



FOUNDATIONS
ADVANCES

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Supporting information for article:

Axial point groups: rank 1, 2, 3 and 4 property tensor tables

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LINE GROUPS

RANK 1,2,3 AND 4 PROPERTY TENSOR TABLES

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Property Tensor Tables

Rank 1 Property Tensor Tables

Table Format:

\vee	1	e	a	ae
	V	eV	aV	aeV

Series	n = 1	n > 1
C_n	$C_1 = 1$	$C_\infty = \infty$
C_{nv}	$C_{1v} = m_y$	$C_{\infty v} = \infty mm$
S_{2n}	$S_2 = \bar{1}$	$C_{\infty h} = \infty/m$
C_{nh}	$C_{1h} = m_z$	$C_{\infty h} = \infty/m$
D_n	$D_1 = 2_y$	$D_\infty = \infty 2$
D_{nd}	$D_{1d} = 2_y/m_y$	$D_{\infty h} = \infty/mmm$
D_{nh}	$D_{1h} = 2_x m_y m_z$	$D_{\infty h} = \infty/mmm$
$C_{2n}(C_n)$	$C_2(C_1) = 2_z(1) = 2_z'$	$C_\infty 1' = \infty 1'$
$S_{2n}(C_n)$	$S_2(C_1) = \bar{1}(1) = \bar{1}'$	$C_{\infty h}(C_\infty) = \infty/m'$
$C_{nh}(C_n)$	$C_{1h}(C_1) = m_z(1) = m_z'$	$C_{\infty h}(C_\infty) = \infty/m'$
$C_{2nh}(S_{2n})$	$C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$	$C_{\infty h} 1' = \infty/m'1'$
$C_{2nh}(C_{nh})$	$C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z'$	$C_{\infty h} 1' = \infty/m'1'$
$D_n(C_n)$	$D_1(C_1) = 2_y(1) = 2_y'$	$D_\infty(C_\infty) = \infty 2'$
$D_{2n}(D_n)$	$D_2(D_1) = 2_x 2_y 2(2_y) = 2_x' 2_y' 2_z'$	$D_\infty 1' = \infty 21'$
$C_{nv}(C_n)$	$C_{1v}(C_1) = m_y(1) = m_y'$	$C_{\infty v}(C_\infty) = \infty m'm'$
$C_{2nv}(C_{nv})$	$C_{2v}(C_{1v}) = 2_z m_x m_y(m_y) = 2_z' m_x' m_y'$	$C_{\infty v} 1' = \infty mm1'$
$D_{nd}(S_{2n})$	$D_{1d}(S_2) = 2_y/m_y(\bar{1}) = 2_y'/m_y'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$
$D_{nd}(D_n)$	$D_{1d}(D_1) = 2_y/m_y(2_y) = 2_y'/m_y'$	$D_{\infty h}(D_\infty) = \infty/m'm'm'$
$D_{nd}(C_{nv})$	$D_{1d}(C_{1v}) = 2_y/m_y(m_y) = 2_y'/m_y'$	$D_{\infty h}(C_{\infty v}) = \infty/m'mm$
$D_{nh}(C_{nh})$	$D_{1h}(C_{1h}) = 2_x m_y m_z(m_z) = 2_x' m_y' m_z'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$
$D_{nh}(D_n)$	$D_{1h}(D_1) = 2_x m_y m_z(2_x) = 2_x' m_y' m_z'$	$D_{\infty h}(D_\infty) = \infty/m'm'm'$
$D_{nh}(C_{nv})$	$D_{1h}(C_{1v}) = 2_x m_y m_z(m_y) = 2_x' m_y' m_z'$	$D_{\infty h}(C_{\infty v}) = \infty/m'mm$
$D_{2nh}(D_{nd})$	$D_{2h}(D_{1d}) = m_x m_y m_z(2_y/m_y) = m_x' m_y' m_z'$	$D_{\infty h} 1' = \infty/mmm1'$
$D_{2nh}(D_{nh})$	$D_{2h}(D_{1h}) = m_x m_y m_z(2_x m_y m_z) = m_x' m_y' m_z'$	$D_{\infty h} 1' = \infty/mmm1'$

Rank 2 Property Tensor Tables

Table Format:

	1	e	a	ae
V^2	V^2	eV^2	aV^2	aeV^2
$[V^2]$	$[V^2]$	$e[V^2]$	$a[V^2]$	$ae[V^2]$
$\{V^2\}$	$\{V^2\}$	$e\{V^2\}$	$a\{V^2\}$	$ae\{V^2\}$

Series	$n = 1$	$n = 2$	$n > 2$
C_n	$C_1 = 1$	$C_2 = 2_z$	$C_\infty = \infty$
C_{nv}	$C_{1v} = m_y$	$C_{2v} = 2_z m_x m_y$	$C_{\infty v} = \infty mm$
S_{2n}	$S_2 = \bar{1}$	$S_4 = \bar{4}_z$	$C_{\infty h} = \infty/m$
C_{nh}	$C_{1h} = m_z$	$C_{2h} = 2/m_z$	$C_{\infty h} = \infty/m$
D_n	$D_1 = 2_y$	$D_2 = 2_x 2_y 2_z$	$D_\infty = \infty 2$
D_{nd}	$D_{1d} = 2_y/m_y$	$D_{2d} = \bar{4}_z m_x 2_{xy}$	$D_{\infty h} = \infty/mmm$
D_{nh}	$D_{1h} = 2_x m_y m_z$	$D_{2h} = m_x m_y m_z$	$D_{\infty h} = \infty/mmm$
$C_{2n}(C_n)$	$C_2(C_1) = 2_z(1) = 2_z'$	$C_4(C_2) = 4_z(2_z) = 4_z'$	$C_\infty 1' = \infty 1'$
$S_{2n}(C_n)$	$S_2(C_1) = \bar{1}(1) = \bar{1}'$	$S_4(C_2) = \bar{4}_z(2_z) = \bar{4}_z'$	$C_{\infty h}(C_\infty) = \infty/m'$
$C_{nh}(C_n)$	$C_{1h}(C_1) = m_z(1) = m_z'$	$C_{2h}(C_2) = 2_z/m_z(2_z) = 2_z/m_z'$	$C_{\infty h}(C_\infty) = \infty/m'$
$C_{2nh}(S_{2n})$	$C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$	$C_{4h}(S_4) = 4_z/m_z(\bar{4}_z) = 4_z'/m_z'$	$C_{\infty h} 1' = \infty/m1'$

Series	n = 1	n = 2	n > 2	Rank 2
	$C_{2nh}(C_{nh})$	$C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z$	$C_{4h}(C_{2h}) = 4_z/m_z(2_z/m_z) = 4_z'/m_z$	
$D_n(C_n)$	$D_1(C_1) = 2_y(1) = 2_y'$	$D_2(C_2) = 2_x2_y2_z(2_z) = 2_x'2_y'2_z$	$D_{\infty}(C_{\infty}) = \infty2'$	
$D_{2n}(D_n)$	$D_2(D_1) = 2_x2_y2(2_y) = 2_x'2_y2_z'$	$D_4(D_2) = 4_z2_x2_{xy}(2_x2_y2_z) = 4_z'2_x2_{xy}'$	$D_{\infty}1' = \infty21'$	
$C_{nv}(C_n)$	$C_{1v}(C_1) = m_y(1) = m_y'$	$C_{2v}(C_2) = 2_zm_xm_y(2_z) = 2_zm_x'm_y'$	$C_{\infty v}(C_{\infty}) = \infty m'm'$	
$C_{2nv}(C_{nv})$	$C_{2v}(C_{1v}) = 2_zm_xm_y(m_y) = 2_z'm_x'm_y$	$C_{4v}(C_{2v}) = 4_zm_xm_{xy}(2_zm_xm_y) = 4_z'm_xm_{xy}'$	$C_{\infty v}1' = \infty mm1'$	
$D_{nd}(S_{2n})$	$D_{1d}(S_2) = 2_y/m_y(\bar{1}) = 2_y'/m_y'$	$D_{2d}(S_4) = \bar{4}_z m_x2_{xy}(\bar{4}_z) = \bar{4}_z m_x'2_{xy}'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$	
$D_{nd}(D_n)$	$D_{1d}(D_1) = 2_y/m_y(2_y) = 2_y'/m_y'$	$D_{2d}(D_2) = \bar{4}_z m_x2_{xy}(2_{xy}\bar{2}_z) = \bar{4}_z'm_x'2_{xy}$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'$	
$D_{nd}(C_{nv})$	$D_{1d}(C_{1v}) = 2_y/m_y(m_y) = 2_y'/m_y$	$D_{2d}(C_{2v}) = \bar{4}_z m_x2_{xy}(2_zm_xm_y) = \bar{4}_z'm_x2_{xy}'$	$D_{\infty h}(C_{\infty v}) = \infty/m'mm$	
$D_{nh}(C_{nh})$	$D_{1h}(C_{1h}) = 2_xm_y m_z(m_z) = 2_x'm_y'm_z$	$D_{2h}(C_{2h}) = m_xm_y m_z(2_z/m_z) = m_x'm_y'm_z$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$	
$D_{nh}(D_n)$	$D_{1h}(D_1) = 2_xm_y m_z(2_x) = 2_x'm_y'm_z'$	$D_{2h}(D_2) = m_xm_y m_z(2_x2_y2_z) = m_x'm_y'm_z'$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'$	
$D_{nh}(C_{nv})$	$D_{1h}(C_{1v}) = 2_xm_y m_z(m_y) = 2_x'm_y m_z'$	$D_{2h}(C_{2v}) = m_xm_y m_z(2_zm_xm_y) = m_xm_y m_z'$	$D_{\infty h}(C_{\infty v}) = \infty/m'mm$	
$D_{2nh}(D_{nd})$	$D_{2h}(D_{1d}) = m_xm_y m_z(2_y/m_y) = m_x'm_y m_z'$	$D_{4h}(D_{2d}) = 4_z/m_z m_x m_{xy}(\bar{4}_z m_x2_{xy}) = 4_z'/m_z'm_x m_{xy}'$	$D_{\infty h}1' = \infty/mmm1'$	
$D_{2nh}(D_{nh})$	$D_{2h}(D_{1h}) = m_xm_y m_z(2_xm_y m_z) = m_x'm_y m_z$	$D_{4h}(D_{2h}) = 4_z/m_z m_x m_{xy}(m_xm_y m_z) = 4_z'/m_z m_x m_{xy}'$	$D_{\infty h}1' = \infty/mmm1'$	

Rank 3 Property Tensor Tables

Table Format:

	1	e	a	ae
V^3	V^3	eV^3	aV^3	aeV^3
$[V^3]$	$[V^3]$	$e[V^3]$	$a[V^3]$	$ae[V^3]$
$V[V^2]$	$V[V^2]$	$eV[V^2]$	$aV[V^2]$	$aeV[V^2]$
$\{V^2\}V$	$\{V^2\}V$	$e\{V^2\}V$	$a\{V^2\}V$	$ae\{V^2\}V$

Series	$n = 1$	$n = 2$	$n = 3$	$n > 3$
C_n	$C_1 = 1$	$C_2 = 2_z$	$C_3 = 3_z$	$C_\infty = \infty$
C_{nv}	$C_{1v} = m_y$	$C_{2v} = 2_z m_x m_y$	$C_{3v} = 3_z m_x$	$C_\infty v = \infty mm$
S_{2n}	$S_2 = \bar{1}$	$S_4 = \bar{4}_z$	$S_6 = \bar{3}_z$	$C_\infty h = \infty / m$
C_{nh}	$C_{1h} = m_z$	$C_{2h} = 2 / m_z$	$C_{3h} = \bar{6}_z$	$C_\infty h = \infty / m$
D_n	$D_1 = 2_y$	$D_2 = 2_x 2_y 2_z$	$D_3 = 3_z 2_x$	$D_\infty = \infty 2$
D_{nd}	$D_{1d} = 2_y / m_y$	$D_{2d} = \bar{4}_z m_x 2_{xy}$	$D_{3d} = \bar{3}_z m_x$	$D_\infty h = \infty / mmm$
D_{nh}	$D_{1h} = 2_x m_y m_z$	$D_{2h} = m_x m_y m_z$	$D_{3h} = \bar{6}_z m_x 2_1$	$D_\infty h = \infty / mmm$

