi^2}{1 + \xi^2}\right\}$$

$$\left\{(mua - mub) \to -\frac{2\sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\right\}$$

$$\left\{mua\hat{}2 \to \left(\frac{1 - AAA^2}{1 + BBB^2} - \frac{2\,AAA\,BBB\ mua}{1 + BBB^2}\right)\right\}\quad \left\{mub\hat{}2 \to \left(\frac{1 - AAA^2}{1 + BBB^2} - \frac{2\,AAA\,BBB\ mub}{1 + BBB^2}\right)\right\}$$

$$\left\{\left(\sqrt{1 + BBB^2}\ \sqrt{(t - mua)\ (mub - t)}\right) \to \sqrt{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2}\right\}$$

$$\left\{\left(\sqrt{1 + BBB^2}\ \sqrt{(t - mua)\ (mub - t)}\right) \to \frac{2\sqrt{1 - AAA^2 + BBB^2}\ \xi}{\sqrt{1 + BBB^2}\ (1 + \xi^2)}\right\}$$

$$\left\{\xi^2 \to \frac{\text{mub}-\text{t}}{\text{t}-\text{mua}} \to \frac{\left(1+\text{BBB}^2\right)\left(1-\text{AAA}^2-2\,\text{AAA BBB t}-\left(1+\text{BBB}^2\right)\text{t}^2\right)}{\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\right)^2}\right\}$$

$$\left\{\sqrt{\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}} \to \frac{\sqrt{1+\text{BBB}^2}\,\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB t}-\left(1+\text{BBB}^2\right)\text{t}^2}}{\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\right)}\right\}$$

$$\left\{\left(1+\xi^2\right) \to \frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{\left(1+\text{BBB}^2\right)(-\text{mua}+\text{t})} \to \frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}\right\}$$

$$\left\{1/\left(1+\xi^2\right) \to \frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}\right\}$$

$$\left\{(\text{t}-\text{mua}) \to \frac{-\text{mua}+\text{mub}}{1+\xi^2} \to \frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}{1+\text{BBB}^2}\right\}$$

$$\left\{(\text{mub}-\text{t}) \to \frac{(-\text{mua}+\text{mub})\,\xi^2}{1+\xi^2} \to \frac{\sqrt{1-\text{AAA}^2+\text{BBB}^2}-\text{AAA BBB}-\left(1+\text{BBB}^2\right)\text{t}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mua}) \to \frac{1+\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mub}) \to \frac{1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mua})\,(1-\text{mub}) \to \frac{(\text{AAA}+\text{BBB})^2}{1+\text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1-\text{mua})\,(1-\text{mub})} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]}{\sqrt{1+\text{BBB}^2}}\right\}$$

$$\left\{\left(\frac{1-\text{mua}}{1-\text{mub}}\right) \to \frac{(\text{AAA}+\text{BBB})^2\left(1+\text{BBB}^2\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\hat{}\,2}\right\}$$

$$\left\{\sqrt{\frac{1-\text{mua}}{1-\text{mub}}} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]\left(\sqrt{1+\text{BBB}^2}\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

$$\left\{(1+\text{mua}) \to \frac{1-\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1+\text{mub}) \to \frac{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1+\text{mua})\ (1+\text{mub}) \to \frac{(\text{AAA}-\text{BBB})^2}{1+\text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1+\text{mua})\ (1+\text{mub})} \to \frac{\text{Abs}[\text{AAA}-\text{BBB}]}{\sqrt{1+\text{BBB}^2}}\right\}$$

$$\left\{\frac{1+\text{mua}}{1+\text{mub}} \to \frac{(\text{AAA}-\text{BBB})^2\ \left(1+\text{BBB}^2\right)}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)^2}\right\}$$

$$\left\{\sqrt{\frac{1+\text{mua}}{1+\text{mub}}} \to \frac{\text{Abs}[\text{AAA}-\text{BBB}]\ \sqrt{1+\text{BBB}^2}}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

$$\left\{\text{Jacob1A} \to -\frac{4\sqrt{1-\text{AAA}^2+\text{BBB}^2}\ \xi}{\left(1+\text{BBB}^2\right)\ \left(1+\xi^2\right)^2}\right\}$$

Since

$$\left(\left(\text{Jacob1A}\Big/\sqrt{1-\text{AAA}^2-2\ \text{AAA BBB t}-\text{t}^2-\text{BBB}^2\ \text{t}^2}\right)\ /.\ \left\{\frac{1}{\sqrt{1-\text{AAA}^2-2\ \text{AAA BBB t}-\text{t}^2-\text{BBB}^2\ \text{t}^2}} \to \frac{\sqrt{1+\text{BBB}^2}\ \left(1+\xi^2\right)}{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}\ \xi},\ \text{Jacob1A} \to -\frac{4\sqrt{1-\text{AAA}^2+\text{BBB}^2}\ \xi}{\left(1+\text{BBB}^2\right)\ \left(1+\xi^2\right)^2}\right\}\right)$$

$$-\frac{2}{\sqrt{1+\text{BBB}^2}\ \left(1+\xi^2\right)}$$

the full integrand, expressed in terms of ξ and t, becomes

$$(\text{NumRmnIntgrndF3Ab}[\text{t}]\ /\ \text{DenRmnIntgrndF3Ab}[\text{t}])\ *\ \left(-\frac{2}{\sqrt{1+\text{BBB}^2}\ \left(1+\xi^2\right)}\right)$$

$$-\frac{\text{t}^3\ (\text{BBB}+\text{AAA t})\ \text{Cot}[\beta]\ (3\ \text{bp r t}+4\ \text{ap Sin}[\beta])}{6\sqrt{1+\text{BBB}^2}\ \left(1-\text{t}^2\right)\ \left(1+\xi^2\right)}$$

$$\text{newintegrandF3AbCsiT}[\xi\_,\ \text{t}\_] := -\frac{\text{t}^3\ (\text{BBB}+\text{AAA t})\ \text{Cot}[\beta]\ (3\ \text{bp r t}+4\ \text{ap Sin}[\beta])}{6\sqrt{1+\text{BBB}^2}\ \left(1-\text{t}^2\right)\ \left(1+\xi^2\right)};$$

```
NumRmnIntgrndF3Ab[t]
DenRmnIntgrndF3Ab[t]
```

$\frac{1}{12} t^3 (BBB + AAA\, t) \, Cot[\beta] \, (3\, bp\, r\, t + 4\, ap\, Sin[\beta])$

$1 - t^2$

```
Simplify[(- 2 NumRmnIntgrndF3Ab[t] / (Sqrt[1 + BBB^2] (1 + ξ^2) DenRmnIntgrndF3Ab[t]))
  - newintegrandF3AbCsiT[ξ, t]]
```

0

We write
NumRmnIntgrndF3Ab[t]/DenRmnIntgrndF3Ab[t]
as

$$\frac{\text{NumRmnIntgrndF3Ab}[t]}{\text{DenRmnIntgrndF3Ab}[t]} = P[t] + \frac{Q1}{1-t} + \frac{Q2}{1+t}$$

with P[t_] = p0 + p1 t + p2 t^2

we have that

$\int$rmnIntgrndF3Ab[t] dt =

$$\int \frac{\text{NumRmnIntgrndF3Ab}[t]}{\text{DenRmnIntgrndF3Ab}[t] * \sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}}\,dt = \int \frac{P[t]+\frac{Q1}{1-t}+\frac{Q2}{1+t}}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}}\,dt =$$

$$\int \left(P[t] + \frac{Q1}{1-t} + \frac{Q2}{1+t}\right)\left(-\frac{2}{\sqrt{1+BBB^2}\,(1+\xi^2)}\right)d\xi \quad \text{where} \quad t = t[\xi]\,.$$

The last integrals is written as

$$\frac{-2}{\sqrt{1+BBB^2}}\left(\int (P[t[\xi]])\left(\frac{1}{(1+\xi^2)}\right)d\xi + \int\left(\frac{Q1}{1-t[\xi]}\right)\left(\frac{1}{(1+\xi^2)}\right)d\xi + \int\left(\frac{Q2}{1+t[\xi]}\right)\left(\frac{1}{(1+\xi^2)}\right)d\xi\right) =$$

$$\frac{-2}{\sqrt{1+BBB^2}}(\text{ contr11}[\xi[t]] + \text{contr22}[\xi[t]] + \text{contr33}[\xi[t]] =$$

$$\frac{-2}{\sqrt{1+BBB^2}}(\text{ CONTR11NOTSIMPL}[t] + \text{CONTR22NOTSIMPL}[t] + \text{ CONTR22NOTSIMPL}[t]]\ )$$

The t-derivative of the last sum yields

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR11NOTSIMPL[t],t]} = \frac{-2}{\sqrt{1+BBB^2}}\,(P[t])\left(\frac{1}{(1+\xi[t]^2)}\right) D[\xi[t], t] = \frac{P[t]}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR22NOTSIMPL[t],t]} = \frac{-2}{\sqrt{1+BBB^2}}\left(\frac{Q1}{1-t}\right)\left(\frac{1}{(1+\xi[t]^2)}\right) D[\xi[t], t] = \frac{\frac{Q1}{1-t}}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR33NOTSIMPL[t],t]} = \frac{-2}{\sqrt{1+BBB^2}}\left(\frac{Q2}{1=t}\right)\left(\frac{1}{(1+\xi[t]^2)}\right) D[\xi[t], t] = \frac{\frac{Q2}{1+t}}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}}.$$

We finally have

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR11NOTSIMPL[t],t]} = \frac{P[t]}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR22NOTSIMPL[t],t]} = \frac{\frac{Q1}{1-t}}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}}$$

$$\frac{-2}{\sqrt{1+BBB^2}}\text{D[CONTR33NOTSIMPL[t],t]} = \frac{\frac{Q2}{1+t}}{\sqrt{1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2}}$$

```
P[t_] = p0 + p1 t + p2 t^2 + p3 t^3;
cf3A = Simplify[
   CoefficientList[P[t] (1 - t^2) + Q1 (1 + t) + Q2 (1 - t) - NumRmnIntgrndF3Ab[t], t]] ;
cf3A[[
 6]]
```

$- p3 - \frac{1}{4} AAA\ bp\ r\ Cot[\beta]$

```
Simplify[TrigExpand[
  Simplify[Solve[{cf3A[[1]] == 0 && cf3A[[2]] == 0 && cf3A[[3]] == 0 && cf3A[[4]] == 0 &&
      cf3A[[5]] == 0 && cf3A[[6]] == 0}, {p0, p1, p2, p3, Q1, Q2}]]]]
```

$\{p0 \to -\frac{1}{12} Cot[\beta]\ (3\ BBB\ bp\ r + 4\ AAA\ ap\ Sin[\beta]),$
$p1 \to -\frac{1}{12} Cot[\beta]\ (3\ AAA\ bp\ r + 4\ ap\ BBB\ Sin[\beta]),\ p2 \to -\frac{1}{12} Cot[\beta]\ (3\ BBB\ bp\ r + 4\ AAA\ ap\ Sin[\beta]),$
$p3 \to -\frac{1}{4} AAA\ bp\ r\ Cot[\beta],\ Q1 \to \frac{1}{24} (AAA + BBB)\ Cot[\beta]\ (3\ bp\ r + 4\ ap\ Sin[\beta]),$
$Q2 \to \frac{1}{24} (AAA - BBB)\ Cot[\beta]\ (-3\ bp\ r + 4\ ap\ Sin[\beta])\}$

checks

```
Simplify[NumRmnIntgrndF3Ab[t] / DenRmnIntgrndF3Ab[t] -
  ((P[t] + Q1 / (1 - t) + Q2 / (1 + t)) /. {p0 -> -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β]), p1 ->
      -1/12 Cot[β] (3 AAA bp r + 4 ap BBB Sin[β]), p2 -> -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β]),
     p3 -> -1/4 AAA bp r Cot[β], Q1 -> 1/24 (AAA + BBB) Cot[β] (3 bp r + 4 ap Sin[β]),
     Q2 -> 1/24 (AAA - BBB) Cot[β] (-3 bp r + 4 ap Sin[β])})]
```

