Merano October/November, 2019

In[9]:=
```
SetDirectory[
  "/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH"];
Directory[]
```

Out[10]=
```
/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH
```

FILE CONTENT:
- the prmitives of CF1ATot[t] and CF1BTot[t] are evaluated,written in the most compact form (I have succeeded to do) as well as numerically checked comparing them to the numerival values of the integrals

The following functions have been copied from the file: "/Users/salvino/Desktop/WORK_IN_PRGS/TWO_TRIANGLES_CLD/POLYHEDRON_BASIC_MATH/Integrand_Formulae.nb"

Expressions of the final t-integrands once we set

$$\phi = \text{fff} + \text{ArcSin}\left[\frac{\text{AAA}+\text{BBB t}}{\sqrt{1-t^2}}\right]$$

```
CF1Aa[t_] := fff * (-a t^2 Cos[β] - b r t^3 Cos[β] Cot[β] - 1/2 b r t (1 - t^2) Sin[β]);

CF1Ab[t_] := (-a t^2 Cos[β] - b r t^3 Cos[β] Cot[β] - 1/2 b r t (1 - t^2) Sin[β]);

CF1ATot[t_] := CF1Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF1Ab[t];


CF1Ba[t_] := 1/2 t Sin[fff] (4 b r t (AAA + BBB t) Cos[β] +
    (2 a (AAA + BBB t) - b r (-1 + 2 AAA^2 + 4 AAA BBB t + t^2 + 2 BBB^2 t^2) Cos[fff]) Sin[β]);

CF1Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] * (1/2 t (b r (AAA + BBB t) Cos[fff]^2 Sin[β] -
      b r (AAA + BBB t) Sin[fff]^2 Sin[β] - 2 Cos[fff] (2 b r t Cos[β] + a Sin[β])));

CF1BTot[t_] := CF1Ba[t] + CF1Bb[t]; CF1BTot[t];
```

```
CF3Aa[t_] := fff * ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3Ab[t_] := ((-ap t^2 Cos[β] - bp r t^3 Cot[β]));
CF3ATot[t_] := CF3Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF3Ab[t]; CF3ATot[t];
CF3Ba[t_] := -1/2 t (-2 r t (AAA + BBB t) Cos[fff] Cos[β] + r (AAA + BBB t)^2 Cos[fff]^2 Sin[β] +
     Sin[fff] (-2 bp r t (AAA + BBB t) + (-2 ap (AAA + BBB t) + r (1 - t^2 - (AAA + BBB t)^2) Sin[fff])
         Sin[β])); CF3Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] *
   (-t (-r t Cos[β] Sin[fff] + Cos[fff] (bp r t + (ap + r (AAA + BBB t) Sin[fff]) Sin[β])));
CF3BTot[t_] := CF3Ba[t] + CF3Bb[t]; CF3BTot[t];

CF2Aa[t_] := ((CF1Aa[t]) /. {a → A, b → A}); CF2Ab[t_] := ((CF1Ab[t]) /. {a → A, b → A});
CF2ATot[t_] := CF2Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF2Ab[t];
Simplify[CF2ATot[t] - ((CF1ATot[t]) /. {a → A, b → A})]
CF2Ba[t_] := ((CF1Ba[t]) /. {a → A, b → B}); CF2Bb[t_] := ((CF1Bb[t]) /. {a → A, b → B});
CF2BTot[t_] := CF2Ba[t] + CF2Bb[t];
CF2BTot[t]; Simplify[(CF2BTot[t] - ((CF1BTot[t]) /. {a → A, b → B}))]

 CF4Aa[t_] := ((CF3Aa[t]) /. {ap → Ap, bp → Ap});
CF4Ab[t_] := ((CF3Ab[t]) /. {ap → Ap, bp → Ap});
CF4ATot[t_] := CF4Aa[t] + ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]] * CF4Ab[t];
Simplify[CF4ATot[t] - ((CF3ATot[t]) /. {ap → Ap, bp → Ap})]
CF4Ba[t_] := ((CF3Ba[t]) /. {ap → Ap, bp → Bp});
CF4Bb[t_] := ((CF3Bb[t]) /. {ap → Ap, bp → Bp});
CF4BTot[t_] := CF4Ba[t] + CF4Bb[t];
CF4BTot[t]; Simplify[(CF4BTot[t] - ((CF3BTot[t]) /. {ap → Ap, bp → Bp}))]
```

0

0

0

0

EVALUATION of the PRIMITIVE of CF1Aa[t]

```
Simplify[Integrate[CF1Aa[t], t]]
```

$$-\frac{1}{24}\,\text{fff}\,t^2\left(2\,t\,\text{Cos}[\beta]\,(4\,a+3\,b\,r\,t\,\text{Cot}[\beta])-3\,b\,r\left(-2+t^2\right)\text{Sin}[\beta]\right)$$

```
PrmitiveF1Aa[t_] := - 1/24 fff t^2 (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β]);
```

EVALUATION of the PRIMITIVE of $\text{ArcSin}\left[\frac{\text{AAA}+\text{BBB}\,t}{\sqrt{1-t^{\wedge}2}}\right] * \text{CF1Ab}[t]$

We proceed integrating by parts.
The first contribution is

```
Integrate[CF1Ab[t], t] ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]
```

```
PrmitiveF1AbIPP[t_] :=
  (- 1/3 a t^3 Cos[β] - 1/4 b r t^4 Cos[β] Cot[β] - 1/4 b r t^2 Sin[β] + 1/8 b r t^4 Sin[β])
   ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]];
```

The remaining integrand is (including the minus sign)

```
rmnIntgrndF1Ab[t_] :=
 Simplify[Together[-Integrate[CF1Ab[t], t] D[ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]], t]],
  Assumptions → {-1 < t < 1}]; rmnIntgrndF1Ab[t]
```

```
NumRmnIntgrndF1Ab[t_] :=
  (t^2 (BBB + AAA t) (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β])) / 24;
DenRmnIntgrndF1Ab[t_] := (1 - t^2);
```

```
Simplify[NumRmnIntgrndF1Ab[t] /
   (DenRmnIntgrndF1Ab[t] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]) - rmnIntgrndF1Ab[t]]
```

0

$$\text{rmnIntgrndF1Ab[t]} = \frac{\text{NumRmnIntgrndF1Ab}[t]}{\text{DenRmnIntgrndF1Ab}[t]\,\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}$$

```
rmnIntgrndF1Ab[t]
```

```
CfNum1A = Simplify[CoefficientList[NumRmnIntgrndF1Ab[t], t]];
CfNum1A[[6]];
```

```
Simplify[Sum[CfNum1A[[j]] * t^(j - 1), {j, 3, 6}] - NumRmnIntgrndF1Ab[t]]
```

0

I use Caccioppoli's substitution defined by the formulae in the below magenta frame

$$\{(1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2) \to \Delta 1\}$$

$$\text{Simplify}[1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2 - (1 - AAA^2 - 2\,AAA\,BBB\,t - (1 + BBB^2)\,t^2)]$$

0

$$\text{Reduce}[\{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2 > 0\ \&\&\ -1 < t < 1\}, \{BBB, AAA, t\}, \text{Reals}]$$

$$\left\{\left\{mua \to \frac{-AAA\,BBB - \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mua\ *)\right\},\ \left\{mub \to \frac{-AAA\,BBB + \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\ (* \to mub\ *)\right\}\right\}$$

$$\text{Simplify}[\text{Solve}[\{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2 == 0\}, t]]$$

$$\left\{(mua - mub) \to -\frac{2\sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\right\}$$

$$\text{Simplify}\left[\left((mua - mub)\ /.\ \left\{mua \to \frac{-AAA\,BBB - \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2},\ mub \to \frac{-AAA\,BBB + \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\right\}\right) - \left(-\frac{2\sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\right)\right]$$

0

$$\left\{mua\verb|^|2 \to \left(\frac{1 - AAA^2}{1 + BBB^2} - \frac{2\,AAA\,BBB\,mua}{1 + BBB^2}\right)\right\}$$

$$\text{Simplify}\left[\left(mua\verb|^|2 - \left(\frac{1 - AAA^2}{1 + BBB^2} - \frac{2\,AAA\,BBB\,mua}{1 + BBB^2}\right)\right)\ /.\ \left\{mua \to \frac{-AAA\,BBB - \sqrt{1 - AAA^2 + BBB^2}}{1 + BBB^2}\right\}\right]$$

0

$$\left\{mub\verb|^|2 \to \left(\frac{1 - AAA^2}{1 + BBB^2} - \frac{2\,AAA\,BBB\,mub}{1 + BBB^2}\right)\right\}$$

```
Simplify[(mub^2 - ((1 - AAA^2)/(1 + BBB^2) - (2 AAA BBB mub)/(1 + BBB^2))) /. {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}]
```

0

$$\left\{\xi \to \sqrt{\frac{mub - t}{t - mua}},\quad t \to \frac{mub + mua\,\xi^2}{1 + \xi^2}\right\}$$

```
Solve[{Sqrt[(mub - t)/(t - mua)] == ξ}, t]
```

$$\left\{\left\{t \to \frac{mub + mua\,\xi^2}{1 + \xi^2}\right\}\right\}$$

$$\left\{\left(\sqrt{1 + BBB^2}\ \sqrt{(t - mua)\ (mub - t)}\right) \to \sqrt{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2}\right\}$$

```
Simplify[ExpandAll[((1 + BBB^2) ((t - mua) (mub - t)) /.
    {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2), mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)})] -
  (1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2)]
Simplify[ExpandAll[1/((1 + BBB^2) ((t - mua) (mub - t)) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2),
      mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)})] - 1/(1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2)]
```

0

0

$$\left\{\left(\sqrt{1 + BBB^2}\ \sqrt{(t - mua)\ (mub - t)}\right) \to \frac{2\sqrt{1 - AAA^2 + BBB^2}\ \xi}{\sqrt{1 + BBB^2}\ \left(1 + \xi^2\right)}\right\}$$

$$\texttt{Simplify}\left[\left(\left(\texttt{Simplify}\left[\left(\left(\sqrt{1+\texttt{BBB}^2}\ \sqrt{(\texttt{t}-\texttt{mua})\ (\texttt{mub}-\texttt{t})}\right)\ /.\ \left\{\texttt{t}\to\frac{\texttt{mub}+\texttt{mua}\ \xi^2}{1+\xi^2}\right\}\right),\right.\right.\right.\right.$$
$$\left.\left.\texttt{Assumptions}\to\{\xi>0\ \&\&\ -1<\texttt{mua}<\texttt{mub}<1\}\right]\right)\ /.\ \left\{\texttt{mua}\to\frac{-\texttt{AAA BBB}-\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2},\right.$$
$$\left.\left.\texttt{mub}\to\frac{-\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2}\right\}\right)-\frac{2\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}\ \xi}{\sqrt{1+\texttt{BBB}^2}\ (1+\xi^2)}\Big]$$

$$\texttt{Simplify}\left[\left(\left(\texttt{Simplify}\left[\left(\left(1\Big/\left(\sqrt{1+\texttt{BBB}^2}\ \sqrt{(\texttt{t}-\texttt{mua})\ (\texttt{mub}-\texttt{t})}\right)\right)\ /.\ \left\{\texttt{t}\to\frac{\texttt{mub}+\texttt{mua}\ \xi^2}{1+\xi^2}\right\}\right),\right.\right.\right.\right.$$
$$\left.\left.\texttt{Assumptions}\to\{\xi>0\ \&\&\ -1<\texttt{mua}<\texttt{mub}<1\}\right]\right)\ /.\ \left\{\texttt{mua}\to\frac{-\texttt{AAA BBB}-\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2},\right.$$
$$\left.\left.\texttt{mub}\to\frac{-\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2}\right\}\right)-\frac{\sqrt{1+\texttt{BBB}^2}\ (1+\xi^2)}{2\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}\ \xi}\Big]$$

0

0

$$\left\{\xi^2\to\frac{\texttt{mub}-\texttt{t}}{\texttt{t}-\texttt{mua}}\to\frac{(1+\texttt{BBB}^2)\ (1-\texttt{AAA}^2-2\ \texttt{AAA BBB t}-(1+\texttt{BBB}^2)\ \texttt{t}^2)}{\left(\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}+\texttt{t}+\texttt{BBB}^2\ \texttt{t}\right)^2}\right\}$$

$$\texttt{Simplify}\left[\left(\left(\left(((\xi^2))\ /.\ \left\{\xi\to\sqrt{\frac{\texttt{mub}-\texttt{t}}{\texttt{t}-\texttt{mua}}}\right\}\right)\ /.\right.\right.\right.$$
$$\left.\left.\left\{\texttt{mua}\to\frac{-\texttt{AAA BBB}-\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2},\ \texttt{mub}\to\frac{-\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}}{1+\texttt{BBB}^2}\right\}\right)\right)-$$
$$\left.\left(\frac{(1+\texttt{BBB}^2)\ (1-\texttt{AAA}^2-2\ \texttt{AAA BBB t}-(1+\texttt{BBB}^2)\ \texttt{t}^2)}{\left(\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}+\texttt{t}+\texttt{BBB}^2\ \texttt{t}\right)^2}\right)\right]$$

0

from this follows that since the denominator is the square of a positive quantity

$$\left\{\sqrt{\frac{\texttt{mub}-\texttt{t}}{\texttt{t}-\texttt{mua}}}\to\frac{\sqrt{1+\texttt{BBB}^2}\ \sqrt{1-\texttt{AAA}^2-2\ \texttt{AAA BBB t}-(1+\texttt{BBB}^2)\ \texttt{t}^2}}{\left(\texttt{AAA BBB}+\sqrt{1-\texttt{AAA}^2+\texttt{BBB}^2}+\texttt{t}+\texttt{BBB}^2\ \texttt{t}\right)}\right\}$$

```
Reduce[{AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t < 0 && -Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
  -AAA BBB/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -AAA BBB/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}, {BBB, AAA, t}, Reals]
```

False

$$\left\{(1+\xi^2) \to \frac{2\sqrt{1-AAA^2+BBB^2}}{(1+BBB^2)\,(-mua+t)} \to \frac{2\sqrt{1-AAA^2+BBB^2}}{AAA\,BBB+\sqrt{1-AAA^2+BBB^2}+t+BBB^2\,t}\right\}$$

```
Simplify[((Together[(1 + ξ^2) /. {ξ^2 → (mub - t)/(t - mua)}]) /. {(-mua + mub) → 2 Sqrt[1 - AAA^2 + BBB^2]/(1 + BBB^2)}) -
  2 Sqrt[1 - AAA^2 + BBB^2]/((1 + BBB^2) (-mua + t))]
```

0

```
Simplify[((((1 + ξ^2)) /. {ξ → Sqrt[(mub - t)/(t - mua)]}) /. {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2),
    mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) - 2 Sqrt[1 - AAA^2 + BBB^2]/(AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
```

0

```
Simplify[2 Sqrt[1 - AAA^2 + BBB^2]/(AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t) -
  ((2 Sqrt[1 - AAA^2 + BBB^2]/((1 + BBB^2) (-mua + t))) /. {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)})]
```

0

$$\left\{1/(1+\xi^2) \to \frac{AAA\,BBB+\sqrt{1-AAA^2+BBB^2}+t+BBB^2\,t}{2\sqrt{1-AAA^2+BBB^2}}\right\}$$

$$\text{Simplify}\left[\left(\left(\left(1/\left(1+\xi^2\right)\right)\ /.\ \left\{\xi\to\sqrt{\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}}\right\}\right)\ /.\ \left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2},\ \text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}}{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}\right]$$

0

$$\left\{(\text{t}-\text{mua})\to\frac{-\text{mua}+\text{mub}}{1+\xi^2}\to\frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}}{1+\text{BBB}^2}\right\}$$

$$\text{Simplify}\left[\text{Together}\left[(\text{t}-\text{mua})\ /.\ \left\{\text{t}\to\frac{\text{mub}+\text{mua}\ \xi^2}{1+\xi^2}\right\}\right]-\frac{-\text{mua}+\text{mub}}{1+\xi^2}\right]$$

0

$$\text{Simplify}\left[\text{Together}\left[(\text{t}-\text{mua})\ /.\ \left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right]-\frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}}{1+\text{BBB}^2}\right]$$