Series	n = 1	n = 2	n = 3	n > 3	Rank 3
$C_{2n}(C_n)$	$C_2(C_1) = 2_z(1) = 2_z'$	$C_4(C_2) = 4_z(2_z) = 4_z'$	$C_6(C_3) = 6_z(3_z) = 6_z'$	$C_\infty 1' = \infty 1'$	
$S_{2n}(C_n)$	$S_2(C_1) = \bar{1}(1) = \bar{1}'$	$S_4(C_2) = \bar{4}_z(2_z) = \bar{4}_z'$	$S_6(C_3) = \bar{3}_z(3_z) = \bar{3}_z'$	$C_{\infty h}(C_\infty) = \infty/m'$	
$C_{nh}(C_n)$	$C_{1h}(C_1) = m_z(1) = m_z'$	$C_{2h}(C_2) = 2_z/m_z(2_z) = 2_z/m_z'$	$C_{3h}(C_3) = \bar{6}_z(3_z) = \bar{6}_z'$	$C_{\infty h}(C_\infty) = \infty/m'$	
$C_{2nh}(S_{2n})$	$C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$	$C_{4h}(S_4) = 4_z/m_z(\bar{4}_z) = 4_z'/m_z'$	$C_{6h}(S_6) = 6_z/m_z(\bar{3}_z) = 6_z'/m_z'$	$C_{\infty h} 1' = \infty/m 1'$	
$C_{2nh}(C_{nh})$	$C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z$	$C_{4h}(C_{2h}) = 4_z/m_z(2_z/m_z) = 4_z'/m_z$	$C_{6h}(C_{3h}) = 6_z/m_z(3_z/m_z) = 6_z'/m_z$	$C_{\infty h} 1' = \infty/m 1'$	
$D_n(C_n)$	$D_1(C_1) = 2_y(1) = 2_y'$	$D_2(C_2) = 2_x 2_y 2_z(2_z) = 2_x' 2_y' 2_z'$	$D_3(C_3) = 3_z 2_x(3_z) = 3_z 2_x'$	$D_\infty(C_\infty) = \infty 2'$	
$D_{2n}(D_n)$	$D_2(D_1) = 2_x 2_y 2(2_y) = 2_x' 2_y 2_z'$	$D_4(D_2) = 4_z 2_x 2_{xy}(2_x 2_y 2_z) = 4_z' 2_x 2_{xy}'$	$D_6(D_3) = 6_z 2_x 2_1(3_z 2_x) = 6_z' 2_x 2_1'$	$D_\infty 1' = \infty 2 1'$	
$C_{nv}(C_n)$	$C_{1v}(C_1) = m_y(1) = m_y'$	$C_{2v}(C_2) = 2_z m_x m_y(2_z) = 2_z m_x' m_y'$	$C_{3v}(C_3) = 3_z m_x(3_z) = 3_z m_x'$	$C_{\infty v}(C_\infty) = \infty m' m'$	
$C_{2nv}(C_{nv})$	$C_{2v}(C_{1v}) = 2_z m_x m_y(m_y) = 2_z' m_x' m_y$	$C_{4v}(C_{2v}) = 4_z m_x m_{xy}(2_z m_x m_y) = 4_z' m_x m_{xy}'$	$C_{6v}(C_{3v}) = 6_z m_x m_1(3_z m_x) = 6_z' m_x m_1'$	$C_{\infty v} 1' = \infty m m 1'$	
$D_{nd}(S_{2n})$	$D_{1d}(S_2) = 2_y/m_y(\bar{1}) = 2_y'/m_y'$	$D_{2d}(S_4) = \bar{4}_z m_x 2_{xy}(\bar{4}_z) = \bar{4}_z m_x' 2_{xy}'$	$D_{3d}(S_6) = \bar{3}_z m_x(\bar{3}_z) = \bar{3}_z m_x'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$	

Series	n = 1	n = 2	n = 3	n > 3	Rank 3
$D_{nd}(D_n)$	$D_{1d}(D_1) = 2_y/m_y(2_y) = 2_y/m_y'$	$D_{2d}(D_2) = \bar{4}_z m_x 2_{xy}(2_{xy} \bar{2}_{xy} 2_z) = \bar{4}_z 'm_x' 2_{xy}$	$D_{3d}(D_3) = \bar{3}_z m_x(3_z 2_x) = \bar{3}_z 'm_x'$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'$	
$D_{nd}(C_{nv})$	$D_{1d}(C_{1v}) = 2_y/m_y(m_y) = 2_y'/m_y$	$D_{2d}(C_{2v}) = \bar{4}_z m_x 2_{xy}(2_z m_x m_y) = \bar{4}_z 'm_x 2_{xy}'$	$D_{3d}(C_{3v}) = \bar{3}_z m_x(3_z m_x) = \bar{3}_z 'm_x$	$D_{\infty h}(C_{\infty v}) = \infty/m'm'mm$	
$D_{nh}(C_{nh})$	$D_{1h}(C_{1h}) = 2_x m_y m_z(m_z) = 2_x 'm_y' m_z$	$D_{2h}(C_{2h}) = m_x m_y m_z(2_z/m_z) = m_x 'm_y' m_z$	$D_{3h}(C_{3h}) = \bar{6}_z m_x 2_1(\bar{6}_z) = \bar{6}_z m_x' 2_1'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'm'$	
$D_{nh}(D_n)$	$D_{1h}(D_1) = 2_x m_y m_z(2_x) = 2_x m_y' m_z'$	$D_{2h}(D_2) = m_x m_y m_z(2_x 2_y 2_z) = m_x 'm_y' m_z'$	$D_{3h}(D_3) = \bar{6}_z m_x 2_1(3_z 2_1) = \bar{6}_z 'm_x' 2_1'$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'm'$	
$D_{nh}(C_{nv})$	$D_{1h}(C_{1v}) = 2_x m_y m_z(m_y) = 2_x 'm_y m_z'$	$D_{2h}(C_{2v}) = m_x m_y m_z(2_z m_x m_y) = m_x m_y m_z'$	$D_{3h}(C_{3v}) = \bar{6}_z m_x 2_1(3_z m_x) = \bar{6}_z 'm_x 2_1'$	$D_{\infty h}(C_{\infty v}) = \infty/m'm'mm$	
$D_{2nh}(D_{nd})$	$D_{2h}(D_{1d}) = m_x m_y m_z(2_y/m_y) = m_x 'm_y m_z'$	$D_{4h}(D_{2d}) = 4_z/m_z m_x m_{xy}(\bar{4}_z m_x 2_{xy}) = 4_z'/m_z' m_x m_{xy}'$	$D_{6h}(D_{3d}) = 6_z/m_z m_x m_1(\bar{3}_z m_x) = 6_z'/m_z' m_x m_1'$	$D_{\infty h} 1' = \infty/mmm1'$	
$D_{2nh}(D_{nh})$	$D_{2h}(D_{1h}) = m_x m_y m_z(2_x m_y m_z) = m_x 'm_y m_z$	$D_{4h}(D_{2h}) = 4_z/m_z m_x m_{xy}(m_x m_y m_z) = 4_z'/m_z m_x m_{xy}'$	$D_{6h}(D_{3h}) = 6_z/m_z m_x m_1(\bar{6}_z m_x 2_1) = 6_z'/m_z m_x m_1'$	$D_{\infty h} 1' = \infty/mmm1'$	

Rank 4 Property Tensor Tables

		1	e	a	ae
Table Format:	V^4	V^4	eV^4	aV^4	aeV^4
	$[V^4]$	$[V^4]$	$e[V^4]$	$a[V^4]$	$ae[V^4]$
	$V[V^3]$	$V[V^3]$	$eV[V^3]$	$aV[V^3]$	$aeV[V^3]$
	$[[V^2]^2]$	$[[V^2]^2]$	$e[[V^2]^2]$	$a[[V^2]^2]$	$ae[[V^2]^2]$
	$[V^2]^2$	$[V^2]^2$	$e[V^2]^2$	$a[V^2]^2$	$ae[V^2]^2$
	$[(V^2)]^2$	$[(V^2)]^2$	$e[(V^2)]^2$	$a[(V^2)]^2$	$ae[(V^2)]^2$
	$[V^2]V^2$	$[V^2]V^2$	$e[V^2]V^2$	$a[V^2]V^2$	$ae[V^2]V^2$

Series	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n > 4$
C_n	$C_1 = 1$	$C_2 = 2_z$	$C_3 = 3_z$	$C_4 = 4_z$	$C_\infty = \infty$
C_{nv}	$C_{1v} = m_y$	$C_{2v} = 2_z m_x m_y$	$C_{3v} = 3_z m_x$	$C_{4v} = 4_z m_x m_{xy}$	$C_{\infty v} = \infty mm$
S_{2n}	$S_2 = \bar{1}$	$S_4 = \bar{4}_z$	$S_6 = \bar{3}_z$	$S_8 = \bar{8}_z$	$C_{\infty h} = \infty/m$

Series	n = 1	n = 2	n = 3	n = 4	n > 4	Rank 4
C_{nh}	$C_{1h} = m_z$	$C_{2h} = 2/m_z$	$C_{3h} = \bar{6}_z$	$C_{4h} = 4_z/m_z$	$C_{\infty h} = \infty/m$	
D_n	$D_1 = 2_y$	$D_2 = 2_x 2_y 2_z$	$D_3 = 3_z 2_x$	$D_4 = 4_z 2_x 2_{xy}$	$D_{\infty} = \infty 2$	
D_{nd}	$D_{1d} = 2_y/m_y$	$D_{2d} = \bar{4}_z m_x 2_{xy}$	$D_{3d} = \bar{3}_z m_x$	$D_{4d} = \bar{8}_z m_x 2_1$	$D_{\infty h} = \infty/mmm$	
D_{nh}	$D_{1h} = 2_x m_y m_z$	$D_{2h} = m_x m_y m_z$	$D_{3h} = \bar{6}_z m_x 2_1$	$D_{4h} = 4_z/m_z m_x m_{xy}$	$D_{\infty h} = \infty/mmm$	
$C_{2n}(C_n)$	$C_2(C_1) = 2_z(1) = 2_z'$	$C_4(C_2) = 4_z(2_z) = 4_z'$	$C_6(C_3) = 6_z(3_z) = 6_z'$	$C_8(C_4) = 8_z(4_z) = 8_z'$	$C_{\infty} 1' = \infty 1'$	
$S_{2n}(C_n)$	$S_2(C_1) = \bar{1}(1) = \bar{1}'$	$S_4(C_2) = \bar{4}_z(2_z) = \bar{4}_z'$	$S_6(C_3) = \bar{3}_z(3_z) = \bar{3}_z'$	$S_8(C_4) = \bar{8}_z(4_z) = \bar{8}_z'$	$C_{\infty h}(C_{\infty}) = \infty/m'$	
$C_{nh}(C_n)$	$C_{1h}(C_1) = m_z(1) = m_z'$	$C_{2h}(C_2) = 2_z/m_z(2_z) = 2_z/m_z'$	$C_{3h}(C_3) = \bar{6}_z(3_z) = \bar{6}_z'$	$C_{4h}(C_4) = 4_z/m_z(4_z) = 4_z/m_z'$	$C_{\infty h}(C_{\infty}) = \infty/m'$	

Series	n = 1	n = 2	n = 3	n = 4	n > 4	Rank 4
$C_{2nh}(S_{2n})$		$C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$	$C_{4h}(S_4) = 4_z/m_z(\bar{4}_z) = 4_z'/m_z'$	$C_{6h}(S_6) = 6_z/m_z(\bar{3}_z) = 6_z'/m_z'$	$C_{8h}(S_8) = 8_z/m_z(\bar{8}_z) = 8_z'/m_z'$	$C_{\infty h} 1' = \infty/m1'$
$C_{2nh}(C_{nh})$		$C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z$	$C_{4h}(C_{2h}) = 4_z/m_z(2_z/m_z) = 4_z'/m_z$	$C_{6h}(C_{3h}) = 6_z/m_z(3_z/m_z) = 6_z'/m_z$	$C_{8h}(C_{4h}) = 8_z/m_z(4_z/m_z) = 8_z'/m_z$	$C_{\infty h} 1' = \infty/m1'$
$D_n(C_n)$		$D_1(C_1) = 2_y(1) = 2_y'$	$D_2(C_2) = 2_x 2_y 2_z(2_z) = 2_x' 2_y' 2_z'$	$D_3(C_3) = 3_z 2_x(3_z) = 3_z 2_x'$	$D_4(C_4) = 4_z 2_x 2_{xy}(4_z) = 4_z 2_x' 2_{xy}'$	$D_{\infty}(C_{\infty}) = \infty 2'$
$D_{2n}(D_n)$		$D_2(D_1) = 2_x 2_y 2(2_y) = 2_x' 2_y 2_z'$	$D_4(D_2) = 4_z 2_x 2_{xy}(2_x 2_y 2_z) = 4_z' 2_x 2_{xy}'$	$D_6(D_3) = 6_z 2_x 2_1(3_z 2_x) = 6_z' 2_x 2_1'$	$D_8(D_4) = 8_z 2_{x1} 2_1(4_z 2_{x1} 2_{x2}) = 8_z' 2_{x1} 2_1'$	$D_{\infty} 1' = \infty 21'$
$C_{nv}(C_n)$		$C_{1v}(C_1) = m_y(1) = m_y'$	$C_{2v}(C_2) = 2_z m_x m_y(2_z) = 2_z m_x' m_y'$	$C_{3v}(C_3) = 3_z m_x(3_z) = 3_z m_x'$	$C_{4v}(C_4) = 4_z m_x m_{xy}(4_z) = 4_z m_x' m_{xy}'$	$C_{\infty v}(C_{\infty}) = \infty m'm'$
$C_{2nv}(C_{nv})$		$C_{2v}(C_{1v}) = 2_z m_x m_y(m_y) = 2_z' m_x' m_y$	$C_{4v}(C_{2v}) = 4_z m_x m_{xy}(2_z m_x m_y) = 4_z' m_x m_{xy}'$	$C_{6v}(C_{3v}) = 6_z m_x m_1(3_z m_x) = 6_z' m_x m_1'$	$C_{8v}(C_{4v}) = 8_z m_{x1} m_1(4_z m_x m_{xy}) = 8_z' m_{x1} m_1'$	$C_{\infty v} 1' = \infty mm1'$
$D_{nd}(S_{2n})$		$D_{1d}(S_2) = 2_y/m_y(\bar{1}) = 2_y'/m_y'$	$D_{2d}(S_4) = \bar{4}_z m_x 2_{xy}(\bar{4}_z) = \bar{4}_z m_x' 2_{xy}'$	$D_{3d}(S_6) = \bar{3}_z m_x(\bar{3}_z) = \bar{3}_z m_x'$	$D_{4d}(S_8) = \bar{8}_z m_{x1} 2_1(\bar{8}_z) = \bar{8}_z m_{x1}' 2_1'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$