0

The integrand takes the form

$$-\frac{2}{\sqrt{1+BBB^2}} * \left(\frac{1}{(1+\xi^2)} P[t] + \frac{1}{(1+\xi^2)} \frac{Q1}{1-t} + \frac{1}{(1+\xi^2)} \frac{Q2}{1+t}\right)$$

```
Simplify[(((-2/Sqrt[1 + BBB^2]) * (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /.
    {p0 → -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β]), p1 →
      -1/12 Cot[β] (3 AAA bp r + 4 ap BBB Sin[β]), p2 → -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β]),
      p3 → -1/4 AAA bp r Cot[β], Q1 → 1/24 (AAA + BBB) Cot[β] (3 bp r + 4 ap Sin[β]),
      Q2 → 1/24 (AAA - BBB) Cot[β] (-3 bp r + 4 ap Sin[β])}) -
   (-NumRmnIntgrndF3Ab[t] / DenRmnIntgrndF3Ab[t]) * 2/(Sqrt[1 + BBB^2] (1 + ξ^2))]
```

0

We evaluate the integrals of the three contributions putting aside the factor $-\frac{2}{\sqrt{1+\text{BBB}^2}}$

1st integral

```
Integrate[((1/(1 + ξ^2) P[t]) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}), ξ]

contr11NotS[ξ_] := 1/48 (-(8 (mua - mub)^3 p3 ξ)/(1 + ξ^2)^3 + (2 (mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(1 + ξ^2)^2 -
    1/(1 + ξ^2) 3 (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ +
    3 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) + mua (8 p1 + 4 mub p2 + 3 mub^2 p3) +
       mub (8 p1 + 6 mub p2 + 5 mub^2 p3)) ArcTan[ξ]);
```

```
Simplify[D[contr11NotS[ξ], ξ] - ((((1/(1 + ξ^2)) P[t]) /. {t → (mub + mua ξ^2)/(1 + ξ^2)})),
 Assumptions → {ξ > 0 && -1 < mua < mub < 1} ]
```

0

the integral is separated into a contribution independent of the ArcTan plus a contribution proportional to ArcTan

```
CoefficientList[contr11NotS[ξ], ArcTan[ξ]]
```

$$\left\{-\frac{(\text{mua}-\text{mub})^3\,\text{p3}\,\xi}{6\left(1+\xi^2\right)^3}+\frac{(\text{mua}-\text{mub})^2\,(6\,\text{p2}+13\,\text{mua}\,\text{p3}+5\,\text{mub}\,\text{p3})\,\xi}{24\left(1+\xi^2\right)^2}-\frac{1}{16\left(1+\xi^2\right)}(\text{mua}-\text{mub})\left(8\,\text{p1}+10\,\text{mua}\,\text{p2}+6\,\text{mub}\,\text{p2}+11\,\text{mua}^2\,\text{p3}+8\,\text{mua}\,\text{mub}\,\text{p3}+5\,\text{mub}^2\,\text{p3}\right)\xi,\right.$$
$$\left.\frac{1}{16}\left(16\,\text{p0}+5\,\text{mua}^3\,\text{p3}+3\,\text{mua}^2\,(2\,\text{p2}+\text{mub}\,\text{p3})+\text{mua}\left(8\,\text{p1}+4\,\text{mub}\,\text{p2}+3\,\text{mub}^2\,\text{p3}\right)+\text{mub}\left(8\,\text{p1}+6\,\text{mub}\,\text{p2}+5\,\text{mub}^2\,\text{p3}\right)\right)\right\}$$

```
contr11NotSAA[ξ_] := -((mua - mub)^3 p3 ξ)/(6 (1 + ξ^2)^3) + ((mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(24 (1 + ξ^2)^2) -
   1/(16 (1 + ξ^2)) (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ;
contr11NotSBB := 1/16 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) +
    mua (8 p1 + 4 mub p2 + 3 mub^2 p3) + mub (8 p1 + 6 mub p2 + 5 mub^2 p3));
```

contr11NotSBB does NOT depend on ξ

```
contr11[ξ_] := (contr11NotSAA[ξ]) + (contr11NotSBB * ArcTan[ξ] ); contr11[ξ]
```

```
contr11[ξ_] := -((mua - mub)^3 p3 ξ)/(6 (1 + ξ^2)^3) + ((mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(24 (1 + ξ^2)^2) -
   1/(16 (1 + ξ^2)) (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ +
   1/16 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) +
      mua (8 p1 + 4 mub p2 + 3 mub^2 p3) + mub (8 p1 + 6 mub p2 + 5 mub^2 p3)) ArcTan[ξ];
```

Derivative's check

```
Simplify[contr11NotS[ξ] - contr11[ξ]]
Simplify[D[contr11NotS[ξ], ξ] - (((1/(1 + ξ^2)) P[t]) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}),
 Assumptions → {ξ > 0 && 1 > mub && 1 > mua} ]
```

0

0

2nd integral

```
Simplify[Together[(((1/((1 + ξ^2))) (Q1/(1 - t))) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)})] - Q1/(1 - mub + (1 - mua) ξ^2)]
```

0

```
Integrate[Q1/(1 - mub + (1 - mua) ξ^2), ξ, Assumptions -> {ξ > 0 && 1 > mub && 1 > mua}]

Simplify[Integrate[(((1/((1 + ξ^2))) (Q1/(1 - t))) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}), ξ,
  Assumptions -> {ξ > 0 && 1 > mub && 1 > mua} ], Assumptions -> {ξ > 0 && 1 > mub && 1 > mua} ]

FullSimplify[- (Q1 ArcTan[(Sqrt[-1+mua] ξ)/Sqrt[-1+mub]])/(Sqrt[-1 + mua] Sqrt[-1 + mub]) - (Q1 ArcTan[Sqrt[(1-mua)/(1-mub)] ξ])/Sqrt[(1 - mua) (1 - mub)],
 Assumptions -> {ξ > 0 && 1 > mub && 1 > mua}]
```

0

```
contr22NotS[ξ_] := (Q1 ArcTan[Sqrt[(1-mua)/(1-mub)] ξ])/Sqrt[(1 - mua) (1 - mub)];
```

Derivative' s check

```
Simplify[D[contr22NotS[ξ], ξ] - (((1/((1 + ξ^2))) (Q1/(1 - t))) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}),
 Assumptions -> {ξ > 0 && 1 > mub && 1 > mua} ]
FullSimplify[D[contr22NotS[ξ], ξ] - (((1/((1 + ξ^2))) (Q1/(1 - t))) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}),
 Assumptions -> {ξ > 0 && 1 > mub && 1 > mua} ]
```

0

3rd integral

```
Integrate[(((1/((1 + ξ^2))) (Q2/(1 + t))) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}),
 ξ, Assumptions -> {ξ > 0 && 1 > mub > -1 && 1 > mua > -1}]
```

```
contr33NotS[ξ_] := (Q2 ArcTan[(Sqrt[1+mua] ξ)/Sqrt[1+mub]])/Sqrt[(1 + mua) (1 + mub)];
```

FULL DERIVATIVE's check [OK]

```
Simplify[
 D[contr11NotSAA[ξ] + contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ], ξ] -
  ((((1/(1 + ξ^2)) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t)) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)})),
 Assumptions -> {-1 < mua < mub < 1}]
FullSimplify[
 D[contr11NotSAA[ξ] + contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ], ξ] -
  ((((1/(1 + ξ^2)) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t)) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)})),
 Assumptions -> {-1 < mua < mub < 1}]
```

$$\frac{\left(-\sqrt{\frac{-1+mua^2}{-1+mub}}+\sqrt{(-1+mua^2)(-1+mub)}+\sqrt{\frac{-1+mua^2}{-1+mub}}\,mub\right)Q1}{\sqrt{(-1+mua^2)(-1+mub)}\,\left(-1+mub+(-1+mua)\,\xi^2\right)}$$

0

The full ξ-primitive of rmnIntgrndF3Ab[t]

```
CsiprmtvF3Ab[ξ_] := (-2/Sqrt[1 + BBB^2]) * (contr11NotSAA[ξ] +
    contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ]); CsiprmtvF3Ab[ξ]
```

CsiprmtvF3Ab[ξ_] :=

$$-\frac{1}{\sqrt{1+BBB^2}}\,2\left(-\frac{(mua-mub)^3\,p3\,\xi}{6\,(1+\xi^2)^3}+\frac{(mua-mub)^2\,(6\,p2+13\,mua\,p3+5\,mub\,p3)\,\xi}{24\,(1+\xi^2)^2}-\right.$$
$$\frac{1}{16\,(1+\xi^2)}\,(mua-mub)\,\left(8\,p1+10\,mua\,p2+6\,mub\,p2+11\,mua^2\,p3+8\,mua\,mub\,p3+5\,mub^2\,p3\right)\xi+$$
$$\frac{1}{16}\left(16\,p0+5\,mua^3\,p3+3\,mua^2\,(2\,p2+mub\,p3)+\right.$$
$$mua\,\left(8\,p1+4\,mub\,p2+3\,mub^2\,p3\right)+mub\,\left(8\,p1+6\,mub\,p2+5\,mub^2\,p3\right)\Big)$$
$$\left.ArcTan[\xi]+\frac{Q1\,ArcTan\left[\sqrt{\frac{1-mua}{1-mub}}\,\xi\right]}{\sqrt{(1-mua)\,(1-mub)}}+\frac{Q2\,ArcTan\left[\frac{\sqrt{1+mua}\,\xi}{\sqrt{1+mub}}\right]}{\sqrt{(1+mua)\,(1+mub)}}\right);$$

DERIVATIVE CHECK [it is OK, even though MATHEMATICA must be helped to find out the result!!]

```
FullSimplify[D[CsiprmtvF3Ab[ξ], ξ] -
  ((-2/Sqrt[1 + BBB^2]) ((((1/(1 + ξ^2)) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t)) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}))),
 Assumptions -> {-1 < mua < 1 && -1 < mub < 1 && mua < mub && ξ > 0 && 1 + BBB^2 > 0}]
```

The result is multiplied by

$$\frac{1}{2\,Q1}\left(\sqrt{\left(1+BBB^2\right)\left(-1+mua^2\right)(-1+mub)}\;\left(-1+mub+(-1+mua)\,\xi^2\right)\right)$$

$$\left(-\left(2\left(-\sqrt{\frac{-1+mua^2}{-1+mub}}+\sqrt{\left(-1+mua^2\right)(-1+mub)}+\sqrt{\frac{-1+mua^2}{-1+mub}}\;mub\right)Q1\right)\Bigg/\right.$$

$$\left.\left(\sqrt{\left(1+BBB^2\right)\left(-1+mua^2\right)(-1+mub)}\;\left(-1+mub+(-1+mua)\,\xi^2\right)\right)\right)*$$

$$\left(\frac{1}{2\,Q1}\left(\sqrt{\left(1+BBB^2\right)\left(-1+mua^2\right)(-1+mub)}\;\left(-1+mub+(-1+mua)\,\xi^2\right)\right)\right)$$

$$\sqrt{\frac{-1+mua^2}{-1+mub}}-\sqrt{\left(-1+mua^2\right)(-1+mub)}-\sqrt{\frac{-1+mua^2}{-1+mub}}\;mub$$

The above expression is equal to zero, even though MATHEMATICA seems unable to realize this property. In fact,

$$\text{Simplify}\Big[\left(\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)(-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\ \text{mub}\right)-$$
$$\left(\sqrt{\frac{1-\text{mua}^2}{1-\text{mub}}}-\sqrt{\left(1-\text{mua}^2\right)(1-\text{mub})}-\sqrt{\frac{1-\text{mua}^2}{1-\text{mub}}}\ \text{mub}\right),$$
$$\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]$$

$$\text{Simplify}\Big[\left(\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)(-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\ \text{mub}\right)-$$
$$\sqrt{1+\text{mua}}\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{(1-\text{mua})(1-\text{mub})}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\ \text{mub}\right),$$
$$\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]$$

$$\text{Simplify}\Big[$$
$$\sqrt{1+\text{mua}}*\left(\text{Simplify}\Big[\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\ \text{mub}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]-\right.$$
$$\left.\sqrt{(1-\text{mua})(1-\text{mub})}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]$$

$$\text{Simplify}\Big[\left(\text{Simplify}\Big[\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\ \text{mub}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]-\right.$$
$$\left.\sqrt{(1-\text{mua})(1-\text{mub})}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\Big]$$

0

$$\sqrt{1+\text{mua}}\left(-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub})-\sqrt{(-1+\text{mua})\ (-1+\text{mub})}\right)$$

$$-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub})-\sqrt{(-1+\text{mua})\ (-1+\text{mub})}$$

In the above expression the first addend is positive because 1>mu and 1>ma and can be written as

$$-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub})==\sqrt{(-1+\text{mua})\ (-1+\text{mub})}$$

