

A

Overview of Orientation Relationship Models

A.1 Nishiyama-Wassermann (NW)

The transformation $T_{\text{NW}1}$ is uniquely defined through our unified approach (cf. Section 2) as the transformation that:

- leaves the normal $\mathbf{n} = (111)_\gamma$ and the direction $\mathbf{v} = [10\bar{1}]_\gamma$ unrotated,
- has pure stretch component U_2 .

The resulting transformation strain is

$$T_{\text{NW}1} = R_2 U_2 = R[\phi(r), [10\bar{1}]] U_2,$$

where $\phi(r) = \arccos\left(\frac{1+\sqrt{2}r}{\sqrt{3}\sqrt{1+r^2}}\right)$. The corresponding OR matrix is

$$O_{\text{NW}1} = R[-45^\circ, \mathbf{e}_2] R[-\phi(r), [10\bar{1}]]$$

which yields the OR

$$(111)_\gamma \parallel (01r)_{\alpha'} \text{ and } [10\bar{1}]_\gamma \parallel [100]_{\alpha'}.$$

The application of \mathcal{P}^{24} yields the remaining eleven NW ORs (cf. Table A1). Note that, unlike Table 1, Table A1 takes the tetragonality of the b.c.t. lattice into account

and the b.c.t. vectors are given in a way that is consistent with the transformation strains and not only up to crystallographic equivalence.

Tab. A1. The NW orientation relationships. The corresponding transformation strain in each row is given by $T_{\text{NW}j} = R[\phi(r), P_{2j-1}[10\bar{1}]] U_j$.

OR ^a	f.c.c. plane ^b	b.c.c. plane ^c	f.c.c. direction ^d	b.c.c. direction ^e	Bain Variant ^f
NW1	(111) $_{\gamma}$	(01r) $_{\alpha'}$	[101] $_{\gamma}$	[100] $_{\alpha'}$	U_2
NW2	(111) $_{\gamma}$	(r01) $_{\alpha'}$	[\bar{1}10] $_{\gamma}$	[010] $_{\alpha'}$	U_3
NW3	(111) $_{\gamma}$	(1r0) $_{\alpha'}$	[0\bar{1}1] $_{\gamma}$	[001] $_{\alpha'}$	U_1
NW4	(\bar{1}11) $_{\gamma}$	(\bar{r}10) $_{\alpha'}$	[101] $_{\gamma}$	[001] $_{\alpha'}$	U_2
NW5	(\bar{1}11) $_{\gamma}$	(0r1) $_{\alpha'}$	[\bar{1}\bar{1}0] $_{\gamma}$	[\bar{1}00] $_{\alpha'}$	U_3
NW6	(\bar{1}11) $_{\gamma}$	(\bar{1}0r) $_{\alpha'}$	[01\bar{1}] $_{\gamma}$	[010] $_{\alpha'}$	U_1
NW7	(1\bar{1}1) $_{\gamma}$	(r\bar{1}0) $_{\alpha'}$	[\bar{1}01] $_{\gamma}$	[001] $_{\alpha'}$	U_2
NW8	(1\bar{1}1) $_{\gamma}$	(0\bar{r}1) $_{\alpha'}$	[110] $_{\gamma}$	[100] $_{\alpha'}$	U_3
NW9	(1\bar{1}1) $_{\gamma}$	(10\bar{r}) $_{\alpha'}$	[0\bar{1}\bar{1}] $_{\gamma}$	[0\bar{1}0] $_{\alpha'}$	U_1
NW10	(11\bar{1}) $_{\gamma}$	(r10) $_{\alpha'}$	[\bar{1}0\bar{1}] $_{\gamma}$	[00\bar{1}] $_{\alpha'}$	U_2
NW11	(11\bar{1}) $_{\gamma}$	(0r\bar{1}) $_{\alpha'}$	[1\bar{1}0] $_{\gamma}$	[100] $_{\alpha'}$	U_3
NW12	(11\bar{1}) $_{\gamma}$	(10\bar{r}) $_{\alpha'}$	[011] $_{\gamma}$	[010] $_{\alpha'}$	U_1

^a NW_j

^b $P_{2j-1}(111)_{\gamma}$

^c $P_{2j-1}(01r)_{\alpha'}$

^d $P_{2j-1}[10\bar{1}]_{\gamma}$

^e $P_{2j-1}(100)_{\alpha'}$

^f $U_j = P_{2j-1}U_2P_{2j-1}^T$

A.2 Kurdjumov-Sachs (KS)

The transformation $T_{\text{KS}1}$ is uniquely defined through our unified approach (cf. Section 2) as the transformation that:

- leaves the normal $\mathbf{n} = (111)_\gamma$ and the direction $\mathbf{v} = [10\bar{1}]_\gamma$ unrotated,
- has pure stretch component U_3 .