Series	n = 1	n = 2	n = 3	n = 4	n > 4	Rank 4
$D_{nd}(D_n)$	$D_{1d}(D_1) = 2_y/m_y(2_y) = 2_y/m_y'$	$D_{2d}(D_2) = \bar{4}_z m_x 2_{xy}(2_{xy} 2_{xy}) = \bar{4}_z 'm_x' 2_{xy}$	$D_{3d}(D_3) = \bar{3}_z m_x(3_z 2_x) = \bar{3}_z 'm_x'$	$D_{4d}(D_4) = \bar{8}_z m_x 2_1(4_z 2_x 2_{xy}) = \bar{8}_z 'm_x' 2_1$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'$	
$D_{nd}(C_{nv})$	$D_{1d}(C_{1v}) = 2_y/m_y(m_y) = 2_y'/m_y$	$D_{2d}(C_{2v}) = \bar{4}_z m_x 2_{xy}(2_z m_x m_y) = \bar{4}_z 'm_x' 2_{xy}'$	$D_{3d}(C_{3v}) = \bar{3}_z m_x(3_z m_x) = \bar{3}_z 'm_x'$	$D_{4d}(C_{4v}) = \bar{8}_z m_x 2_1(4_z m_x 1 m_{x3}) = \bar{8}_z 'm_x' 2_1$	$D_{\infty h}(C_{\infty v}) = \infty/m'm'm'$	
$D_{nh}(C_{nh})$	$D_{1h}(C_{1h}) = 2_x m_y m_z(m_z) = 2_x 'm_y' m_z$	$D_{2h}(C_{2h}) = m_x m_y m_z(2_z/m_z) = m_x 'm_y' m_z$	$D_{3h}(C_{3h}) = \bar{6}_z m_x 2_1(\bar{6}_z) = \bar{6}_z m_x '2_1'$	$D_{4h}(C_{4h}) = 4_z/m_z m_x m_{xy}(4_z/m_z) = 4_z/m_z m_x 'm_{xy}'$	$D_{\infty h}(C_{\infty h}) = \infty/mm'm'$	
$D_{nh}(D_n)$	$D_{1h}(D_1) = 2_x m_y m_z(2_x) = 2_x m_y 'm_z'$	$D_{2h}(D_2) = m_x m_y m_z(2_x 2_y 2_z) = m_x 'm_y' m_z'$	$D_{3h}(D_3) = \bar{6}_z m_x 2_1(3_z 2_1) = \bar{6}_z 'm_x' 2_1$	$D_{4h}(D_4) = 4_z/m_z m_x m_{xy}(4_z 2_x 2_{xy}) = 4_z/m_z m_x 'm_{xy}'$	$D_{\infty h}(D_{\infty}) = \infty/m'm'm'$	
$D_{nh}(C_{nv})$	$D_{1h}(C_{1v}) = 2_x m_y m_z(m_y) = 2_x 'm_y' m_z'$	$D_{2h}(C_{2v}) = m_x m_y m_z(2_z m_x m_y) = m_x m_y m_z'$	$D_{3h}(C_{3v}) = \bar{6}_z m_x 2_1(3_z m_x) = \bar{6}_z 'm_x' 2_1'$	$D_{4h}(C_{4v}) = 4_z/m_z m_x m_{xy}(4_z m_x m_{xy}) = 4_z/m_z m_x m_{xy}'$	$D_{\infty h}(C_{\infty v}) = \infty/m'm'm'$	
$D_{2nh}(D_{nd})$	$D_{2h}(D_{1d}) = m_x m_y m_z(2_y/m_y) = m_x 'm_y' m_z'$	$D_{4h}(D_{2d}) = 4_z/m_z m_x m_{xy}(\bar{4}_z m_x 2_{xy}) = 4_z'/m_z m_x m_{xy}'$	$D_{6h}(D_{3d}) = 6_z/m_z m_x m_1(\bar{3}_z m_x) = 6_z'/m_z m_x m_1'$	$C_{8h}(D_{4d}) = 8_z/m_z m_x 1 m_1(\bar{8}_z m_x 2_1) = 8_z'/m_z m_x 1 m_1'$	$D_{\infty h} 1' = \infty/mmm1'$	
$D_{2nh}(D_{nh})$	$D_{2h}(D_{1h}) = m_x m_y m_z(2_x m_y m_z) = m_x 'm_y' m_z$	$D_{4h}(D_{2h}) = 4_z/m_z m_x m_{xy}(m_x m_y m_z) = 4_z'/m_z m_x m_{xy}'$	$D_{6h}(D_{3h}) = 6_z/m_z m_x m_1(\bar{6}_z m_x 2_1) = 6_z'/m_z m_x m_1'$	$D_{8h}(D_{4h}) = 8_z/m_z m_x 1 m_1(4_z/m_z m_x 1 m_{x2}) = 8_z'/m_z m_x 1 m_1'$	$D_{\infty h} 1' = \infty/mmm1'$	

Rank 1, 2, 3, and 4 Property Tensors

Rank 1:

V	eV	aV	aeV
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Rank 2:

V^2	eV^2	aV^2	aeV^2	
$[V^2]$	$e[V^2]$	$a[V^2]$	$ae[V^2]$	$ij=ji$
$\{V^2\}$	$e\{V^2\}$	$a\{V^2\}$	$ae\{V^2\}$	$ij=-ji$

Rank 3:

V^3	eV^3	aV^3	aeV^3	
$[V^3]$	$e[V^3]$	$a[V^3]$	$ae[V^3]$	symmetric wrt all indicies
$V[V^2]$	$eV[V^2]$	$aV[V^2]$	$aeV[V^2]$	$ilk=ikl$
$\{V^2\}V$	$e\{V^2\}V$	$a\{V^2\}V$	$ae\{V^2\}V$	$jik=-ijk$

Rank 4:

V^4	eV^4	aV^4	aeV^4	
$[V^4]$	$e[V^4]$	$a[V^4]$	$ae[V^4]$	symmetric wrt all indicies
$V[V^3]$	$eV[V^3]$	$aV[V^3]$	$aeV[V^3]$	symmetric wrt to last three indicies
$[[V^2]^2]$	$e[[V^2]^2]$	$a[[V^2]^2]$	$ae[[V^2]^2]$	$ijkl= jikl=ijlk=klj$
$[V^2]^2$	$e[V^2]^2$	$a[V^2]^2$	$ae[V^2]^2$	$ijkl=jikl=ijlk$
$[(V^2)]^2$	$e[(V^2)]^2$	$a[(V^2)]^2$	$ae[(V^2)]^2$	$klj=ijkl$
$[V^2]V^2$	$e[V^2]V^2$	$a[V^2]V^2$	$ae[V^2]V^2$	$jikl=ijkl$

Printed Matrix Format of Rank 1, 2, 3, and 4 Property Tensors

Rank 1 tensors:

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Rank 2 tensors:

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V^2 \quad \begin{pmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [V^2] \quad \begin{pmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{pmatrix} \quad ij = ji$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} \{V^2\} \quad \begin{pmatrix} 0 & xy & xz \\ -xy & 0 & yz \\ -xz & -yz & 0 \end{pmatrix} \quad ij = -ji$$

Rank 3 tensors:

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V^3 \quad \begin{pmatrix} xxx & xyy & xzz & xyz & xzx & xxy & xzy & xxz & xyx \\ yxx & yyy & yzz & yyz & yzx & yxy & yzy & yxz & yyx \\ zxx & zyy & zzz & zyz & zzx & zxy & zzy & zxz & zyx \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [V^3] \quad \begin{pmatrix} xxx & xyy & xzz & xyz \\ yxx & yyy & yzz & \\ zxx & zyy & zzz & \end{pmatrix} \quad \text{symmetric with respect to all indices}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V[V^2] \begin{pmatrix} xxx & xyy & xzz & xyz & xxz & xxy \\ yxx & yyy & yzz & yyz & yxz & yxy \\ zxx & zyy & zzz & zyz & zxz & zxy \end{pmatrix} \quad \text{ilk} = \text{ikl}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} \{V^2\}V \begin{pmatrix} yzx & yzy & yzz \\ zxx & zxy & zxz \\ xyx & xyy & xyz \end{pmatrix} \quad \text{jik} = -\text{ijk}$$

Rank 4 tensors:

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V^4 \begin{pmatrix} xxxx & xxyy & xxzz & xxyz & xxzx & xxxy & xxzy & xxxz & xxyx \\ yyxx & yyyy & yyzz & yyyz & yyzx & yyxy & yyzy & yyxz & yyyy \\ zzxx & zzyy & zzzz & zzyz & zzzx & zzxy & zzzz & zzxz & zzyx \\ yzxx & yzyy & yzzz & yzyz & yzzx & yzxy & yzzy & yzxx & yzyx \\ zxxx & zxxy & zxzz & zxyz & zxzx & zxyx & zxzy & zxxz & zxyx \\ xyxx & xyxy & xyzz & xyxz & xyzx & xyxy & xyzy & xyxz & xyxy \\ zyxx & zyxy & zyzz & zyyz & zyzx & zyxy & zyzy & zyxz & zyxy \\ xzxx & xzyy & xzzz & xzyz & xzzx & xzxy & xzzy & xzxz & xzyx \\ yxxx & yxyy & yxzz & yxyz & yxzx & yxyx & yxzy & yxxz & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [V^4] \begin{pmatrix} xxxx & xyyy & xzzz & yyzz & xxyz \\ yxxx & yyyy & yzzz & zzxx & yyxz \\ zxxx & zyyy & zzzz & xxyy & zzxy \end{pmatrix} \quad \text{symmetric with respect to all indices}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} V[V^3] \begin{pmatrix} xxxx & xyyy & xzzz & xxyy & xyzz & xzxx & xxzz & xyxx & xzyy & xxyz \\ yxxx & yyyy & yzzz & yxyy & yyzz & yzxx & yxzz & yyxx & yzyy & yxyz \\ zxxx & zyyy & zzzz & zxyy & zyzz & zzxx & zxzz & zyxx & zzyy & zxyz \end{pmatrix}$$

symmetric with respect to last three indices

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [[V^2]^2] \begin{pmatrix} xxxx & xxyy & xxzz & xxyz & xxxz & xxxy \\ & yyyy & yyzz & yyyz & yyxz & yyxy \\ & & zzzz & zzyz & zzzz & zzxy \\ & & & yzyz & yzxx & yzxy \\ & & & & xzxz & xzxy \\ & & & & & xyxy \end{pmatrix} \quad ijkl = jikl = ijlk = klij$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [V^2]^2 \begin{pmatrix} xxxx & xxyy & xxzz & xxyz & xxxz & xxxy \\ yyxx & yyyy & yyzz & yyyz & yyxz & yyxy \\ zzxx & zzyy & zzzz & zzyz & zzzz & zzxy \\ yzxx & yzyy & yzzz & yzyz & yzxx & yzxy \\ xzxx & xzyy & xzzz & xzyz & xzxz & xzxy \\ xyxx & xyxy & xyzz & xyxz & xyxz & xyxy \end{pmatrix} \quad ijkl = jikl = ijlk$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & xxyz & xxzx & xxxy & xxzy & xxxz & xxyx \\ & yyyy & yyzz & yyyz & yyzx & yyxy & yyzy & yyxz & yyyx \\ & & zzzz & zzyz & zzzx & zzxy & zzzz & zzzz & zzyx \\ & & & yzyz & yzzx & yzxy & yzzy & yzxz & yzyx \\ & & & & zxzx & zxyy & zxzy & zxxz & zxyx \\ & & & & & xyxy & xzyy & xyxz & xyyx \\ & & & & & & zyzy & zyxz & zyyx \\ & & & & & & & xzxz & xzyx \\ & & & & & & & & yxyx \end{pmatrix} \quad \text{klij} = \text{ijkl}$$

$$\begin{pmatrix} 1 \\ e \\ a \\ ae \end{pmatrix} [V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & xxyz & xxzx & xxxy & xxzy & xxxz & xxyx \\ yyxx & yyyy & yyzz & yyyz & yyzx & yyxy & yyzy & yyxz & yyyx \\ zzxx & zzyy & zzzz & zzyz & zzzx & zzxy & zzzz & zzzz & zzyx \\ yzxx & yzyy & yzzz & yzyz & yzzx & yzxy & yzzy & yzxz & yzyx \\ zxxx & zxyy & zxzz & zxyz & zxzx & zxyy & zxzy & zxxz & zxyx \\ xyxx & xyyy & xyzz & xyyz & xyzx & xyxy & xzyy & xyxz & xyyx \end{pmatrix} \quad \text{jikl} = \text{ijkl}$$