Squaring one gets

$$\left(-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub})\right)\hat{}\,2 == \left(\sqrt{(-1+\text{mua})\ (-1+\text{mub})}\right)\hat{}\,2$$

True

It is surprising the MATHEMATICA does not well handle radicals

$$\left\{\left(-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}+\sqrt{(-1+\text{mua})\ (-1+\text{mub})}+\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ \text{mub}\right)\to 0\right\}$$

$$\left\{\frac{\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)\ (-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\ \text{mub}}{\sqrt{1-\text{mua}}}\to 0\right\}$$

If we set $\xi=\xi[t]$ inside CsiprmtvF3Ab[ξ], we get CsiprmtvF3Ab[ξ[t]]. The t-derivative yields D[CsiprmtvF3Ab[ξ[t]],t]=D[CsiprmtvF3Ab[ξ],ξ]$|_{\xi=\xi[t]}$ D[ξ[t],t]. In this way we find that :

$$\text{D[CsiprmtvF3Ab}[\xi],\xi]|_{\xi=\xi[t]} = \frac{D[\text{CsiprmtvF3Ab}[\xi[t]],t]}{D[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\text{P}[t]+\frac{1}{(1+\xi^2)}\frac{\text{Q1}}{1-t}+\frac{1}{(1+\xi^2)}\frac{\text{Q2}}{1+t}\right)\right)\ \Big|\ _{\xi=\xi[t]}$$

The first equality

$$\text{D[CsiprmtvF3Ab}[\xi],\xi]|_{\xi=\xi[t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\text{P}[t]+\frac{1}{(1+\xi^2)}\frac{\text{Q1}}{1-t}+\frac{1}{(1+\xi^2)}\frac{\text{Q2}}{1+t}\right)\right)\ \Big|\ _{\xi=\xi[t]}$$

is verified.

```
FullSimplify[(D[CsiprmtvF3Ab[ξ], ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
 Assumptions → {-1 < mua < t < mub < 1}]
```

```
FullSimplify[(-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
 Assumptions → {-1 < mua < t < mub < 1}]
```

```
Simplify[FullSimplify[(D[CsiprmtvF3Ab[ξ], ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}] - FullSimplify[
   (-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}]]
```

0

The second equality

$$\frac{D[\text{CsiprmtvF3Ab}[\xi[t]],t]}{D[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\text{P}[t] + \frac{1}{(1+\xi^2)}\frac{\text{Q1}}{1-t} + \frac{1}{(1+\xi^2)}\frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

is somewhat more involved to be verified.
We write the equality under the form

$$D[\text{CsiprmtvF3Ab}[\xi[t]], t] = \left(\left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\text{P}[t] + \frac{1}{(1+\xi^2)}\frac{\text{Q1}}{1-t} + \frac{1}{(1+\xi^2)}\frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}\right)*$$

D[ξ[t], t]

```
FullSimplify[
 D[Simplify[(CsiprmtvF3Ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}], t],
 Assumptions → {-1 < mua < t < mub < 1}]

FullSimplify[
 (FullSimplify[D[Sqrt[(mub - t)/(t - mua)], t], Assumptions → {-1 < ma < t < mub < 1}]) * FullSimplify[
   (-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}], Assumptions → {-1 < mua < t < mub < 1}]
```

```
FullSimplify[FullSimplify[
  D[Simplify[(CsiprmtvF3Ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}],
   t], Assumptions → {-1 < mua < t < mub < 1}] -
  FullSimplify[(FullSimplify[D[Sqrt[(mub - t)/(t - mua)], t], Assumptions → {-1 < ma < t < mub < 1}]) *
   FullSimplify[(-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /.
     {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}],
   Assumptions → {-1 < mua < t < mub < 1}], Assumptions → {-1 < mua < t < mub < 1}]
```

The above expression is equal to zero because the factor

$$\left(\left(\frac{-mub+t}{mua-t}\right)^{3/2}-\sqrt{\frac{(-mub+t)^3}{(mua-t)^3}}\right)$$

is equal to sero since $\sqrt{\frac{(-mub+t)^3}{(mua-t)^3}}$ can be written as $\left(\frac{-mub+t}{mua-t}\right)^{3/2}$

IN CONCLUSION WE HAVE HAVE VERIFIED THAT

$$D[CsiprmtvF3Ab[\xi],\xi]|_{\xi=\xi[t]} = \frac{D[CsiprmtvF3Ab[\xi[t]],t]}{D[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+BBB^2}}\left(\frac{1}{(1+\xi^2)}P[t]+\frac{1}{(1+\xi^2)}\frac{Q1}{1-t}+\frac{1}{(1+\xi^2)}\frac{Q2}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

HOLDS TRUE.

We go back to variable t using the transformations $\xi \to \sqrt{\frac{mub-t}{t-mua}}$

```
tprmtvF3AbNotSimpl[t_] :=
  Simplify[(CsiprmtvF3Ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}];
ReducedtprmtvF3AbNotSimpl[t_] :=
 (((tprmtvF3AbNotSimpl[t]) /. {ArcTan[Sqrt[(-mub + t)/(mua - t)]] → atanaa}) /.
   {ArcTan[Sqrt[((-1 + mua) (-mub + t))/((-1 + mub) (mua - t))]] → atanbb}) /.
  {ArcTan[Sqrt[((1 + mua) (-mub + t))/((1 + mub) (mua - t))]] → atancc};
```

```
cfausxx = CoefficientList[ReducedtprmtvF3AbNotSimpl[t], atancc];
cfausxx[[2]]
```

$$-\frac{2\,Q2}{\sqrt{1+BBB^2}\;\sqrt{(1+mua)\;(1+mub)}}$$

```
cfausyy = CoefficientList[cfausxx[[1]], atanbb];
cfausyy[[2]]
```

$$-\frac{2\,Q1}{\sqrt{1+BBB^2}\;\sqrt{(-1+mua)\;(-1+mub)}}$$

```
cfauszz = Simplify[CoefficientList[cfausyy[[1]], atanaa]];     cfauszz[[2]]
cfauszz[[1]]
```

$$\text{f00[t\_] := }\frac{1}{24\sqrt{1+BBB^2}}\,(mua-t)\,\sqrt{\frac{-mub+t}{mua-t}}\,\big(24\,p1+18\,mub\,p2+15\,mua^2\,p3+15\,mub^2\,p3+12\,p2\,t+10\,mub\,p3\,t+8\,p3\,t^2+2\,mua\,(9\,p2+7\,mub\,p3+5\,p3\,t)\big);$$

$$\text{faa[t\_] := FullSimplify}\Big[-\frac{1}{8\sqrt{1+BBB^2}}\,\big(16\,p0+5\,mua^3\,p3+3\,mua^2\,(2\,p2+mub\,p3)+mua\,\big(8\,p1+4\,mub\,p2+3\,mub^2\,p3\big)+mub\,\big(8\,p1+6\,mub\,p2+5\,mub^2\,p3\big)\big)\Big]*\text{ArcTan}\Big[\sqrt{\frac{-mub+t}{mua-t}}\Big];$$

$$\text{fbb[t\_] := }-\frac{2\,Q1}{\sqrt{1+BBB^2}\;\sqrt{(-1+mua)\;(-1+mub)}}*\text{ArcTan}\Big[\sqrt{\frac{(-1+mua)\;(-mub+t)}{(-1+mub)\;(mua-t)}}\Big];$$

$$\text{fcc[t\_] := Simplify}\Big[\Big(-\frac{2\,Q2}{\sqrt{1+BBB^2}\;\sqrt{(1+mua)\;(1+mub)}}\Big)\Big]*\text{ArcTan}\Big[\sqrt{\frac{(1+mua)\;(-mub+t)}{(1+mub)\;(mua-t)}}\Big];$$

```
Simplify[tprmtvF3AbNotSimpl[t] - (f00[t] + faa[t] + fbb[t] + fcc[t])]
```

0

```
f00[t]
faa[t]
fbb[t]
fcc[t]
```

PrmitiveF3AbNotSimpl00[t_] :=

$$\frac{-1}{24\sqrt{1+BBB^2}}\sqrt{(mub-t)\ (t-mua)}\ \left(24\ p1+18\ mua\ p2+18\ mub\ p2+15\ mua^2\ p3+14\ mua\ mub\ p3+15\ mub^2\ p3+2\ (6\ p2+5\ (mua+mub)\ p3)\ t+8\ p3\ t^2\right);$$

PrmitiveF3AbNotSimplaa[t_] :=

$$-\frac{1}{8\sqrt{1+BBB^2}}\left(16\ p0+5\ mua^3\ p3+3\ mua^2\ (2\ p2+mub\ p3)+mua\ \left(8\ p1+4\ mub\ p2+3\ mub^2\ p3\right)+mub\ \left(8\ p1+6\ mub\ p2+5\ mub^2\ p3\right)\right)\ ArcTan\left[\sqrt{\frac{-mub+t}{mua-t}}\right];$$

PrmitiveF3AbNotSimplbb[t_] :=

$$-\frac{2\ Q1\ ArcTan\left[\sqrt{\frac{(-1+mua)\ (-mub+t)}{(-1+mub)\ (mua-t)}}\right]}{\sqrt{1+BBB^2}\ \sqrt{(-1+mua)\ (-1+mub)}};$$

PrmitiveF3AbNotSimplcc[t_] :=

$$-\frac{2\ Q2\ ArcTan\left[\sqrt{\frac{(1+mua)\ (-mub+t)}{(1+mub)\ (mua-t)}}\right]}{\sqrt{1+BBB^2}\ \sqrt{(1+mua)\ (1+mub)}};$$

```
FullSimplify[((FullSimplify[
    PrmitiveF3AbNotSimpl00[t] - f00[t], Assumptions → {-1 < mua < t < mub < 1}]) /.
  {(mua - t) Sqrt[(-mub + t)/(mua - t)] → -Sqrt[(t - mua) (mub - t)]}), Assumptions → {-1 < mua < t < mub < 1}]
FullSimplify[PrmitiveF3AbNotSimplaa[t] - faa[t], Assumptions → {-1 < mua < t < mub < 1}]
FullSimplify[PrmitiveF3AbNotSimplbb[t] - fbb[t], Assumptions → {-1 < mua < t < mub < 1}]
FullSimplify[PrmitiveF3AbNotSimplcc[t] - fcc[t], Assumptions → {-1 < mua < t < mub < 1}]
```

0

0

0

0

$$\left\{\left(\sqrt{1+\mathrm{BBB}^2}\ \sqrt{(t-\mathrm{mua})\ (\mathrm{mub}-t)}\right) \to \sqrt{1-\mathrm{AAA}^2-2\ \mathrm{AAA}\ \mathrm{BBB}\ t-t^2-\mathrm{BBB}^2\ t^2}\right\}$$

$$\left\{\mathrm{mua} \to \frac{-\mathrm{AAA}\ \mathrm{BBB}-\sqrt{1-\mathrm{AAA}^2+\mathrm{BBB}^2}}{1+\mathrm{BBB}^2}\ (* \to \mathrm{mua}\ *)\right\},$$

$$\left\{\mathrm{mub} \to \frac{-\mathrm{AAA}\ \mathrm{BBB}+\sqrt{1-\mathrm{AAA}^2+\mathrm{BBB}^2}}{1+\mathrm{BBB}^2}\ (* \to \mathrm{mub}\ *)\right\}$$

$$\left\{\mathrm{p0} \to -\frac{1}{12}\mathrm{Cot}[\beta]\ (3\ \mathrm{BBB}\ \mathrm{bp}\ r+4\ \mathrm{AAA}\ \mathrm{ap}\ \mathrm{Sin}[\beta]),\right.$$
$$\mathrm{p1} \to -\frac{1}{12}\mathrm{Cot}[\beta]\ (3\ \mathrm{AAA}\ \mathrm{bp}\ r+4\ \mathrm{ap}\ \mathrm{BBB}\ \mathrm{Sin}[\beta]),\ \mathrm{p2} \to -\frac{1}{12}\mathrm{Cot}[\beta]\ (3\ \mathrm{BBB}\ \mathrm{bp}\ r+4\ \mathrm{AAA}\ \mathrm{ap}\ \mathrm{Sin}[\beta]),$$
$$\mathrm{p3} \to -\frac{1}{4}\mathrm{AAA}\ \mathrm{bp}\ r\ \mathrm{Cot}[\beta],\ \mathrm{Q1} \to \frac{1}{24}(\mathrm{AAA}+\mathrm{BBB})\ \mathrm{Cot}[\beta]\ (3\ \mathrm{bp}\ r+4\ \mathrm{ap}\ \mathrm{Sin}[\beta]),$$
$$\left.\mathrm{Q2} \to \frac{1}{24}(\mathrm{AAA}-\mathrm{BBB})\ \mathrm{Cot}[\beta]\ (-3\ \mathrm{bp}\ r+4\ \mathrm{ap}\ \mathrm{Sin}[\beta])\right\},\ \left\{-\sqrt{1+\mathrm{BBB}^2}<\mathrm{AAA}<\sqrt{1+\mathrm{BBB}^2}\ \&\&\right.$$
$$\left.-\frac{\mathrm{AAA}\ \mathrm{BBB}}{1+\mathrm{BBB}^2}-\sqrt{\frac{1-\mathrm{AAA}^2+\mathrm{BBB}^2}{\left(1+\mathrm{BBB}^2\right)^2}}<t<-\frac{\mathrm{AAA}\ \mathrm{BBB}}{1+\mathrm{BBB}^2}+\sqrt{\frac{1-\mathrm{AAA}^2+\mathrm{BBB}^2}{\left(1+\mathrm{BBB}^2\right)^2}}\right\}$$