The resulting transformation strain is

$$T_{\text{KS}1} = R[\theta(r), [111]] R[\phi(r), [\bar{1}10]] U_3,$$

where $\theta(r) = \arccos\left(\frac{\sqrt{3}\sqrt{r^2+1}+1}{2\sqrt{r^2+2}}\right)$, The corresponding OR matrix is

$$O_{\text{KS}1} = R[45^\circ, \mathbf{e}_3] R[-\phi(r), [\bar{1}10]] R[-\theta(r), [111]]$$

which yields the OR

$$(111)_\gamma \parallel (0r1)_{\alpha'} \text{ and } [10\bar{1}]_\gamma \parallel [11\bar{r}]_{\alpha'}.$$

The application of \mathcal{P}^{24} yields the remaining 23 KS ORs (cf. Table A2). Note that, unlike Table 2, Table A2 takes the tetragonality of the b.c.t. lattice into account and the b.c.t. vectors are given in a way that is consistent with the transformation strains and not only up to crystallographic equivalence.

Tab. A2. The KS orientation relationships. The corresponding transformation strain in each row is given by $T_{KSj} = R[\theta(r), P_j[111]] R[\phi(r), P_j[\bar{1}10]] U_j$.

OR ^a	f.c.c. plane ^b	b.c.c. plane ^c	f.c.c. direction ^d	b.c.c. direction ^e	Bain Variant ^f
KS1	(111) $_{\gamma}$	(0r1) $_{\alpha'}$	[10 $\bar{1}$] $_{\gamma}$	[11 \bar{r}] $_{\alpha'}$	U_3
KS2	($\bar{1}\bar{1}\bar{1}$) $_{\gamma}$	($\bar{1}\bar{r}0$) $_{\alpha'}$	[10 $\bar{1}$] $_{\gamma}$	[$r\bar{1}\bar{1}$] $_{\alpha'}$	U_1
KS3	(111) $_{\gamma}$	(10r) $_{\alpha'}$	[$\bar{1}10$] $_{\gamma}$	[$\bar{r}11$] $_{\alpha'}$	U_1
KS4	($\bar{1}\bar{1}\bar{1}$) $_{\gamma}$	(0 $\bar{1}\bar{r}$) $_{\alpha'}$	[$\bar{1}10$] $_{\gamma}$	[$\bar{1}r\bar{1}$] $_{\alpha'}$	U_2
KS5	(111) $_{\gamma}$	(r10) $_{\alpha'}$	[0 $\bar{1}1$] $_{\gamma}$	[1 $\bar{r}1$] $_{\alpha'}$	U_2
KS6	($\bar{1}\bar{1}\bar{1}$) $_{\gamma}$	($\bar{r}0\bar{1}$) $_{\alpha'}$	[0 $\bar{1}1$] $_{\gamma}$	[$\bar{1}\bar{1}r$] $_{\alpha'}$	U_3
KS7	($\bar{1}11$) $_{\gamma}$	($\bar{1}r0$) $_{\alpha'}$	[101] $_{\gamma}$	[$r11$] $_{\alpha'}$	U_1
KS8	(1 $\bar{1}\bar{1}$) $_{\gamma}$	(0 $\bar{r}\bar{1}$) $_{\alpha'}$	[101] $_{\gamma}$	[1 $\bar{1}r$] $_{\alpha'}$	U_3
KS9	($\bar{1}11$) $_{\gamma}$	(01r) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[$\bar{1}\bar{r}1$] $_{\alpha'}$	U_2
KS10	(1 $\bar{1}\bar{1}$) $_{\gamma}$	(10 \bar{r}) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[$\bar{r}\bar{1}\bar{1}$] $_{\alpha'}$	U_1
KS11	($\bar{1}11$) $_{\gamma}$	($\bar{r}01$) $_{\alpha'}$	[01 $\bar{1}$] $_{\gamma}$	[$\bar{1}1\bar{r}$] $_{\alpha'}$	U_3
KS12	(1 $\bar{1}\bar{1}$) $_{\gamma}$	(r $\bar{1}0$) $_{\alpha'}$	[01 $\bar{1}$] $_{\gamma}$	[1 $r\bar{1}$] $_{\alpha'}$	U_2
KS13	(1 $\bar{1}1$) $_{\gamma}$	(1 $\bar{r}0$) $_{\alpha'}$	[$\bar{1}01$] $_{\gamma}$	[$\bar{r}\bar{1}1$] $_{\alpha'}$	U_1
KS14	($\bar{1}1\bar{1}$) $_{\gamma}$	(0r $\bar{1}$) $_{\alpha'}$	[$\bar{1}01$] $_{\gamma}$	[$\bar{1}1r$] $_{\alpha'}$	U_3
KS15	(1 $\bar{1}1$) $_{\gamma}$	(0 $\bar{1}r$) $_{\alpha'}$	[110] $_{\gamma}$	[1 $r1$] $_{\alpha'}$	U_2
KS16	($\bar{1}1\bar{1}$) $_{\gamma}$	($\bar{1}0r$) $_{\alpha'}$	[110] $_{\gamma}$	[$r\bar{1}\bar{1}$] $_{\alpha'}$	U_1
KS17	(1 $\bar{1}1$) $_{\gamma}$	(r01) $_{\alpha'}$	[01 $\bar{1}$] $_{\gamma}$	[1 $\bar{1}\bar{r}$] $_{\alpha'}$	U_3
KS18	($\bar{1}1\bar{1}$) $_{\gamma}$	($\bar{r}10$) $_{\alpha'}$	[01 $\bar{1}$] $_{\gamma}$	[$\bar{1}\bar{r}\bar{1}$] $_{\alpha'}$	U_2
KS19	(11 $\bar{1}$) $_{\gamma}$	(1r0) $_{\alpha'}$	[$\bar{1}0\bar{1}$] $_{\gamma}$	[$\bar{r}1\bar{1}$] $_{\alpha'}$	U_1
KS20	($\bar{1}\bar{1}1$) $_{\gamma}$	(0 $\bar{r}1$) $_{\alpha'}$	[$\bar{1}0\bar{1}$] $_{\gamma}$	[$\bar{1}\bar{1}\bar{r}$] $_{\alpha'}$	U_3
KS21	(11 $\bar{1}$) $_{\gamma}$	(01 \bar{r}) $_{\alpha'}$	[1 $\bar{1}0$] $_{\gamma}$	[1 $\bar{r}\bar{1}$] $_{\alpha'}$	U_2
KS22	($\bar{1}\bar{1}1$) $_{\gamma}$	($\bar{1}0r$) $_{\alpha'}$	[1 $\bar{1}0$] $_{\gamma}$	[$r\bar{1}1$] $_{\alpha'}$	U_1
KS23	(11 $\bar{1}$) $_{\gamma}$	(r0 $\bar{1}$) $_{\alpha'}$	[011] $_{\gamma}$	[11r] $_{\alpha'}$	U_3
KS24	($\bar{1}\bar{1}1$) $_{\gamma}$	($\bar{r}\bar{1}0$) $_{\alpha'}$	[011] $_{\gamma}$	[$\bar{1}r1$] $_{\alpha'}$	U_2