0

$$\left\{(\text{mub}-\text{t})\to\frac{(-\text{mua}+\text{mub})\ \xi^2}{1+\xi^2}\to\frac{\sqrt{1-\text{AAA}^2+\text{BBB}^2}-\text{AAA BBB}-\left(1+\text{BBB}^2\right)\ \text{t}}{1+\text{BBB}^2}\right\}$$

$$\text{Simplify}\left[\text{Together}\left[(\text{mub}-\text{t})\ /.\ \left\{\text{t}\to\frac{\text{mub}+\text{mua}\ \xi^2}{1+\xi^2}\right\}\right]-\frac{(-\text{mua}+\text{mub})\ \xi^2}{1+\xi^2}\right]$$

0

$$\text{Simplify}\left[\text{Together}\left[(\text{mub}-\text{t})\ /.\ \left\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right]-\frac{\sqrt{1-\text{AAA}^2+\text{BBB}^2}-\text{AAA BBB}-\left(1+\text{BBB}^2\right)\ \text{t}}{1+\text{BBB}^2}\right]$$

0

$$\left\{(1-\text{mua})\to\frac{1+\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

```
Simplify[Together[(1 - mua) /. {mua → (-AAA BBB - √(1 - AAA² + BBB²))/(1 + BBB²)}] -
  (1 + AAA BBB + BBB² + √(1 - AAA² + BBB²))/(1 + BBB²)]
```

0

$$\left\{(1-\text{mub}) \to \frac{1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

```
Simplify[Together[(1 - mub) /. {mub → (-AAA BBB + √(1 - AAA² + BBB²))/(1 + BBB²)}] -
  (1 + AAA BBB + BBB² - √(1 - AAA² + BBB²))/(1 + BBB²)]
```

0

$$\left\{(1-\text{mua})\,(1-\text{mub}) \to \frac{(\text{AAA}+\text{BBB})^2}{1+\text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1-\text{mua})\,(1-\text{mub})} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]}{\sqrt{1+\text{BBB}^2}}\right\}$$

```
Simplify[Together[ExpandAll[((1 - mua) (1 - mub)) /.
     {mua → (-AAA BBB - √(1 - AAA² + BBB²))/(1 + BBB²), mub → (-AAA BBB + √(1 - AAA² + BBB²))/(1 + BBB²)}]] - (AAA + BBB)²/(1 + BBB²)]
```

0

$$\left\{\left(\frac{1-\text{mua}}{1-\text{mub}}\right) \to \frac{(\text{AAA}+\text{BBB})^2\left(1+\text{BBB}^2\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\hat{}\,2}\right\}$$

$$\left\{\sqrt{\frac{1-\text{mua}}{1-\text{mub}}} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]\left(\sqrt{1+\text{BBB}^2}\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

```
Simplify[Simplify[Together[ExpandAll[
      (((1 - mua)/(1 - mub)) /. {mua → (-AAA BBB - √(1 - AAA² + BBB²))/(1 + BBB²), mub → (-AAA BBB + √(1 - AAA² + BBB²))/(1 + BBB²)})]]] -
  ((AAA + BBB)² (1 + BBB²))/((1 + AAA BBB + BBB² - √(1 - AAA² + BBB²)) ^ 2)]
```

0

Simplify[Expand[$\left(1 + \text{AAA BBB} + \text{BBB}^2 + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}\right)\left(1 + \text{AAA BBB} + \text{BBB}^2 - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}\right)$]]

$(\text{AAA} + \text{BBB})^2\,(1 + \text{BBB}^2)$

**Reduce[{$1 + \text{AAA BBB} + \text{BBB}^2 + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2} < 0$ && $-\sqrt{1 + \text{BBB}^2} < \text{AAA} < \sqrt{1 + \text{BBB}^2}$},
{BBB, AAA}, Reals]**

**Reduce[{$1 + \text{AAA BBB} + \text{BBB}^2 - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2} < 0$ && $-\sqrt{1 + \text{BBB}^2} < \text{AAA} < \sqrt{1 + \text{BBB}^2}$},
{BBB, AAA}, Reals]**

False

False

Reduce[{AAA + BBB > 0 && $-\sqrt{1 + \text{BBB}^2} < \text{AAA} < \sqrt{1 + \text{BBB}^2}$}, {BBB, AAA}, Reals]

$-\text{BBB} < \text{AAA} < \sqrt{1 + \text{BBB}^2}$

$$\left\{(1 + \text{mua}) \to \frac{1 - \text{AAA BBB} + \text{BBB}^2 - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}$$

**Simplify[Together[(1 + mua) /. {mua → $\frac{-\text{AAA BBB} - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}$}] -
$\frac{1 - \text{AAA BBB} + \text{BBB}^2 - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}$]**

0

$$\left\{(1 + \text{mub}) \to \frac{1 - \text{AAA BBB} + \text{BBB}^2 + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}$$

**Simplify[Together[(1 + mub) /. {mub → $\frac{-\text{AAA BBB} + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}$}] -
$\frac{1 - \text{AAA BBB} + \text{BBB}^2 + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}$]**

0

$$\left\{(1 + \text{mua})\,(1 + \text{mub}) \to \frac{(\text{AAA} - \text{BBB})^2}{1 + \text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1 + \text{mua})\,(1 + \text{mub})} \to \frac{\text{Abs}[\text{AAA} - \text{BBB}]}{\sqrt{1 + \text{BBB}^2}}\right\}$$

$$\text{Simplify}\left[\left(((1+\text{mua})\ (1+\text{mub}))-\frac{(\text{AAA}-\text{BBB})^2}{1+\text{BBB}^2}\right)\ /.\ \left\{(1+\text{mub})\to\frac{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2},\ (1+\text{mua})\to\frac{1-\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right]$$

0

$$\left\{\frac{1+\text{mua}}{1+\text{mub}}\to\frac{(\text{AAA}-\text{BBB})^2\ \left(1+\text{BBB}^2\right)}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)^2}\right\}$$

$$\left\{\sqrt{\frac{1+\text{mua}}{1+\text{mub}}}\to\frac{\text{Abs}[\text{AAA}-\text{BBB}]\ \sqrt{1+\text{BBB}^2}}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

$$\text{Simplify}\left[\left(\left(\frac{1+\text{mua}}{1+\text{mub}}\right)\ /.\ \left\{(1+\text{mua})\to\frac{1-\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2},\ (1+\text{mub})\to\frac{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\frac{(\text{AAA}-\text{BBB})^2\ \left(1+\text{BBB}^2\right)}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)^2}\right]$$

0

$$\text{Reduce}\left[\left\{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}<0\ \&\&\ -\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\right\},\ \{\text{BBB},\ \text{AAA}\},\ \text{Reals}\right]$$

False

$$\text{Reduce}\left[\left\{1-\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}<0\ \&\&\ -\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\right\},\ \{\text{BBB},\ \text{AAA}\},\ \text{Reals}\right]$$

False

$$\text{Reduce}\left[\left\{\text{AAA}-\text{BBB}<0\ \&\&\ -\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\right\},\ \{\text{BBB},\ \text{AAA}\},\ \text{Reals}\right]$$

$$-\sqrt{1+\text{BBB}^2}<\text{AAA}<\text{BBB}$$

$$\left\{\text{Jacob1A}\to-\frac{4\sqrt{1-\text{AAA}^2+\text{BBB}^2}\ \xi}{\left(1+\text{BBB}^2\right)\ \left(1+\xi^2\right)^2}\right\}$$

$$\text{FullSimplify}\left[\left(\left(D\left[\frac{\text{mub}+\text{mua}\,\xi^2}{1+\xi^2},\xi\right]\right)\;/.\;\left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2},\ \text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\left(-\frac{4\sqrt{1-\text{AAA}^2+\text{BBB}^2}\;\xi}{\left(1+\text{BBB}^2\right)\left(1+\xi^2\right)^2}\right),\ \text{Assumptions}\to\left\{\text{mua}<\text{mub}\;\&\&\;\xi>0\;\&\&\;1-\text{AAA}^2+\text{BBB}^2>0\right\}\right]$$

0

TABLE OF THE IDENTITIES

$$\left\{\left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\ \ (*\ \to\ \text{mua}\ *)\right\},\right.$$

$$\left.\left\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\ \ (*\ \to\ \text{mub}\ *)\right\}\right\}$$

$$\left\{\xi\to\sqrt{\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}}\ ,\qquad \text{t}\to\frac{\text{mub}+\text{mua}\,\xi^2}{1+\xi^2}\right\}$$

$$\left\{(\text{mua}-\text{mub})\to-\frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{\text{mua}\,\hat{}\,2\to\left(\frac{1-\text{AAA}^2}{1+\text{BBB}^2}-\frac{2\ \text{AAA BBB mua}}{1+\text{BBB}^2}\right)\right\}\quad\left\{\text{mub}\,\hat{}\,2\to\left(\frac{1-\text{AAA}^2}{1+\text{BBB}^2}-\frac{2\ \text{AAA BBB mub}}{1+\text{BBB}^2}\right)\right\}$$

$$\left\{\left(\sqrt{1+\text{BBB}^2}\ \sqrt{(\text{t}-\text{mua})\ (\text{mub}-\text{t})}\right)\to\sqrt{1-\text{AAA}^2-2\ \text{AAA BBB t}-\text{t}^2-\text{BBB}^2\,\text{t}^2}\right\}$$

$$\left\{\left(\sqrt{1+\text{BBB}^2}\ \sqrt{(\text{t}-\text{mua})\ (\text{mub}-\text{t})}\right)\to\frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}\ \xi}{\sqrt{1+\text{BBB}^2}\ \left(1+\xi^2\right)}\right\}$$

$$\left\{\xi^2\ \to\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}\ \to\frac{\left(1+\text{BBB}^2\right)\left(1-\text{AAA}^2-2\ \text{AAA BBB t}-\left(1+\text{BBB}^2\right)\text{t}^2\right)}{\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\right)^2}\right\}$$

$$\left\{\sqrt{\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}}\ \to\frac{\sqrt{1+\text{BBB}^2}\ \sqrt{1-\text{AAA}^2-2\ \text{AAA BBB t}-\left(1+\text{BBB}^2\right)\text{t}^2}}{\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\right)}\right\}$$

$$\left\{\left(1+\xi^2\right) \to \frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{\left(1+\text{BBB}^2\right)\,(-\text{mua}+\text{t})} \to \frac{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}\right\}$$

$$\left\{1\Big/\left(1+\xi^2\right) \to \frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}{2\sqrt{1-\text{AAA}^2+\text{BBB}^2}}\right\}$$

$$\left\{(\text{t}-\text{mua}) \to \frac{-\text{mua}+\text{mub}}{1+\xi^2} \to \frac{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}}{1+\text{BBB}^2}\right\}$$

$$\left\{(\text{mub}-\text{t}) \to \frac{(-\text{mua}+\text{mub})\,\xi^2}{1+\xi^2} \to \frac{\sqrt{1-\text{AAA}^2+\text{BBB}^2}-\text{AAA BBB}-\left(1+\text{BBB}^2\right)\,\text{t}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mua}) \to \frac{1+\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mub}) \to \frac{1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1-\text{mua})\,(1-\text{mub}) \to \frac{(\text{AAA}+\text{BBB})^2}{1+\text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1-\text{mua})\,(1-\text{mub})} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]}{\sqrt{1+\text{BBB}^2}}\right\}$$

$$\left\{\left(\frac{1-\text{mua}}{1-\text{mub}}\right) \to \frac{(\text{AAA}+\text{BBB})^2\,\left(1+\text{BBB}^2\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\text{^}2}\right\}$$

$$\left\{\sqrt{\frac{1-\text{mua}}{1-\text{mub}}} \to \frac{\text{Abs}[\text{AAA}+\text{BBB}]\,\left(\sqrt{1+\text{BBB}^2}\right)}{\left(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

$$\left\{(1+\text{mua}) \to \frac{1-\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1+\text{mub}) \to \frac{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}$$

$$\left\{(1+\text{mua})\,(1+\text{mub}) \to \frac{(\text{AAA}-\text{BBB})^2}{1+\text{BBB}^2}\right\}$$

$$\left\{\sqrt{(1+\text{mua})\,(1+\text{mub})}\to\frac{\text{Abs}[\text{AAA}-\text{BBB}]}{\sqrt{1+\text{BBB}^2}}\right\}$$

$$\left\{\frac{1+\text{mua}}{1+\text{mub}}\to\frac{(\text{AAA}-\text{BBB})^2\,(1+\text{BBB}^2)}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)^2}\right\}$$

$$\left\{\sqrt{\frac{1+\text{mua}}{1+\text{mub}}}\to\frac{\text{Abs}[\text{AAA}-\text{BBB}]\,\sqrt{1+\text{BBB}^2}}{\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)}\right\}$$

$$\left\{\text{Jacob1A}\to-\frac{4\,\sqrt{1-\text{AAA}^2+\text{BBB}^2}\;\xi}{(1+\text{BBB}^2)\,(1+\xi^2)^2}\right\}$$

Since

$$\left(\left(\text{Jacob1A}\Big/\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB t}-\text{t}^2-\text{BBB}^2\,\text{t}^2}\right)\;/.\right.$$

$$\left.\left\{\frac{1}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB t}-\text{t}^2-\text{BBB}^2\,\text{t}^2}}\to\frac{\sqrt{1+\text{BBB}^2}\,(1+\xi^2)}{2\,\sqrt{1-\text{AAA}^2+\text{BBB}^2}\;\xi},\;\text{Jacob1A}\to-\frac{4\,\sqrt{1-\text{AAA}^2+\text{BBB}^2}\;\xi}{(1+\text{BBB}^2)\,(1+\xi^2)^2}\right\}\right)$$

the full integrand, expressed in terms of ξ and t, becomes

$$(\text{NumRmnIntgrndF1Ab}[\text{t}]\;/\;\text{DenRmnIntgrndF1Ab}[\text{t}])\,*\left(-\frac{2}{\sqrt{1+\text{BBB}^2}\,(1+\xi^2)}\right)$$

```
newintegrandF1AbCsiT[ξ_, t_] :=
  -(t^2 (BBB + AAA t) (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β])) /
   (12 Sqrt[1 + BBB^2] (1 - t^2) (1 + ξ^2));

DenRmnIntgrndF1Ab[t]
```

We write
NumRmnIntgrndF1Ab[t]/DenRmnIntgrndF1Ab[t]
as

$$\frac{\text{NumRmnIntgrndF1Ab}[t]}{\text{DenRmnIntgrndF1Ab}[t]}=\text{P[t]}+\frac{\text{Q1}}{1-t}+\frac{\text{Q2}}{1+t}$$

with P[t_] = p0 + p1 t + p2 t^2 + p3 t^3.

we have that

$\int$rmnIntgrndF1Ab[t] dt =

$$\int \frac{\text{NumRmnIntgrndF1Ab}[t]}{\text{DenRmnIntgrndF1Ab}[t] * \sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}\,dt = \int \frac{P[t]+\frac{\text{Q1}}{1-t}+\frac{\text{Q2}}{1+t}}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}\,dt=$$

$$\int \left(P[t] + \frac{\text{Q1}}{1-t} + \frac{\text{Q2}}{1+t}\right)\left(-\frac{2}{\sqrt{1+\text{BBB}^2}\,(1+\xi^2)}\right) d\xi \quad \text{where} \quad t = t[\xi]\ .$$

The last integrals is written as

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\left(\int (P[t[\xi]])\left(\frac{1}{(1+\xi^2)}\right) d\xi + \int\left(\frac{\text{Q1}}{1-t[\xi]}\right)\left(\frac{1}{(1+\xi^2)}\right) d\xi + \int\left(\frac{\text{Q2}}{1+t[\xi]}\right)\left(\frac{1}{(1+\xi^2)}\right) d\xi\right) =$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}(\text{ contr11}[\xi[t]] + \text{contr22}[\xi[t]] + \text{contr33}[\xi[t]] =$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}(\text{ CONTR11NOTSIMPL}[t] + \text{CONTR22NOTSIMPL}[t] + \text{ CONTR22NOTSIMPL}[t]]\ )$$