Line Group Series and Line Groups for $n = 1, 2, 3, 4$ and ∞

C_n	$C_1 = 1$ $C_2 = 2_z$ $C_3 = 3_z$ $C_4 = 4_z$ $C_\infty = \infty$
C_{nv}	$C_{1v} = m_y$ $C_{2v} = 2_z m_x m_y$ $C_{3v} = 3_z m_x$ $C_{4v} = 4_z m_x m_{xy}$ $C_{\infty v} = \infty mm$
C_{nh}	$C_{1h} = m_z$ $C_{2h} = 2_z / m_z$ $C_{3h} = \bar{6}_z$ $C_{4h} = 4_z / m_z$ $C_{\infty h} = \infty / m$
S_{2n}	$S_2 = \bar{1}$ $S_4 = \bar{4}_z$ $S_6 = \bar{3}_z$ $S_8 = \bar{8}_z$ $C_{\infty h} = \infty / m$
D_n	$D_1 = 2_y$ $D_2 = 2_x 2_y 2_z$ $D_3 = 3_z 2_x$ $D_4 = 4_z 2_x 2_{xy}$ $D_\infty = \infty 2$
D_{nd}	$D_{1d} = 2_y / m_y$ $D_{2d} = \bar{4}_z m_x 2_{xy}$ $D_{3d} = \bar{3}_z m_x$ $D_{4d} = \bar{8}_z m_x 2_1$ $D_{\infty h} = \infty / mmm$
D_{nh}	$D_{1h} = 2_x m_y m_z$ $D_{2h} = m_x m_y m_z$ $D_{3h} = \bar{6}_z m_x 2_1$ $D_{4h} = 4_z / m_z m_x m_{xy}$ $D_{\infty h} = \infty / mmm$

$$\begin{aligned}
C_{2n}(C_n) \quad & C_2(C_1) = 2_z(1) = 2_z' \\
& C_4(C_2) = 4_z(2_z) = 4_z' \\
& C_6(C_3) = 6_z(3_z) = 6_z' \\
& C_8(C_4) = 8_z(4_z) = 8_z' \\
& C_\infty(1) = \infty 1'
\end{aligned}$$

$$\begin{aligned}
S_{2n}(C_n) \quad & S_2(C_1) = \bar{1}(1) = \bar{1}' \\
& S_4(C_2) = \bar{4}_z(2_z) = \bar{4}_z' \\
& S_6(C_3) = \bar{6}_z(3_z) = \bar{6}_z' \\
& S_8(C_4) = \bar{8}_z(4_z) = \bar{8}_z' \\
& C_{\infty h}(C_\infty) = \infty/m'
\end{aligned}$$

$$\begin{aligned}
C_{nh}(C_n) \quad & C_{1h}(C_1) = m_z(1) = m_z' \\
& C_{2h}(C_2) = 2_z/m_z(2_z) = 2_z'/m_z' \\
& C_{3h}(C_3) = \bar{6}_z(3_z) = \bar{6}_z' \\
& C_{4h}(C_4) = 4_z/m_z(4_z) = 4_z'/m_z' \\
& C_{\infty h}(C_\infty) = \infty/m'
\end{aligned}$$

$$\begin{aligned}
C_{2nh}(S_{2n}) \quad & C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z' \\
& C_{4h}(S_4) = 4_z/m_z(\bar{4}_z) = 4_z'/m_z' \\
& C_{6h}(S_6) = 6_z/m_z(\bar{6}_z) = 6_z'/m_z' \\
& C_{8h}(S_8) = 8_z/m_z(\bar{8}_z) = 8_z'/m_z' \\
& C_{\infty h}(1) = \infty/m1'
\end{aligned}$$

$$\begin{aligned}
C_{2nh}(C_{nh}) \quad & C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z \\
& C_{4h}(C_{2h}) = 4_z/m_z(2_z/m_z) = 4_z'/m_z \\
& C_{6h}(C_{3h}) = 6_z/m_z(3_z/m_z) = 6_z'/m_z \\
& C_{8h}(C_{4h}) = 8_z/m_z(4_z/m_z) = 8_z'/m_z \\
& C_{\infty h}(1) = \infty/m1'
\end{aligned}$$

$$\begin{aligned}
D_n(C_n) \quad & D_1(C_1) = 2_y(1) = 2_y' \\
& D_2(C_2) = 2_x 2_y 2_z(2_z) = 2_x' 2_y' 2_z' \\
& D_3(C_3) = 3_z 2_x(3_z) = 3_z 2_x' \\
& D_4(C_4) = 4_z 2_x 2_{xy}(4_z) = 4_z 2_x' 2_{xy}' \\
& D_\infty(C_\infty) = \infty 2'
\end{aligned}$$

$$\begin{aligned}
D_{2n}(D_n) \quad & D_2(D_1) = 2_x 2_y 2(2_y) = 2_x' 2_y 2_z' \\
& D_4(D_2) = 4_z 2_x 2_{xy}(2_x 2_y 2_z) = 4_z' 2_x 2_{xy}' \\
& D_6(D_3) = 6_z 2_x 2_1(3_z 2_x) = 6_z' 2_x 2_1' \\
& D_8(D_4) = 8_z 2_{x1} 2_1(4_z 2_{x1} 2_{x2}) = 8_z' 2_{x1} 2_1' \\
& D_\infty(1) = \infty 21'
\end{aligned}$$

$$\begin{aligned}
C_{nv}(C_n) \quad & C_{1v}(C_1) = m_y(1) = m_y' \\
& C_{2v}(C_2) = 2_z m_x m_y(2_z) = 2_z m_x' m_y' \\
& C_{3v}(C_3) = 3_z m_x(3_z) = 3_z m_x' \\
& C_{4v}(C_4) = 4_z m_x m_{xy}(4_z) = 4_z m_x' m_{xy}' \\
& C_{\infty v}(C_{\infty}) = \infty m' m'
\end{aligned}$$

$$\begin{aligned}
C_{2nv}(C_{nv}) \quad & C_{2v}(C_{1v}) = 2_z m_x m_y(m_y) = 2_z' m_x' m_y \\
& C_{4v}(C_{2v}) = 4_z m_x m_{xy}(2_z m_x m_y) = 4_z' m_x m_{xy}' \\
& C_{6v}(C_{3v}) = 6_z m_x m_1(3_z m_x) = 6_z' m_x m_1' \\
& C_{8v}(C_{4v}) = 8_z m_{x1} m_1(4_z m_x m_{xy}) = 8_z' m_{x1} m_1' \\
& C_{\infty v} 1' = \infty m m 1'
\end{aligned}$$

$$\begin{aligned}
D_{nd}(S_{2n}) \quad & D_{1d}(S_2) = 2_y/m_y(\bar{1}) = 2_y'/m_y' \\
& D_{2d}(S_4) = \bar{4}_z m_x 2_{xy}(\bar{4}_z) = \bar{4}_z m_x' 2_{xy}' \\
& D_{3d}(S_6) = \bar{3}_z m_x(\bar{3}_z) = \bar{3}_z m_x' \\
& D_{4d}(S_8) = \bar{8}_z m_{x1} 2_1(\bar{8}_z) = \bar{8}_z m_{x1}' 2_1' \\
& D_{\infty h}(C_{\infty h}) = \infty/mm'm'
\end{aligned}$$

$$\begin{aligned}
D_{nd}(D_n) \quad & D_{1d}(D_1) = 2_y/m_y(2_y) = 2_y'/m_y' \\
& D_{2d}(D_2) = \bar{4}_z m_x 2_{xy}(2_{xy} 2_{xy} 2_z) = \bar{4}_z' m_x' 2_{xy}' \\
& D_{3d}(D_3) = \bar{3}_z m_x(3_z 2_x) = \bar{3}_z' m_x' \\
& D_{4d}(D_4) = \bar{8}_z m_{x1} 2_1(4_z 2_x 2_{xy}) = \bar{8}_z' m_{x1}' 2_1' \\
& D_{\infty h}(D_{\infty}) = \infty/m'm'm'
\end{aligned}$$

$$\begin{aligned}
D_{nd}(C_{nv}) \quad & D_{1d}(C_{1v}) = 2_y/m_y(m_y) = 2_y'/m_y \\
& D_{2d}(C_{2v}) = \bar{4}_z m_x 2_{xy}(2_z m_x m_y) = \bar{4}_z' m_x 2_{xy}' \\
& D_{3d}(C_{3v}) = \bar{3}_z m_x(3_z m_x) = \bar{3}_z' m_x \\
& D_{4d}(C_{4v}) = \bar{8}_z m_{x1} 2_1(4_z m_{x1} m_{x3}) = \bar{8}_z' m_{x1} 2_1 \\
& D_{\infty h}(C_{\infty v}) = \infty/m'mm
\end{aligned}$$

$$\begin{aligned}
D_{nh}(C_{nh}) \quad & D_{1h}(C_{1h}) = 2_x m_y m_z(m_z) = 2_x' m_y' m_z \\
& D_{2h}(C_{2h}) = m_x m_y m_z(2_z/m_z) = m_x' m_y' m_z \\
& D_{3h}(C_{3h}) = \bar{6}_z m_x 2_1(\bar{6}_z) = \bar{6}_z m_x' 2_1' \\
& D_{4h}(C_{4h}) = 4_z/m_z m_x m_{xy}(4_z/m_z) = 4_z/m_z m_x' m_{xy}' \\
& D_{\infty h}(C_{\infty h}) = \infty/mm'm'
\end{aligned}$$

$$\begin{aligned}
D_{nh}(D_n) \quad & D_{1h}(D_1) = 2_x m_y m_z(2_x) = 2_x' m_y' m_z' \\
& D_{2h}(D_2) = m_x m_y m_z(2_x 2_y 2_z) = m_x' m_y' m_z' \\
& D_{3h}(D_3) = \bar{6}_z m_x 2_1(3_z 2_1) = \bar{6}_z' m_x' 2_1' \\
& D_{4h}(D_4) = 4_z/m_z m_x m_{xy}(4_z 2_x 2_{xy}) = 4_z/m_z m_x' m_{xy}' \\
& D_{\infty h}(D_{\infty}) = \infty/m'm'm'
\end{aligned}$$

$$\begin{aligned}
D_{nh}(C_{nv}) \quad & D_{1h}(C_{1v}) = 2_x m_y m_z (m_y) = 2_x' m_y m_z' \\
& D_{2h}(C_{2v}) = m_x m_y m_z (2_z m_x m_y) = m_x m_y m_z' \\
& D_{3h}(C_{3v}) = \bar{6}_z m_x 2_1 (3_z m_x) = \bar{6}_z' m_x 2_1' \\
& D_{4h}(C_{4v}) = 4_z / m_z m_x m_{xy} (4_z m_x m_{xy}) = 4_z' / m_z' m_x m_{xy}' \\
& D_{\infty h}(C_{\infty v}) = \infty / m' m m \\
\\
D_{2nh}(D_{nd}) \quad & D_{2h}(D_{1d}) = m_x m_y m_z (2_y / m_y) = m_x' m_y m_z' \\
& D_{4h}(D_{2d}) = 4_z / m_z m_x m_{xy} (\bar{4}_z m_x 2_{xy}) = 4_z' / m_z' m_x m_{xy}' \\
& D_{6h}(D_{3d}) = 6_z / m_z m_x m_1 (\bar{3}_z m_x) = 6_z' / m_z' m_x m_1' \\
& C_{8h}(D_{4d}) = 8_z / m_z m_{x1} m_1 (\bar{8}_z m_{x1} 2_1) = 8_z' / m_z' m_{x1} m_1' \\
& D_{\infty h} 1' = \infty / m m m 1' \\
\\
D_{2nh}(D_{nh}) \quad & D_{2h}(D_{1h}) = m_x m_y m_z (2_x m_y m_z) = m_x' m_y m_z \\
& D_{4h}(D_{2h}) = 4_z / m_z m_x m_{xy} (m_x m_y m_z) = 4_z' / m_z m_x m_{xy}' \\
& D_{6h}(D_{3h}) = 6_z / m_z m_x m_1 (\bar{6}_z m_x 2_1) = 6_z' / m_z m_x m_1' \\
& D_{8h}(D_{4h}) = 8_z / m_z m_{x1} m_1 (4_z / m_z m_{x1} m_{x2}) = 8_z' / m_z m_{x1} m_1' \\
& D_{\infty h} 1' = \infty / m m m 1'
\end{aligned}$$