^a KSj ^b $P_j(111)_{\gamma}$ ^c $P_j(0r1)_{\alpha'}$ ^d $P_j[10\bar{1}]_{\gamma}$ ^e $P_j[11\bar{r}]_{\alpha'}$ ^f $U_j = P_j U_3 P_j^T$

A.3 Pitsch (PT)

The transformation T_{P1} is uniquely defined through our unified approach (cf. Section 2) as the transformation that:

- leaves the normal $\mathbf{n} = (1\ 1\ 0)_\gamma$ and the direction $\mathbf{v} = [0\ 0\ 1]_\gamma$ unrotated,
- has pure stretch component U_2 .

The resulting transformation strain is

$$T_{P1} = R[\psi(r), [1\ 0\ 0]] U_2,$$

where $\psi(r) = -\arccos\left(\frac{\sqrt{2+r}}{\sqrt{2+r^2}}\right)$. The corresponding OR matrix is

$$O_{P1} = R[45^\circ, \mathbf{e}_2] R[-\psi(r), [1\ 0\ 0]]$$

which yields the OR

$$(0\ 1\ \bar{1})_\gamma \parallel (\bar{r}\ 2\ \bar{r})_{\alpha'} \text{ and } [1\ 0\ 0]_\gamma \parallel [1\ 0\ \bar{1}]_{\alpha'}$$

The application of \mathcal{P}^{24} yields the remaining eleven P ORs (cf. Table A3).

Remark

O_{P1} also yields the parallelism $[0\ 1\ 1]_\gamma \parallel [1\ r\ 1]_{\alpha'}$ stated in “The martensite transformation in thin foils of iron-nitrogen alloys” by Pitsch (for $r = 1$).

Tab. A3. The Pitsch orientation relationships. The corresponding transformation strain in each row is given by $T_{Pj} = R[\psi(r), P_{2j-1}[1\ 0\ 0]] U_j$.