$$\left\{\sqrt{\frac{\mathrm{mub}-t}{t-\mathrm{mua}}} \to \frac{\sqrt{1+\mathrm{BBB}^2}\ \sqrt{1-\mathrm{AAA}^2-2\ \mathrm{AAA}\ \mathrm{BBB}\ t-\left(1+\mathrm{BBB}^2\right)\ t^2}}{\left(\mathrm{AAA}\ \mathrm{BBB}+\sqrt{1-\mathrm{AAA}^2+\mathrm{BBB}^2}+t+\mathrm{BBB}^2\ t\right)}\right\}$$

```
PrmitiveF3AbNotSimplbb[t]
```

we use the following three identities:

$$\left\{\sqrt{(\text{mub}-\text{t})\ (\text{t}-\text{mua})}\ \rightarrow\right\}\left\{\sqrt{\frac{-\text{mub}+\text{t}}{\text{mua}-\text{t}}}\ \rightarrow\left(\frac{\sqrt{1+\text{BBB}^2}\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}}{\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}}\right)\right\}$$

$$\left\{\sqrt{\frac{(-1+\text{mua})\ (-\text{mub}+\text{t})}{(-1+\text{mub})\ (\text{mua}-\text{t})}}\ \rightarrow \left(\sqrt{(\text{AAA}+\text{BBB})^2}\ \left(1+\text{BBB}^2\right)\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}\right)\Big/ \left(\left(1+\text{AAA}\ \text{BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\left(\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}\right)\right)\right\}$$

$$\left\{\sqrt{\frac{(1+\text{mua})\ (-\text{mub}+\text{t})}{(1+\text{mub})\ (\text{mua}-\text{t})}}\ \rightarrow\left(\sqrt{(\text{AAA}-\text{BBB})^2}\ \left(1+\text{BBB}^2\right)\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}\right)\Big/ \left(\left(1-\text{AAA}\ \text{BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\left(\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}\ (\text{AAA}+\text{BBB}\ \text{t})\right)\right)\right\}$$

$$\left\{\text{p0}\rightarrow-\frac{1}{12}\ \text{Cot}[\beta]\ (3\ \text{BBB}\ \text{bp}\ \text{r}+4\ \text{AAA}\ \text{ap}\ \text{Sin}[\beta]),\right.$$
$$\text{p1}\rightarrow-\frac{1}{12}\ \text{Cot}[\beta]\ (3\ \text{AAA}\ \text{bp}\ \text{r}+4\ \text{ap}\ \text{BBB}\ \text{Sin}[\beta]),\ \text{p2}\rightarrow-\frac{1}{12}\ \text{Cot}[\beta]\ (3\ \text{BBB}\ \text{bp}\ \text{r}+4\ \text{AAA}\ \text{ap}\ \text{Sin}[\beta]),$$
$$\text{p3}\rightarrow-\frac{1}{4}\ \text{AAA}\ \text{bp}\ \text{r}\ \text{Cot}[\beta],\ \text{Q1}\rightarrow\frac{1}{24}\ (\text{AAA}+\text{BBB})\ \text{Cot}[\beta]\ (3\ \text{bp}\ \text{r}+4\ \text{ap}\ \text{Sin}[\beta]),$$
$$\left.\text{Q2}\rightarrow\frac{1}{24}\ (\text{AAA}-\text{BBB})\ \text{Cot}[\beta]\ (-3\ \text{bp}\ \text{r}+4\ \text{ap}\ \text{Sin}[\beta])\right\}$$

```
Simplify[
 ((((((((PrmitiveF3AbNotSimpl00[t]) /. {Sqrt[(mub - t) (-mua + t)] -> (Sqrt[(1 - AAA^2 - 2 AAA BBB t -
          t^2 - BBB^2 t^2)] / (Sqrt[1 + BBB^2]))}) /. {mub ->
          (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
      {p0 -> -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β])}) /.
     {p1 -> -1/12 Cot[β] (3 AAA bp r + 4 ap BBB Sin[β])}) /.
    {p2 -> -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β])}) /. {p3 -> -1/4 AAA bp r Cot[β]}) /.
   {Q1 -> 1/24 (AAA + BBB) Cot[β] * (3 bp r + 4 ap Sin[β])}) /.
  {Q2 -> 1/24 (AAA - BBB) Cot[β] (-3 bp r + 4 ap Sin[β])}),
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
Simplify[
 (((((((((PrmitiveF3AbNotSimplaa[t]) /. {Sqrt[(-mub + t)/(mua - t)] → (Sqrt[1 + BBB^2] Sqrt[(1 - AAA^2 - 2 AAA BBB t -
              (1 + BBB^2) t^2)] / (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t))}) /.
          {mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
       {p0 → -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β])}) /.
      {p1 → -1/12 Cot[β] (3 AAA bp r + 4 ap BBB Sin[β])}) /.
     {p2 → -1/12 Cot[β] (3 BBB bp r + 4 AAA ap Sin[β])}) /.
    {p3 → -1/4 AAA bp r Cot[β]}), Assumptions →
  {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
FullSimplify[
 ((((PrmitiveF3AbNotSimplbb[t]) /. {{Sqrt[((-1 + mua) (-mub + t))/((-1 + mub) (mua - t))] -> (Sqrt[(AAA + BBB)^2] (1 + BBB^2)
         Sqrt[(1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2)]) / ((1 + AAA BBB + BBB^2 -
          Sqrt[1 - AAA^2 + BBB^2]) (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t))}}) /.
     {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
   {Q1 -> 1/24 (AAA + BBB) Cot[β] (3 bp r + 4 ap Sin[β])}),
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]

FullSimplify[
 ((((PrmitiveF3AbNotSimplcc[t]) /. {Sqrt[((1 + mua) (-mub + t))/((1 + mub) (mua - t))] -> (Sqrt[(AAA - BBB)^2] (1 + BBB^2)
         Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) / ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2])
         (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))}) /.
     {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
   {Q2 -> 1/24 (AAA - BBB) Cot[β] (-3 bp r + 4 ap Sin[β])}),
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

The sum of the following four functions is the primitive of rmnIntgrndF3Ab[t]

```
PrmitiveF3Ab00[t_] := 1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
    Cot[β] (bp r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t +
          3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (5 + t^2) + BBB^2 (-3 + 2 t^2))) +
       4 ap (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[β]);
PrmitiveF3Ab11[t_] := (ArcTan[(Sqrt(-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)))/
        (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)] Cot[β]
      (3 BBB (AAA^4 (3 - 2 BBB^2) - 6 AAA^2 (1 + BBB^2) + (1 + BBB^2)^2 (3 + 2 BBB^2)) bp r +
        4 AAA ap (1 + BBB^2) (3 (1 + BBB^2) + AAA^2 (-1 + 2 BBB^2)) Sin[β]))/ (12 (1 + BBB^2)^(7/2));
PrmitiveF3Ab22[t_] := 1/12 ArcTan[((AAA + BBB) (1 + BBB^2)
        Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r + 4 ap Sin[β]);
PrmitiveF3Ab33[t_] := 1/12 ArcTan[
     ((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r - 4 ap Sin[β]);
```

CHECK OF THE DERIVATIVE OF THE FINAL PRIMITIVE

```
Simplify[(D[PrmitiveF3Ab00[t], t] + D[PrmitiveF3Ab11[t], t] +
    D[PrmitiveF3Ab22[t], t] + D[PrmitiveF3Ab33[t], t]) -
  rmnIntgrndF3Ab[t], Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -AAA BBB/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -AAA BBB/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

Simplify::time :
Time spent on a transformation exceeded 300 seconds, and the transformation was aborted. Increasing the value of TimeConstraint option may improve the result of simplification. ≫

0

We check the above result evaluating the derivatives step by step

The derivative of PrmitiveF3Ab00[t] is:
cf00[[1]] $\sqrt{\Delta 1}$ + cf00[[2]] / $\sqrt{\Delta 1}$

```
cf000NS = Simplify[CoefficientList[
   ((Expand[((D[PrmitiveF3Ab00[t], t]) /. {Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] → Sqrt[Δ1]}) /.
        {1 / Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] → 1 / Sqrt[Δ1]}]) /. {1/Sqrt[Δ1] → XXXX}), XXXX]];
cf00 = {cf000NS[[1]] / Sqrt[Δ1], cf000NS[[2]]};

Simplify[D[PrmitiveF3Ab00[t], t] -
  ((cf00[[1]] Sqrt[Δ1] + cf00[[2]] / Sqrt[Δ1]) /. {Sqrt[Δ1] → Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2],
     1 / Sqrt[Δ1] → 1 / Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]})]
```

0

The derivative of PrmitiveF3Ab11[t] is $\frac{\mathrm{cf11a}}{\sqrt{\Delta 1}}+\mathrm{cf11b}\sqrt{\Delta 1} = \frac{\mathrm{cf11}}{\sqrt{\Delta 1}}$

```
FullSimplify[(Simplify[Sqrt[Δ1] (((D[PrmitiveF3Ab11[t], t]) /.
         {1 / √((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)) → 1/(Sqrt[Δ1] Sqrt[1 + BBB^2])}) /.
       {√((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)) → Sqrt[Δ1] Sqrt[1 + BBB^2]})]) /.
  {Δ1 → 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2}]

cf11 = -1/(24 (1 + BBB^2)^3) (4 AAA ap (1 + BBB^2) (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Cos[β] +
    3 BBB (3 (-1 + AAA^2)^2 - 2 (-4 + 3 AAA^2 + AAA^4) BBB^2 + 7 BBB^4 + 2 BBB^6) bp r Cot[β]);
```

```
FullSimplify[
 D[PrmitiveF3Ab11[t], t] - (((cf11/Sqrt[Δ1]) /. {1/Sqrt[Δ1] → 1/Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}),
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

The derivative of PrmitiveF3Ab22[t] is

$$\frac{(\text{AAA}+\text{BBB})\ \text{Cot}[\beta]\ (3\ \text{bp}\ \text{r}+4\ \text{ap}\ \text{Sin}[\beta])}{24\ (1-\text{t})\ \sqrt{\Delta 1}}$$

```
FullSimplify[((Simplify[
    Sqrt[Δ1] (((D[PrmitiveF3Ab22[t], t]) /. {Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → Sqrt[Δ1]}) /.
       {1/Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → 1/Sqrt[Δ1]})]) /.
   {Δ1 → 1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2})]

cf22 = -((AAA + BBB) Cot[β] (3 bp r + 4 ap Sin[β]))/(24 (-1 + t));

Simplify[
 D[PrmitiveF3Ab22[t], t] - (((cf22/Sqrt[Δ1]) /. {1/Sqrt[Δ1] → 1/Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}),
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

The derivative of PrmitiveF3Ab33[t] is

$$\frac{(\text{AAA}-\text{BBB})\ \text{Cot}[\beta]\ (-3\ \text{bp}\ \text{r}+4\ \text{ap}\ \text{Sin}[\beta])}{24\ (1+\text{t})\ \sqrt{\Delta 1}}$$

```
FullSimplify[(Simplify[
    (Sqrt[Δ1] (((D[PrmitiveF3Ab33[t], t]) /. {Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → Sqrt[Δ1]}) /.
       {1/Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → 1/Sqrt[Δ1]}))] /.
   {Δ1 → 1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2}), Assumptions → {1 - AAA^2 + BBB^2 > 0}]

cf33 = ((AAA - BBB) Cot[β] (-3 bp r + 4 ap Sin[β]))/(24 (1 + t));
```

```
Simplify[
 D[PrmitiveF3Ab33[t], t] - ((cf33/Sqrt[Δ1]) /. {1/Sqrt[Δ1] → 1/Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}),
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -AAA BBB/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -AAA BBB/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

Summing up the four contributions we get