The t-derivative of the last sum yields

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR11NOTSIMPL[t],t]} = \frac{-2}{\sqrt{1+\text{BBB}^2}}\,(\text{P}[t])\left(\frac{1}{(1+\xi[t]^2)}\right)\text{D}[\xi[t], t] = \frac{P[t]}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR22NOTSIMPL[t],t]}= \frac{-2}{\sqrt{1+\text{BBB}^2}}\left(\frac{\text{Q1}}{1-t}\right)\left(\frac{1}{(1+\xi[t]^2)}\right)\text{D}[\xi[t], t]=\frac{\frac{\text{Q1}}{1-t}}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR33NOTSIMPL[t],t]} = \frac{-2}{\sqrt{1+\text{BBB}^2}}\left(\frac{\text{Q2}}{1=t}\right)\left(\frac{1}{(1+\xi[t]^2)}\right)\text{D}[\xi[t], t]=\frac{\frac{\text{Q2}}{1+t}}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}.$$

We finally have

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR11NOTSIMPL[t],t]}= \frac{P[t]}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}},$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR22NOTSIMPL[t],t]}=\frac{\frac{\text{Q1}}{1-t}}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}$$

$$\frac{-2}{\sqrt{1+\text{BBB}^2}}\text{D[CONTR33NOTSIMPL[t],t]} =\frac{\frac{\text{Q2}}{1+t}}{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}$$

```
P[t_] = p0 + p1 t + p2 t^2 + p3 t^3;
cf1a = Simplify[
   CoefficientList[P[t] (1 - t^2) + Q1 (1 + t) + Q2 (1 - t) - NumRmnIntgrndF1Ab[t], t]] ;
cf1a[[
  6]]
```

$$\frac{1}{8}\,(-8\,\text{p3} - 2\,\text{AAA b r Cos}[\beta]\,\text{Cot}[\beta] + \text{AAA b r Sin}[\beta])$$

```
Simplify[TrigExpand[
  Simplify[Solve[{cf1a[[1]] == 0 && cf1a[[2]] == 0 && cf1a[[3]] == 0 && cf1a[[4]] == 0 &&
      cf1a[[5]] == 0 && cf1a[[6]] == 0}, {p0, p1, p2, p3, Q1, Q2}]]]]
```

$$\left\{p0 \to -\frac{1}{48} \text{Csc}[\beta]\ (9\ b\ BBB\ r + 3\ b\ BBB\ r\ \text{Cos}[2\ \beta] + 8\ a\ AAA\ \text{Sin}[2\ \beta]),\right.$$
$$p1 \to -\frac{1}{48} \text{Csc}[\beta]\ (9\ AAA\ b\ r + 3\ AAA\ b\ r\ \text{Cos}[2\ \beta] + 8\ a\ BBB\ \text{Sin}[2\ \beta]),$$
$$p2 \to -\frac{1}{48} \text{Csc}[\beta]\ (3\ b\ BBB\ r + 9\ b\ BBB\ r\ \text{Cos}[2\ \beta] + 8\ a\ AAA\ \text{Sin}[2\ \beta]),$$
$$p3 \to -\frac{1}{16} AAA\ b\ r\ (1 + 3\ \text{Cos}[2\ \beta])\ \text{Csc}[\beta],$$
$$Q1 \to \frac{1}{96} (AAA + BBB)\ \text{Csc}[\beta]\ (9\ b\ r + 3\ b\ r\ \text{Cos}[2\ \beta] + 8\ a\ \text{Sin}[2\ \beta]),$$
$$\left. Q2 \to -\frac{1}{96} (AAA - BBB)\ \text{Csc}[\beta]\ (9\ b\ r + 3\ b\ r\ \text{Cos}[2\ \beta] - 8\ a\ \text{Sin}[2\ \beta])\right\}$$

checks

```
Simplify[NumRmnIntgrndF1Ab[t] / DenRmnIntgrndF1Ab[t] - ((P[t] + Q1 / (1 - t) + Q2 / (1 + t)) /.
    {p0 -> -1/48 Csc[β] (9 b BBB r + 3 b BBB r Cos[2 β] + 8 a AAA Sin[2 β]),
     p1 -> -1/48 Csc[β] (9 AAA b r + 3 AAA b r Cos[2 β] + 8 a BBB Sin[2 β]),
     p2 -> -1/48 Csc[β] (3 b BBB r + 9 b BBB r Cos[2 β] + 8 a AAA Sin[2 β]),
     p3 -> -1/16 AAA b r (1 + 3 Cos[2 β]) Csc[β],
     Q1 -> 1/96 (AAA + BBB) Csc[β] (9 b r + 3 b r Cos[2 β] + 8 a Sin[2 β]),
     Q2 -> -1/96 (AAA - BBB) Csc[β] (9 b r + 3 b r Cos[2 β] - 8 a Sin[2 β])})]
```

0

The integrand takes the form

$$-\frac{2}{\sqrt{1+BBB^2}} * \left(\frac{1}{(1+\xi^2)} P[t] + \frac{1}{(1+\xi^2)} \frac{Q1}{1-t} + \frac{1}{(1+\xi^2)} \frac{Q2}{1+t}\right)$$

```
Simplify[((-2/Sqrt[1 + BBB^2] * (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /.
   {p0 → -1/48 Csc[β] (9 b BBB r + 3 b BBB r Cos[2 β] + 8 a AAA Sin[2 β]),
    p1 → -1/48 Csc[β] (9 AAA b r + 3 AAA b r Cos[2 β] + 8 a BBB Sin[2 β]),
    p2 → -1/48 Csc[β] (3 b BBB r + 9 b BBB r Cos[2 β] + 8 a AAA Sin[2 β]),
    p3 → -1/16 AAA b r (1 + 3 Cos[2 β]) Csc[β],
    Q1 → 1/96 (AAA + BBB) Csc[β] (9 b r + 3 b r Cos[2 β] + 8 a Sin[2 β]),
    Q2 → -1/96 (AAA - BBB) Csc[β] (9 b r + 3 b r Cos[2 β] - 8 a Sin[2 β])}) -
  (-NumRmnIntgrndF1Ab[t] / DenRmnIntgrndF1Ab[t]) * 2/(Sqrt[1 + BBB^2] (1 + ξ^2))]
```

0

We evaluate the integrals of the three contributions putting aside the factor $-\frac{2}{\sqrt{1+BBB^2}}$

1st integral

```
Integrate[((1/(1 + ξ^2) P[t]) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}), ξ]

contr1NotS[ξ_] := 1/48 (-(8 (mua - mub)^3 p3 ξ)/(1 + ξ^2)^3 + (2 (mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(1 + ξ^2)^2 -
    1/(1 + ξ^2) 3 (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ +
    3 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) + mua (8 p1 + 4 mub p2 + 3 mub^2 p3) +
       mub (8 p1 + 6 mub p2 + 5 mub^2 p3)) ArcTan[ξ]);
```

```
Simplify[D[contr11NotS[ξ], ξ] - (((1/(1 + ξ^2) P[t]) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)})),
 Assumptions -> {ξ > 0 && -1 < mua < mub < 1}]
```

0

the integral is separated into a contribution independent of the ArcTan plus a contribution proportional to ArcTan

```
CoefficientList[contr11NotS[ξ], ArcTan[ξ]]
```

```
contr11NotSAA[ξ_] := -((mua - mub)^3 p3 ξ)/(6 (1 + ξ^2)^3) + ((mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(24 (1 + ξ^2)^2) -
   1/(16 (1 + ξ^2)) (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ;
contr11NotSBB := 1/16 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) +
    mua (8 p1 + 4 mub p2 + 3 mub^2 p3) + mub (8 p1 + 6 mub p2 + 5 mub^2 p3));
```

contr11NotSBB does NOT depend on ξ

```
contr11[ξ_] := (contr11NotSAA[ξ]) + (contr11NotSBB * ArcTan[ξ] );
```

```
contr11[ξ_] := (-((mua - mub)^3 p3 ξ)/(6 (1 + ξ^2)^3) + ((mua - mub)^2 (6 p2 + 13 mua p3 + 5 mub p3) ξ)/(24 (1 + ξ^2)^2) -
    1/(16 (1 + ξ^2)) (mua - mub) (8 p1 + 10 mua p2 + 6 mub p2 + 11 mua^2 p3 + 8 mua mub p3 + 5 mub^2 p3) ξ) +
  1/16 (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) + mua (8 p1 + 4 mub p2 + 3 mub^2 p3) +
    mub (8 p1 + 6 mub p2 + 5 mub^2 p3)) ArcTan[ξ]
```

Derivative's check

```
Simplify[contr11NotS[ξ] - contr11[ξ]]
Simplify[D[contr11NotS[ξ], ξ] - ((1/(1 + ξ^2) P[t]) /. {t -> (mub + mua ξ^2)/(1 + ξ^2)}),
 Assumptions -> {ξ > 0 && 1 > mub && 1 > mua}]
```

0

0

2nd integral

```
Simplify[Together[(((1/(1 + ξ^2)) (Q1/(1 - t))) /. {t → (mub + mua ξ^2)/(1 + ξ^2)})] - Q1/(1 - mub + (1 - mua) ξ^2)]
```

0

```
Integrate[Q1/(1 - mub + (1 - mua) ξ^2), ξ, Assumptions → {ξ > 0 && 1 > mub && 1 > mua}]
```

```
Simplify[Integrate[(((1/(1 + ξ^2)) (Q1/(1 - t))) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}), ξ,
  Assumptions → {ξ > 0 && 1 > mub && 1 > mua} ], Assumptions → {ξ > 0 && 1 > mub && 1 > mua} ]
```

```
FullSimplify[- (Q1 ArcTan[(Sqrt[-1+mua] ξ)/Sqrt[-1+mub]])/(Sqrt[-1 + mua] Sqrt[-1 + mub]) - (Q1 ArcTan[Sqrt[(1-mua)/(1-mub)] ξ])/Sqrt[(1 - mua) (1 - mub)],
 Assumptions → {ξ > 0 && 1 > mub && 1 > mua}]
```

0

```
contr22NotS[ξ_] := (Q1 ArcTan[Sqrt[(1-mua)/(1-mub)] ξ])/Sqrt[(1 - mua) (1 - mub)];
```

Derivative' s check

```
Simplify[D[contr22NotS[ξ], ξ] - ((1/(1 + ξ^2) Q1/(1 - t)) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}),
  Assumptions → {ξ > 0 && 1 > mub && 1 > mua} ]
FullSimplify[D[contr22NotS[ξ], ξ] - ((1/(1 + ξ^2) Q1/(1 - t)) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}),
  Assumptions → {ξ > 0 && 1 > mub && 1 > mua} ]
```

0

3rd integral

```
Integrate[((1/(1 + ξ^2) Q2/(1 + t)) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}),
  ξ, Assumptions → {ξ > 0 && 1 > mub > -1 && 1 > mua > -1}]
```

```
contr33NotS[ξ_] := (Q2 ArcTan[(Sqrt[1+mua] ξ)/Sqrt[1+mub]])/Sqrt[(1 + mua) (1 + mub)];
```

FULL DERIVATIVE's check [OK]

```
Simplify[
  D[contr11NotSAA[ξ] + contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ], ξ] -
   (((1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t)) /. {t → (mub + mua ξ^2)/(1 + ξ^2)})),
  Assumptions → {-1 < mua < mub < 1}]
FullSimplify[
  D[contr11NotSAA[ξ] + contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ], ξ] -
   (((1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t)) /. {t → (mub + mua ξ^2)/(1 + ξ^2)})),
  Assumptions → {-1 < mua < mub < 1}]
```

The full ξ-primitive of rmnIntgrndF1Ab[t]

```
CsiprmtvF1ab[ξ_] := (-2/Sqrt[1 + BBB^2]) * (contr11NotSAA[ξ] +
     contr11NotSBB * ArcTan[ξ] + contr22NotS[ξ] + contr33NotS[ξ]); CsiprmtvF1ab[ξ]
```

```
CsiprmtvF1ab[ξ_] :=
```

$$-\frac{2}{\sqrt{1+BBB^2}}*\left(-\frac{(mua-mub)^3\,p3\,\xi}{6\,(1+\xi^2)^3}+\frac{(mua-mub)^2\,(6\,p2+13\,mua\,p3+5\,mub\,p3)\,\xi}{24\,(1+\xi^2)^2}-\frac{1}{16\,(1+\xi^2)}(mua-mub)\,(8\,p1+10\,mua\,p2+6\,mub\,p2+11\,mua^2\,p3+8\,mua\,mub\,p3+5\,mub^2\,p3)\,\xi+\frac{1}{16}(16\,p0+5\,mua^3\,p3+3\,mua^2\,(2\,p2+mub\,p3)+mua\,(8\,p1+4\,mub\,p2+3\,mub^2\,p3)+mub\,(8\,p1+6\,mub\,p2+5\,mub^2\,p3))\,ArcTan[\xi]+\frac{Q1\,ArcTan\left[\sqrt{\frac{1-mua}{1-mub}}\,\xi\right]}{\sqrt{(1-mua)\,(1-mub)}}+\frac{Q2\,ArcTan\left[\frac{\sqrt{1+mua}\,\xi}{\sqrt{1+mub}}\right]}{\sqrt{(1+mua)\,(1+mub)}}\right);$$

DERIVATIVE CHECK [it is OK, even though MATHEMATICA must be helped to find out the result!!]

$$FullSimplify\left[D[CsiprmtvF1ab[\xi],\xi]-\left(-\frac{2}{\sqrt{1+BBB^2}}\left(\left(\left(\frac{1}{(1+\xi^2)}P[t]+\frac{1}{(1+\xi^2)}\frac{Q1}{1-t}+\frac{1}{(1+\xi^2)}\frac{Q2}{1+t}\right)/.\left\{t\to\frac{mub+mua\,\xi^2}{1+\xi^2}\right\}\right)\right)\right),\right.$$
$$\left.Assumptions\to\{-1<mua<1\,\&\&\,-1<mub<1\,\&\&\,mua<mub\,\&\&\,\xi>0\,\&\&\,1+BBB^2>0\}\right]$$

The result is multiplied by

$$\frac{1}{2\,Q1}\left(\sqrt{(1+BBB^2)\,(-1+mua^2)\,(-1+mub)}\,\left(-1+mub+(-1+mua)\,\xi^2\right)\right)$$

$$\left(-\left(2\left(-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}+\sqrt{\left(-1+\text{mua}^2\right)\,(-1+\text{mub})}+\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\;\text{mub}\right)\text{Q1}\right)\Big/\right.$$

$$\left.\left(\sqrt{\left(1+\text{BBB}^2\right)\left(-1+\text{mua}^2\right)(-1+\text{mub})}\;\left(-1+\text{mub}+(-1+\text{mua})\,\xi^2\right)\right)\right)*$$

$$\left(\frac{1}{2\,\text{Q1}}\left(\sqrt{\left(1+\text{BBB}^2\right)\left(-1+\text{mua}^2\right)(-1+\text{mub})}\;\left(-1+\text{mub}+(-1+\text{mua})\,\xi^2\right)\right)\right)$$

$$\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)\,(-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\;\text{mub}$$

The above expression is equal to zero, even though MATHEMATICA seems unable to realize this property. In fact,

$$\text{Simplify}\left[\left(\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)(-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\;\text{mub}\right)-\right.$$

$$\sqrt{1+\text{mua}}\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{(1-\text{mua})\,(1-\text{mub})}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\;\text{mub}\right),$$

$$\left.\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]$$

$$\text{Simplify}\left[\left(\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)(-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\;\text{mub}\right)-\right.$$

$$\left(\sqrt{\frac{1-\text{mua}^2}{1-\text{mub}}}-\sqrt{\left(1-\text{mua}^2\right)(1-\text{mub})}-\sqrt{\frac{1-\text{mua}^2}{1-\text{mub}}}\;\text{mub}\right),$$

$$\left.\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]$$

$$\text{Simplify}\left[\sqrt{1+\text{mua}}\;*\right.$$

$$\left(\text{Simplify}\left[\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\;\text{mub}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]-\right.$$

$$\left.\left.\sqrt{(1-\text{mua})\,(1-\text{mub})}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]$$

$$\text{Simplify}\left[\left(\text{Simplify}\left[\left(\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}-\sqrt{\frac{1-\text{mua}}{1-\text{mub}}}\;\text{mub}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]-\right.\right.$$

$$\left.\left.\sqrt{(1-\text{mua})\,(1-\text{mub})}\right),\text{Assumptions}\to\{-1<\text{mua}<\text{mub}<1\}\right]$$

In the above expression the first addend is positive because 1>mu and 1>ma and can be written as

$$-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub}) == \sqrt{(-1+\text{mua})\ (-1+\text{mub})}$$