OR ^a	f.c.c. plane ^b	b.c.c. plane ^c	f.c.c. direction ^d	b.c.c. direction ^e	Bain Variant ^f
P1	$(0\ 1\ \bar{1})_\gamma$	$(\bar{r}\ 2\ \bar{r})_{\alpha'}$	$[1\ 0\ 0]_\gamma$	$[1\ 0\ \bar{1}]_{\alpha'}$	U_2
P2	$(\bar{1}\ 0\ 1)_\gamma$	$(\bar{r}\ \bar{r}\ 2)_{\alpha'}$	$[0\ 1\ 0]_\gamma$	$[\bar{1}\ 1\ 0]_{\alpha'}$	U_3
P3	$(1\ \bar{1}\ 0)_\gamma$	$(2\ \bar{r}\ \bar{r})_{\alpha'}$	$[0\ 0\ 1]_\gamma$	$[0\ \bar{1}\ 1]_{\alpha'}$	U_1
P4	$(1\ 1\ 0)_\gamma$	$(r\ 2\ \bar{r})_{\alpha'}$	$[0\ 0\ 1]_\gamma$	$[1\ 0\ 1]_{\alpha'}$	U_2
P5	$(0\ \bar{1}\ 1)_\gamma$	$(r\ \bar{r}\ 2)_{\alpha'}$	$[\bar{1}\ 0\ 0]_\gamma$	$[\bar{1}\ \bar{1}\ 0]_{\alpha'}$	U_3
P6	$(\bar{1}\ 0\ \bar{1})_\gamma$	$(\bar{2}\ \bar{r}\ \bar{r})_{\alpha'}$	$[0\ 1\ 0]_\gamma$	$[0\ 1\ \bar{1}]_{\alpha'}$	U_1
P7	$(\bar{1}\ \bar{1}\ 0)_\gamma$	$(\bar{r}\ \bar{2}\ \bar{r})_{\alpha'}$	$[0\ 0\ 1]_\gamma$	$[\bar{1}\ 0\ 1]_{\alpha'}$	U_2
P8	$(0\ 1\ 1)_\gamma$	$(\bar{r}\ r\ 2)_{\alpha'}$	$[1\ 0\ 0]_\gamma$	$[1\ 1\ 0]_{\alpha'}$	U_3
P9	$(1\ 0\ \bar{1})_\gamma$	$(2\ r\ \bar{r})_{\alpha'}$	$[0\ \bar{1}\ 0]_\gamma$	$[0\ \bar{1}\ \bar{1}]_{\alpha'}$	U_1
P10	$(\bar{1}\ 1\ 0)_\gamma$	$(\bar{r}\ 2\ r)_{\alpha'}$	$[0\ 0\ \bar{1}]_\gamma$	$[\bar{1}\ 0\ \bar{1}]_{\alpha'}$	U_2
P11	$(0\ \bar{1}\ \bar{1})_\gamma$	$(\bar{r}\ \bar{r}\ \bar{2})_{\alpha'}$	$[1\ 0\ 0]_\gamma$	$[1\ \bar{1}\ 0]_{\alpha'}$	U_3
P12	$(1\ 0\ 1)_\gamma$	$(2\ \bar{r}\ r)_{\alpha'}$	$[0\ 1\ 0]_\gamma$	$[0\ 1\ 1]_{\alpha'}$	U_1

^a P_j

^b $P_{2j-1}(0\ 1\ \bar{1})_\gamma$

^c $P_{2j-1}(\bar{r}\ 2\ \bar{r})_{\alpha'}$

^d $P_{2j-1}[1\ 0\ 0]_\gamma$

^e $P_{2j-1}(1\ 0\ \bar{1})_{\alpha'}$

^f $U_j = P_{2j-1} U_2 P_{2j-1}^T$

A.4 Greninger-Troiano (GT)

The transformation T_{GT1} is uniquely defined through our unified approach (cf. Section 2) as the transformation that:

- leaves the normal $\mathbf{n} = (111)_\gamma$ and the direction $\mathbf{v} = [\bar{5}17\bar{1}\bar{2}]_\gamma$ unrotated,
- has pure stretch component U_3 .

The resulting transformation strain is

$$T_{\text{GT1}} = R[\xi(r), [111]] R[\phi(r), [\bar{1}10]] U_3,$$

where $\xi(r) = \arccos\left(\frac{7^2+17^2\sqrt{3}\sqrt{1+r^2}}{\sqrt{2}\sqrt{5^2+12^2+17^2}\sqrt{7^2+17^2+17^2r^2}}\right)$. The corresponding OR matrix is

$$O_{\text{GT1}} = R[45^\circ, \mathbf{e}_3] R[-\phi(r), [\bar{1}10]] R[-\xi(r), [111]]$$

which yields the OR

$$(111)_\gamma \parallel (0r1)_{\alpha'} \text{ and } [\bar{1}\bar{2}\bar{5}17]_\gamma \parallel [\bar{7}\bar{1}\bar{7}17r]_{\alpha'}.$$

The application of \mathcal{P}^{24} yields the remaining 23 GT ORs (cf. Table A4).