```
(cf00[[1]] Sqrt[Δ1] + cf00[[2]]/Sqrt[Δ1]) + (cf11/Sqrt[Δ1]) + (cf22/Sqrt[Δ1]) + (cf33/Sqrt[Δ1])
```

```
Simplify[
 (Simplify[Together[(cf00[[1]] Sqrt[Δ1] + cf00[[2]]/Sqrt[Δ1]) + (cf11/Sqrt[Δ1]) + (cf22/Sqrt[Δ1]) + (cf33/Sqrt[Δ1])]]) /.
  {Δ1 → 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2}]
```

$$-\frac{t^3\,(BBB + AAA\,t)\,Cot[\beta]\,(3\,bp\,r\,t + 4\,ap\,Sin[\beta])}{12\,(-1 + t^2)\,\sqrt{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2}}$$

and, finally, subtracting to this result the outset integrand rmnIntgrndF3Ab[t]
we get zero

```
Simplify[(-(t^3 (BBB + AAA t) Cot[β] (3 bp r t + 4 ap Sin[β]))/(12 (-1 + t^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])) - rmnIntgrndF3Ab[t]]
```

0

quiqui

IN CONCLUSION THE PRIMITVE OF THE CONTRIBUTION:

```
CF3ATot[t_] := CF3Aa[t] + ArcSin[(AAA+BBB t)/Sqrt[1-t^2]] * CF3Ab[t]
```

IS THE SUM OF THE FOLLOWING SIX FUNCTIONS

```
PrmitiveF3Aa[t_] := -1/12 fff t^3 Cot[β] (3 bp r t + 4 ap Sin[β]);
PrmitiveF3AbIPP[t_] := ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] (-1/3 ap t^3 Cos[β] - 1/4 bp r t^4 Cot[β]);
PrmitiveF3Ab00[t_] :=
  1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Cot[β] (bp r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (5 + t^2) + BBB^2 (-3 + 2 t^2))) +
     4 ap (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[β]);

PrmitiveF3Ab11[t_] := (ArcTan[(Sqrt(-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))) /
       (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)] Cot[β]
     (3 BBB (AAA^4 (3 - 2 BBB^2) - 6 AAA^2 (1 + BBB^2) + (1 + BBB^2)^2 (3 + 2 BBB^2)) bp r +
       4 AAA ap (1 + BBB^2) (3 (1 + BBB^2) + AAA^2 (-1 + 2 BBB^2)) Sin[β])) / (12 (1 + BBB^2)^(7/2));
PrmitiveF3Ab22[t_] := 1/12 ArcTan[((AAA + BBB) (1 + BBB^2)
       Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r + 4 ap Sin[β]);
PrmitiveF3Ab33[t_] := 1/12 ArcTan[
     ((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r - 4 ap Sin[β]);
```

EVALUATION OF THE PRIMITIVE OF
CF3BTot[t] = CF3Ba[t] + CF3Bb[t]

```
CF3Ba[t_] :=
 -1/2 t (-2 r t (AAA + BBB t) Cos[fff] Cos[β] + r (AAA + BBB t)^2 Cos[fff]^2 Sin[β] + Sin[fff]
     (-2 bp r t (AAA + BBB t) + (-2 ap (AAA + BBB t) + r (1 - t^2 - (AAA + BBB t)^2) Sin[fff]) Sin[β]));
CF3Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] *
   (-t (-r t Cos[β] Sin[fff] + Cos[fff] (bp r t + (ap + r (AAA + BBB t) Sin[fff]) Sin[β])));
CF3BTot[t_] := CF3Ba[t] + CF3Bb[t]; CF3BTot[t];
```

Integration of the first contribution CF3Ba[t]

```
Integrate[CF3Ba[t], t]
```

PRIMITIVE OF CF3Ba[t]

```
PrmitiveF3Ba[t_] := -1/48 t^2 (-4 r t (4 AAA + 3 BBB t) Cos[fff] Cos[β] -
     3 r (-2 + t^2) Sin[β] + r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]^2 Sin[β] -
     r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Sin[fff]^2 Sin[β] -
     4 Sin[fff] (bp r t (4 AAA + 3 BBB t) + 2 ap (3 AAA + 2 BBB t) Sin[β]));
```

```
Simplify[D[PrmitiveF3Ba[t], t] - CF3Ba[t]]
```

0

Integration of the second contribution CF3Bb[t].

By the Cacciopoli transformation we find

```
CF3Bb[t]
```

$$-t\sqrt{1-t^2-(AAA+BBB\,t)^2}\,\left(-r\,t\,Cos[\beta]\,Sin[fff]+Cos[fff]\,(bp\,r\,t+(ap+r\,(AAA+BBB\,t)\,Sin[fff])\,Sin[\beta])\right)$$

```
Expand[1 - t^2 - (AAA + BBB t)^2]
```

$$1-AAA^2-2\,AAA\,BBB\,t-t^2-BBB^2\,t^2$$

```
NewIntgrnd3Bb[ξ_] := FullSimplify[
  (((((CF3Bb[t]) /. {Sqrt[1 - t^2 - (AAA + BBB t)^2] → Sqrt[1 + BBB^2] Sqrt[(t - mua) (mub - t)]}) /.
        {Sqrt[(mub - t) (-mua + t)] → ξ (t - mua)}) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}) * Jacob1A) /.
   {Jacob1A → -(4 Sqrt[1 - AAA^2 + BBB^2] ξ)/((1 + BBB^2) (1 + ξ^2)^2)}, Assumptions → {-1 < mua < t < mub < 1 &&
    ξ > 0}]; NewIntgrnd3Bb[ξ]
```

```
num3Bb[ξ_] := -(4 Sqrt[1 - AAA^2 + BBB^2] (mua - mub)
    ξ^2 (mub + mua ξ^2) (r (mub + mua ξ^2) (bp Cos[fff] - Cos[β] Sin[fff]) +
      Cos[fff] (ap (1 + ξ^2) + r (AAA + BBB mub + (AAA + BBB mua) ξ^2) Sin[fff]) Sin[β])) /
  (Sqrt[1 + BBB^2]); Simplify[num3Bb[ξ] / (1 + ξ^2)^5 - NewIntgrnd3Bb[ξ]]
```

0

```
cf3Bb = Simplify[CoefficientList[num3Bb[ξ], ξ]]; cf3Bb[[7]] ;
cf3Bb[[1]] ;
```

check

```
Simplify[cf3Bb[[3]] ξ^2 + cf3Bb[[5]] ξ^4 + cf3Bb[[7]] ξ^6 - num3Bb[ξ]]
```

0

```
integralBb[ξ_] := (cf3Bb[[3]] Integrate[ξ^2/(1 + ξ^2)^5, ξ]) +
  (cf3Bb[[5]] Integrate[ξ^4/(1 + ξ^2)^5, ξ]) + (cf3Bb[[7]] Integrate[ξ^6/(1 + ξ^2)^5, ξ]); integralBb[ξ]
```

Check

```
Simplify[D[integralBb[ξ], ξ] - NewIntgrnd3Bb[ξ]]
```

0

```
(((integralBb[ξ]) /. {ArcTan[ξ] → atan}) /. {1/(1 + ξ^2)^4 → DD4}))

CFcsiPrmtv = Simplify[
   CoefficientList[(((integralBb[ξ]) /. {ArcTan[ξ] → atan}) /. {1/(1 + ξ^2)^4 → DD4})), atan]];

(CFcsiPrmtv[[1]]) /. {DD4 → 1/(1 + ξ^2)^4};

(CFcsiPrmtv[[2]]) /. {DD4 → 1/(1 + ξ^2)^4};

ausPrimtv3BbAA[ξ_] := ((CFcsiPrmtv[[1]]) /. {DD4 → 1/(1 + ξ^2)^4});

ausPrimtv3BbBB[ξ_] := Simplify[((CFcsiPrmtv[[2]]) /. {DD4 → 1/(1 + ξ^2)^4})] * ArcTan[ξ];

ausPrimtv3BbAA[ξ]
ausPrimtv3BbBB[ξ]
```

the ξ-primitive is the sum of the following two contributions.

```
ausPrimtv3BbAA[ξ_] :=
 (Sqrt[1 - AAA^2 + BBB^2] (mua - mub) ξ (-mua (-15 - 55 ξ^2 - 73 ξ^4 + 15 ξ^6) (-mua r Cos[β] Sin[fff] +
        Cos[fff] (bp mua r + (ap + (AAA + BBB mua) r Sin[fff]) Sin[β])) -
     mub (-15 + 73 ξ^2 + 55 ξ^4 + 15 ξ^6) (-mub r Cos[β] Sin[fff] + Cos[fff]
          (bp mub r + (ap + (AAA + BBB mub) r Sin[fff]) Sin[β])) - 3 (-3 - 11 ξ^2 + 11 ξ^4 + 3 ξ^6)
      (Cos[fff] (2 bp mua mub r + (mua + mub) (ap + AAA r Sin[fff]) Sin[β]) +
        mua mub r (-2 Cos[β] Sin[fff] + BBB Sin[2 fff] Sin[β])))) /
  (96 Sqrt[1 + BBB^2] (1 + ξ^2)^4); ausPrimtv3BbBB[ξ_] := -1/(32 Sqrt[1 + BBB^2])
  Sqrt[1 - AAA^2 + BBB^2]
  (mua - mub)
  ArcTan[ξ]
  (Cos[fff] (bp (5 mua^2 + 6 mua mub + 5 mub^2) r +
        (8 ap (mua + mub) + (8 AAA (mua + mub) + 5 BBB (mua^2 + mub^2)) r Sin[fff]) Sin[β]) -
    r ((5 mua^2 + 6 mua mub + 5 mub^2) Cos[β] Sin[fff] - 3 BBB mua mub Sin[2 fff] Sin[β]));
```

Checks

```
Simplify[ausPrimtv3BbAA[ξ] + ausPrimtv3BbBB[ξ] - integralBb[ξ]]

Simplify[D[ausPrimtv3BbAA[ξ], ξ] + D[ausPrimtv3BbBB[ξ], ξ] - NewIntgrnd3Bb[ξ]]
```

0

0

F3Bb: We go back to variable t

$$\text{FullSimplify}\left[\left(\left(\left(\left((\text{ausPrimtv3BbAA}[\xi]) \;/.\; \left\{\xi \to \sqrt{\frac{\text{mub} - t}{t - \text{mua}}}\right\}\right) \;/.\; \left\{\sqrt{\frac{\text{mub} - t}{-\text{mua} + t}} \to \left(\sqrt{1 + \text{BBB}^2}\,\sqrt{1 - \text{AAA}^2 - 2\,\text{AAA BBB}\,t - (1 + \text{BBB}^2)\,t^2}\right) \Big/ \left(\text{AAA BBB} + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2} + t + \text{BBB}^2\,t\right)\right\}\right) \;/.\; \left\{\text{mua} \to \frac{-\text{AAA BBB} - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}\right) \;/.\; \left\{\text{mub} \to \frac{-\text{AAA BBB} + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}\right),\ \text{Assumptions} \to \left\{-\sqrt{1 + \text{BBB}^2} < \text{AAA} < \sqrt{1 + \text{BBB}^2}\ \&\&\ -\frac{\text{AAA BBB}}{1 + \text{BBB}^2} - \sqrt{\frac{1 - \text{AAA}^2 + \text{BBB}^2}{(1 + \text{BBB}^2)^2}} < t < -\frac{\text{AAA BBB}}{1 + \text{BBB}^2} + \sqrt{\frac{1 - \text{AAA}^2 + \text{BBB}^2}{(1 + \text{BBB}^2)^2}}\right\}\right]$$

$$\text{Primtv3BbAANotS}[t\_] := \frac{1}{24\,(1 + \text{BBB}^2)^3}\sqrt{1 - \text{AAA}^2 - 2\,\text{AAA BBB}\,t - t^2 - \text{BBB}^2\,t^2}\;\Big(-\text{bp}\,r\,\big(\text{AAA}^3\,\text{BBB}\,(-13 + 2\,\text{BBB}^2) + \text{AAA}^2\,(3 + \text{BBB}^2 - 2\,\text{BBB}^4)\,t + 3\,(1 + \text{BBB}^2)^2\,t\,(-1 + 2\,(1 + \text{BBB}^2)\,t^2) + \text{AAA BBB}\,(1 + \text{BBB}^2)\,(13 + 2\,(1 + \text{BBB}^2)\,t^2)\big)\,\text{Cos}[\text{fff}] + r\,\big(\text{AAA}^3\,\text{BBB}\,(-13 + 2\,\text{BBB}^2) + \text{AAA}^2\,(3 + \text{BBB}^2 - 2\,\text{BBB}^4)\,t + 3\,(1 + \text{BBB}^2)^2\,t\,(-1 + 2\,(1 + \text{BBB}^2)\,t^2) + \text{AAA BBB}\,(1 + \text{BBB}^2)\,(13 + 2\,(1 + \text{BBB}^2)\,t^2)\big)\,\text{Cos}[\beta]\,\text{Sin}[\text{fff}] + \text{Cos}[\text{fff}]\,\big(4\,\text{ap}\,(1 + \text{BBB}^2)\,(\text{AAA}^2\,(-2 + \text{BBB}^2) + 2\,(1 + \text{BBB}^2) - \text{AAA}\,(\text{BBB} + \text{BBB}^3)\,t - 2\,(1 + \text{BBB}^2)^2\,t^2) + r\,(\text{AAA}^3\,(-8 + 9\,\text{BBB}^2 + 2\,\text{BBB}^4) - \text{AAA}^2\,\text{BBB}\,(7 + 9\,\text{BBB}^2 + 2\,\text{BBB}^4)\,t + 3\,\text{BBB}\,(1 + \text{BBB}^2)^2\,t\,(1 - 2\,(1 + \text{BBB}^2)\,t^2) - \text{AAA}\,(1 + \text{BBB}^2)\,(-8 + 5\,\text{BBB}^2 + 2\,(4 + 9\,\text{BBB}^2 + 5\,\text{BBB}^4)\,t^2))\,\text{Sin}[\text{fff}]\big)\,\text{Sin}[\beta]\Big);$$