Squaring one gets

$$\left(-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ (-1+\text{mub})\right)\text{^}2 == \left(\sqrt{(-1+\text{mua})\ (-1+\text{mub})}\right)\text{^}2$$

True

It is surprising the MATHEMATICA does not well handle radicals

$$\left\{\left(-\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}+\sqrt{(-1+\text{mua})\ (-1+\text{mub})}+\sqrt{\frac{-1+\text{mua}}{-1+\text{mub}}}\ \text{mub}\right)\to 0\right\}$$

$$\left\{\frac{\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}-\sqrt{\left(-1+\text{mua}^2\right)\ (-1+\text{mub})}-\sqrt{\frac{-1+\text{mua}^2}{-1+\text{mub}}}\ \text{mub}}{\sqrt{1-\text{mua}}}\to 0\right\}$$

If we set $\xi = \xi[t]$ inside CsiprmtvF1ab[ξ], we get CsiprmtvF1ab[ξ[t]]. The t-derivative yields D[CsiprmtvF1ab[ξ[t]],t]=D[CsiprmtvF1ab[ξ],ξ]$|_{\xi=\xi[t]}$ D[ξ[t],t]. In this way we find that :

$$\text{D[CsiprmtvF1ab}[\xi],\xi]|_{\xi=\xi[t]} = \frac{\text{D[CsiprmtvF1ab}[\xi[t]],t]}{\text{D}[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\ \text{P[t]}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q1}}{1-t}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

The first equality

$$\text{D[CsiprmtvF1ab}[\xi],\xi]|_{\xi=\xi[t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\ \text{P[t]}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q1}}{1-t}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

is verified.

$$\text{FullSimplify}\left[(\text{D[CsiprmtvF1ab}[\xi],\ \xi])\ /.\ \left\{\xi\to\sqrt{\frac{\text{mub}-t}{t-\text{mua}}}\right\},\ \text{Assumptions}\to\{-1<\text{mua}<t<\text{mub}<1\}\right]$$

$$\text{FullSimplify}\left[\left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\ \text{P[t]}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q1}}{1-t}+\frac{1}{(1+\xi^2)}\ \frac{\text{Q2}}{1+t}\right)\right)\ /.\ \left\{\xi\to\sqrt{\frac{\text{mub}-t}{t-\text{mua}}}\right\},\ \text{Assumptions}\to\{-1<\text{mua}<t<\text{mub}<1\}\right]$$

```
Simplify[FullSimplify[(D[CsiprmtvF1ab[ξ], ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}] - FullSimplify[
   (-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}]]
```

0

The second equality

$$\frac{D[\text{CsiprmtvF1ab}[\xi[t]],t]}{D[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\,\text{P[t]} + \frac{1}{(1+\xi^2)}\,\frac{\text{Q1}}{1-t} + \frac{1}{(1+\xi^2)}\,\frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

is somewhat more involved to be verified.

We write the equality under the form

$$D[\text{CsiprmtvF1ab}[\xi[t]], t] = \left(\left(-\frac{2}{\sqrt{1+\text{BBB}^2}}\left(\frac{1}{(1+\xi^2)}\,\text{P[t]} + \frac{1}{(1+\xi^2)}\,\frac{\text{Q1}}{1-t} + \frac{1}{(1+\xi^2)}\,\frac{\text{Q2}}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}\right)*$$

D[ξ[t], t]

```
FullSimplify[
 D[Simplify[(CsiprmtvF1ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}], t],
 Assumptions → {-1 < mua < t < mub < 1}]
```

```
FullSimplify[
 (FullSimplify[D[Sqrt[(mub - t)/(t - mua)], t], Assumptions → {-1 < ma < t < mub < 1}]) * FullSimplify[
   (-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /. {ξ → Sqrt[(mub - t)/(t - mua)]},
   Assumptions → {-1 < mua < t < mub < 1}], Assumptions → {-1 < mua < t < mub < 1}]
```

```
FullSimplify[FullSimplify[
  D[Simplify[(CsiprmtvF1ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}],
   t], Assumptions → {-1 < mua < t < mub < 1}] -
  FullSimplify[(FullSimplify[D[Sqrt[(mub - t)/(t - mua)], t], Assumptions → {-1 < ma < t < mub < 1}]) *
    FullSimplify[(-2/Sqrt[1 + BBB^2] (1/(1 + ξ^2) P[t] + 1/(1 + ξ^2) Q1/(1 - t) + 1/(1 + ξ^2) Q2/(1 + t))) /.
     {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}],
   Assumptions → {-1 < mua < t < mub < 1}], Assumptions → {-1 < mua < t < mub < 1}]
```

The above expression is equal to zero because the factor

$$\left(\left(\frac{-mub+t}{mua-t}\right)^{3/2}-\sqrt{\frac{(-mub+t)^3}{(mua-t)^3}}\right)$$

is equal to sero since $\sqrt{\frac{(-mub+t)^3}{(mua-t)^3}}$ can be written as $\left(\frac{-mub+t}{mua-t}\right)^{3/2}$

IN CONCLUSION WE HAVE HAVE VERIFIED THAT

$$D[CsiprmtvF1ab[\xi],\xi]|_{\xi=\xi[t]} = \frac{D[CsiprmtvF1ab[\xi[t]],t]}{D[\xi[t],t]} = \left(-\frac{2}{\sqrt{1+BBB^2}}\left(\frac{1}{(1+\xi^2)}P[t]+\frac{1}{(1+\xi^2)}\frac{Q1}{1-t}+\frac{1}{(1+\xi^2)}\frac{Q2}{1+t}\right)\right)\Bigg|_{\xi=\xi[t]}$$

HOLDS TRUE.

We go back to variable t using the transformations $\xi \to \sqrt{\frac{mub-t}{t-mua}}$

```
tprmtvF1abNotSimpl[t_] :=
  Simplify[(CsiprmtvF1ab[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}, Assumptions → {-1 < mua < t < mub < 1}];
ReducedtprmtvF1abNotSimpl[t_] :=
  (((tprmtvF1abNotSimpl[t]) /. {ArcTan[Sqrt[(-mub + t)/(mua - t)]] → atanaa}) /.
     {ArcTan[Sqrt[((-1 + mua) (-mub + t))/((-1 + mub) (mua - t))]] → atanbb}) /.
   {ArcTan[Sqrt[((1 + mua) (-mub + t))/((1 + mub) (mua - t))]] → atancc};
cfausxx = CoefficientList[ReducedtprmtvF1abNotSimpl[t], atancc];
cfausxx[[2]]
```

```
cfausyy = CoefficientList[cfausxx[[1]], atanbb];
cfausyy[[2]]

cfauszz = Simplify[CoefficientList[cfausyy[[1]], atanaa]];      cfauszz[[2]]
cfauszz[[1]]

f00[t_] :=
  FullSimplify[1/(24 Sqrt[1 + BBB^2]) (24 p1 + 18 mub p2 + 15 mua^2 p3 + 15 mub^2 p3 + 12 p2 t + 10 mub p3 t +
      8 p3 t^2 + 2 mua (9 p2 + 7 mub p3 + 5 p3 t)) (mua - t) Sqrt[(-mub + t)/(mua - t)]];

faa[t_] := FullSimplify[-1/(8 Sqrt[1 + BBB^2]) (16 p0 + 5 mua^3 p3 + 3 mua^2 (2 p2 + mub p3) +
      mua (8 p1 + 4 mub p2 + 3 mub^2 p3) + mub (8 p1 + 6 mub p2 + 5 mub^2 p3))] * ArcTan[Sqrt[(-mub + t)/(mua - t)]];

fbb[t_] := -(2 Q1)/(Sqrt[1 + BBB^2] Sqrt[(-1 + mua) (-1 + mub)]) * ArcTan[Sqrt[((-1 + mua) (-mub + t))/((-1 + mub) (mua - t))]];

fcc[t_] := Simplify[(-(2 Q2)/(Sqrt[1 + BBB^2] Sqrt[(1 + mua) (1 + mub)]))] * ArcTan[Sqrt[((1 + mua) (-mub + t))/((1 + mub) (mua - t))]];
```

```
Simplify[tprmtvF1abNotSimpl[t] - (f00[t] + faa[t] + fbb[t] + fcc[t])]
```

0

```
fcc[t]
```

$$\text{PrmitiveF1AbNotSimpl00[t\_] :=} \ \frac{-1}{24\sqrt{1+\text{BBB}^2}}\sqrt{(\text{mub}-t)\,(t-\text{mua})}\,\Big(24\,\text{p1}+18\,\text{mua}\,\text{p2}+18\,\text{mub}\,\text{p2}+15\,\text{mua}^2\,\text{p3}+14\,\text{mua}\,\text{mub}\,\text{p3}+15\,\text{mub}^2\,\text{p3}+2\,(6\,\text{p2}+5\,(\text{mua}+\text{mub})\,\text{p3})\,t+8\,\text{p3}\,t^2\Big);$$

$$\text{PrmitiveF1AbNotSimplaa[t\_] :=} -\frac{1}{8\sqrt{1+\text{BBB}^2}}\Big(16\,\text{p0}+5\,\text{mua}^3\,\text{p3}+3\,\text{mua}^2\,(2\,\text{p2}+\text{mub}\,\text{p3})+\text{mua}\,\big(8\,\text{p1}+4\,\text{mub}\,\text{p2}+3\,\text{mub}^2\,\text{p3}\big)+\text{mub}\,\big(8\,\text{p1}+6\,\text{mub}\,\text{p2}+5\,\text{mub}^2\,\text{p3}\big)\Big)\,\text{ArcTan}\Big[\sqrt{\frac{-\text{mub}+t}{\text{mua}-t}}\Big];$$

$$\text{PrmitiveF1AbNotSimplbb[t\_] :=} -\frac{2\,\text{Q1}\,\text{ArcTan}\Big[\sqrt{\frac{(-1+\text{mua})\,(-\text{mub}+t)}{(-1+\text{mub})\,(\text{mua}-t)}}\Big]}{\sqrt{1+\text{BBB}^2}\,\sqrt{(-1+\text{mua})\,(-1+\text{mub})}};$$

$$\text{PrmitiveF1AbNotSimplcc[t\_] :=} -\frac{2\,\text{Q2}\,\text{ArcTan}\Big[\sqrt{\frac{(1+\text{mua})\,(-\text{mub}+t)}{(1+\text{mub})\,(\text{mua}-t)}}\Big]}{\sqrt{1+\text{BBB}^2}\,\sqrt{(1+\text{mua})\,(1+\text{mub})}};$$

$$\left\{\left(\sqrt{1+\text{BBB}^2}\ \sqrt{(\text{t}-\text{mua})\ (\text{mub}-\text{t})}\right)\to\sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\text{t}^2-\text{BBB}^2\ \text{t}^2}\right\}$$

$$\left\{\text{mua}\to\frac{-\text{AAA}\ \text{BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\ \ (*\to\text{mua}\ *)\right\},$$

$$\left\{\text{mub}\to\frac{-\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\ (*\to\text{mub}\ *)\right\}$$

$$\left\{\text{p0}\to-\frac{1}{48}\text{Csc}[\beta]\ (9\ \text{b}\ \text{BBB}\ \text{r}+3\ \text{b}\ \text{BBB}\ \text{r}\ \text{Cos}[2\ \beta]+8\ \text{a}\ \text{AAA}\ \text{Sin}[2\ \beta]),\right.$$

$$\text{p1}\to-\frac{1}{48}\text{Csc}[\beta]\ (9\ \text{AAA}\ \text{b}\ \text{r}+3\ \text{AAA}\ \text{b}\ \text{r}\ \text{Cos}[2\ \beta]+8\ \text{a}\ \text{BBB}\ \text{Sin}[2\ \beta]),$$

$$\text{p2}\to-\frac{1}{48}\text{Csc}[\beta]\ (3\ \text{b}\ \text{BBB}\ \text{r}+9\ \text{b}\ \text{BBB}\ \text{r}\ \text{Cos}[2\ \beta]+8\ \text{a}\ \text{AAA}\ \text{Sin}[2\ \beta]),$$

$$\text{p3}\to-\frac{1}{16}\text{AAA}\ \text{b}\ \text{r}\ (1+3\ \text{Cos}[2\ \beta])\ \text{Csc}[\beta],$$

$$\text{Q1}\to\frac{1}{96}(\text{AAA}+\text{BBB})\ \text{Csc}[\beta]\ (9\ \text{b}\ \text{r}+3\ \text{b}\ \text{r}\ \text{Cos}[2\ \beta]+8\ \text{a}\ \text{Sin}[2\ \beta]),$$

$$\left.\text{Q2}\to-\frac{1}{96}(\text{AAA}-\text{BBB})\ \text{Csc}[\beta]\ (9\ \text{b}\ \text{r}+3\ \text{b}\ \text{r}\ \text{Cos}[2\ \beta]-8\ \text{a}\ \text{Sin}[2\ \beta])\right\},$$

$$\left\{-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\ \&\&\right.$$

$$\left.-\frac{\text{AAA}\ \text{BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}<\text{t}<-\frac{\text{AAA}\ \text{BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}\right\}$$

$$\left\{\sqrt{\frac{\text{mub}-\text{t}}{\text{t}-\text{mua}}}\to\frac{\sqrt{1+\text{BBB}^2}\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}}{\left(\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}\right)}\right\}$$

PrmitiveF1AbNotSimplbb[t]

we use the following three identities:

$$\left\{\sqrt{(\text{mub}-\text{t})\ (\text{t}-\text{mua})}\to\right\}\left\{\sqrt{\frac{-\text{mub}+\text{t}}{\text{mua}-\text{t}}}\to\left(\frac{\sqrt{1+\text{BBB}^2}\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}}{\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}}\right)\right\}$$

$$\left\{\sqrt{\frac{(-1+\text{mua})\ (-\text{mub}+\text{t})}{(-1+\text{mub})\ (\text{mua}-\text{t})}}\to\right.$$
$$\left(\sqrt{(\text{AAA}+\text{BBB})^2}\ \left(1+\text{BBB}^2\right)\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}\right)\Big/$$
$$\left.\left(\left(1+\text{AAA}\ \text{BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\left(\text{AAA}\ \text{BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\ \text{t}\right)\right)\right\}$$

$$\left\{\sqrt{\frac{(1+\text{mua})\ (-\text{mub}+\text{t})}{(1+\text{mub})\ (\text{mua}-\text{t})}}\to\left(\sqrt{(\text{AAA}-\text{BBB})^2}\ \left(1+\text{BBB}^2\right)\ \sqrt{1-\text{AAA}^2-2\ \text{AAA}\ \text{BBB}\ \text{t}-\left(1+\text{BBB}^2\right)\ \text{t}^2}\right)\Big/\right.$$
$$\left.\left(\left(1-\text{AAA}\ \text{BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\left(\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}\ (\text{AAA}+\text{BBB}\ \text{t})\right)\right)\right\}$$

$$\texttt{Simplify}\left[\left(\left(\left(\left(\sqrt{(\text{mub}-t)\ (-\text{mua}+t)}\right)\texttt{^}2\right)\ \texttt{/.}\ \left\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)\ \texttt{/.}\ \left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\left(\frac{\sqrt{1-\text{AAA}^2-2\ \text{AAA BBB}\ t-t^2-\text{BBB}^2\ t^2}}{\sqrt{1+\text{BBB}^2}}\right)\texttt{^}2\right]$$

0

$$\texttt{Simplify}\left[\left(\left(\left(\sqrt{\frac{-\text{mub}+t}{\text{mua}-t}}\right)\texttt{^}2\ \texttt{/.}\ \left\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)\ \texttt{/.}\ \left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\left(\frac{\sqrt{1+\text{BBB}^2}\ \sqrt{1-\text{AAA}^2-2\ \text{AAA BBB}\ t-\left(1+\text{BBB}^2\right)t^2}}{\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}^2\ t}\right)\texttt{^}2\right]$$