C_n	$C_1 = 1$	$n = 1$	$C_{\infty} = \infty$	$n > 1;$
		$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \end{matrix}$	

C_{nv}	$C_{1v} = m_y$	$n = 1$	$C_{\infty v} = \infty mm$	$n > 1$
		$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} x \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} & \begin{pmatrix} x \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{matrix}$	

S_{2n}	$S_2 = \bar{1}$	$n = 1$	$C_{\infty h} = \infty/m$	$n > 1$
		$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \end{matrix}$	

C_{nh}	$C_{1h} = m_z$	$n = 1$	$C_{\infty h} = \infty/m$	$n > 1$
		$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \end{matrix}$	

D_n	$n = 1$	$n > 1$
	$D_1 = 2_y$	$D_\infty = \infty 2$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$

D_{nd}	$n = 1$	$n > 1$
	$D_{1d} = 2_y/m_y$	$D_{\infty h} = \infty/mmm$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$

D_{nh}	$n = 1$	$n > 1$
	$D_{1h} = 2_x m_y m_z$	$D_{\infty h} = \infty/mmm$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} x \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$

$C_{2n}(C_n)$	$n = 1$	$n > 1$
	$C_2(C_1) = 2_z(1) = 2_z'$	$C_\infty 1' = \infty 1'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ y \\ 0 \end{smallmatrix} \right) \end{matrix}$	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$

	$n = 1$		$n > 1$
$S_{2n}(C_n)$	$S_2(C_1) = \bar{1}(1) = \bar{1}'$		$C_{\infty h}(C_{\infty}) = \infty/m'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} x \\ y \\ z \end{matrix} \right) \left(\begin{matrix} x \\ y \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \end{matrix}$

	$n = 1$		$n > 1$
$C_{nh}(C_n)$	$C_{1h}(C_1) = m_z(1) = m_z'$		$C_{\infty h}(C_{\infty}) = \infty/m'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} x \\ y \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} x \\ y \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \end{matrix}$

	$n = 1$		$n > 1$
$C_{2nh}(S_{2n})$	$C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$		$C_{\infty h} 1' = \infty/m1'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} x \\ y \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$

	$n = 1$		$n > 1$
$C_{2nh}(C_{nh})$	$C_{2h}(C_{1h}) = 2_z/m_z(m_z) = 2_z'/m_z$		$C_{\infty h} 1' = \infty/m1'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} x \\ y \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$

$$\mathbf{D}_n(\mathbf{C}_n) \quad n = 1 \quad \mathbf{D}_1(\mathbf{C}_1) = \mathbf{2}_y(\mathbf{1}) = \mathbf{2}_y'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ 0 \\ z \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{D}_\infty(\mathbf{C}_\infty) = \infty \mathbf{2}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{D}_{2n}(\mathbf{D}_n) \quad n = 1 \quad \mathbf{D}_2(\mathbf{D}_1) = \mathbf{2}_x \mathbf{2}_y \mathbf{2}'(\mathbf{2}_y) = \mathbf{2}_x' \mathbf{2}_y \mathbf{2}_z'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{D}_\infty \mathbf{1}' = \infty \mathbf{2} \mathbf{1}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{C}_{nv}(\mathbf{C}_n) \quad n = 1 \quad \mathbf{C}_{1v}(\mathbf{C}_1) = \mathbf{m}_y(\mathbf{1}) = \mathbf{m}_y'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} x \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ 0 \\ z \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{C}_{\infty v}(\mathbf{C}_\infty) = \infty \mathbf{m}' \mathbf{m}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{C}_{2nv}(\mathbf{C}_{nv}) \quad n = 1 \quad \mathbf{C}_{2v}(\mathbf{C}_{1v}) = \mathbf{2}_z \mathbf{m}_x \mathbf{m}_y'(\mathbf{m}_y) = \mathbf{2}_z' \mathbf{m}_x' \mathbf{m}_y$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} x \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ y \\ 0 \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{C}_{\infty v} \mathbf{1}' = \infty \mathbf{m} \mathbf{m} \mathbf{1}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{smallmatrix} 0 \\ 0 \\ z \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) & \left(\begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right) \end{matrix}$$

$$\mathbf{D}_{nd}(\mathbf{S}_{2n}) \quad n = 1 \quad \mathbf{D}_{1d}(\mathbf{S}_2) = \mathbf{2}_y / \mathbf{m}_y(\bar{1}) = \mathbf{2}_y' / \mathbf{m}_y'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} x \\ 0 \\ z \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{\infty h}(\mathbf{C}_{\infty h}) = \infty / \mathbf{m} \mathbf{m}' \mathbf{m}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{nd}(\mathbf{D}_n) \quad n = 1 \quad \mathbf{D}_{1d}(\mathbf{D}_1) = \mathbf{2}_y / \mathbf{m}_y(\mathbf{2}_y) = \mathbf{2}_y' / \mathbf{m}_y'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{\infty h}(\mathbf{D}_{\infty}) = \infty / \mathbf{m}' \mathbf{m}' \mathbf{m}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{nd}(\mathbf{C}_{nv}) \quad n = 1 \quad \mathbf{D}_{1d}(\mathbf{C}_{1v}) = \mathbf{2}_y / \mathbf{m}_y(\mathbf{m}_y) = \mathbf{2}_y' / \mathbf{m}_y'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) & \left(\begin{matrix} x \\ 0 \\ z \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{\infty h}(\mathbf{C}_{\infty v}) = \infty / \mathbf{m}' \mathbf{m} \mathbf{m} \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{nh}(\mathbf{C}_{nh}) \quad n = 1 \quad \mathbf{D}_{1h}(\mathbf{C}_{1h}) = \mathbf{2}_x \mathbf{m}_y \mathbf{m}_z(\mathbf{m}_z) = \mathbf{2}_x' \mathbf{m}_y' \mathbf{m}_z'$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} x \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \end{matrix}$$

$$\mathbf{D}_{\infty h}(\mathbf{C}_{\infty h}) = \infty / \mathbf{m} \mathbf{m}' \mathbf{m}' \quad n > 1$$

$$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) \end{matrix}$$

$D_{nh}(D_n)$	$n = 1$	$D_{1h}(D_1) = 2x m_y m_z (2x) = 2x' m_y' m_z'$	$n > 1$	$D_{\infty h}(D_{\infty}) = \infty / m' m' m'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} x \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} x \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$	

$D_{nh}(C_{nv})$	$n = 1$	$D_{1h}(C_{1v}) = 2x m_y m_z (m_y) = 2x' m_y' m_z'$	$n > 1$	$D_{\infty h}(C_{\infty v}) = \infty / m' m m$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} x \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) & \left(\begin{matrix} x \\ y \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ z \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$	

$D_{2nh}(D_{nd})$	$n = 1$	$D_{2h}(D_{1d}) = m_x m_y m_z (2y/m_y) = m_x' m_y' m_z'$	$n > 1$	$D_{\infty h} 1' = \infty / m m m 1'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ y \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$	

$D_{2nh}(D_{nh})$	$n = 1$	$D_{2h}(D_{1h}) = m_x m_y m_z (2x m_y m_z) = m_x' m_y' m_z'$	$n > 1$	$D_{\infty h} 1' = \infty / m m m 1'$
	$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} x \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$		$V \begin{matrix} 1 & e & a & ae \\ \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) & \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \end{matrix}$	

	\mathbf{C}_n	$n = 1$ $\mathbf{C}_1 = \mathbf{1}$	$n = 2$ $\mathbf{C}_2 = \mathbf{2}_z$	$n > 2;$ $\mathbf{C}_{\infty} = \infty$
		1 e a ae	1 e a ae	1 e a ae
V^2		$\begin{pmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ yx & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ -xy & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ xy & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} 0 & xy & xz \\ -xy & 0 & yz \\ -xz & -yz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

	\mathbf{C}_{nv}	$n = 1$ $\mathbf{C}_{1v} = \mathbf{m}_y$	$n = 2$ $\mathbf{C}_{2v} = \mathbf{2}_z \mathbf{m}_x \mathbf{m}_y$	$n > 2$ $\mathbf{C}_{\infty v} = \infty \mathbf{m} \mathbf{m}$
		1 e a ae	1 e a ae	1 e a ae
V^2		$\begin{pmatrix} xx & 0 & xz \\ 0 & yy & 0 \\ zx & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} xx & 0 & xz \\ 0 & yy & 0 \\ xz & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & 0 \\ -xz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

	\mathbf{S}_{2n}	$n = 1$ $\mathbf{S}_2 = \bar{\mathbf{1}}$	$n = 2$ $\mathbf{S}_4 = \bar{\mathbf{4}}_z$	$n > 2$ $\mathbf{C}_{\infty h} = \infty / \mathbf{m}$
		1 e a ae	1 e a ae	1 e a ae
V^2		$\begin{pmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ -xy & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ -xy & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$		$\begin{pmatrix} 0 & xy & xz \\ -xy & 0 & yz \\ -xz & -yz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

\mathbf{C}_{nh}	$n=1$ $\mathbf{C}_{1h} = m_z$	$n=2$ $\mathbf{C}_{2h} = 2/m_z$	$n > 2$ $\mathbf{C}_{\infty h} = \infty/m$
	1 e a ae	1 e a ae	1 e a ae
V^2	$\begin{pmatrix} xx & xy & 0 \\ yx & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & yz \\ zx & zy & 0 \end{pmatrix}$	$\begin{pmatrix} xx & xy & 0 \\ yx & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$
$[V^2]$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & yz \\ zx & zy & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & yz \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & yz \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ -xy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

\mathbf{D}_n	$n=1$ $\mathbf{D}_1 = 2_y$	$n=2$ $\mathbf{D}_2 = 2_x 2_y 2_z$	$n > 2$ $\mathbf{D}_{\infty} = \infty 2$
	1 e a ae	1 e a ae	1 e a ae
V^2	$\begin{pmatrix} xx & 0 & xz \\ 0 & yy & 0 \\ zx & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$[V^2]$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & yy & 0 \\ xz & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & yy & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & 0 \\ -xz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

\mathbf{D}_{nd}	$n=1$ $\mathbf{D}_{1d} = 2_y/m_y$	$n=2$ $\mathbf{D}_{2d} = \bar{4}_z m_x 2_{xy}$	$n > 2$ $\mathbf{D}_{\infty h} = \infty/mmm$
	1 e a ae	1 e a ae	1 e a ae
V^2	$\begin{pmatrix} xx & 0 & xz \\ 0 & yy & 0 \\ zx & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$[V^2]$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & yy & 0 \\ xz & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ xy & 0 & 0 \\ 0 & 0 & zz \end{pmatrix}$	$\begin{pmatrix} 0 & xy & 0 \\ 0 & xx & 0 \\ 0 & 0 & zz \end{pmatrix}$
$\{V^2\}$	$\begin{pmatrix} 0 & 0 & xz \\ 0 & 0 & 0 \\ -xz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$C_{2nh}(S_{2n})$

$$n=1 \quad C_{2h}(S_2) = 2_z/m_z(\bar{1}) = 2_z'/m_z'$$

	e	a	ae
V^3	$\begin{pmatrix} 0 & 0 & 0 & xyz & xzx & 0 & xzy & xxz & 0 \\ 0 & 0 & 0 & yyz & yzx & 0 & yzy & yxz & 0 \\ zxx & zyy & zzz & 0 & 0 & zxy & 0 & 0 & zyx \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & 0 & xxy & 0 & 0 & xyx \\ yxx & yyy & yzz & 0 & 0 & yxy & 0 & 0 & yyx \\ 0 & 0 & 0 & zyz & zzx & 0 & zzy & zxz & 0 \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & \\ zxx & zyy & zzz & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 \\ yxx & yyy & yzz & \\ 0 & 0 & 0 & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & xyz & xxz & 0 \\ 0 & 0 & 0 & yyz & yxz & 0 \\ zxx & zyy & zzz & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & 0 & xxy \\ yxx & yyy & yzz & 0 & 0 & yxy \\ 0 & 0 & 0 & zyz & zxz & 0 \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} yzx & yzy & 0 \\ zxx & zxy & 0 \\ 0 & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & yzz \\ 0 & 0 & zxz \\ xyx & xyy & 0 \end{pmatrix}$

$$n=2 \quad C_{4h}(S_4) = 4_z/m_z(\bar{4}_z) = 4_z'/m_z'$$

	e	a	ae
V^3	$\begin{pmatrix} 0 & 0 & 0 & xyz & xzx & 0 & -yzx & yyz & 0 \\ 0 & 0 & 0 & yyz & yzx & 0 & xzx & -xyz & 0 \\ zxx & zyy & zzz & 0 & 0 & zxy & 0 & 0 & -zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & xzx & 0 & yzx & -yyz & 0 \\ 0 & 0 & 0 & yyz & yzx & 0 & -xzx & xyz & 0 \\ zxx & -zxx & 0 & 0 & 0 & zxy & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & zxx & zzz & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & \\ zxx & -zxx & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & xyz & xxz & 0 \\ 0 & 0 & 0 & xxz & -xyz & 0 \\ zxx & zyy & zzz & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & xxz & 0 \\ 0 & 0 & 0 & -xxz & xyz & 0 \\ zxx & -zxx & 0 & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} yzx & yzy & 0 \\ -yzy & yzx & 0 \\ 0 & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} yzx & yzy & 0 \\ yzy & -yzx & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$C_{2nv}(C_{nv})$