```
FullSimplify[(((((ausPrimtv3BbAA[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}) /.
      {Sqrt[(mub - t)/(-mua + t)] → (Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
          (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)}) /.
    {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) -
  Primtv3BbAANotS[t], Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

```
Simplify[(((((ausPrimtv3BbBB[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}) /.
    {Sqrt[(mub - t)/(-mua + t)] → (Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
       (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)}) /. {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
  {mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}), Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

$$-\frac{1}{4\left(1+\mathrm{BBB}^2\right)^{7/2}}\left(-1+\mathrm{AAA}^2-\mathrm{BBB}^2\right)\mathrm{ArcTan}\left[\frac{\sqrt{-\left(1+\mathrm{BBB}^2\right)\left(-1+\mathrm{AAA}^2+2\,\mathrm{AAA}\,\mathrm{BBB}\,t+\left(1+\mathrm{BBB}^2\right)t^2\right)}}{\mathrm{AAA}\,\mathrm{BBB}+\sqrt{1-\mathrm{AAA}^2+\mathrm{BBB}^2}+t+\mathrm{BBB}^2\,t}\right]\left(-\left(1+\mathrm{BBB}^2+\mathrm{AAA}^2\left(-1+4\,\mathrm{BBB}^2\right)\right)r\,\mathrm{Cos}[\beta]\,\mathrm{Sin}[\mathrm{fff}]+\mathrm{Cos}[\mathrm{fff}]\left(\left(1+\mathrm{BBB}^2+\mathrm{AAA}^2\left(-1+4\,\mathrm{BBB}^2\right)\right)\mathrm{bp}\,r+\mathrm{BBB}\left(-4\,\mathrm{AAA}\,\mathrm{ap}\left(1+\mathrm{BBB}^2\right)+\left(1-5\,\mathrm{AAA}^2+\mathrm{BBB}^2\right)r\,\mathrm{Sin}[\mathrm{fff}]\right)\mathrm{Sin}[\beta]\right)\right)$$

```
Primtv3BbBBNotS[t_] := -1/(4 (1 + BBB^2)^(7/2)) (-1 + AAA^2 - BBB^2)
   ArcTan[Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)]/
     (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
   (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] + Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2))
        bp r + BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β]));
```

```
Simplify[(((((ausPrimtv3BbBB[ξ]) /. {ξ -> Sqrt[(mub - t)/(t - mua)]}) /.
     {Sqrt[(mub - t)/(-mua + t)] -> (Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
        (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)}) /.
    {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) -
  Primtv3BbBBNotS[t], Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

THE PRIMITIVE OF CF3Bb[T] is the sum of the following two functions

```
Primtv3BbAANotS[t_] := 1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
  (-bp r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
       3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[fff] +
    r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
       AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] Sin[fff] + Cos[fff]
    (4 ap (1 + BBB^2) (AAA^2 (-2 + BBB^2) + 2 (1 + BBB^2) - AAA (BBB + BBB^3) t - 2 (1 + BBB^2)^2 t^2) +
       r (AAA^3 (-8 + 9 BBB^2 + 2 BBB^4) - AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t +
          3 BBB (1 + BBB^2)^2 t (1 - 2 (1 + BBB^2) t^2) -
          AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Sin[fff]) Sin[β]);

Primtv3BbBBNotS[t_] := -1/(4 (1 + BBB^2)^(7/2)) (-1 + AAA^2 - BBB^2)
  ArcTan[
   Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)] /
    (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
  (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] +
   Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r +
      BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β]));
```

CHECK OF THE FINAL t-DERIVATIVE [OK]

```
Simplify[D[Primtv3BbAANotS[t], t], Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
DrvPrimtv3BbAANotS[t_] :=
  (2 (1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2) (-bp r (AAA^2 (3 + BBB^2 - 2 BBB^4) +
          4 AAA BBB (1 + BBB^2)^2 t + 3 (1 + BBB^2)^2 (-1 + 6 (1 + BBB^2) t^2)) Cos[fff] +
        r (AAA^2 (3 + BBB^2 - 2 BBB^4) + 4 AAA BBB (1 + BBB^2)^2 t + 3 (1 + BBB^2)^2 (-1 + 6 (1 + BBB^2) t^2))
         Cos[\[Beta]] Sin[fff] + (1 + BBB^2) Cos[fff] (-4 ap (1 + BBB^2) (AAA BBB + 4 (1 + BBB^2) t) -
          r (16 AAA t + 36 AAA BBB^2 t + 20 AAA BBB^4 t + 18 BBB^5 t^2 +
             BBB (-3 + 7 AAA^2 + 18 t^2) + BBB^3 (-3 + 2 AAA^2 + 36 t^2)) Sin[fff]) Sin[\[Beta]]) -
     2 (AAA BBB + t + BBB^2 t) (-bp r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
          3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2))
        Cos[fff] + r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
          3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2))
        Cos[\[Beta]] Sin[fff] + Cos[fff] (4 ap (1 + BBB^2) (AAA^2 (-2 + BBB^2) + 2 (1 + BBB^2) -
             AAA (BBB + BBB^3) t - 2 (1 + BBB^2)^2 t^2) + r (AAA^3 (-8 + 9 BBB^2 + 2 BBB^4) -
             AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t + 3 BBB (1 + BBB^2)^2 t (1 - 2 (1 + BBB^2) t^2) -
             AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Sin[fff]) Sin[\[Beta]]))/
   (48 (1 + BBB^2)^3 Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]);
```

```
(Simplify[Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)] D[Primtv3BbBBNotS[t], t]])/
 (Sqrt[1 + BBB^2] * Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])
```

```
DrvPrimtv3BbBBNotS[t_] :=
  ((-1 + AAA^2 - BBB^2) (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[\[Beta]] Sin[fff] +
       Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r +
          BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[\[Beta]])))/
   (8 (1 + BBB^2)^3 Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]);
```

```
Simplify[Simplify[D[Primtv3BbBBNotS[t], t]] - DrvPrimtv3BbBBNotS[t],
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
0
```

```
Simplify[(Simplify[Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] * DrvPrimtv3BbAANotS[t]] +
   Simplify[Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] * DrvPrimtv3BbBBNotS[t]]  -
   Simplify[CF3Bb[t] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]],
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -AAA BBB/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -AAA BBB/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

THE PRIMITIVE OF CF3BTot[t]   is the sum of following functions.
The first is the primitive of CF3Ba[t] and the sum of the remainig two ones is the primitive of CF3Bb[t].

Note:
Primtv3BbAA[t] = Primtv3BbAANotS[t]
and
Primtv3BbBB[t] = Primtv3BbBBNotS[t]

```
qui
```

```
PrmitiveF3Ba[t_] := -1/48 t^2 (-4 r t (4 AAA + 3 BBB t) Cos[fff] Cos[β] -
    3 r (-2 + t^2) Sin[β] + r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]^2 Sin[β] -
    r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Sin[fff]^2 Sin[β] -
    4 Sin[fff] (bp r t (4 AAA + 3 BBB t) + 2 ap (3 AAA + 2 BBB t) Sin[β]));
Primtv3BbAA[t_] := 1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
  (-bp r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
        3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[fff] +
    r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
        AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] Sin[fff] + Cos[fff]
     (4 ap (1 + BBB^2) (AAA^2 (-2 + BBB^2) + 2 (1 + BBB^2) - AAA (BBB + BBB^3) t - 2 (1 + BBB^2)^2 t^2) +
        r (AAA^3 (-8 + 9 BBB^2 + 2 BBB^4) - AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t +
           3 BBB (1 + BBB^2)^2 t (1 - 2 (1 + BBB^2) t^2) -
           AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Sin[fff]) Sin[β]);
Primtv3BbBB[t_] := -1/(4 (1 + BBB^2)^(7/2)) (-1 + AAA^2 - BBB^2) ArcTan[
   Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)]/
     (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
  (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] +
    Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r +
       BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β]));
```

In conclusion, the primitves of the two terms, whose sum yields the integrand CF3TOT[t], is given by the following functions to give the integrand CF1, i.e. CF1ATot[t] and CF3BTot[t] are respectively equal to
PrmitiveF1Aa[t] + PrmitiveF1AbIPP[t] + PrmitiveF1Ab00[t] + PrmitiveF1Ab11[t] + PrmitiveF1Ab22[t] + PrmitiveF1Ab33[t]
and to
PrmitiveF3Ba[t] + Primtv3BbAA[t] + Primtv3BbBB[t].
The functions are reported below.

FINAL PRIMITIVE EXPRESSIONS of CF3ATot[t] AND CF3BTot[t]

the CF3ATot[t] primitive is the sum of of the following yellow expressions

```
PrmitiveF3Aa[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -1/12 fff t^3 Cot[β] (3 bp r t + 4 ap Sin[β]);
PrmitiveF3AbIPP[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] (-1/3 ap t^3 Cos[β] - 1/4 bp r t^4 Cot[β]);
PrmitiveF3Ab00[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Cot[β] (bp r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (5 + t^2) + BBB^2 (-3 + 2 t^2))) +
     4 ap (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[β]);

PrmitiveF3Ab11[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
   (ArcTan[(√(-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))) /
        (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)] Cot[β]
      (3 BBB (AAA^4 (3 - 2 BBB^2) - 6 AAA^2 (1 + BBB^2) + (1 + BBB^2)^2 (3 + 2 BBB^2)) bp r +
        4 AAA ap (1 + BBB^2) (3 (1 + BBB^2) + AAA^2 (-1 + 2 BBB^2)) Sin[β])) / (12 (1 + BBB^2)^(7/2));
PrmitiveF3Ab22[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
   1/12 ArcTan[((AAA + BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
     Cot[β] (3 bp r + 4 ap Sin[β]);
PrmitiveF3Ab33[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
   1/12 ArcTan[((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
     Cot[β] (3 bp r - 4 ap Sin[β]);
```

the CF3BTot[t] primitive is the sum of
of the following orange expressions