Example

Let $r = 1.045$ (as in “The mechanism of martensite formation” by Greninger and Troiano) then $(111)_\gamma : (011)_{\alpha'} \approx 1.26^\circ$, $[11\bar{2}]_\gamma : [01\bar{1}]_{\alpha'} \approx 2.82^\circ$, $[10\bar{1}]_\gamma : [11\bar{1}]_{\alpha'} \approx 2.94^\circ$ and $[0\bar{1}1]_\gamma : [1\bar{1}1]_{\alpha'} \approx 7.86^\circ$.

Tab. A4. The GT orientation relationships. The corresponding transformation strain in each row is given by $T_{GTj} = R[\xi(r), P_j[1\ 1\ 1]] R[\phi(r), P_j[\bar{1}\ 1\ 0]] U_j$.

OR ^a	f.c.c. plane ^b	b.c.c. plane ^c	f.c.c. direction ^d	b.c.c. direction ^e	Bain Variant ^f
GT1	(1 1 1) _γ	(0 r 1) _{α'}	[12 5 17] _γ	[7 17 17r] _{α'}	U_3
GT2	(1 1 1) _γ	(1 ̄r 0) _{α'}	[17 5 12] _γ	[17r 17 7] _{α'}	U_1
GT3	(1 1 1) _γ	(1 0 r) _{α'}	[17 12 5] _γ	[17r 7 17] _{α'}	U_1
GT4	(1 1 1) _γ	(0 1 ̄r) _{α'}	[12 17]5 _γ	[7 17r 17] _{α'}	U_2
GT5	(1 1 1) _γ	(r 1 0) _{α'}	[5 17 12] _γ	[17 17r 7] _{α'}	U_2
GT6	(1 1 1) _γ	(̄r 0 1) _{α'}	[5 12 17] _γ	[17 7 17r] _{α'}	U_3
GT7	(1 1 1) _γ	(1 ̄r 0) _{α'}	[17 5 12] _γ	[17r 17 7] _{α'}	U_1
GT8	(1 1 1) _γ	(0 ̄r 1) _{α'}	[12 5 17] _γ	[7 17 17r] _{α'}	U_3
GT9	(1 1 1) _γ	(0 1 r) _{α'}	[12 17 5] _γ	[7 17r 17] _{α'}	U_2
GT10	(1 1 1) _γ	(1 0 ̄r) _{α'}	[17 12 5] _γ	[17r 7 17] _{α'}	U_1
GT11	(1 1 1) _γ	(̄r 0 1) _{α'}	[5 12 17] _γ	[17 7 17r] _{α'}	U_3
GT12	(1 1 1) _γ	(r 1 0) _{α'}	[5 17 12] _γ	[17 17r 7] _{α'}	U_2
GT13	(1 1 1) _γ	(1 ̄r 0) _{α'}	[17 5 12] _γ	[17r 17 7] _{α'}	U_1
GT14	(1 1 1) _γ	(0 r 1) _{α'}	[12 5 17] _γ	[7 17 17r] _{α'}	U_3
GT15	(1 1 1) _γ	(0 1 ̄r) _{α'}	[12 17 5] _γ	[7 17r 17] _{α'}	U_2
GT16	(1 1 1) _γ	(1 0 r) _{α'}	[17 12 5] _γ	[17r 7 17] _{α'}	U_1
GT17	(1 1 1) _γ	(r 0 1) _{α'}	[5 12 17] _γ	[17 7 17r] _{α'}	U_3
GT18	(1 1 1) _γ	(̄r 1 0) _{α'}	[5 17 12] _γ	[17 17r 7] _{α'}	U_2
GT19	(1 1 1) _γ	(1 r 0) _{α'}	[17 5 12] _γ	[17r 17 7] _{α'}	U_1
GT20	(1 1 1) _γ	(0 ̄r 1) _{α'}	[12 5 17] _γ	[7 17 17r] _{α'}	U_3
GT21	(1 1 1) _γ	(0 1 ̄r) _{α'}	[12 17 5] _γ	[7 17r 17] _{α'}	U_2
GT22	(1 1 1) _γ	(1 0 r) _{α'}	[17 12 5] _γ	[17r 7 17] _{α'}	U_1
GT23	(1 1 1) _γ	(r 0 1) _{α'}	[5 12 17] _γ	[17 7 17r] _{α'}	U_3
GT24	(1 1 1) _γ	(̄r 1 0) _{α'}	[5 17 12] _γ	[17 17r 7] _{α'}	U_2

$$^a \text{GT}j \quad ^b P_j(1\ 1\ 1)_\gamma \quad ^c P_j(0\ r\ 1)_{\alpha'} \quad ^d P_j[12\ 5\ 17]_\gamma \quad ^e P_j[7\ 17\ 17r]_{\alpha'} \quad ^f U_j = P_j U_3 P_j^T$$

A.5 Inverse Greninger-Troiano (GT')

The transformation $T_{\text{GT}'1}$ is uniquely defined through our unified approach (cf. Section 2) as the transformation that:

- leaves the normal $\mathbf{n} = (\bar{1}\bar{7}\bar{7}17)_\gamma$ and the direction $\mathbf{v} = [101]_\gamma$ unrotated,
- has pure stretch component U_3 .