$n=1 \quad C_{2v}(C_{1v}) = 2_z m_x m_y (m_y) = 2_z' m_x' m_y'$

	1	e	a	ae
V^3	$\begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & yzy & 0 & 0 \\ zxx & zyy & zzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & xzy & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & yxz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & zyx \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yxy & 0 & 0 & yyx \\ 0 & 0 & 0 & 0 & zzx & 0 & 0 & zxz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxy & 0 & 0 & xyx \\ yxx & yyy & yzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & zyz & 0 & 0 & zzy & 0 & 0 \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & zyy & zzz & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ yxx & yyy & yzz & \\ 0 & 0 & 0 & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 \\ zxx & zyy & zzz & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & yxz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yxy \\ 0 & 0 & 0 & 0 & zxz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxy \\ yxx & yyy & yzz & 0 & 0 & 0 \\ 0 & 0 & 0 & zyz & 0 & 0 \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} 0 & yzy & 0 \\ zxx & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} yzx & 0 & 0 \\ 0 & zxy & 0 \\ 0 & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & zxz \\ 0 & xyy & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & yzz \\ 0 & 0 & 0 \\ xyx & 0 & 0 \end{pmatrix}$

$n=2 \quad C_{4v}(C_{2v}) = 4_z m_x m_{xy} (2_z m_x m_y) = 4_z' m_x m_{xy}'$

	1	e	a	ae
V^3	$\begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & xzx & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & -yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & -zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & -yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & -xzx & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & zxy \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & zyy & zzz & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & -zxx & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & xxz & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & -xxz & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} 0 & yzy & 0 \\ -yzy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} yzx & 0 & 0 \\ 0 & yzx & 0 \\ 0 & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} 0 & yzy & 0 \\ yzy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} yzx & 0 & 0 \\ 0 & -yzx & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$D_{nd}(S_{2n})$

$$n=1 \quad D_{1d}(S_2) = \bar{2}_y / m_y(\bar{1}) = \bar{2}_y' / m_y'$$

	e	a	ae
V^3	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & xxy & xzy & 0 & xyx \\ yxx & yyy & yzz & 0 & yzx & 0 & 0 & yxz & 0 \\ 0 & 0 & 0 & zyz & 0 & zxy & zzy & 0 & zyx \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & xzx & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & yxy & yzy & 0 & yyx \\ zxx & zyy & zzz & 0 & zzx & 0 & 0 & zxz & 0 \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ yxx & yyy & yzz & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 \\ 0 & 0 & 0 & \\ zxx & zyy & zzz & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & xxy \\ yxx & yyy & yzz & 0 & yxz & 0 \\ 0 & 0 & 0 & zyz & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} xxx & xyy & xzz & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & yxy \\ zxx & zyy & zzz & 0 & zxz & 0 \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} yzx & 0 & yzz \\ 0 & zxy & 0 \\ xyx & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & yzy & 0 \\ zxx & 0 & zxz \\ 0 & xyy & 0 \end{pmatrix}$

$$n=2 \quad D_{2d}(S_4) = \bar{4}_z m_x \bar{2}_{xy}(\bar{4}_z) = \bar{4}_z m_x' \bar{2}_{xy}'$$

	e	a	ae	
V^3	$\begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & -yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & -xzx & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & -yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & -zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & xzx & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$[V^3]$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & -zxx & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ zxx & zyy & zzz & \end{pmatrix}$
$V[V^2]$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & -xxz & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & xxz & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 \end{pmatrix}$
$\{V^2\}V$	$\begin{pmatrix} yzx & 0 & 0 \\ 0 & yzx & 0 \\ 0 & 0 & xyz \end{pmatrix}$	$\begin{pmatrix} yzx & 0 & 0 \\ 0 & -yzx & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & yzy & 0 \\ -yzy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	

$D_{nd}(C_{nv})$

$$n=1 \quad D_{1d}(C_{1v}) = \frac{2y}{m_y(m_y)} = \frac{2y'}{m_y}$$

 1

$$V^3 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & xyz & 0 & xxy & xzy & 0 & xyx \\ yxx & yyy & yzz & 0 & yzx & 0 & 0 & yxz & 0 \\ 0 & 0 & 0 & zyz & 0 & zxy & zzy & 0 & zyx \end{pmatrix}$$

$$a \begin{pmatrix} xxx & xyy & xzz & 0 & xzx & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & yxy & yzy & 0 & yyx \\ zxx & zyy & zzz & 0 & zzx & 0 & 0 & zxz & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$[V^3] \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xyz \\ yxx & yyy & yzz & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxx & xyy & xzz & 0 \\ 0 & 0 & 0 & 0 \\ zxx & zyy & zzz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V[V^2] \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & xxy \\ yxx & yyy & yzz & 0 & yxz & 0 \\ 0 & 0 & 0 & zyz & 0 & zxy \end{pmatrix}$$

$$\begin{pmatrix} xxx & xyy & xzz & 0 & xxz & 0 \\ 0 & 0 & 0 & yyz & 0 & yxy \\ zxx & zyy & zzz & 0 & zxz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\{V^2\}V \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} yzx & 0 & yzz \\ 0 & zxy & 0 \\ xyx & 0 & xyz \end{pmatrix}$$

$$\begin{pmatrix} 0 & yzy & 0 \\ zxx & 0 & zxz \\ 0 & xyy & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$n=2 \quad D_{2d}(C_{2v}) = \frac{\bar{4}_z m_x 2_{xy}(2_z m_x m_y)}{1} = \frac{\bar{4}_z m_x 2_{xy}'}{1}$$

 1

$$V^3 \begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & -yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & -xzx & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & -yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & -zxy \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & 0 & 0 & xzx & 0 & 0 & yyz & 0 \\ 0 & 0 & 0 & yyz & 0 & 0 & xzx & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 & yzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzx & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy & 0 & 0 & zxy \end{pmatrix}$$

$$[V^3] \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ zxx & -zxx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ zxx & zyy & zzz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xyz \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V[V^2] \begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & -xxz & 0 & 0 \\ zxx & -zxx & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & -xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxz & 0 \\ 0 & 0 & 0 & xxz & 0 & 0 \\ zxx & zxx & zzz & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xyz & 0 \\ 0 & 0 & 0 & 0 & 0 & zxy \end{pmatrix}$$

$$\{V^2\}V \begin{pmatrix} 0 & yzy & 0 \\ yzy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} yzx & 0 & 0 \\ 0 & yzx & 0 \\ 0 & 0 & xyz \end{pmatrix}$$

$$\begin{pmatrix} 0 & yzy & 0 \\ -yzy & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} yzx & 0 & 0 \\ 0 & -yzx & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$n=4 \quad C_8(C_4) = 8_z(4_z) = 8_z'$

$C_{2n}(C_n)$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$; $yyxy-xxyy=2xyxx$

e

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$; $yyxy-xxyy=2xyxx$

a

$$\begin{pmatrix} xxxx & -xxxx & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & xxxx & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & -xxxx & 0 & 0 & -xxxx \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & -xxxx \end{pmatrix}$$

ae

$$\begin{pmatrix} xxxx & -xxxx & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & xxxx & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & -xxxx & 0 & 0 & -xxxx \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & -xxxx \end{pmatrix}$$

$$\begin{pmatrix} xxxx & -xxxx & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ -xxxx & xxxx & 0 & 0 & 0 & xyyy & 0 & 0 & xyyy \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -xyyy & xyyy & 0 & 0 & 0 & -xxxx & 0 & 0 & -xxxx \end{pmatrix}$$

$C_{nh}(C_n)$

$n=3 \quad C_{3h}(C_3) = \bar{6}_z(3_z) = \bar{6}_z'$

1

e

a

ae

$[(V^2)]^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & -xxzx & xxyz & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & -xxzx & xxyz & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$[V^2]V^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$; $yyxy-xxyy=2xyxx$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ 0 & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ 0 & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$; $yyxy-xxyy=2xyxx$

$$n=3 \quad C_{6h}(S_6) = 6_z/m_z(\bar{3}_z) = 6_z'/m_z'$$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xxyy \end{pmatrix}$$

$$\text{and } xxxx = xxyy + yzyz + zyzy$$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$C_{2nh}(S_{2n})$

a

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & -xxzx & xxyz & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$[V^2]V^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

$$\text{and } xxxx = xxyy + 2xyxy ; \quad yyxy - xxyy = 2xyxx$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ & 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$n=1 \quad \mathbf{C}_{2h}(\mathbf{C}_{1h}) = \mathbf{2}_z/\mathbf{m}_z(\mathbf{m}_z) = \mathbf{2}_z'/\mathbf{m}_z$$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & xxyx \\ & yyyy & yyzz & 0 & 0 & yyxy & 0 & 0 & yyyx \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & zzyx \\ & & & yzyz & yzzx & 0 & yzzy & yzxz & 0 \\ & & & & zxzx & 0 & zxzy & zxxz & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zyzy & zyxz & 0 \\ & & & & & & & xzxz & 0 \\ & & & & & & & & yxyx \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & xxyx \\ yyxx & yyyy & yyzz & 0 & 0 & yyxy & 0 & 0 & yyyx \\ zzzx & zzyy & zzzz & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & yzyz & yzzx & 0 & yzzy & yzxz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & zxzy & zxxz & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & xyxy & 0 & 0 & xyyx \end{pmatrix}$$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbf{C}_{2nh}(\mathbf{C}_{nh})$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & yyyz & yyzx & 0 & yyzy & yyxz & 0 \\ & & 0 & zzyz & zzzx & 0 & zzzy & zzzx & 0 \\ & & & 0 & 0 & yzxy & 0 & 0 & yzyx \\ & & & & 0 & zxyx & 0 & 0 & zxyx \\ & & & & & 0 & xyzy & xyxz & 0 \\ & & & & & & 0 & 0 & zyyx \\ & & & & & & & 0 & xzyx \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ 0 & 0 & 0 & yyyz & yyzx & 0 & yyzy & yyxz & 0 \\ 0 & 0 & 0 & zzyz & zzzx & 0 & zzzy & zzzx & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zxzz & 0 & 0 & zxyx & 0 & 0 & zxyx \\ 0 & 0 & 0 & xxyz & xyxz & 0 & xyzy & xyxz & 0 \end{pmatrix}$$

$$n=3 \quad \mathbf{C}_{6h}(\mathbf{C}_{3h}) = \mathbf{6}_z/m_z(\mathbf{3}_z/m_z) = \mathbf{6}_z'/m_z$$