0

$$\texttt{Simplify}\Big[-\texttt{Expand}\left[\left(\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}^2\ t\right)*\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}^2\ t\right)\right]\Big/\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}^2\ t\right)\texttt{^}2\Big]$$

$$\frac{\left(1+\text{BBB}^2\right)\left(1-\text{AAA}^2-2\ \text{AAA BBB}\ t-\left(1+\text{BBB}^2\right)t^2\right)}{\left(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}^2\ t\right)^2}$$

$$\texttt{FullSimplify}\Big[\left(\left(\left(\sqrt{\frac{(1+\text{mua})\ (-\text{mub}+t)}{(1+\text{mub})\ (\text{mua}-t)}}\right)\texttt{^}2\ \texttt{/.}\ \left\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)\ \texttt{/.}\ \left\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\right\}\right)-\left(\left(\sqrt{(\text{AAA}-\text{BBB})^2}\ \left(1+\text{BBB}^2\right)\sqrt{1-\text{AAA}^2-2\ \text{AAA BBB}\ t-\left(1+\text{BBB}^2\right)t^2}\right)\Big/\left(\left(1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}\right)\left(\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}\ (\text{AAA}+\text{BBB}\ t)\right)\right)\right)\texttt{^}2,$$
$$\texttt{Assumptions}\to\left\{-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\ \texttt{\&\&}\ -\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}<t<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}\right\}\Big]$$

0

$$\text{FullSimplify}\Big[\Big(\Big(\Big(\sqrt{\frac{(-1+\text{mua})\,(-\text{mub}+\text{t})}{(-1+\text{mub})\,(\text{mua}-\text{t})}}\Big)\,\hat{}\,2\;/.\;\Big\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)\;/.\;\Big\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)-\Big(\Big(\sqrt{(\text{AAA}+\text{BBB})^2}\,\big(1+\text{BBB}^2\big)\,\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB t}-\big(1+\text{BBB}^2\big)\,\text{t}^2}\Big)\Big/\Big(\Big(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\Big)\Big(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\Big)\Big)\Big)\,\hat{}\,2,\;\text{Assumptions}\to\Big\{-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\;\&\&\;-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}<\text{t}<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}\Big\}\Big]$$

0

$$\text{FullSimplify}\Big[\Big(\Big(\Big(\sqrt{\frac{(-1+\text{mua})\,(-\text{mub}+\text{t})}{(-1+\text{mub})\,(\text{mua}-\text{t})}}\Big)\,\hat{}\,2\;/.\;\Big\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)\;/.\;\Big\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)-\Big(\Big(\sqrt{(\text{AAA}+\text{BBB})^2}\,\big(1+\text{BBB}^2\big)\,\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB t}-\big(1+\text{BBB}^2\big)\,\text{t}^2}\Big)\Big/\Big(\Big(1+\text{AAA BBB}+\text{BBB}^2-\sqrt{1-\text{AAA}^2+\text{BBB}^2}\Big)\Big(\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}+\text{t}+\text{BBB}^2\,\text{t}\Big)\Big)\Big)\,\hat{}\,2,\;\text{Assumptions}\to\Big\{-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\;\&\&\;-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}<\text{t}<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}\Big\}\Big]$$

0

$$\text{Reduce}\Big[\Big\{1-\text{AAA}^2-2\,\text{AAA BBB t}-\big(1+\text{BBB}^2\big)\,\text{t}^2<0\;\&\&-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\;\&\&\;-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}<\text{t}<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}\Big\},\;\{\text{BBB},\text{AAA},\text{t}\},\;\text{Reals}\Big]$$

False

$$\text{Reduce}\Big[\Big\{1-\text{AAA BBB}+\text{BBB}^2+\sqrt{1-\text{AAA}^2+\text{BBB}^2}<0\;\&\&-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\;\&\&\;-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}<\text{t}<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\big(1+\text{BBB}^2\big)^2}}\Big\},\;\{\text{BBB},\text{AAA},\text{t}\},\;\text{Reals}\Big]$$

False

$$\text{Reduce}\left[\left\{1+\text{BBB}\ (\text{AAA}+\text{BBB})-\sqrt{1-\text{AAA}^2+\text{BBB}^2}<0\ \&\&\ -\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\ \&\&\ -\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}<t<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}\right\},\ \{\text{BBB},\ \text{AAA},\ t\},\ \text{Reals}\right]$$

False

$$\text{Reduce}\Big[\Big\{\sqrt{1-\text{AAA}^2+\text{BBB}^2}+t+\text{BBB}\,(\text{AAA}+\text{BBB}\,t)<0\ \&\&\ -\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\ \&\&$$
$$-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}<t<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}\Big\},\ \{\text{BBB},\text{AAA},t\},\ \text{Reals}\Big]$$

False

$$\text{Simplify}\Big[\Big(\Big(\Big(\Big(\Big(\Big(\Big((\text{PrmitiveF1AbNotSimpl00}[t])\ /.$$
$$\Big\{\sqrt{(\text{mub}-t)\,(-\text{mua}+t)}\to\frac{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2\,t^2}}{\sqrt{1+\text{BBB}^2}}\Big\}\Big)\ /.$$
$$\Big\{\text{mub}\to\frac{-\text{AAA BBB}+\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)\ /.\ \Big\{\text{mua}\to\frac{-\text{AAA BBB}-\sqrt{1-\text{AAA}^2+\text{BBB}^2}}{1+\text{BBB}^2}\Big\}\Big)\ /.$$
$$\Big\{\text{p0}\to-\frac{1}{48}\,\text{Csc}[\beta]\,(9\,\text{b BBB r}+3\,\text{b BBB r}\,\text{Cos}[2\,\beta]+8\,\text{a AAA}\,\text{Sin}[2\,\beta])\Big\}\Big)\ /.$$
$$\Big\{\text{p1}\to-\frac{1}{48}\,\text{Csc}[\beta]\,(9\,\text{AAA b r}+3\,\text{AAA b r}\,\text{Cos}[2\,\beta]+8\,\text{a BBB}\,\text{Sin}[2\,\beta])\Big\}\Big)\ /.$$
$$\Big\{\text{p2}\to-\frac{1}{48}\,\text{Csc}[\beta]\,(3\,\text{b BBB r}+9\,\text{b BBB r}\,\text{Cos}[2\,\beta]+8\,\text{a AAA}\,\text{Sin}[2\,\beta])\Big\}\Big)\ /.$$
$$\Big\{\text{p3}\to-\frac{1}{16}\,\text{AAA b r}\,(1+3\,\text{Cos}[2\,\beta])\,\text{Csc}[\beta]\Big\}\Big),$$
$$\text{Assumptions}\to\Big\{-\sqrt{1+\text{BBB}^2}<\text{AAA}<\sqrt{1+\text{BBB}^2}\ \&\&$$
$$-\frac{\text{AAA BBB}}{1+\text{BBB}^2}-\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}<t<-\frac{\text{AAA BBB}}{1+\text{BBB}^2}+\sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{\left(1+\text{BBB}^2\right)^2}}\Big\}\Big]$$

```
Simplify[
 (((((((PrmitiveF1AbNotSimplaa[t]) /. {Sqrt[(-mub + t)/(mua - t)] ->
          (Sqrt[1 + BBB^2] Sqrt[(1 - AAA^2 - 2 AAA BBB t -
                (1 + BBB^2) t^2)]) / (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)}) /.
       {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
     {p0 -> -1/48 Csc[β] (9 b BBB r + 3 b BBB r Cos[2 β] + 8 a AAA Sin[2 β])}) /.
    {p1 -> -1/48 Csc[β] (9 AAA b r + 3 AAA b r Cos[2 β] + 8 a BBB Sin[2 β])}) /.
   {p2 -> -1/48 Csc[β] (3 b BBB r + 9 b BBB r Cos[2 β] + 8 a AAA Sin[2 β])}) /.
  {p3 -> -1/16 AAA b r (1 + 3 Cos[2 β]) Csc[β]}),
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
FullSimplify[
 ((((((PrmitiveF1AbNotSimplbb[t]) /. {{Sqrt[((-1 + mua) (-mub + t))/((-1 + mub) (mua - t))] -> (Sqrt[(AAA + BBB)^2] (1 + BBB^2)
          Sqrt[(1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2)]) / (((1 + AAA BBB + BBB^2 -
             Sqrt[1 - AAA^2 + BBB^2]) (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)))}}) /.
      {mub -> (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua -> (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
   {Q1 -> 1/96 (AAA + BBB) Csc[β] (9 b r + 3 b r Cos[2 β] + 8 a Sin[2 β])}),
 Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
Simplify[((1/48) ArcTan[((AAA + BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β])) -
  (((AAA + BBB) ArcTan[((1 + BBB^2) √(- (AAA + BBB)^2 (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))) /
        ((-1 - AAA BBB - BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t))]
      Csc[β] (9 b r + 3 b r Cos[2 β] + 8 a Sin[2 β])) /
    (48 Abs[AAA + BBB])), Assumptions → {AAA + BBB < 0}]
```

0

```
FullSimplify[
 (((((PrmitiveF1AbNotSimplcc[t]) /. {Sqrt[((1 + mua) (-mub + t))/((1 + mub) (mua - t))] → (Sqrt[(AAA - BBB)^2] (1 + BBB^2)
           Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) / ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2])
           (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))}) /.
      {mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /.
   {Q2 → -(1/96) (AAA - BBB) Csc[β] (9 b r + 3 b r Cos[2 β] - 8 a Sin[2 β])}),
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] }]
```

```
PrmitiveF1Ab00[t_] := 1/(96 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]
    Csc[β] (b r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t +
          3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (11 + t^2) + BBB^2 (9 + 2 t^2))) +
      3 b r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t +
          AAA (1 + BBB^2) (2 (3 + t^2) + BBB^2 (-7 + 2 t^2))) Cos[2 β] +
      8 a (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[2 β]);
PrmitiveF1Ab11[t_] := (ArcTan[(Sqrt[((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))]) /
       (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
      (16 a AAA (1 + BBB^2) (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Cos[β] + 3 b BBB (1 - AAA^2 + BBB^2) r
         (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
         Csc[β])) / (48 (1 + BBB^2)^(7/2));
PrmitiveF1Ab22[t_] := 1/48 ArcTan[((AAA + BBB) (1 + BBB^2)
         Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]);
PrmitiveF1Ab33[t_] := 1/48 ArcTan[
      ((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β]);
```

CHECK OF THE DERIVATIVE OF THE FINAL PRIMITIVE

$$\text{Simplify}\Big[(\text{D[PrmitiveF1Ab00[t], t]} + \text{D[PrmitiveF1Ab11[t], t]} + \text{D[PrmitiveF1Ab22[t], t]} + \text{D[PrmitiveF1Ab33[t], t]}) - \text{rmnIntgrndF1Ab[t]}, \text{Assumptions} \to \Big\{-\sqrt{1+\text{BBB}^2} < \text{AAA} < \sqrt{1+\text{BBB}^2} \;\&\&\; -\frac{\text{AAA BBB}}{1+\text{BBB}^2} - \sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{(1+\text{BBB}^2)^2}} < t < -\frac{\text{AAA BBB}}{1+\text{BBB}^2} + \sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{(1+\text{BBB}^2)^2}}\Big\}\Big]$$

0

We check the above result evaluating the derivatives step by step

The derivative of PrmitiveF1Ab00[t] is:

$$\text{cf00[[1]]}\sqrt{\Delta 1} + \text{cf000[[2]]}\Big/\sqrt{\Delta 1}$$

$$\text{cf000NS} = \text{Simplify}\Big[\text{CoefficientList}\Big[\Big(\Big(\text{Expand}\Big[\Big((\text{D[PrmitiveF1Ab00[t], t]}) \,/.\, \Big\{\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2 t^2} \to \sqrt{\Delta 1}\Big\}\Big) \,/.\, \Big\{1\Big/\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2 t^2} \to 1\Big/\sqrt{\Delta 1}\Big\}\Big]\Big) \,/.\, \Big\{\frac{1}{\sqrt{\Delta 1}} \to \text{XXXX}\Big\}\Big), \text{XXXX}\Big]\Big];$$

$$\text{cf00} = \Big\{\text{cf000NS[[1]]}\Big/\sqrt{\Delta 1},\ \text{cf000NS[[2]]}\Big\};$$

$$\text{Simplify}\Big[\text{D[PrmitiveF1Ab00[t], t]} - \Big(\Big(\text{cf00[[1]]}\sqrt{\Delta 1} + \text{cf000[[2]]}\Big/\sqrt{\Delta 1}\Big) \,/.\, \Big\{\sqrt{\Delta 1} \to \sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2 t^2},\ 1\Big/\sqrt{\Delta 1} \to 1\Big/\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2 t^2}\Big\}\Big)\Big]$$

0

The derivative of PrmitiveF1Ab11[t] is $-\frac{\text{cf11}}{2\sqrt{\Delta 1}}$

$$\text{cf11} = \Big(16\,a\,\text{AAA}\,(1+\text{BBB}^2)\,(3-\text{AAA}^2+(3+2\,\text{AAA}^2)\,\text{BBB}^2)\,\text{Cos}[\beta] + 3\,b\,\text{BBB}\,(1-\text{AAA}^2+\text{BBB}^2)\,r\,\big(7-3\,\text{AAA}^2+(13+2\,\text{AAA}^2)\,\text{BBB}^2+6\,\text{BBB}^4+(5-9\,\text{AAA}^2+(7+6\,\text{AAA}^2)\,\text{BBB}^2+2\,\text{BBB}^4)\,\text{Cos}[2\beta]\big)\,\text{Csc}[\beta]\Big)\Big/\Big(48\,(1+\text{BBB}^2)^3\Big);$$

$$\text{Simplify}\Big[\text{D[PrmitiveF1Ab11[t], t]} - \Big(\Big(-\frac{\text{cf11}}{2\sqrt{\Delta 1}}\Big) \,/.\, \Big\{1\Big/\sqrt{\Delta 1} \to 1\Big/\sqrt{1-\text{AAA}^2-2\,\text{AAA BBB}\,t-t^2-\text{BBB}^2 t^2}\Big\}\Big), \text{Assumptions} \to \Big\{-\sqrt{1+\text{BBB}^2} < \text{AAA} < \sqrt{1+\text{BBB}^2} \;\&\&\; -\frac{\text{AAA BBB}}{1+\text{BBB}^2} - \sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{(1+\text{BBB}^2)^2}} < t < -\frac{\text{AAA BBB}}{1+\text{BBB}^2} + \sqrt{\frac{1-\text{AAA}^2+\text{BBB}^2}{(1+\text{BBB}^2)^2}}\Big\}\Big]$$

0

The derivative of PrmitiveF1Ab22[t] is

$$\left(\frac{\text{Csc}[\beta]\,(3\,b\,r\,(3+\text{Cos}[2\beta])+8\,a\,\text{Sin}[2\beta])}{96}\right)\frac{\text{AAA}+\text{BBB}}{(1-t)\sqrt{\Delta 1}}$$

```
Simplify[
 D[PrmitiveF1Ab22[t], t] - ((((Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]))/96) (AAA + BBB)/((1 - t) Sqrt[Δ1])) /.
   {1/Sqrt[Δ1] → 1/Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}),
 Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
   -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

$$\mathtt{Simplify}\left[\mathtt{D[PrmitiveF1Ab22[t],t]} - \left(\left(\left(\frac{\mathrm{Csc}[\beta]\,(3\,b\,r\,(3+\mathrm{Cos}[2\,\beta]) + 8\,a\,\mathrm{Sin}[2\,\beta])}{96}\right)\frac{AAA+BBB}{(1-t)\sqrt{\Delta 1}}\right) /. \left\{1\Big/\sqrt{\Delta 1} \to 1\Big/\sqrt{1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2}\right\}\right),\right.$$
$$\left.\mathtt{Assumptions} \to \left\{-\sqrt{1+BBB^2} < AAA < \sqrt{1+BBB^2}\ \&\&\ -\frac{AAA\,BBB}{1+BBB^2} - \sqrt{\frac{1-AAA^2+BBB^2}{\left(1+BBB^2\right)^2}} < t < -\frac{AAA\,BBB}{1+BBB^2} + \sqrt{\frac{1-AAA^2+BBB^2}{\left(1+BBB^2\right)^2}}\right\}\right]$$

0

The derivative of PrmitiveF1Ab33[t] is

$$\frac{\left(\mathrm{Csc}[\beta]\,\left(3\,b\,r\,(3+\mathrm{Cos}[2\,\beta]) - 8\,a\,\mathrm{Sin}[2\,\beta]\right)\right)\,(BBB-AAA)}{96\,(1+t)\,\sqrt{\Delta 1}}$$