```
PrmitiveF3Ba[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
 -1/48 t^2 (-4 r t (4 AAA + 3 BBB t) Cos[fff] Cos[β] - 3 r (-2 + t^2) Sin[β] +
    r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]^2 Sin[β] -
    r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Sin[fff]^2 Sin[β] -
    4 Sin[fff] (bp r t (4 AAA + 3 BBB t) + 2 ap (3 AAA + 2 BBB t) Sin[β]));
Primtv3BbAA[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
 1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
  (-bp r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
        3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[fff] +
    r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
        AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] Sin[fff] + Cos[fff]
     (4 ap (1 + BBB^2) (AAA^2 (-2 + BBB^2) + 2 (1 + BBB^2) - AAA (BBB + BBB^3) t - 2 (1 + BBB^2)^2 t^2) +
       r (AAA^3 (-8 + 9 BBB^2 + 2 BBB^4) - AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t +
          3 BBB (1 + BBB^2)^2 t (1 - 2 (1 + BBB^2) t^2) -
          AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Sin[fff]) Sin[β]);
Primtv3BbBB[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
 -1/(4 (1 + BBB^2)^(7/2))
  (-1 + AAA^2 - BBB^2)
  ArcTan[Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)]/
     (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
  (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] +
    Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r +
       BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β]));
```

NUMERICAL CHECKS

```
Reduce[{-1 < (AAA + BBB t)/Sqrt[1 - t^2] < 1 && -Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}, {BBB, AAA, t}, Reals]

CF3ATot[t]

With[{r = 1, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, ap = 2, bp = 1},

 bounds = {N[-(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2], 30], N[-(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2], 30]}]
```

NUMERICAL CHECKS FOR CF3ATot

```
val = N[(CF3ATot[1 / 10]) /. {r → 1, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, ap → 2, bp → 1}];
val
```

```
-0.0180895
```

```
With[{r = 1 / 2, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, ap = 2, bp = 1},
 Pr3Aa[t_] := PrmitiveF3Aa[r, β, ap, bp, fff, AAA, BBB, t] ;
 Pr3APP[t_] := PrmitiveF3AbIPP[r, β, ap, bp, fff, AAA, BBB, t];
 Pr3A00[t_] := PrmitiveF3Ab00[r, β, ap, bp, fff, AAA, BBB, t];
 Pr3A11[t_] := PrmitiveF3Ab11[r, β, ap, bp, fff, AAA, BBB, t] ;
 Pr3A22[t_] := PrmitiveF3Ab22[r, β, ap, bp, fff, AAA, BBB, t] ;
 Pr3A33[t_] := PrmitiveF3Ab33[r, β, ap, bp, fff, AAA, BBB, t];]
CF3AT[x_] :=
  ((CF3ATot[x]) /. {r → 1 / 2, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, ap → 2, bp → 1});
```

```
Pr3Aa[t]
Pr3APP[t]
Pr3A00[t]
Pr3A11[t]
Pr3A22[t]
Pr3A33[t]
CF3AT[x]
```

```
N[Pr3Aa[0]]
N[Pr3APP[0]]
N[Pr3A00[0]]
N[Pr3A11[0]]
N[Pr3A22[0]]
N[Pr3A33[0]]
N[CF3AT[0]]
```

```
step = -1 / 100; Clear[t];
Do[t = J * step;
  valaa = N[Pr3Aa[t] - Pr3Aa[0], 30];
  valPP = N[Pr3APP[t] - Pr3APP[0], 30];
  val00 = N[Pr3A00[t] - Pr3A00[0], 30];
  val11 = N[Pr3A11[t] - Pr3A11[0], 30];
  val22 = N[Pr3A22[t] - Pr3A22[0], 30];
  val33 = N[Pr3A33[t] - Pr3A33[0], 30];
  val = NIntegrate[CF3AT[x], {x, 0, t}, PrecisionGoal → 20, WorkingPrecision → 30];
  valtot = valaa + valPP + val00 + val11 + val22 + val33;
  diff = valtot - val;
  If[Abs[diff] > 10^(-30),
   Print[PaddedForm[valaa, {3, 7}], " ,", PaddedForm[valPP, {3, 7}],
    " ,", PaddedForm[val00, {3, 7}], " ,", PaddedForm[val11, {3, 7}],
    " ,", PaddedForm[val22, {3, 7}], " ,", PaddedForm[val33, {3, 7}]];
   Print[J, " ,", PaddedForm[val, {3, 7}], " ,", PaddedForm[valtot, {3, 7}],
    " ,", PaddedForm[diff, {3, 7}]]];, {J, 1, 20}];
Clear[
  t];
```

NUMERICAL CHECKS FOR CF3BTot

```
With[{r = 1 / 2, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, ap = 2, bp = 1},
 Pr3Ba[t_] := PrmitiveF3Ba[r, β, ap, bp, fff, AAA, BBB, t] ;
 Pr3BAA[t_] := PrmitiveF3BbAA[r, β, ap, bp, fff, AAA, BBB, t];
 Pr3BBB[t_] := PrmitiveF3BbBB[r, β, ap, bp, fff, AAA, BBB, t]; ]
CF3BT[x_] :=
  ((CF3BTot[x]) /. {r → 1 / 2, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, ap → 2, bp → 1});
```

```
step = -1 / 100; Clear[t];
Do[t = J * step;
  valBaa = N[Pr3Ba[t] - Pr3Ba[0], 30];
  valBAA = N[Pr3BAA[t] - Pr3BAA[0], 30];
  valBBB = N[Pr3BBB[t] - Pr3BBB[0], 30];
  val = NIntegrate[CF3BT[x], {x, 0, t}, PrecisionGoal → 20, WorkingPrecision → 30];
  valtot = valBaa + valBAA + valBBB;
  diff = valtot - val;
  If[Abs[diff] > 10^(-30), Print[PaddedForm[valBaa, {3, 7}],
     " ,", PaddedForm[valBAA, {3, 7}], " ,", PaddedForm[valBBB, {3, 7}]];
   Print[J, " ,", PaddedForm[val, {3, 7}], " ,", PaddedForm[valtot, {3, 7}],
     " ,", PaddedForm[diff, {3, 7}]]];, {J, 1, 20}];
Clear[
  t];
```

Compactification of the primitives

```
PrmitiveF3Aa[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -1/12 fff t^3 Cot[β] (3 bp r t + 4 ap Sin[β]);
PrmitiveF3AbIPP[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] (-1/3 ap t^3 Cos[β] - 1/4 bp r t^4 Cot[β]);
PrmitiveF3Ab00[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Cot[β] (bp r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (5 + t^2) + BBB^2 (-3 + 2 t^2))) +
     4 ap (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[β]);

PrmitiveF3Ab11[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  (ArcTan[(Sqrt[(-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))]) /
        (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)] Cot[β]
     (3 BBB (AAA^4 (3 - 2 BBB^2) - 6 AAA^2 (1 + BBB^2) + (1 + BBB^2)^2 (3 + 2 BBB^2)) bp r +
        4 AAA ap (1 + BBB^2) (3 (1 + BBB^2) + AAA^2 (-1 + 2 BBB^2)) Sin[β])) / (12 (1 + BBB^2)^(7/2));
PrmitiveF3Ab22[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/12 ArcTan[((AAA + BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r + 4 ap Sin[β]);
PrmitiveF3Ab33[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/12 ArcTan[((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Cot[β] (3 bp r - 4 ap Sin[β]);
```

```
{Δ → √(1 - t²), Δ1 → √(1 - AAA² - 2 AAA BBB t - t² - BBB² t²),
 Δ2 → √(1 + BBB²), Δ3 → √(1 - AAA² + BBB²),
 BBB² → Δ2² - 1,
 AAA² → Δ2² - Δ3²}
```

```
PrmitiveF3AaNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -(t^3 Cos[β])/12 fff (4 ap + (3 bp r t)/Sin[β]);
```

```
Simplify[((PrmitiveF3AaNEW[r, β, ap, bp, fff, AAA, BBB, t]) /. {Δ → √(1 - t²)}) -
  PrmitiveF3Aa[r, β, ap, bp, fff, AAA, BBB, t]]
```

0

```
PrmitiveF3AbIPPNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -(t^3 Cos[β])/12 ArcSin[(AAA + BBB t)/Δ] (4 ap + (3 bp r t)/Sin[β]) ;
```

```
Simplify[((PrmitiveF3AbIPPNEW[r, β, ap, bp, fff, AAA, BBB, t]) /. {Δ → √(1 - t²)}) -
  PrmitiveF3AbIPP[r, β, ap, bp, fff, AAA, BBB, t]]
```

0

```
a300 = AAA (-2 Δ2² + 8 Δ2⁴ + 15 Δ3² - 11 Δ2² Δ3²);
a301 = -BBB Δ2² (2 Δ2² - 5 Δ3²);
a302 = 2 AAA Δ2⁴; PrmitiveF3Ab00NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
 (Δ1 Cos[β])/(24 Δ2⁶) ((r bp (a300 + a301 t + a302 t^2))/Sin[β] + 4 ap Δ2^2 (BBB (2 Δ2² - 3 AAA²) + AAA Δ2² t));
```

```
FullSimplify[PrmitiveF3Ab00[r, β, ap, bp, fff, AAA, BBB, t] -
  ((((PrmitiveF3Ab00NEW[r, β, ap, bp, fff, AAA, BBB, t]) /. {Δ2 → √(1 + BBB²)}) /.
     {Δ1 → √(1 - AAA² - 2 AAA BBB t - t² - BBB² t²)}) /. {Δ3 → √(1 + BBB² - AAA²)})]
```

0

```
arg3A1 = (Δ1 Δ2)/(AAA BBB + t Δ2² + Δ3); a3A10 = 4 AAA ap Δ2² (2 Δ2⁴ + 3 Δ3² - 2 Δ2² Δ3²);
a3A11 = 3 BBB bp r Δ3² (4 Δ2⁴ + 5 Δ3² - 2 Δ2² (2 + Δ3²));
PrmitiveF3Ab11NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
 Cos[β]/(12 Δ2⁷) ArcTan[arg3A1] (a3A10 + a3A11/Sin[β]);
```

```
Simplify[((PrmitiveF3Ab11[r, β, ap, bp, fff, AAA, BBB, t]) /.
    {Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)] →
      Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}) -
  ((((PrmitiveF3Ab11NEW[r, β, ap, bp, fff, AAA, BBB, t]
        ) /. {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}) /.
      {Δ2 → Sqrt[1 + BBB^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]}),
 Assumptions → {1 + BBB^2 > 0 && 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2 > 0}]
```

0

```
arg3A2 = ((AAA + BBB) Δ1) / (AAA BBB + Δ2^2 + Δ3 - Δ3^2 - 1 + t (AAA BBB + Δ2^2 - Δ3));
PrmitiveF3Ab22NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -Cos[β]/12 ArcTan[arg3A2] (4 ap + (3 bp r)/Sin[β]);
```

```
/. {Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}
FullSimplify[ExpandAll[((PrmitiveF3Ab22[r, β, ap, bp, fff, AAA, BBB, t])) -
    ((((PrmitiveF3Ab22NEW[r, β, ap, bp, fff, AAA, BBB, t]) /.
          {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /.
        {Δ2 → Sqrt[1 + BBB^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]})],
 Assumptions → {1 + BBB^2 > 0 && 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2 > 0}]
```

0

```
arg3A3 = ((AAA - BBB) Δ1) / (1 + AAA BBB - Δ2^2 + Δ3 + Δ3^2 + t (Δ2^2 + Δ3 - AAA BBB));
PrmitiveF3Ab33NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -Cos[β]/12 ArcTan[arg3A3] (4 ap - (3 bp r)/Sin[β]);
```

```
FullSimplify[ExpandAll[((PrmitiveF3Ab33[r, β, ap, bp, fff, AAA, BBB, t])) -
    (((((PrmitiveF3Ab33NEW[r, β, ap, bp, fff, AAA, BBB, t]) /.
          {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /.
        {Δ2 → Sqrt[1 + BBB^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]})))],
  Assumptions → {1 + BBB^2 > 0 && 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2 > 0}]
```

0

```
{Δ → Sqrt[1 - t^2], Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2],
  Δ2 → Sqrt[1 + BBB^2], Δ3 → Sqrt[1 - AAA^2 + BBB^2],
  BBB^2 → Δ2^2 - 1,
  AAA^2 → Δ2^2 - Δ3^2}
```

```
PrmitiveF3Ba[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -1/48 t^2 (-4 r t (4 AAA + 3 BBB t) Cos[fff] Cos[β] - 3 r (-2 + t^2) Sin[β] +
     r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]^2 Sin[β] -
     r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Sin[fff]^2 Sin[β] -
     4 Sin[fff] (bp r t (4 AAA + 3 BBB t) + 2 ap (3 AAA + 2 BBB t) Sin[β]));
Primtv3BbAA[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/(24 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
   (-bp r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t +
        3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[fff] +
     r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
        AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] Sin[fff] + Cos[fff]
      (4 ap (1 + BBB^2) (AAA^2 (-2 + BBB^2) + 2 (1 + BBB^2) - AAA (BBB + BBB^3) t - 2 (1 + BBB^2)^2 t^2) +
        r (AAA^3 (-8 + 9 BBB^2 + 2 BBB^4) - AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t +
           3 BBB (1 + BBB^2)^2 t (1 - 2 (1 + BBB^2) t^2) -
           AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Sin[fff]) Sin[β]);
Primtv3BbBB[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  -1/(4 (1 + BBB^2)^(7/2))
   (-1 + AAA^2 - BBB^2)
   ArcTan[Sqrt[-(1 + BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2)] /
      (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
   (-(1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] +
     Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r +
        BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β]));
```