The resulting transformation strain is

$$T_{\text{GT}'1} = R[\iota(r), [101]] R[-\psi(r), [010]] U_3 = R[\iota(r), [101]] R_{P2} U_3$$

where $\iota(r) = \arccos\left(\frac{17^2\sqrt{2}\sqrt{2+r^2}+7^2r}{\sqrt{17^2+17^2+7^2}\sqrt{2\cdot17^2+7^2r^2+17^2r^2}}\right)$. The corresponding OR matrix is

$$O_{\text{GT}'1} = R[45^\circ, [001]] R[\psi(r), [010]] R[-\iota(r), [101]]$$

which yields the OR

$$(\bar{1}\bar{7}\bar{7}17)_\gamma \parallel (\bar{5}r\bar{1}\bar{2}r17)_{\alpha'} \text{ and } [101]_\gamma \parallel [11r]_{\alpha'}.$$

The application of \mathcal{P}^{24} yields the remaining 23 GT' ORs (cf. Table A5).

Tab. A5. The GT' orientation relationships. The corresponding transformation strain in each row is given by $T_{GT'j} = R[\iota(r), P_j[1\ 0\ 1]] R[-\psi(r), P_j[0\ 1\ 0]] U_j$.

OR ^a	f.c.c. plane ^b	b.c.c. plane ^c	f.c.c. direction ^d	b.c.c. direction ^e	Bain Variant ^f
GT'1	($\bar{1}\bar{7}\bar{7}17$) $_{\gamma}$	($\bar{5}r\bar{12}r17$) $_{\alpha'}$	[1 0 1] $_{\gamma}$	[1 1 r] $_{\alpha'}$	U_3
GT'2	($\bar{1}\bar{7}717$) $_{\gamma}$	($\bar{1}\bar{7}12r5r$) $_{\alpha'}$	[$\bar{1}0\bar{1}$] $_{\gamma}$	[$\bar{r}\bar{1}\bar{1}$] $_{\alpha'}$	U_1
GT'3	($17\bar{1}\bar{7}\bar{7}$) $_{\gamma}$	($17\bar{5}r\bar{12}r$) $_{\alpha'}$	[1 1 0] $_{\gamma}$	[$r11$] $_{\alpha'}$	U_1
GT'4	($17\bar{1}\bar{7}7$) $_{\gamma}$	($5\bar{1}\bar{7}r12r$) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[$\bar{1}\bar{r}\bar{1}$] $_{\alpha'}$	U_2
GT'5	($\bar{7}17\bar{1}\bar{7}$) $_{\gamma}$	($\bar{1}2r17\bar{5}r$) $_{\alpha'}$	[0 1 1] $_{\gamma}$	[1 r 1] $_{\alpha'}$	U_2
GT'6	($717\bar{1}\bar{7}$) $_{\gamma}$	($12r5r\bar{1}\bar{7}$) $_{\alpha'}$	[0 $\bar{1}\bar{1}$] $_{\gamma}$	[$\bar{1}\bar{1}\bar{r}$] $_{\alpha'}$	U_3
GT'7	($\bar{1}\bar{7}\bar{7}\bar{1}\bar{7}$) $_{\gamma}$	($\bar{1}\bar{7}\bar{12}r\bar{5}r$) $_{\alpha'}$	[$\bar{1}01$] $_{\gamma}$	[$\bar{r}11$] $_{\alpha'}$	U_1
GT'8	($\bar{1}\bar{7}7\bar{1}\bar{7}$) $_{\gamma}$	($\bar{5}r12r\bar{1}\bar{7}$) $_{\alpha'}$	[1 0 $\bar{1}$] $_{\gamma}$	[1 $\bar{1}r$] $_{\alpha'}$	U_3
GT'9	($1717\bar{7}$) $_{\gamma}$	($5r17\bar{1}\bar{2}r$) $_{\alpha'}$	[$\bar{1}10$] $_{\gamma}$	[$\bar{1}r1$] $_{\alpha'}$	U_2