```
Simplify[D[PrmitiveF1Ab33[t], t] -
  (((Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β])) (BBB - AAA)) / (96 (1 + t) √Δ1)) /.
   {1 / √Δ1 → 1 / √(1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2)}),
 Assumptions → {-√(1 + BBB^2) < AAA < √(1 + BBB^2) &&
   -AAA BBB/(1 + BBB^2) - √((1 - AAA^2 + BBB^2)/(1 + BBB^2)^2) < t < -AAA BBB/(1 + BBB^2) + √((1 - AAA^2 + BBB^2)/(1 + BBB^2)^2)}]
```

0

Summing up the four contributions we get

$$\left(\text{cf00}[[1]]\sqrt{\Delta 1}+\text{cf000}[[2]]\Big/\sqrt{\Delta 1}\right)+\left(-\frac{\text{cf11}}{2\sqrt{\Delta 1}}\right)+\left(\frac{\text{Csc}[\beta]\,(3\,b\,r\,(3+\text{Cos}[2\beta])+8\,a\,\text{Sin}[2\beta])}{96}*\frac{\text{AAA}+\text{BBB}}{(1-t)\sqrt{\Delta 1}}\right)+\frac{\text{Csc}[\beta]\,(3\,b\,r\,(3+\text{Cos}[2\beta])-8\,a\,\text{Sin}[2\beta])\,(\text{BBB}-\text{AAA})}{96\,(1+t)\sqrt{\Delta 1}}$$

```
Simplify[(Simplify[Together[(cf00[[1]] √Δ1 + cf000[[2]] / √Δ1) +
     (-cf11/(2 √Δ1)) + ((Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]))/96 * (AAA + BBB)/((1 - t) √Δ1)) +
     (Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β]) (BBB - AAA)) / (96 (1 + t) √Δ1)]]) /.
  {Δ1 → 1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2}]
```

and, finally, subtracting to this result the outset integrand rmnIntgrndF1Ab[t]
we get zero

```
Simplify[(-(t^2 (BBB + AAA t) Csc[β] (3 b r (2 + t^2) + 3 b r (-2 + 3 t^2) Cos[2 β] + 8 a t Sin[2 β]))/
    (48 (-1 + t^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])) - rmnIntgrndF1Ab[t]]
```

0

IN CONCLUSION THE PRIMITVE OF THE CONTRIBUTION:

```
CF1ATot[t_] := CF1Aa[t] + ArcSin[(AAA+BBB t)/Sqrt[1-t^2]] * CF1Ab[t]
```

IS THE SUM OF THE FOLLOWING SIX FUNCTIONS

```
PrmitiveF1Aa[t_] := -1/24 fff t^2 (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β]);
PrmitiveF1AbIPP[t_] :=
  (-1/3 a t^3 Cos[β] - 1/4 b r t^4 Cos[β] Cot[β] - 1/4 b r t^2 Sin[β] + 1/8 b r t^4 Sin[β]) *
   ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]; PrmitiveF1Ab00[t_] :=
  1/(96 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Csc[β]
   (b r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t +
        AAA (1 + BBB^2) (2 (11 + t^2) + BBB^2 (9 + 2 t^2))) + 3 b r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (3 + t^2) + BBB^2 (-7 + 2 t^2)))
      Cos[2 β] + 8 a (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[2 β]);

PrmitiveF1Ab11[t_] :=
  (ArcTan[(Sqrt[((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))])/ (AAA BBB +
          Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)] (16 a AAA (1 + BBB^2) (3 - AAA^2 + (3 + 2 AAA^2) BBB^2)
        Cos[β] + 3 b BBB (1 - AAA^2 + BBB^2) r (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 +
           (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β]) Csc[β]))/ (48 (1 + BBB^2)^(7/2));
PrmitiveF1Ab22[t_] := 1/48 ArcTan[((AAA + BBB) (1 + BBB^2)
        Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]);
PrmitiveF1Ab33[t_] := 1/48 ArcTan[
     ((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β]);
```

EVALUATION OF THE PRIMITIVE OF CF1BTot[t]

```
CF1Ba[t_] := 1/2 t Sin[fff] (4 b r t (AAA + BBB t) Cos[β] +
    (2 a (AAA + BBB t) - b r (-1 + 2 AAA^2 + 4 AAA BBB t + t^2 + 2 BBB^2 t^2) Cos[fff]) Sin[β]);
CF1Bb[t_] := Sqrt[1 - t^2 - (AAA + BBB t)^2] * (1/2 t (b r (AAA + BBB t) Cos[fff]^2 Sin[β] -
      b r (AAA + BBB t) Sin[fff]^2 Sin[β] - 2 Cos[fff] (2 b r t Cos[β] + a Sin[β])));
CF1BTot[t_] := CF1Ba[t] + CF1Bb[t]; CF1BTot[t];
```

Integration of the first contribution CF1Ba[t]

```
Integrate[CF1Ba[t], t]
```

```
PrmitiveF1Ba[t_] := 1/24 t^2 Sin[fff] (4 b r t (4 AAA + 3 BBB t) Cos[β] +
    (4 a (3 AAA + 2 BBB t) - b r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]) Sin[β]);
```

Integration of the second contribution CF1Bb[t].
By the Cacciopoli transformation we find

```
CF1Bb[t]

Expand[1 - t^2 - (AAA + BBB t)^2]
```

$1 - AAA^2 - 2\,AAA\,BBB\,t - t^2 - BBB^2\,t^2$

```
NewIntgrnd1Bb[ξ_] :=
  Simplify[(((((CF1Bb[t]) /. {Sqrt[1 - t^2 - (AAA + BBB t)^2] → Sqrt[1 + BBB^2] Sqrt[(t - mua) (mub - t)]}) /.
          {Sqrt[(mub - t) (-mua + t)] → ξ (t - mua)}) /. {t → (mub + mua ξ^2)/(1 + ξ^2)}) * Jacob1A) /.
     {Jacob1A → -(4 Sqrt[1 - AAA^2 + BBB^2] ξ)/((1 + BBB^2) (1 + ξ^2)^2)}, Assumptions → {-1 < mua < t < mub < 1 &&
      ξ > 0}]; NewIntgrnd1Bb[ξ]

num1Bb[ξ_] :=
  (2 Sqrt[1 - AAA^2 + BBB^2] (mua - mub) ξ^2 (mub + mua ξ^2) (b r (AAA + BBB mub + AAA ξ^2 + BBB mua ξ^2)
        Cos[fff]^2 Sin[β] - b r (AAA + BBB mub + AAA ξ^2 + BBB mua ξ^2) Sin[fff]^2 Sin[β] -
       2 Cos[fff] (2 b r (mub + mua ξ^2) Cos[β] + a (1 + ξ^2) Sin[β]))) / (Sqrt[1 + BBB^2]);
Simplify[num1Bb[ξ] / (1 + ξ^2)^5 - NewIntgrnd1Bb[ξ]]
```

0

```
cf1Bb = Simplify[CoefficientList[num1Bb[ξ], ξ]]; cf1Bb[[7]]
cf1Bb[[1]]
```

check

```
Simplify[cf1Bb[[3]] ξ^2 + cf1Bb[[5]] ξ^4 + cf1Bb[[7]] ξ^6 - num1Bb[ξ]]
```

0

```
integralBb[ξ_] := (cf1Bb[[3]] Integrate[ξ^2/(1 + ξ^2)^5, ξ]) +
   (cf1Bb[[5]] Integrate[ξ^4/(1 + ξ^2)^5, ξ]) + (cf1Bb[[7]] Integrate[ξ^6/(1 + ξ^2)^5, ξ]); integralBb[ξ]
```

Check

```
Simplify[D[integralBb[ξ], ξ] - NewIntgrnd1Bb[ξ]]
```

0

```
(((integralBb[ξ]) /. {ArcTan[ξ] → atan}) /. {1/(1 + ξ^2)^4 → DD4})

CFcsiPrmtv = Simplify[
  CoefficientList[(((integralBb[ξ]) /. {ArcTan[ξ] → atan}) /. {1/(1 + ξ^2)^4 → DD4}), atan]]

(CFcsiPrmtv[[1]]) /. {DD4 → 1/(1 + ξ^2)^4};

(CFcsiPrmtv[[2]]) /. {DD4 → 1/(1 + ξ^2)^4};

ausPrimtv1BbAA[ξ_] := ((CFCsiPrmtv[[1]]) /. {DD4 → 1/(1 + ξ^2)^4});

ausPrimtv1BbBB[ξ_] := Simplify[((CFCsiPrmtv[[2]]) /. {DD4 → 1/(1 + ξ^2)^4})] * ArcTan[ξ];
```

the ξ-primitive is the sum of the following two contributions.

```
ausPrimtv1BbAA[ξ_] := (Sqrt[1 - AAA^2 + BBB^2] (mua - mub) ξ
    (mua (-15 - 55 ξ^2 - 73 ξ^4 + 15 ξ^6) (b (AAA + BBB mua) r Cos[fff]^2 Sin[β] -
        b (AAA + BBB mua) r Sin[fff]^2 Sin[β] - 2 Cos[fff] (2 b mua r Cos[β] + a Sin[β])) +
     mub (-15 + 73 ξ^2 + 55 ξ^4 + 15 ξ^6) (b (AAA + BBB mub) r Cos[fff]^2 Sin[β] -
        b (AAA + BBB mub) r Sin[fff]^2 Sin[β] - 2 Cos[fff] (2 b mub r Cos[β] + a Sin[β])) +
     3 (-3 - 11 ξ^2 + 11 ξ^4 + 3 ξ^6) (b (2 BBB mua mub + AAA (mua + mub)) r Cos[fff]^2 Sin[β] -
        b (2 BBB mua mub + AAA (mua + mub)) r Sin[fff]^2 Sin[β] -
        2 Cos[fff] (4 b mua mub r Cos[β] + a (mua + mub) Sin[β])))) /
  (192 Sqrt[1 + BBB^2] (1 + ξ^2)^4); ausPrimtv1BbBB[ξ_] := 1/(64 Sqrt[1 + BBB^2])
  Sqrt[1 - AAA^2 + BBB^2]
  (mua - mub)
  ArcTan[ξ]
  (b (8 AAA (mua + mub) + BBB (5 mua^2 + 6 mua mub + 5 mub^2)) r Cos[fff]^2 Sin[β] -
    b (8 AAA (mua + mub) + BBB (5 mua^2 + 6 mua mub + 5 mub^2)) r Sin[fff]^2 Sin[β] -
    4 Cos[fff] (b (5 mua^2 + 6 mua mub + 5 mub^2) r Cos[β] + 4 a (mua + mub) Sin[β]));
```

Checks

```
Simplify[ausPrimtv1BbAA[ξ] + ausPrimtv1BbBB[ξ] - integralBb[ξ]]

Simplify[D[ausPrimtv1BbAA[ξ], ξ] + D[ausPrimtv1BbBB[ξ], ξ] - NewIntgrnd1Bb[ξ]]
```

0

0

F1Bb: We go back to variable t

$$(\text{ausPrimtv1BbAA}[\xi]) \;/.\; \left\{\xi \to \sqrt{\frac{\text{mub} - t}{t - \text{mua}}}\right\}$$

$$\text{FullSimplify}\Bigg[\Bigg(\Bigg(\Bigg(\Bigg((\text{ausPrimtv1BbAA}[\xi]) \;/.\; \left\{\xi \to \sqrt{\frac{\text{mub} - t}{t - \text{mua}}}\right\}\Bigg) \;/.$$

$$\left\{\sqrt{\frac{\text{mub} - t}{-\text{mua} + t}} \to \left(\sqrt{1 + \text{BBB}^2}\,\sqrt{1 - \text{AAA}^2 - 2\,\text{AAA BBB}\,t - \left(1 + \text{BBB}^2\right) t^2}\right) \Big/ \left(\text{AAA BBB} + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2} + t + \text{BBB}^2\, t\right)\right\}\Bigg) \;/.$$

$$\left\{\text{mua} \to \frac{-\text{AAA BBB} - \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}\Bigg) \;/.\; \left\{\text{mub} \to \frac{-\text{AAA BBB} + \sqrt{1 - \text{AAA}^2 + \text{BBB}^2}}{1 + \text{BBB}^2}\right\}\Bigg) -$$

$$\text{Primtv1BbAANotS}[t],\ \text{Assumptions} \to \left\{-\sqrt{1 + \text{BBB}^2} < \text{AAA} < \sqrt{1 + \text{BBB}^2}\ \&\&\ -\frac{\text{AAA BBB}}{1 + \text{BBB}^2} - \sqrt{\frac{1 - \text{AAA}^2 + \text{BBB}^2}{\left(1 + \text{BBB}^2\right)^2}} < t < -\frac{\text{AAA BBB}}{1 + \text{BBB}^2} + \sqrt{\frac{1 - \text{AAA}^2 + \text{BBB}^2}{\left(1 + \text{BBB}^2\right)^2}}\right\}\Bigg]$$

0

```
Simplify[(((((ausPrimtv1BbBB[ξ]) /. {ξ → Sqrt[(mub - t)/(t - mua)]}) /.
      {Sqrt[(mub - t)/(-mua + t)] → (Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2])/
          (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)}) /.
    {mua → (-AAA BBB - Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) /. {mub → (-AAA BBB + Sqrt[1 - AAA^2 + BBB^2])/(1 + BBB^2)}) -
  Primtv1BbBBNotS[t], Assumptions → {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]
```

0

```
PrimtvF1BbAANotS[t_] :=
  Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]/(48 (1 + BBB^2)^3) (b r (AAA^3 (8 - 9 BBB^2 - 2 BBB^4) + AAA^2
          BBB (7 + 9 BBB^2 + 2 BBB^4) t + 3 BBB (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
         AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Cos[2 fff] Sin[β] -
      4 Cos[fff] (b r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 +
                 2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] + 2 a (1 + BBB^2)
          (-AAA^2 (-2 + BBB^2) + AAA BBB (1 + BBB^2) t + 2 (1 + BBB^2) (-1 + (1 + BBB^2) t^2)) Sin[β]));
PrimtvF1BbBBNotS[t_] := -1/(8 (1 + BBB^2)^(7/2)) (1 - AAA^2 + BBB^2)
    (-4 b (1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[fff] Cos[β] +
      8 a AAA (BBB + BBB^3) Cos[fff] Sin[β] + b BBB (1 - 5 AAA^2 + BBB^2) r Cos[2 fff] Sin[β])
    ArcTan[(Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])/
      (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)];
```

CHECK OF THE FINAL t-DERIVATIVE [OK]

```
Simplify[Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] * D[PrimtvF1BbAANotS[t], t]]/
  Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2];
Simplify[D[PrimtvF1BbBBNotS[t], t]];
```

```
Simplify[(Simplify[Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] * D[PrimtvF1BbAANotS[t], t]] /
    Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]) + Simplify[D[PrimtvF1BbBBNotS[t], t]] -
  ((Simplify[CF1Bb[t] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2],
      Assumptions -> {-Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] && -AAA BBB/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] <
         t < -AAA BBB/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}]) / Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])]
```

0

In conclusion, the primitves of the two terms, adding up to give the integrand CF1, i.e. CF1ATot[t] and CF1BTot[t] are respectively equal to
PrmitiveF1Aa[t] + PrmitiveF1AbIPP[t] + PrmitiveF1Ab00[t] + PrmitiveF1Ab11[t] + PrmitiveF1Ab22[t] + PrmitiveF1Ab33[t]
and to
PrmitiveF1Ba[t] + Primtv1BbAA[t] + Primtv1BbBB[t].
The functions are reported below.