```
Simplify[Factor[CoefficientList[PrmitiveF3Ba[r, β, ap, bp, fff, AAA, BBB, t], t]]]
```

```
Factor[FullSimplify[CoefficientList[
   Simplify[Factor[CoefficientList[PrmitiveF3Ba[r, β, ap, bp, fff, AAA, BBB, t], t]]], r]]]
```

```
a3Ba3 = Sin[β]/8 (4 AAA ap Sin[fff] - r (1 - Cos[2 fff] + 2 AAA^2 Cos[2 fff]) );
a3Ba4 =  (ap BBB Sin[fff] Sin[β] +
     r ( AAA (Cos[fff] Cos[β] + bp Sin[fff] - BBB Cos[2 fff] Sin[β]) ) ) / 3;
a3Ba5 = r/16 (4 BBB Cos[fff] Cos[β] + 4 BBB bp Sin[fff] + Sin[β] (1 - (1 + 2 BBB^2) Cos[2 fff] ));
PrmitiveF3BaNew[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  t^2 (a3Ba3  + a3Ba4 t + a3Ba5 t^2);
```

```
FullSimplify[PrmitiveF3BaNew[r, β, ap, bp, fff, AAA, BBB, t] -
  PrmitiveF3Ba[r, β, ap, bp, fff, AAA, BBB, t]]
```

0

```
{Δ → Sqrt[1 - t^2] , Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] ,
 Δ2 → Sqrt[1 + BBB^2] , Δ3 → Sqrt[1 - AAA^2 + BBB^2] ,  1 + BBB^2 → Δ2^2,
 BBB^2  →  Δ2^2  - 1,  BBB^3 → BBB (Δ2^2 - 1), BBB^4 → (Δ2^2 - 1) ^2,
 AAA^2 → Δ2^2 - Δ3^2,  AAA^3 → AAA (Δ2^2 - Δ3^2)}
```

```
Cf3BAA = {{4 ap Δ2^2 (AAA^2 (-2 + BBB^2) + 2 Δ2^2) Cos[fff] Sin[β],
    AAA BBB (AAA^2 (- 13 + 2 BBB^2) + 13 Δ2^2) (-bp Cos[fff] + Cos[β] Sin[fff]) +
     AAA (8 + 3 BBB^2 - 5 BBB^4 + AAA^2 (- 8 + 9 BBB^2 + 2 BBB^4)) Cos[fff] Sin[fff] Sin[β]}, {- 4 AAA ap
     BBB Δ2^4 Cos[fff] Sin[β], Δ2^2 ((AAA^2 (- 3 + 2 BBB^2) + 3 Δ2^2) (bp Cos[fff] - Cos[β] Sin[fff]) +
       BBB (-AAA^2 (7 + 2 BBB^2) + 3 Δ2^2) Cos[fff] Sin[fff] Sin[β])}, {- 8 ap Δ2^6 Cos[fff] Sin[β],
   - 2 AAA Δ2^4 (-BBB Cos[β] Sin[fff] + Cos[fff] (BBB bp + (4 + 5 BBB^2) Sin[fff] Sin[β]))},
  {0, - 6 Δ2^6 (-Cos[β] Sin[fff] + Cos[fff] (bp + BBB Sin[fff] Sin[β]))}};
Primtv3BbAANEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/(24 Δ2^6) Δ1 * (Cf3BAA[[1, 1]] + r Cf3BAA[[1, 2]] +
    t (Cf3BAA[[2, 1]] + r Cf3BAA[[2, 2]]) +
    t^2 (Cf3BAA[[3, 1]] + r Cf3BAA[[3, 2]]) + t^3 (Cf3BAA[[4, 1]] + r Cf3BAA[[4, 2]]));
```

```
Simplify[Primtv3BbAA[r, β, ap, bp, fff, AAA, BBB, t] -
  (((Primtv3BbAA[r, β, ap, bp, fff, AAA, BBB, t]) /.
      {√(1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2) → Δ1}) /. {√(1 - t^2) → Δ}),
 Assumptions → { 1 - t^2 > 0 && 1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2 > 0}]
```

0

```
Primtv3BbBB[r, β, ap, bp, fff, AAA, BBB, t]
```

```
Cf3BbBB = Simplify[CoefficientList[
   ((-1 - BBB^2 - AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] + Cos[fff] ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2))
        bp r + BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2) r Sin[fff]) Sin[β])), r]]
```

```
{Δ → √(1 - t^2) , Δ1 → √(1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2) ,
 Δ2 → √(1 + BBB^2) , Δ3 → √(1 - AAA^2 + BBB^2) ,
 BBB^2 → Δ2^2 - 1,
 AAA^2 → Δ2^2 - Δ3^2}
```

```
FullSimplify[
 ((Simplify[CoefficientList[((-1 - BBB^2 - AAA^2 (-1 + 4 BBB^2)) r Cos[β] Sin[fff] + Cos[fff]
          ((1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) bp r + BBB (-4 AAA ap (1 + BBB^2) + (1 - 5 AAA^2 + BBB^2)
                r Sin[fff]) Sin[β])), r]]) /. {AAA^2 → Δ2^2 - Δ3^2}) /. {BBB^2 → Δ2^2 - 1}]
```

```
arg3BbBB = (Δ2 Δ1 / (AAA BBB + Δ3 + Δ2^2 t));
Cf3BbBB = {-4 AAA ap BBB Δ2^2 Cos[fff] Sin[β], (4 Δ2^4 + 5 Δ3^2 - 4 Δ2^2 (1 + Δ3^2))
    (bp Cos[fff] - Cos[β] Sin[fff]) + BBB (-4 Δ2^2 + 5 Δ3^2) Cos[fff] Sin[fff] Sin[β]};
Primtv3BbBBNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  Δ3^2 / (4 (Δ2)^7) ArcTan[arg3BbBB] (Cf3BbBB [[1]] + Cf3BbBB[[2]] r);
```

```
Simplify[Primtv3BbBB[r, β, ap, bp, fff, AAA, BBB, t] -
  (((((Primtv3BbBBNEW[r, β, ap, bp, fff, AAA, BBB, t]) /.
         {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /. {Δ → Sqrt[1 - t^2]}) /.
      {Δ2 → Sqrt[1 + BBB^2]}) /. Δ3 → Sqrt[1 - AAA^2 + BBB^2]), Assumptions →
  {1 - t^2 > 0 && 1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2 > 0}]
```

0

FINAL COMPACT EXPRESSIONS

```
PrmitiveF3AaNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  - (t^3 Cos[β])/12 fff (4 ap + (3 bp r t)/Sin[β]);
```

```
PrmitiveF3AbIPPNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  - (t^3 Cos[β])/12 ArcSin[(AAA + BBB t)/Δ] (4 ap + (3 bp r t)/Sin[β]) ;
```

```
a300 = AAA (-2 Δ2^2 + 8 Δ2^4 + 15 Δ3^2 - 11 Δ2^2 Δ3^2);
a301 = -BBB Δ2^2 (2 Δ2^2 - 5 Δ3^2);
a302 = 2 AAA Δ2^4; PrmitiveF3Ab00NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  (Δ1 Cos[β])/(24 Δ2^6) ((r bp (a300 + a301 t + a302 t^2))/Sin[β] + 4 ap Δ2^2 (BBB (2 Δ2^2 - 3 AAA^2) + AAA Δ2^2 t));
```

```
arg3A1 = (Δ1 Δ2)/(AAA BBB + t Δ2^2 + Δ3); a3A10 = 4 AAA ap Δ2^2 (2 Δ2^4 + 3 Δ3^2 - 2 Δ2^2 Δ3^2);
a3A11 = 3 BBB bp r Δ3^2 (4 Δ2^4 + 5 Δ3^2 - 2 Δ2^2 (2 + Δ3^2));
PrmitiveF3Ab11NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  Cos[β]/(12 Δ2^7) ArcTan[arg3A1] (a3A10 + a3A11/Sin[β]);
```

```
arg3A2 = ((AAA + BBB) Δ1) / (AAA BBB + Δ2^2 + Δ3 - Δ3^2 - 1 + t (AAA BBB + Δ2^2 - Δ3));
PrmitiveF3Ab22NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  - Cos[β]/12 ArcTan[arg3A2] (4 ap + (3 bp r)/Sin[β]);
```

```
arg3A3 = ((AAA - BBB) Δ1) / (1 + AAA BBB - Δ2^2 + Δ3 + Δ3^2 + t (Δ2^2 + Δ3 - AAA BBB));
PrmitiveF3Ab33NEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  - Cos[β]/12 ArcTan[arg3A3] (4 ap - (3 bp r)/Sin[β]);
```

```
a3Ba3 = Sin[β]/8 (4 AAA ap Sin[fff] - r (1 - Cos[2 fff] + 2 AAA^2 Cos[2 fff]) );
a3Ba4 =  (ap BBB Sin[fff] Sin[β] +
    r ( AAA (Cos[fff] Cos[β] + bp Sin[fff] - BBB Cos[2 fff] Sin[β]) )) / 3;
a3Ba5 = r/16 (4 BBB Cos[fff] Cos[β] + 4 BBB bp Sin[fff] + Sin[β] (1 - (1 + 2 BBB^2) Cos[2 fff] ));
PrmitiveF3BaNew[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  t^2 (a3Ba3 + a3Ba4 t + a3Ba5 t^2);
```

```
Cf3BAA = {{4 ap Δ2^2 (AAA^2 (-2 + BBB^2) + 2 Δ2^2) Cos[fff] Sin[β],
    AAA BBB (AAA^2 (-13 + 2 BBB^2) + 13 Δ2^2) (-bp Cos[fff] + Cos[β] Sin[fff]) +
     AAA (8 + 3 BBB^2 - 5 BBB^4 + AAA^2 (-8 + 9 BBB^2 + 2 BBB^4)) Cos[fff] Sin[fff] Sin[β]}, {-4 AAA ap
     BBB Δ2^4 Cos[fff] Sin[β], Δ2^2 ((AAA^2 (-3 + 2 BBB^2) + 3 Δ2^2) (bp Cos[fff] - Cos[β] Sin[fff]) +
       BBB (-AAA^2 (7 + 2 BBB^2) + 3 Δ2^2) Cos[fff] Sin[fff] Sin[β])}, {-8 ap Δ2^6 Cos[fff] Sin[β],
   -2 AAA Δ2^4 (-BBB Cos[β] Sin[fff] + Cos[fff] (BBB bp + (4 + 5 BBB^2) Sin[fff] Sin[β]))},
  {0, -6 Δ2^6 (-Cos[β] Sin[fff] + Cos[fff] (bp + BBB Sin[fff] Sin[β]))}};
Primtv3BbAANEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  1/(24 Δ2^6) Δ1 * (Cf3BAA[[1, 1]] + r Cf3BAA[[1, 2]] +
    t (Cf3BAA[[2, 1]] + r Cf3BAA[[2, 2]]) +
    t^2 (Cf3BAA[[3, 1]] + r Cf3BAA[[3, 2]]) + t^3 (Cf3BAA[[4, 1]] + r Cf3BAA[[4, 2]]));
```

```
arg3BbBB = ((Δ2 Δ1)/(AAA BBB + Δ3 + Δ2^2 t));
Cf3BbBB = {-4 AAA ap BBB Δ2^2 Cos[fff] Sin[β], (4 Δ2^4 + 5 Δ3^2 - 4 Δ2^2 (1 + Δ3^2))
    (bp Cos[fff] - Cos[β] Sin[fff]) + BBB (-4 Δ2^2 + 5 Δ3^2) Cos[fff] Sin[fff] Sin[β]};
Primtv3BbBBNEW[r_, β_, ap_, bp_, fff_, AAA_, BBB_, t_] :=
  Δ3^2/(4 (Δ2)^7) ArcTan[arg3BbBB] (Cf3BbBB [[1]] + Cf3BbBB[[2]] r);
```