 $\mathbf{C}_{2nh}(\mathbf{C}_{nh})$

1

e

a

ae

$$V^4 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ yyxx & xxxx & yyzz & 0 & 0 & yyxy & 0 & 0 & -xxxxy \\ zzxx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & yzzy & yzxz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & zxzy & zxxz & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & xyxy & 0 & 0 & xyyx \\ 0 & 0 & 0 & zxxz & -zxzy & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & -yzxz & yzzy & 0 & -yzzx & yzyz & 0 \\ -xyyy & -xyxx & -xyzz & 0 & 0 & xyyx & 0 & 0 & xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & xxzy & xxxz & 0 \\ 0 & 0 & 0 & 0 & -xxzx & 0 & -xxzy & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -xzxx & 0 & 0 & -zxzx \\ zxzx & -zxzx & 0 & 0 & 0 & zyxx & 0 & 0 & zyxx \\ 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz & 0 \\ zyxx & -zyxx & 0 & 0 & 0 & -zxxx & 0 & 0 & -zxxx \\ xzxx & -xzxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz & 0 \end{pmatrix}$$

and: $xxxx=xxyy+xxyx+xxyx$; $xxxxy-yyxy=xyyy-xyxx$

$$[V^4] \begin{pmatrix} 3xxyy & 0 & 0 & yyzz & 0 \\ 0 & 3xxyy & 0 & yyzz & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & 0 & 0 & -zxxx \\ zxzx & -xxyz & 0 & 0 & 0 \end{pmatrix}$$

$$V[V^3] \begin{pmatrix} 3xxyy & 3xyxx & 0 & xxyy & xyzz & 0 & yyzz & xyxx & 0 & yyxx \\ -3xyxx & 3xxyy & 0 & -xyxx & yyzz & 0 & -xyzz & xxyy & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzxx & 0 & 0 & zzxx & zxyz \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xzxx & 0 & 0 & -xzxx & yzxx \\ 0 & 0 & 0 & 0 & 0 & yzxx & 0 & 0 & -yzxx & -xzxx \\ zxzx & -zyxx & 0 & -zxxx & 0 & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$$

$$[[V^2]^2] \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & yzyz & 0 \\ & & & & & xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & -yyxz & 0 \\ & 0 & 0 & -xxyz & yyxz & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & yyxz \\ & & & & 0 & xxyz \\ & & & & & 0 \end{pmatrix}$$

and $xyxy = \frac{1}{2}(xxxx - xxyy)$

$$[V^2]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy \\ xxyy & xxxx & xxzz & 0 & 0 & -xxxxy \\ zzxx & zzxx & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & yzxz & 0 \\ 0 & 0 & 0 & yzxz & yzyz & 0 \\ -xxxxy & xxyy & 0 & 0 & 0 & xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & -yyxz & 0 \\ 0 & 0 & 0 & -xxyz & yyxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & xzyy \\ -xzyy & xzyy & 0 & 0 & 0 & yzxx \\ 0 & 0 & 0 & yyxz & xxyz & 0 \end{pmatrix}$$

and $xyxy = \frac{1}{2}(xxxx - xxyy)$

$$n=3 \quad \mathbf{C}_{6h}(\mathbf{C}_{3h}) = \mathbf{6}_z/m_z(\mathbf{3}_z/m_z) = \mathbf{6}_z'/m_z$$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & xxxx & xxzz & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & yzyz & yzzx & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zxzx & -yzzx & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

$$\text{and } xxxx=xxyy+yzyz+zyzy$$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$\mathbf{C}_{2nh}(\mathbf{C}_{nh})$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & -xxzx & xxyz & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

$[V^2]V^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ xxyy & xxxx & xxzz & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ zzzx & zzxx & zzzz & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & yzyz & yzzx & 0 & zxzx & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & zxzx & 0 & -yzzx & yzyz & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \end{pmatrix}$$

$$\text{and } xxxx=xxyy+2xyxy ; \quad yyxy-xxyy=2xyxx$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & xxzx & 0 & xxzy & xxxz & 0 \\ 0 & 0 & 0 & -xxyz & -xxzx & 0 & -xxzy & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & -xxxz & xxzy & 0 & -xxzx & xxyz & 0 \end{pmatrix}$$

$$n=3 \quad \mathbf{C}_{3v}(\mathbf{C}_3) = \mathbf{3}_z \mathbf{m}_x(\mathbf{3}_z) = \mathbf{3}_z \mathbf{m}_x'$$

 $\mathbf{C}_{nv}(\mathbf{C}_n)$

1

e

a

ae

 $[(V^2)]^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & xxxx & xxzz & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & xxzy & 0 & yzzy & xxzy \\ & & & & & xyxy & 0 & xxyz & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & xxyz \\ & & & & & & & & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & xxyy & 0 & xxxz & -xxyy \\ & 0 & 0 & 0 & -xxzx & xxyy & 0 & -xxxz & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & -yzzx & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & xxyy & 0 & xxxz & -xxyy \\ & 0 & 0 & 0 & -xxzx & xxyy & 0 & -xxxz & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & -yzzx & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & xxxx & xxzz & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & xxzy & 0 & yzzy & xxzy \\ & & & & & xyxy & 0 & xxyz & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & xxyz \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$ and $xxxx=xxyy+yzyz+zyzy$
 $[V^2]V^2$

$$\begin{pmatrix} xxxx & xxyy & xxzz & xxyz & 0 & 0 & xxzy & 0 & 0 \\ xxyy & xxxx & xxzz & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ zzzx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & yzxx & 0 & yzyz & yzxx \\ 0 & 0 & 0 & 0 & xxzy & xyxy & 0 & xxyz & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & xxyy & 0 & xxxz & -yyxy \\ 0 & 0 & 0 & 0 & -xxzx & yyxy & 0 & -xxxz & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & -zxxx & 0 & -zxyz & -zxxx \\ zxxx & -zxxx & 0 & zxyz & 0 & 0 & -yzzx & 0 & 0 \\ xyxx & -xyxx & 0 & -xxxz & 0 & 0 & -xxzx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & xxyy & 0 & xxxz & -yyxy \\ 0 & 0 & 0 & 0 & -xxzx & yyxy & 0 & -xxxz & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & -zxxx & 0 & -zxyz & -zxxx \\ zxxx & -zxxx & 0 & zxyz & 0 & 0 & -yzzx & 0 & 0 \\ xyxx & -xyxx & 0 & -xxxz & 0 & 0 & -xxzx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & xxyz & 0 & 0 & xxzy & 0 & 0 \\ xxyy & xxxx & xxzz & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ zzzx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & yzxx & 0 & yzyz & yzxx \\ 0 & 0 & 0 & 0 & xxzy & xyxy & 0 & xxyz & xyxy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$ and: $yyxy-xxxz=2xyxx$ and: $yyxy-xxxz=2xyxx$ and $xxxx=xxyy+2xyxy$

$C_{nv}(C_n)$

$n > 4 \quad C_{\infty v}(C_{\infty}) = \infty m' m'$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx = xxyy + yzyz + zyzy$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx = xxyy + 2xyxy$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

and: $yyxy - xxyy = 2xyxx$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

and: $yyxy - xxyy = 2xyxx$

ae

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx = xxyy + yzyz + zyzy$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx = xxyy + 2xyxy$

$$n=1 \quad \mathbf{C}_{2v}(\mathbf{C}_{1v}) = \mathbf{2}_z \mathbf{m}_x \mathbf{m}_y (\mathbf{m}_y) = \mathbf{2}_z' \mathbf{m}_x' \mathbf{m}_y$$

$$\mathbf{C}_{2nv}(\mathbf{C}_{nv})$$

1

e

a

ae

$$V^4 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \\ 0 & 0 & 0 & zyyz & 0 & 0 & zyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & xzzx & 0 & 0 & xzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & yxyy & 0 & 0 & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zyzx & 0 & 0 & zyxz & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 & xzzy & 0 & 0 \\ yxxx & yxyy & yxzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzzz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxzx & zxxy & zxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 & xyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zyxy & 0 & 0 & zyyx \\ xzxx & xzyy & xzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yxyz & 0 & 0 & yxzy & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzzz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxxy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & xyzx & 0 & 0 & xxyz & 0 \\ zyxx & zyyy & zyyz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy & 0 & 0 & xzyx \\ 0 & 0 & 0 & 0 & yxzx & 0 & 0 & yxxz & 0 \end{pmatrix}$$

$$[V^4] \begin{pmatrix} xxxx & 0 & 0 & yyzz & 0 \\ 0 & yyyy & 0 & zzxx & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & xyyy & 0 & 0 & 0 \\ yxxx & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zzxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & xzzz & 0 & 0 \\ 0 & 0 & 0 & 0 & yyxz \\ zxxx & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yzzz & 0 & 0 \\ 0 & zyyy & 0 & 0 & 0 \end{pmatrix}$$

$$V[V^3] \begin{pmatrix} xxxx & 0 & 0 & xxyy & 0 & 0 & xxzz & 0 & 0 & 0 \\ 0 & yyyy & 0 & 0 & yyzz & 0 & 0 & yyxx & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzxx & 0 & 0 & zzyy & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & xyyy & 0 & 0 & xyzz & 0 & 0 & xyxx & 0 & 0 \\ yxxx & 0 & 0 & yxyy & 0 & 0 & yxzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & zxyz \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & xzzz & 0 & 0 & xzxx & 0 & 0 & xzyy & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & yxyz \\ zxxx & 0 & 0 & zxyy & 0 & 0 & zxzz & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yzzz & 0 & 0 & yzxx & 0 & 0 & yzyy & 0 \\ 0 & zyyy & 0 & 0 & zyyz & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$$

$$[[V^2]^2] \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & xzxz & 0 \\ & & & & & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ & 0 & 0 & 0 & 0 & yyxy \\ & & 0 & 0 & 0 & zzxy \\ & & & 0 & yzxx & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzzz & 0 \\ & & & 0 & 0 & yzxy \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 \\ & & 0 & zzyz & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & xzxy \\ & & & & & 0 \end{pmatrix}$$

$$[V^2]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & zzxy \\ 0 & 0 & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy \\ xzxx & xzyy & xzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy \\ 0 & 0 & 0 & 0 & xxyz & 0 \end{pmatrix}$$

$$n=1 \quad \mathbf{C}_{2v}(\mathbf{C}_{1v}) = \mathbf{2}_z \mathbf{m}_x \mathbf{m}_y (\mathbf{m}_y) = \mathbf{2}_z' \mathbf{m}_x' \mathbf{m}_y$$

 $\mathbf{C}_{2nv}(\mathbf{C}_{nv})$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & zxxz & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zyzy & 0 & 0 \\ & & & & & & & xzxz & 0 \\ & & & & & & & & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyx \\ & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ & & & 0 & yzzx & 0 & 0 & yzxz & 0 \\ & & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & zyxz & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzzx & 0 & 0 & zzxz & 0 \\ & & & 0 & 0 & yzxy & 0 & 0 & yzyx \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & xyzy & 0 & 0 \\ & & & & & & 0 & 0 & zyyx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ & & 0 & zzyz & 0 & 0 & zzzy & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & zxxy & 0 & 0 & zxyx \\ & & & & & 0 & 0 & xyxz & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & xzyx \\ & & & & & & & & 0 \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxz & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 & xyzy & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzzy & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxxy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & 0 & xyzx & 0 & 0 & xyxz \end{pmatrix}$$

$C_{2nv}(C_{nv})$

$n=2 \quad C_{4v}(C_{2v}) = 4_z m_x m_{xy} (2_z m_x m_y) = 4_z' m_x m_{xy}'$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & 0 & yzzx & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -yzzx & 0 & 0 \\ & & & & & & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & 0 & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & -yzzx & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & -zxzx & 0 & 0 \\ & & & & & & & -yzyz & 0 \\ & & & & & & & & -xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & xxyy \\ & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ & & 0 & yzzx & 0 & 0 & yzzy & 0 \\ & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & yzzx & 0 \\ & & & & & & 0 & 0 \\ & & & & & & & 0 \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzzx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & -yzzx & 0 & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ -xxyy & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & -zzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & -zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & -yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xyxy \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & yzzx & 0 & 0 \\ xyxx & xyxx & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n=3 \quad \mathbf{C}_{6v}(\mathbf{C}_{3v}) = \mathbf{6}_z \mathbf{m}_x \mathbf{m}_1 (\mathbf{3}_z \mathbf{m}_x) = \mathbf{6}_z' \mathbf{m}_x \mathbf{m}_1'$$

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$$[(V^2)]^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \text{yzyz} & 0 & 0 & \text{yzyz} & 0 & 0 \\ & & & & \text{zxzx} & 0 & 0 & \text{yzyz} & 0 \\ & & & & & \text{xyxy} & 0 & 0 & \text{xyxy} \\ & & & & & & \text{zxzx} & 0 & 0 \\ & & & & & & & \text{yzyz} & 0 \\ & & & & & & & & \text{xyxy} \end{pmatrix}$$

$$\text{and } \text{xxxx} = \text{xyxy} + \text{yzyz} + \text{zyzy}$$

$$[V^2]V^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{xyxy} & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{zzxx} & \text{zzxx} & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{yzyz} & 0 & 0 & \text{zxzx} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{zxzx} & 0 & 0 & \text{yzyz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{xyxy} & 0 & 0 & \text{xyxy} \end{pmatrix}$$

$$\text{and } \text{xxxx} = \text{xyxy} + 2\text{xyxy}$$

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$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ & & & 0 & \text{yzzx} & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -\text{yzzx} & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{yyxy} \\ 0 & 0 & 0 & 0 & 0 & \text{yyxy} & 0 & 0 & -\text{xxxy} \\ 0 & 0 & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ 0 & 0 & 0 & 0 & \text{yzzx} & 0 & 0 & -\text{zxyz} & 0 \\ 0 & 0 & 0 & \text{zxyz} & 0 & 0 & -\text{yzzx} & 0 & 0 \\ \text{xyxx} & -\text{xyxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{and } \text{yyxy} - \text{xxxy} = 2\text{xyxx}$$