```
PrmitiveF1Aa[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  -(1/24) fff t^2 (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β]);
PrmitiveF1AbIPP[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  (-(1/3) a t^3 Cos[β] - (1/4) b r t^4 Cos[β] Cot[β] - (1/4) b r t^2 Sin[β] + (1/8) b r t^4 Sin[β]) *
   ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]; PrmitiveF1Ab00[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  1/(96 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Csc[β]
   (b r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t +
        AAA (1 + BBB^2) (2 (11 + t^2) + BBB^2 (9 + 2 t^2))) + 3 b r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (3 + t^2) + BBB^2 (-7 + 2 t^2)))
      Cos[2 β] + 8 a (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[2 β]);

PrmitiveF1Ab11[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  (ArcTan[(√((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))) /
       (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
     (16 a AAA (1 + BBB^2) (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Cos[β] + 3 b BBB (1 - AAA^2 + BBB^2) r
        (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
        Csc[β])) / (48 (1 + BBB^2)^(7/2));
PrmitiveF1Ab22[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
   1/48 ArcTan[((AAA + BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]);
PrmitiveF1Ab33[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
   1/48 ArcTan[((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β]);
```

```
PrmitiveF1Ba[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  1/24 t^2 Sin[fff] (4 b r t (4 AAA + 3 BBB t) Cos[β] +
     (4 a (3 AAA + 2 BBB t) - b r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]) Sin[β]);
PrimtvF1BbAA[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]/(48 (1 + BBB^2)^3) (b r (AAA^3 (8 - 9 BBB^2 - 2 BBB^4) +
        AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t + 3 BBB (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
        AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Cos[2 fff] Sin[β] -
     4 Cos[fff] (b r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 +
              2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] + 2 a (1 + BBB^2)
         (-AAA^2 (-2 + BBB^2) + AAA BBB (1 + BBB^2) t + 2 (1 + BBB^2) (-1 + (1 + BBB^2) t^2)) Sin[β]));
PrimtvF1BbBB[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] := -1/(8 (1 + BBB^2)^(7/2))
   (1 - AAA^2 + BBB^2) (-4 b (1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[fff] Cos[β] +
     8 a AAA (BBB + BBB^3) Cos[fff] Sin[β] + b BBB (1 - 5 AAA^2 + BBB^2) r Cos[2 fff] Sin[β])
   ArcTan[(Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2])/
     (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)];
```

NUMERICAL CHECKS

```
Reduce[{-1 < (AAA + BBB t)/Sqrt[1 - t^2] < 1 && -Sqrt[1 + BBB^2] < AAA < Sqrt[1 + BBB^2] &&
    -(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2] < t < -(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2]}, {BBB, AAA, t}, Reals]

CF1ATot[t]

With[{r = 1, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, a = 2, b = 1},
 bounds = {N[-(AAA BBB)/(1 + BBB^2) - Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2], 30], N[-(AAA BBB)/(1 + BBB^2) + Sqrt[(1 - AAA^2 + BBB^2)/(1 + BBB^2)^2], 30]}]
```

NUMERICAL CHECKS FOR CF3ATot

```
val = N[(CF1ATot[1 / 10]) /. {r → 1, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, a → 2, b → 1}];
val
```

-0.047797

```
With[{r = 1 / 2, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, a = 2, b = 1},
 Pr1Aa[t_] := PrmitiveF1Aa[r, β, a, b, fff, AAA, BBB, t] ;
 Pr1APP[t_] := PrmitiveF1AbIPP[r, β, a, b, fff, AAA, BBB, t];
 Pr1A00[t_] := PrmitiveF1Ab00[r, β, a, b, fff, AAA, BBB, t];
 Pr1A11[t_] := PrmitiveF1Ab11[r, β, a, b, fff, AAA, BBB, t] ;
 Pr1A22[t_] := PrmitiveF1Ab22[r, β, a, b, fff, AAA, BBB, t] ;
 Pr1A33[t_] :=  PrmitiveF1Ab33[r, β, a, b, fff, AAA, BBB, t];]
CF1AT[x_] :=
  ((CF1ATot[x]) /. {r → 1 / 2, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, a → 2, b → 1});


Pr1Aa[t]
Pr1APP[t]
Pr1A00[t]
Pr1A11[t]
Pr1A22[t]
Pr1A33[t]
CF1AT[x]

N[Pr1Aa[0]]
N[Pr1APP[0]]
N[Pr1A00[0]]
N[Pr1A11[0]]
N[Pr1A22[0]]
N[Pr1A33[0]]
N[CF1AT[0]]

step = -1 / 100; Clear[t];
Do[t = J * step;
  valaa = N[Pr1Aa[t] - Pr1Aa[0], 30];
  valPP = N[Pr1APP[t] - Pr1APP[0], 30];
  val00 = N[Pr1A00[t] - Pr1A00[0], 30];
  val11 = N[Pr1A11[t] - Pr1A11[0], 30];
  val22 = N[Pr1A22[t] - Pr1A22[0], 30];
  val33 = N[Pr1A33[t] - Pr1A33[0], 30];
  val = NIntegrate[CF1AT[x], {x, 0, t}, PrecisionGoal → 20, WorkingPrecision → 30];
  valtot = valaa + valPP + val00 + val11 + val22 + val33;
  diff = valtot - val;
  If[Abs[diff] > 10^(-30),
   Print[PaddedForm[valaa, {3, 7}], " ,", PaddedForm[valPP, {3, 7}],
    " ,", PaddedForm[val00, {3, 7}], " ,", PaddedForm[val11, {3, 7}],
    " ,", PaddedForm[val22, {3, 7}], " ,", PaddedForm[val33, {3, 7}]];
   Print[J, " ,", PaddedForm[val, {3, 7}], " ,", PaddedForm[valtot, {3, 7}],
    " ,", PaddedForm[diff, {3, 7}]]];, {J, 1, 20}];
Clear[
  t];
```

NUMERICAL CHECKS FOR CF1BTot

```
With[{r = 1 / 2, β = π / 5, BBB = 2, AAA = 1 / 2, fff = 1 / 4, a = 2, b = 1},
 Pr1Ba[t_] := PrmitiveF1Ba[r, β, a, b, fff, AAA, BBB, t] ;
 Pr1BAA[t_] := PrimtvF1BbAA[r, β, a, b, fff, AAA, BBB, t];
 Pr1BBB[t_] := PrimtvF1BbBB[r, β, a, b, fff, AAA, BBB, t]; ]
CF1BT[x_] :=
  ((CF1BTot[x]) /. {r → 1 / 2, β → π / 5, BBB → 2, AAA → 1 / 2, fff → 1 / 4, a → 2, b → 1});
```

```
step = -1 / 100; Clear[t];
Do[t = J * step;
  valBaa = N[Pr1Ba[t] - Pr1Ba[0], 30];
  valBAA = N[Pr1BAA[t] - Pr1BAA[0], 30];
  valBBB = N[Pr1BBB[t] - Pr1BBB[0], 30];
  val = NIntegrate[CF1BT[x], {x, 0, t}, PrecisionGoal → 20, WorkingPrecision → 30];
  valtot = valBaa + valBAA + valBBB;
  diff = valtot - val;
  If[Abs[diff] > 10^(-30), Print[PaddedForm[valBaa, {3, 7}],
    " ,", PaddedForm[valBAA, {3, 7}], " ,", PaddedForm[valBBB, {3, 7}]];
   Print[J, " ,", PaddedForm[val, {3, 7}], " ,", PaddedForm[valtot, {3, 7}],
    " ,", PaddedForm[diff, {3, 7}]]];, {J, 1, 20}];
Clear[
  t];
```

Compactification of the primitives

FIRST SET OF FUNCTIONS

```
PrmitiveF1Aa[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  -1/24 fff t^2 (2 t Cos[β] (4 a + 3 b r t Cot[β]) - 3 b r (-2 + t^2) Sin[β]);
PrmitiveF1AbIPP[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 (-1/3 a t^3 Cos[β] - 1/4 b r t^4 Cos[β] Cot[β] - 1/4 b r t^2 Sin[β] + 1/8 b r t^4 Sin[β]) *
  ArcSin[(AAA + BBB t)/Sqrt[1 - t^2]]; PrmitiveF1Ab00[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 1/(96 (1 + BBB^2)^3) Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] Csc[β]
  (b r (AAA^3 (-4 + 11 BBB^2) - 5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t +
        AAA (1 + BBB^2) (2 (11 + t^2) + BBB^2 (9 + 2 t^2))) + 3 b r (AAA^3 (-4 + 11 BBB^2) -
        5 AAA^2 BBB (1 + BBB^2) t + 3 BBB (1 + BBB^2)^2 t + AAA (1 + BBB^2) (2 (3 + t^2) + BBB^2 (-7 + 2 t^2)))
     Cos[2 β] + 8 a (1 + BBB^2) ((2 - 3 AAA^2) BBB + 2 BBB^3 + AAA t + AAA BBB^2 t) Sin[2 β]);

PrmitiveF1Ab11[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  (ArcTan[(√((-1 - BBB^2) (-1 + AAA^2 + 2 AAA BBB t + (1 + BBB^2) t^2))) /
        (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)]
     (16 a AAA (1 + BBB^2) (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Cos[β] + 3 b BBB (1 - AAA^2 + BBB^2) r
        (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
        Csc[β])) / (48 (1 + BBB^2)^(7/2));
PrmitiveF1Ab22[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  1/48 ArcTan[((AAA + BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) + 8 a Sin[2 β]);
PrmitiveF1Ab33[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  1/48 ArcTan[((AAA - BBB) (1 + BBB^2) Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]) /
      ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))]
    Csc[β] (3 b r (3 + Cos[2 β]) - 8 a Sin[2 β]);
```

```
{Δ → Sqrt[1 - t^2], Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2],
 Δ2 → Sqrt[1 + BBB^2], Δ3 → Sqrt[1 - AAA^2 + BBB^2],
 BBB^2 → Δ2^2 - 1,
 AAA^2 → Δ2^2 - Δ3^2}
```

```
PrmitiveF1Aa[r, β, a, b, fff, AAA, BBB, t]

Cf1Aa = Factor[FullSimplify[CoefficientList[PrmitiveF1Aa[r, β, a, b, fff, AAA, BBB, t], t]]]
```

```
Cf1Aa = {0, 0, -1/4 b fff r Sin[β], -1/3 a fff Cos[β], -1/16 b fff r (1 + 3 Cos[2 β]) Csc[β]};
PrmitiveF1AaNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Cf1Aa[[3]] t^2 + Cf1Aa[[4]] t^3 + Cf1Aa[[5]] t^4;
```

```
Simplify[
 PrmitiveF1Aa[r, β, a, b, fff, AAA, BBB, t] - PrmitiveF1AaNEW[r, β, a, b, fff, AAA, BBB, t]]
```

0

```
Cf1AbIPP = {-1/4 b r Sin[β], -1/3 a Cos[β], -1/16 b r (1 + 3 Cos[2 β]) Csc[β]};
PrmitiveF1AbIPPNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  ArcSin[(AAA + BBB t)/Δ] * t^2 * (Cf1AbIPP[[1]] + Cf1AbIPP[[2]] t + Cf1AbIPP[[3]] t^2);
```

```
Simplify[PrmitiveF1AbIPP[r, β, a, b, fff, AAA, BBB, t] -
  ((PrmitiveF1AbIPPNEW[r, β, a, b, fff, AAA, BBB, t]) /. {1 / Δ → 1/Sqrt[1 - t^2]})]
```

0

PrmitiveF1Ab00[r, β, a, b, fff, AAA, BBB, t]

```
FullSimplify[
 ((((PrmitiveF1Ab00[r, β, a, b, fff, AAA, BBB, t]) /. {Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] →
         Δ1}) /. {BBB^2 → Δ2^2 - 1}) /. {AAA^2 → Δ2^2 - Δ3^2}) /. {AAA^3 → AAA (Δ2^2 - Δ3^2)}]
```

```
aus = (8 a Δ2^2 (2 BBB^3 + AAA t Δ2^2 + BBB (2 - 3 Δ2^2 + 3 Δ3^2)) Cos[β] +
     b r (2 (BBB t Δ2^2 (-2 Δ2^2 + 5 Δ3^2) + AAA (15 Δ3^2 + Δ2^2 (-2 + 2 (4 + t^2) Δ2^2 - 11 Δ3^2))) Csc[β] -
        3 (BBB t Δ2^2 (-2 Δ2^2 + 5 Δ3^2) + AAA (15 Δ3^2 + Δ2^2 (-2 + 2 (2 + t^2) Δ2^2 - 11 Δ3^2))) Sin[β]));
FullSimplify[CoefficientList[CoefficientList[aus, t], r]]
```

```
CfF1Ab00 = {{8 a BBB Δ2^2 (2 + 2 BBB^2 - 3 Δ2^2 + 3 Δ3^2) Cos[β], AAA b
      (2 (8 Δ2^4 + 15 Δ3^2 - Δ2^2 (2 + 11 Δ3^2)) Csc[β] + 3 (-4 Δ2^4 - 15 Δ3^2 + Δ2^2 (2 + 11 Δ3^2)) Sin[β])},
   {8 a AAA Δ2^4 Cos[β], b BBB Δ2^2 (-2 Δ2^2 + 5 Δ3^2) (2 Csc[β] - 3 Sin[β])},
   {0, AAA b Δ2^4 (1 + 3 Cos[2 β]) Csc[β]}};
PrmitiveF1Ab00NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Δ1/(48 Δ2^6) (CfF1Ab00[[1, 1]] + CfF1Ab00[[1, 2]] r +
     t (CfF1Ab00[[2, 1]] + CfF1Ab00[[2, 2]] r) + t^2 (CfF1Ab00[[3, 1]] + CfF1Ab00[[3, 2]] r));
```

```
FullSimplify[ExpandAll[PrmitiveF1Ab00[r, β, a, b, fff, AAA, BBB, t] -
    ((((PrmitiveF1Ab00NEW[r, β, a, b, fff, AAA, BBB, t]) /.
          {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}) /. {Δ2 → Sqrt[1 + BBB^2]}) /.
       {Δ3 → Sqrt[1 - AAA^2 + BBB^2]})], Assumptions → {1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2 > 0}]
```

0

PrmitiveF1Ab11[r, β, a, b, fff, AAA, BBB, t]

```
(PrmitiveF1Ab11[r, β, a, b, fff, AAA, BBB, t]) /. {1 + BBB^2 → Δ2^2}
```

```
Simplify[CoefficientList[
  (((((16 a AAA (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Δ2^2 Cos[β] + 3 b BBB r (-AAA^2 + Δ2^2) (7 - 3 AAA^2 +
                (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
             Csc[β])) /. {AAA^2 → Δ2^2 - Δ3^2}) /. {BBB^2 → Δ2^2 - 1}) /. {BBB^4 → (Δ2^2 - 1) ^2}), r]]
```

```
CoefficientList[
 (16 a AAA (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Δ2^2 Cos[β] + 3 b BBB r (-AAA^2 + Δ2^2) (7 - 3 AAA^2 +
       (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β]) Csc[β]), r]
```

```
argF1Ab11 = (Δ2 Δ1) / (AAA BBB + Δ3 + Δ2^2 t);
CfF1Ab11 = {16 a AAA (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Δ2^2 Cos[β], 3 b BBB (-AAA^2 + Δ2^2)
    (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
    Csc[β]}; PrmitiveF1Ab11NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  (ArcTan[argF1Ab11] (CfF1Ab11[[1]] + CfF1Ab11[[2]] r)) / (48 Δ2^7);
```

```
Simplify[PrmitiveF1Ab11[r, β, a, b, fff, AAA, BBB, t] -
  ((((PrmitiveF1Ab11NEW[r, β, a, b, fff, AAA, BBB, t]) /. {Δ2 → Sqrt[1 + BBB^2]}) /.
      {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]}),
  Assumptions → {1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2 > 0}]
```

0

PrmitiveF1Ab22[r, β, a, b, fff, AAA, BBB, t]

```
Simplify[(((PrmitiveF1Ab22[r, β, a, b, fff, AAA, BBB, t]) /.
     {Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → Δ1}) /.
   {1 / ((-1 - BBB (AAA + BBB) + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t))) →
     1 / (-(1 + BBB^2)
         (-1 + AAA BBB + AAA^2 + Sqrt[1 - AAA^2 + BBB^2] + (1 + AAA BBB + BBB^2 - Sqrt[1 - AAA^2 + BBB^2]) t))})]
```

```
argF1Ab22 = ((AAA + BBB) Δ1) / (1 - AAA BBB - AAA^2 - Δ3 - (1 + AAA BBB + BBB^2 - Δ3) t);
PrmitiveF1Ab22NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  ArcTan[argF1Ab22]/48 Csc[β] (8 a Sin[2 β] + 3 b r (3 + Cos[2 β]));
```

```
FullSimplify[PrmitiveF1Ab22[r, β, a, b, fff, AAA, BBB, t] -
  (((PrmitiveF1Ab22NEW[r, β, a, b, fff, AAA, BBB, t]) /.
      {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]}),
 Assumptions → {1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2 > 0}]
```

0

PrmitiveF1Ab33[r, β, a, b, fff, AAA, BBB, t]

```
(((PrmitiveF1Ab33[r, β, a, b, fff, AAA, BBB, t]) /.
    {Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2] → Δ1}) /.
  {1 / ((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t))) → 1 /
     ((1 + BBB^2) (1 - AAA^2 + AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t (1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2])))})
```