GT'10	(17177) $_{\gamma}$	($175r12r$) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[$r\bar{1}\bar{1}$] $_{\alpha'}$	U_1
GT'11	($7\bar{1}\bar{7}17$) $_{\gamma}$	($\bar{1}2r\bar{5}r17$) $_{\alpha'}$	[0 1 1] $_{\gamma}$	[$\bar{1}1r$] $_{\alpha'}$	U_3
GT'12	($\bar{7}\bar{1}\bar{7}17$) $_{\gamma}$	($\bar{1}2r\bar{1}\bar{7}5r$) $_{\alpha'}$	[0 $\bar{1}\bar{1}$] $_{\gamma}$	[1 $\bar{r}\bar{1}$] $_{\alpha'}$	U_2
GT'13	($177\bar{1}\bar{7}$) $_{\gamma}$	($1712r\bar{5}r$) $_{\alpha'}$	[1 0 1] $_{\gamma}$	[$r\bar{1}1$] $_{\alpha'}$	U_1
GT'14	($17\bar{7}\bar{1}\bar{7}$) $_{\gamma}$	($5r\bar{12}r\bar{1}\bar{7}$) $_{\alpha'}$	[$\bar{1}0\bar{1}$] $_{\gamma}$	[$\bar{1}1\bar{r}$] $_{\alpha'}$	U_3
GT'15	($\bar{1}\bar{7}\bar{1}\bar{7}\bar{7}$) $_{\gamma}$	($\bar{5}r\bar{1}\bar{7}\bar{1}\bar{2}r$) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[1 $\bar{r}1$] $_{\alpha'}$	U_2
GT'16	($\bar{1}\bar{7}\bar{1}\bar{7}7$) $_{\gamma}$	($\bar{1}\bar{7}\bar{5}r12r$) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[$\bar{r}1\bar{1}$] $_{\alpha'}$	U_1
GT'17	($\bar{7}1717$) $_{\gamma}$	($\bar{1}2r5r17$) $_{\alpha'}$	[0 $\bar{1}1$] $_{\gamma}$	[1 $\bar{1}r$] $_{\alpha'}$	U_3
GT'18	(71717) $_{\gamma}$	($12r175r$) $_{\alpha'}$	[0 1 $\bar{1}$] $_{\gamma}$	[$\bar{1}r\bar{1}$] $_{\alpha'}$	U_2
GT'19	($17\bar{7}17$) $_{\gamma}$	($17\bar{12}r5r$) $_{\alpha'}$	[$\bar{1}01$] $_{\gamma}$	[$r1\bar{1}$] $_{\alpha'}$	U_1
GT'20	(17717) $_{\gamma}$	($5r12r17$) $_{\alpha'}$	[1 0 $\bar{1}$] $_{\gamma}$	[$\bar{1}\bar{1}r$] $_{\alpha'}$	U_3
GT'21	($\bar{1}\bar{7}177$) $_{\gamma}$	($\bar{5}r1712r$) $_{\alpha'}$	[$\bar{1}\bar{1}0$] $_{\gamma}$	[1 $r\bar{1}$] $_{\alpha'}$	U_2
GT'22	($\bar{1}\bar{7}17\bar{7}$) $_{\gamma}$	($\bar{1}\bar{7}5r\bar{1}\bar{2}r$) $_{\alpha'}$	[1 1 0] $_{\gamma}$	[$\bar{r}\bar{1}1$] $_{\alpha'}$	U_1
GT'23	($\bar{7}\bar{1}\bar{7}17$) $_{\gamma}$	($\bar{1}2r\bar{5}r\bar{1}\bar{7}$) $_{\alpha'}$	[0 $\bar{1}1$] $_{\gamma}$	[1 1 \bar{r}] $_{\alpha'}$	U_3
GT'24	($\bar{7}\bar{1}\bar{7}\bar{1}\bar{7}$) $_{\gamma}$	($\bar{1}2r\bar{1}\bar{7}5r$) $_{\alpha'}$	[0 1 $\bar{1}$] $_{\gamma}$	[$\bar{1}\bar{1}1$] $_{\alpha'}$	U_2

^a GT' j ^b $P_j(\bar{1}\bar{7}\bar{7}17)_{\gamma}$ ^c $P_j(\bar{5}r\bar{12}r17)_{\alpha'}$ ^d $P_j[1\ 0\ 1]_{\gamma}$ ^e $P_j[1\ 1\ r]_{\alpha'}$ ^f $U_j = P_j U_3 P_j^T$