```
Simplify[CoefficientList[
  Expand[((1 - AAA BBB + BBB^2 + Sqrt[1 - AAA^2 + BBB^2]) (Sqrt[1 - AAA^2 + BBB^2] + t + BBB (AAA + BBB t)))], t]]
```

```
argF1Ab33 = ((AAA - BBB) Δ1) / (1 - AAA^2 + AAA BBB + Δ3 + (1 - AAA BBB + BBB^2 + Δ3) t);
PrmitiveF1Ab33NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  - ArcTan[argF1Ab33] / 48 Csc[β] (8 a Sin[2 β] - 3 b r (3 + Cos[2 β]));
```

```
FullSimplify[PrmitiveF1Ab33[r, β, a, b, fff, AAA, BBB, t] -
  (((PrmitiveF1Ab33NEW[r, β, a, b, fff, AAA, BBB, t]) /.
      {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]}),
 Assumptions → {1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2 > 0}]
```

0

SECOND SET OF FUNCTIONS

```
PrmitiveF1Ba[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  1/24 t^2 Sin[fff] (4 b r t (4 AAA + 3 BBB t) Cos[β] +
    (4 a (3 AAA + 2 BBB t) - b r (-6 + 12 AAA^2 + 16 AAA BBB t + 3 t^2 + 6 BBB^2 t^2) Cos[fff]) Sin[β]);
PrimtvF1BbAA[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]/(48 (1 + BBB^2)^3) (b r (AAA^3 (8 - 9 BBB^2 - 2 BBB^4) +
       AAA^2 BBB (7 + 9 BBB^2 + 2 BBB^4) t + 3 BBB (1 + BBB^2)^2 t (-1 + 2 (1 + BBB^2) t^2) +
       AAA (1 + BBB^2) (-8 + 5 BBB^2 + 2 (4 + 9 BBB^2 + 5 BBB^4) t^2)) Cos[2 fff] Sin[β] -
    4 Cos[fff] (b r (AAA^3 BBB (-13 + 2 BBB^2) + AAA^2 (3 + BBB^2 - 2 BBB^4) t + 3 (1 + BBB^2)^2 t (-1 +
            2 (1 + BBB^2) t^2) + AAA BBB (1 + BBB^2) (13 + 2 (1 + BBB^2) t^2)) Cos[β] + 2 a (1 + BBB^2)
        (-AAA^2 (-2 + BBB^2) + AAA BBB (1 + BBB^2) t + 2 (1 + BBB^2) (-1 + (1 + BBB^2) t^2)) Sin[β]));
PrimtvF1BbBBB[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] := - 1/(8 (1 + BBB^2)^(7/2))
   (1 - AAA^2 + BBB^2) (-4 b (1 + BBB^2 + AAA^2 (-1 + 4 BBB^2)) r Cos[fff] Cos[β] +
     8 a AAA (BBB + BBB^3) Cos[fff] Sin[β] + b BBB (1 - 5 AAA^2 + BBB^2) r Cos[2 fff] Sin[β])
   ArcTan[(Sqrt[1 + BBB^2] Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]) /
     (AAA BBB + Sqrt[1 - AAA^2 + BBB^2] + t + BBB^2 t)];
```

```
{Δ → Sqrt[1 - t^2], Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2],
 Δ2 → Sqrt[1 + BBB^2], Δ3 → Sqrt[1 - AAA^2 + BBB^2],
 BBB^2 → Δ2^2 - 1,
 AAA^2 → Δ2^2 - Δ3^2}
```

PrmitiveF1Ba[r, β, a, b, fff, AAA, BBB, t]

```
CfF1Ba = FullSimplify[CoefficientList[PrmitiveF1Ba[r, β, a, b, fff, AAA, BBB, t], t]]
```

```
CfF1Ba = {0, 0, Sin[fff] Sin[β]/4 (2 a AAA + (1 - 2 AAA^2) b r Cos[fff]),
  1/3 Sin[fff] (2 AAA b r Cos[β] + BBB (a - 2 AAA b r Cos[fff]) Sin[β]),
  -1/8 b r Sin[fff] ((1 + 2 BBB^2) Cos[fff] Sin[β] - 4 BBB Cos[β])};
PrmitiveF1BaNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 t^2 (CfF1Ba[[3]] + CfF1Ba[[4]] t + CfF1Ba[[5]] t^2);
```

```
Simplify[
 PrmitiveF1Ba[r, β, a, b, fff, AAA, BBB, t] - PrmitiveF1BaNEW[r, β, a, b, fff, AAA, BBB, t]]
```

0

PrimtvF1BbAA[r, β, a, b, fff, AAA, BBB, t]

```
((PrimtvF1BbAA[r, β, a, b, fff, AAA, BBB, t]) /. {Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] → Δ1}) /.
 {1/(1 + BBB^2)^3 → 1/Δ2^6}
```

```
ausF1BbAA = ((((PrimtvF1BbAA[r, β, a, b, fff, AAA, BBB, t]) /.
      {Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2] → Δ1}) /. {1/(1 + BBB^2)^3 → 1/Δ2^6}) / (1/(48 Δ2^6) Δ1));
```

```
(FullSimplify[CoefficientList[FullSimplify[CoefficientList[ausF1BbAA, t]], r]]) /.
 {(1 + BBB^2) → Δ2^2}
```

```
CfF1BbAA = {{8 a (2 - 2 AAA^2 + (2 + AAA^2) BBB^2) Δ2^2 Cos[fff] Sin[β],
    AAA b (-4 BBB (AAA^2 (-13 + 2 BBB^2) + 13 Δ2^2) Cos[fff] Cos[β] -
       (8 + 3 BBB^2 - 5 BBB^4 + AAA^2 (-8 + 9 BBB^2 + 2 BBB^4)) Cos[2 fff] Sin[β])},
   {-8 a AAA BBB Δ2^4 Cos[fff] Sin[β], b Δ2^2 (4 (AAA^2 (-3 + 2 BBB^2) + 3 Δ2^2) Cos[fff] Cos[β] +
       BBB (AAA^2 (7 + 2 BBB^2) - 3 Δ2^2) Cos[2 fff] Sin[β])}, {-16 a Δ2^6 Cos[fff] Sin[β],
    2 AAA b Δ2^4 (-4 BBB Cos[fff] Cos[β] + (4 + 5 BBB^2) Cos[2 fff] Sin[β])},
   {0, 6 b Δ2^6 (-4 Cos[fff] Cos[β] + BBB Cos[2 fff] Sin[β])}};
PrmitvF1BbAANEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
   Δ1/(48 Δ2^6) * Sum[CfF1BbAA[[i, j]] t^(i - 1) r^(j - 1), {i, 1, 4}, {j, 1, 2}];
```

```
Simplify[PrimtvF1BbAA[r, β, a, b, fff, AAA, BBB, t] -
  (((PrmitvF1BbAANEW[r, β, a, b, fff, AAA, BBB, t]) /.
      {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - (1 + BBB^2) t^2]}) /. {Δ2 → Sqrt[1 + BBB^2]})]
```

0

PrimtvF1BbBB[r, β, a, b, fff, AAA, BBB, t]

```
BBB^2 → Δ2^2 - 1,
AAA^2 → Δ2^2 - Δ3^2
```

```
Simplify[((((((PrimtvF1BbBB[r, β, a, b, fff, AAA, BBB, t]) /. {√(1 + BBB²) → Δ2}) /.
        {√(1 - AAA² - 2 AAA BBB t - t² - BBB² t²) → Δ1}) /. {√(1 - AAA² + BBB²) → Δ3}) /.
      {1 + BBB² → Δ2²}) /. {AAA² → Δ2² - Δ3²}) /. {BBB² → Δ2² - 1}) /. {BBB³ → BBB (Δ2² - 1)}]
```

$$\frac{1}{8\,\Delta 2^6\,\sqrt{\Delta 2^2}}\,\Delta 3^2\,\text{ArcTan}\left[\frac{\Delta 1\,\Delta 2}{\text{AAA BBB} + t\,\Delta 2^2 + \Delta 3}\right]\,\left(b\,\text{BBB}\,r\,\left(4\,\Delta 2^2 - 5\,\Delta 3^2\right)\,\text{Cos}[2\,\text{fff}]\,\text{Sin}[\beta] + 4\,\text{Cos}[\text{fff}]\,\left(b\,r\,\left(4\,\Delta 2^4 + 5\,\Delta 3^2 - 4\,\Delta 2^2\,\left(1 + \Delta 3^2\right)\right)\,\text{Cos}[\beta] - 2\,a\,\text{AAA BBB}\,\Delta 2^2\,\text{Sin}[\beta]\right)\right)$$

```
FullSimplify[CoefficientList[(b BBB r (4 Δ2² - 5 Δ3²) Cos[2 fff] Sin[β] +
    4 Cos[fff] (b r (4 Δ2⁴ + 5 Δ3² - 4 Δ2² (1 + Δ3²)) Cos[β] - 2 a AAA BBB Δ2² Sin[β])), r]]
```

```
CfF1BbBB = {-8 a AAA BBB Δ2² Cos[fff] Sin[β],
   4 b (4 Δ2⁴ + 5 Δ3² - 4 Δ2² (1 + Δ3²)) Cos[fff] Cos[β] + b BBB (4 Δ2² - 5 Δ3²) Cos[2 fff] Sin[β]};
PrmitvF1BbBBNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Δ3²/(8 Δ2⁷) ArcTan[(Δ1 Δ2)/(AAA BBB + t Δ2² + Δ3)] (CfF1BbBB[[1]] + CfF1BbBB[[2]] r);
```

```
Simplify[PrimtvF1BbBB[r, β, a, b, fff, AAA, BBB, t] -
  ((((PrmitvF1BbBBNEW[r, β, a, b, fff, AAA, BBB, t]) /.
        {Δ1 → Sqrt[1 - AAA^2 - 2 AAA BBB t - t^2 - BBB^2 t^2]}) /.
      {Δ2 → Sqrt[1 + BBB^2]}) /. {Δ3 → Sqrt[1 - AAA^2 + BBB^2]})]
```

0

FINAL COMPACT EXPRESSIONS

The primitive of CF1ATot[t] is the sum of the following six functions

```
Cf1Aa = {0, 0, -1/4 b fff r Sin[β], -1/3 a fff Cos[β], -1/16 b fff r (1 + 3 Cos[2 β]) Csc[β]};
PrmitiveF1AaNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 Cf1Aa[[3]] t^2 + Cf1Aa[[4]] t^3 + Cf1Aa[[5]] t^4;
```

```
Cf1AbIPP = {-1/4 b r Sin[β], -1/3 a Cos[β], -1/16 b r (1 + 3 Cos[2 β]) Csc[β]};
PrmitiveF1AbIPPNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 ArcSin[(AAA + BBB t)/Δ] * t^2 * (Cf1AbIPP[[1]] + Cf1AbIPP[[2]] t + Cf1AbIPP[[3]] t^2);
```

```
CfF1Ab00 = {{8 a BBB Δ2^2 (2 + 2 BBB^2 - 3 Δ2^2 + 3 Δ3^2) Cos[β], AAA b
      (2 (8 Δ2^4 + 15 Δ3^2 - Δ2^2 (2 + 11 Δ3^2)) Csc[β] + 3 (-4 Δ2^4 - 15 Δ3^2 + Δ2^2 (2 + 11 Δ3^2)) Sin[β])},
   {8 a AAA Δ2^4 Cos[β], b BBB Δ2^2 (-2 Δ2^2 + 5 Δ3^2) (2 Csc[β] - 3 Sin[β])},
   {0, AAA b Δ2^4 (1 + 3 Cos[2 β]) Csc[β]}};
PrmitiveF1Ab00NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 Δ1/(48 Δ2^6) (CfF1Ab00[[1, 1]] + CfF1Ab00[[1, 2]] r +
    t (CfF1Ab00[[2, 1]] + CfF1Ab00[[2, 2]] r) + t^2 (CfF1Ab00[[3, 1]] + CfF1Ab00[[3, 2]] r));
```

```
argF1Ab11 = (Δ2 Δ1)/(AAA BBB + Δ3 + Δ2^2 t);
CfF1Ab11 = {16 a AAA (3 - AAA^2 + (3 + 2 AAA^2) BBB^2) Δ2^2 Cos[β], 3 b BBB (-AAA^2 + Δ2^2)
    (7 - 3 AAA^2 + (13 + 2 AAA^2) BBB^2 + 6 BBB^4 + (5 - 9 AAA^2 + (7 + 6 AAA^2) BBB^2 + 2 BBB^4) Cos[2 β])
    Csc[β]}; PrmitiveF1Ab11NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 (ArcTan[argF1Ab11] (CfF1Ab11[[1]] + CfF1Ab11[[2]] r))/(48 Δ2^7);
```

```
argF1Ab22 = ((AAA + BBB) Δ1) / (1 - AAA BBB - AAA^2 - Δ3 - (1 + AAA BBB + BBB^2 - Δ3) t);
PrmitiveF1Ab22NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 ArcTan[argF1Ab22]/48 Csc[β] (8 a Sin[2 β] + 3 b r (3 + Cos[2 β]));
```

```
argF1Ab33 = ((AAA - BBB) Δ1) / (1 - AAA^2 + AAA BBB + Δ3 + (1 - AAA BBB + BBB^2 + Δ3) t);
PrmitiveF1Ab33NEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 - ArcTan[argF1Ab33]/48 Csc[β] (8 a Sin[2 β] - 3 b r (3 + Cos[2 β]));
```

The primitive of CF1BTot[t] is the sum of the following three functions

```
CfF1Ba = {0, 0, (Sin[fff] Sin[β])/4 (2 a AAA + (1 - 2 AAA^2) b r Cos[fff]) ,
  1/3 Sin[fff] (2 AAA b r Cos[β] + BBB (a - 2 AAA b r Cos[fff]) Sin[β]) ,
  - 1/8 b r Sin[fff] ((1 + 2 BBB^2) Cos[fff] Sin[β] - 4 BBB Cos[β])};
PrmitiveF1BaNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
 t^2 (CfF1Ba[[3]] + CfF1Ba[[4]] t + CfF1Ba[[5]] t^2);
```

```
CfF1BbAA = {{8 a (2 - 2 AAA^2 + (2 + AAA^2) BBB^2) Δ2^2 Cos[fff] Sin[β],
     AAA b (- 4 BBB (AAA^2 (- 13 + 2 BBB^2) + 13 Δ2^2) Cos[fff] Cos[β] -
         (8 + 3 BBB^2 - 5 BBB^4 + AAA^2 (- 8 + 9 BBB^2 + 2 BBB^4)) Cos[2 fff] Sin[β])},
   {- 8 a AAA BBB Δ2^4 Cos[fff] Sin[β], b Δ2^2 (4 (AAA^2 (- 3 + 2 BBB^2) + 3 Δ2^2) Cos[fff] Cos[β] +
        BBB (AAA^2 (7 + 2 BBB^2) - 3 Δ2^2) Cos[2 fff] Sin[β])}, {- 16 a Δ2^6 Cos[fff] Sin[β],
     2 AAA b Δ2^4 (- 4 BBB Cos[fff] Cos[β] + (4 + 5 BBB^2) Cos[2 fff] Sin[β])},
   {0, 6 b Δ2^6 (- 4 Cos[fff] Cos[β] + BBB Cos[2 fff] Sin[β])}};
PrmitvF1BbAANEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Δ1/(48 Δ2^6) * Sum[CfF1BbAA[[i, j]] t^(i - 1) r^(j - 1), {i, 1, 4}, {j, 1, 2}];
```

```
CfF1BbBB = {- 8 a AAA BBB Δ2^2 Cos[fff] Sin[β],
   4 b (4 Δ2^4 + 5 Δ3^2 - 4 Δ2^2 (1 + Δ3^2)) Cos[fff] Cos[β] + b BBB (4 Δ2^2 - 5 Δ3^2) Cos[2 fff] Sin[β]};
PrmitvF1BbBBNEW[r_, β_, a_, b_, fff_, AAA_, BBB_, t_] :=
  Δ3^2/(8 Δ2^7) ArcTan[(Δ1 Δ2)/(AAA BBB + t Δ2^2 + Δ3)] (CfF1BbBB[[1]] + CfF1BbBB[[2]] r );
```