B

The Group \mathcal{P}^{24}

The elements of \mathcal{P}^{24} in the standard Euclidean basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ are given by

$$\begin{aligned}
 P_1 = \mathbf{1} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P_2 = R[180^\circ, \mathbf{e}_1 - \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \\
 P_3 = R[120^\circ, \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & P_4 = R[180^\circ, \mathbf{e}_2 - \mathbf{e}_3] &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \\
 P_5 = R[-120^\circ, \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, & P_6 = R[180^\circ, \mathbf{e}_1 - \mathbf{e}_2] &= \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\
 P_7 = R[-90^\circ, \mathbf{e}_2] &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & P_8 = R[180^\circ, \mathbf{e}_1] &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\
 P_9 = R[180^\circ, \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & P_{10} = R[-120^\circ, \mathbf{e}_1 - \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \\
 P_{11} = R[90^\circ, \mathbf{e}_3] &= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P_{12} = R[120^\circ, \mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3] &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}, \\
 P_{13} = R[180^\circ, \mathbf{e}_1 + \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & P_{14} = R[180^\circ, \mathbf{e}_2] &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\
 P_{15} = R[90^\circ, \mathbf{e}_1] &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, & P_{16} = R[-120^\circ, \mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \\
 P_{17} = R[-90^\circ, \mathbf{e}_3] &= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P_{18} = R[120^\circ, -\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, \\
 P_{19} = R[90^\circ, \mathbf{e}_2] &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, & P_{20} = R[180^\circ, \mathbf{e}_3] &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
 P_{21} = R[-90^\circ, \mathbf{e}_1] &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, & P_{22} = R[-120^\circ, -\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\
 P_{23} = R[180^\circ, \mathbf{e}_1 + \mathbf{e}_2] &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, & P_{24} = R[120^\circ, \mathbf{e}_1 - \mathbf{e}_2 + \mathbf{e}_3] &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}.
 \end{aligned}$$

List of Figures

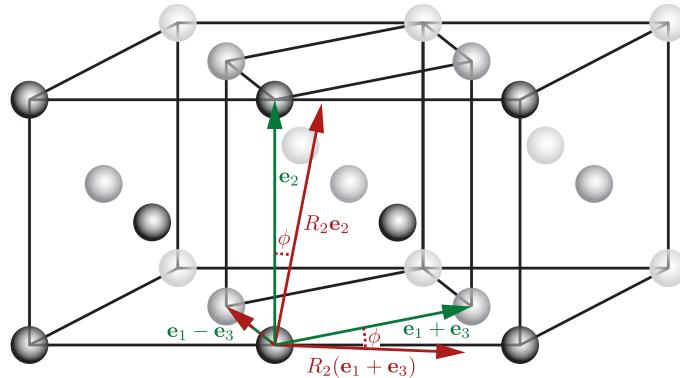


Fig. C.1. The green vectors $e_1 - e_3$, e_2 , $e_1 + e_3$ are along the edges of the tetragonal b.c.t. cell that is contained in the f.c.c. lattice and the red vectors are obtained through the rotation R_2 .

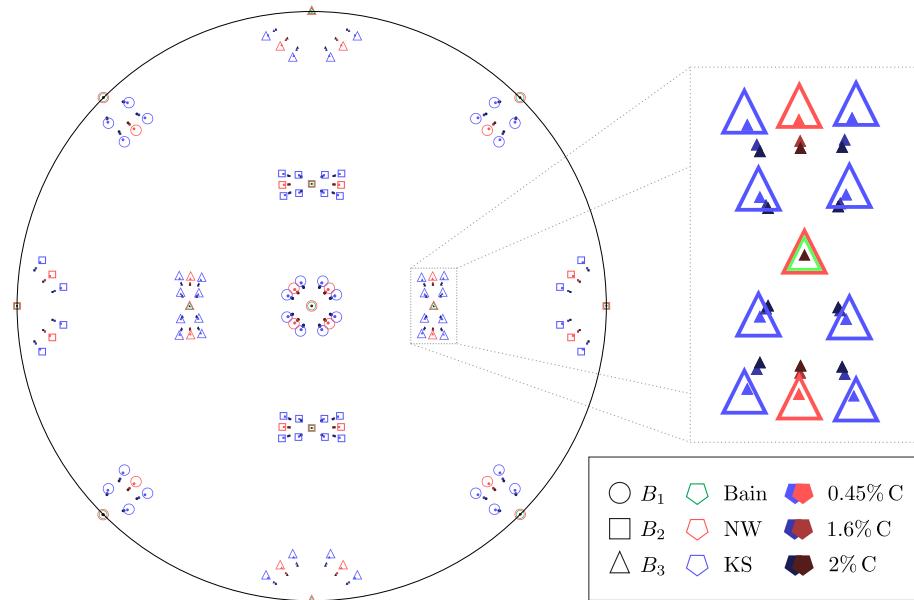


Fig. C.3. $\{100\}$ pole figures showing the change in the ORs with increasing carbon content. Hollow circles, squares and triangles correspond respectively to the f.c.c. to b.c.c. transformations with stretch components B_1 , B_2 and B_3 . The colours blue, red and green correspond respectively to KS, NW and Bain. The solid shapes correspond to increasing carbon content from lighter to darker shading and with values 0.45, 1.6 and 2 wt % C respectively.