$\mathbf{C}_{2nv}(\mathbf{C}_{nv})$

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$$\begin{pmatrix} 0 & 0 & 0 & \text{xyz} & 0 & 0 & \text{xxzy} & 0 & 0 \\ & 0 & 0 & -\text{xyz} & 0 & 0 & -\text{xxzy} & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & \text{xxzy} & 0 & 0 & \text{xxzy} \\ & & & & & 0 & 0 & \text{xyz} & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & \text{xyz} \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & \text{xyz} & 0 & 0 & \text{xxzy} & 0 & 0 \\ 0 & 0 & 0 & -\text{xyz} & 0 & 0 & -\text{xxzy} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{yzxx} & -\text{yzxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{yzxx} & 0 & 0 & \text{yzxx} \\ 0 & 0 & 0 & 0 & \text{xxzy} & 0 & 0 & \text{xyz} & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & 0 & 0 & 0 & \text{xxzx} & 0 & 0 & \text{xxxz} & 0 \\ & 0 & 0 & 0 & -\text{xxzx} & 0 & 0 & -\text{xxxz} & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -\text{xxxz} & 0 & 0 & -\text{xxxz} \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -\text{xxzx} & 0 & 0 \\ & & & & & & 0 & 0 & -\text{xxzx} \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \text{xxzx} & 0 & 0 & \text{xxxz} & 0 \\ 0 & 0 & 0 & 0 & -\text{xxzx} & 0 & 0 & -\text{xxxz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\text{zxxx} \\ \text{zxxx} & -\text{zxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\text{xxxz} & 0 & 0 & -\text{xxzx} & 0 & 0 \end{pmatrix}$$

$$n=4 \quad \mathbf{C}_{8v}(\mathbf{C}_{4v}) = 8_z \mathbf{m}_{x1} \mathbf{m}_1 (4_z \mathbf{m}_x \mathbf{m}_{xy}) = 8_z' \mathbf{m}_{x1} \mathbf{m}_1'$$

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$$[(V^2)]^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \text{yzyz} & 0 & 0 & \text{yzzz} & 0 & 0 \\ & & & & \text{zxzx} & 0 & 0 & \text{yzzz} & 0 \\ & & & & & \text{xyxy} & 0 & 0 & \text{xyyx} \\ & & & & & & \text{zxzx} & 0 & 0 \\ & & & & & & & \text{yzyz} & 0 \\ & & & & & & & & \text{xyxy} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ & & & 0 & \text{yzzx} & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -\text{yzzx} & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

and $\text{xxxx} = \text{xyxy} + \text{yzyz} + \text{zyzy}$

$$[V^2]V^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{xyxy} & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{zzxx} & \text{zzxx} & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{yzyz} & 0 & 0 & \text{zxzx} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{zxzx} & 0 & 0 & \text{yzyz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{xyxy} & 0 & 0 & \text{xyxy} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{yyxy} \\ 0 & 0 & 0 & 0 & 0 & \text{yyxy} & 0 & 0 & -\text{xxxy} \\ 0 & 0 & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ 0 & 0 & 0 & 0 & \text{yzzx} & 0 & 0 & -\text{zxyz} & 0 \\ 0 & 0 & 0 & \text{zxyz} & 0 & 0 & -\text{yzzx} & 0 & 0 \\ \text{xyxx} & -\text{xyxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $\text{xxxx} = \text{xyxy} + 2\text{xyxy}$

and $\text{yyxy} - \text{xxxy} = 2\text{xyxx}$

$\mathbf{C}_{2nv}(\mathbf{C}_{nv})$

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$$\begin{pmatrix} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & -\text{xxxx} & 0 & 0 & -\text{xxxx} \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & -\text{xxxx} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\text{xyyy} & 0 & 0 & -\text{xyyy} \\ & 0 & 0 & 0 & 0 & -\text{xyyy} & 0 & 0 & -\text{xyyy} \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\text{xxxx} & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\text{xxxx} & 0 & 0 & -\text{xxxx} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\text{xyyy} & 0 & 0 & -\text{xyyy} \\ 0 & 0 & 0 & 0 & 0 & \text{xyyy} & 0 & 0 & \text{xyyy} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\text{xyyy} & \text{xyyy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$D_{nd}(S_{2n})$

$n=2 \quad D_{2d}(S_4) = \bar{4}_z m_x 2_{xy}(\bar{4}_z) = \bar{4}_z m_x' 2_{xy}'$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xxyy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yxyx \\ & 0 & 0 & 0 & 0 & yxyx & 0 & 0 & xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ & & & 0 & yzzx & 0 & 0 & yzxx & 0 \\ & & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yxyx \\ & 0 & 0 & 0 & 0 & yxyx & 0 & 0 & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & 0 & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & -yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & -zxzx & 0 & 0 \\ & & & & & & & -yzyz & 0 \\ & & & & & & & & -xyxy \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyyx & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzzx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yxyx \\ 0 & 0 & 0 & 0 & 0 & yxyx & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & yzzx & 0 & 0 \\ xyxx & xyxx & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yxyx \\ 0 & 0 & 0 & 0 & 0 & yxyx & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & -yzzx & 0 & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ -xxyy & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & -zzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & -zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & -yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xyxy \end{pmatrix}$$

$D_{nd}(S_{2n})$

$n=4 \quad D_{4d}(S_8) = \bar{\mathbf{8}}_z \mathbf{m}_{x_1} \mathbf{2}_1(\bar{\mathbf{8}}_z) = \bar{\mathbf{8}}_z \mathbf{m}_{x_1} \mathbf{2}'_1$

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$$[(V^2)]^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \text{yzyz} & 0 & 0 & \text{yzzy} & 0 & 0 \\ & & & & \text{zxzx} & 0 & 0 & \text{yzzy} & 0 \\ & & & & & \text{xyxy} & 0 & 0 & \text{xyyx} \\ & & & & & & \text{zxzx} & 0 & 0 \\ & & & & & & & \text{yzyz} & 0 \\ & & & & & & & & \text{xyxy} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

and $\text{xxxx}=\text{xyxy}+\text{yzyz}+\text{zyzy}$

$$[V^2]V^2 \begin{pmatrix} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{xyxy} & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{zzxx} & \text{zzxx} & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{yzyz} & 0 & 0 & \text{zxzx} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{zxzx} & 0 & 0 & \text{yzyz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{xyxy} & 0 & 0 & \text{xyxy} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ 0 & 0 & 0 & 0 & 0 & \text{xyyy} & 0 & 0 & \text{xyyy} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -xyyy & \text{xyyy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $\text{xxxx}=\text{xyxy}+2\text{xyxy}$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ & & & 0 & \text{yzzx} & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -\text{yzzx} & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{yyxy} \\ 0 & 0 & 0 & 0 & 0 & \text{yyxy} & 0 & 0 & -\text{xxxy} \\ 0 & 0 & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ 0 & 0 & 0 & 0 & \text{yzzx} & 0 & 0 & -\text{zxyz} & 0 \\ 0 & 0 & 0 & \text{zxyz} & 0 & 0 & -\text{yzzx} & 0 & 0 \\ \text{xyxx} & -\text{xyxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $\text{yyxy}-\text{xxxy}=2\text{xyxx}$

ae

$$\begin{pmatrix} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & -\text{xxxx} & 0 & 0 & -\text{xxxx} \\ & & & & & & & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & -\text{xxxx} \end{pmatrix}$$

$$\begin{pmatrix} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\text{xxxx} & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\text{xxxx} & 0 & 0 & -\text{xxxx} \end{pmatrix}$$

$D_{nd}(C_{nv})$

$n=2 \quad D_{2d}(C_{2v}) = \bar{4}_z m_x 2_{xy} (2_z m_x m_y) = \bar{4}_z 'm_x 2_{xy}'$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yyxy \\ & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ & & & 0 & yzzx & 0 & 0 & yzzy & 0 \\ & & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & 0 & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & -yzzy & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & -zxzx & 0 & 0 \\ & & & & & & & -yzyz & 0 \\ & & & & & & & & -xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & yzzx & 0 & 0 \\ xyxx & xyxx & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ -xxyy & -xxxx & -xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & -zzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & -zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & -yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & -xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & -zxyz & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & -yzzx & 0 & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n=4 \quad \mathbf{D}_{4d}(\mathbf{C}_{4v}) = \bar{\mathbf{8}}_z \mathbf{m}_{x1} \mathbf{2}_1 (4_z \mathbf{m}_{x1} \mathbf{m}_{x3}) = \bar{\mathbf{8}}_z ' \mathbf{m}_{x1} \mathbf{2}_1 '$$

$$[(V^2)]^2 \quad \begin{array}{c} 1 \\ \left(\begin{array}{cccccccc} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 \\ & & \text{zzzz} & 0 & 0 & 0 & 0 & 0 \\ & & & \text{yzyz} & 0 & 0 & \text{yzyz} & 0 \\ & & & & \text{zxzx} & 0 & 0 & \text{yzyz} \\ & & & & & \text{xyxy} & 0 & 0 \\ & & & & & & \text{zxzx} & 0 \\ & & & & & & & \text{yzyz} \\ & & & & & & & & \text{xyxy} \end{array} \right) \end{array} \quad \begin{array}{c} e \\ \left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{array} \right) \end{array}$$

and $\text{xxxx} = \text{xyxy} + \text{yzyz} + \text{zyzy}$

$$[V^2]V^2 \quad \begin{array}{c} \left(\begin{array}{cccccccc} \text{xxxx} & \text{xyxy} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{xyxy} & \text{xxxx} & \text{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{zzxx} & \text{zzxx} & \text{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{yzyz} & 0 & 0 & \text{zxzx} & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{zxzx} & 0 & 0 & \text{yzyz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{xyxy} & 0 & 0 & \text{xyxy} \end{array} \right) \end{array} \quad \begin{array}{c} \left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & -xyyy & 0 & 0 & -xyyy \\ 0 & 0 & 0 & 0 & 0 & \text{xyyy} & 0 & 0 & \text{xyyy} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -xyyy & \text{xyyy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

and $\text{xxxx} = \text{xyxy} + 2\text{xyxy}$

$\mathbf{D}_{nd}(\mathbf{C}_{nv})$

$$\quad \begin{array}{c} a \\ \left(\begin{array}{cccccc} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & -\text{xxxx} & 0 & 0 & -\text{xxxx} \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & -\text{xxxx} \end{array} \right) \end{array}$$

$$\quad \begin{array}{c} \left(\begin{array}{cccccc} \text{xxxx} & -\text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\text{xxxx} & \text{xxxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\text{xxxx} & 0 & 0 & -\text{xxxx} \end{array} \right) \end{array}$$

$$\quad \begin{array}{c} ae \\ \left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{xxxy} \\ & & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ & & & 0 & \text{yzzx} & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -\text{yzzx} & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{array} \right) \end{array}$$

$$\quad \begin{array}{c} \left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & \text{xxxy} & 0 & 0 & -\text{yyxy} \\ 0 & 0 & 0 & 0 & 0 & \text{yyxy} & 0 & 0 & -\text{xxxy} \\ 0 & 0 & 0 & 0 & 0 & \text{zzxy} & 0 & 0 & -\text{zzxy} \\ 0 & 0 & 0 & 0 & \text{yzzx} & 0 & 0 & -\text{zxyz} & 0 \\ 0 & 0 & 0 & \text{zxyz} & 0 & 0 & -\text{yzzx} & 0 & 0 \\ \text{xyxx} & -\text{xyxx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

and $\text{yyxy} - \text{xxxy} = 2\text{xyxx}$

$$n=1 \quad \mathbf{D}_{1h}(\mathbf{C}_{1h}) = \mathbf{2}_x \mathbf{m}_y \mathbf{m}_z (\mathbf{m}_z) = \mathbf{2}_x' \mathbf{m}_y' \mathbf{m}_z$$

$$\mathbf{D}_{nh}(\mathbf{C}_{nh})$$

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$$V^4 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & xzxx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \\ 0 & 0 & 0 & zyyz & 0 & 0 & zyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & xzzx & 0 & 0 & xzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & yxyy & 0 & 0 & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxxy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & xyzx & 0 & 0 & yxz & 0 \\ zyxx & zyyy & zyyz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy & 0 & 0 & xzyx \\ 0 & 0 & 0 & 0 & yxzx & 0 & 0 & yxxz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & zxyz & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & yzxx & 0 & 0 & zyxx & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 & xzzy & 0 & 0 \\ yxxx & yxyy & yxzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 & xyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zyxy & 0 & 0 & zyyx \\ xzxx & xzyy & xzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yxyz & 0 & 0 & yxzy & 0 & 0 \end{pmatrix}$$

$$[V^4] \begin{pmatrix} xxxx & 0 & 0 & yyyz & 0 \\ 0 & yyyy & 0 & zzzx & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yzzz & 0 & 0 \\ 0 & zyyy & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & xyyy & 0 & 0 & 0 \\ yxxx & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zzyy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & xzzz & 0 & 0 \\ 0 & 0 & 0 & 0 & yyxz \\ zxxx & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$V[V^3] \begin{pmatrix} xxxx & 0 & 0 & xxyy & 0 & 0 & xxzz & 0 & 0 & 0 \\ 0 & yyyy & 0 & 0 & yyzz & 0 & 0 & yyxx & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzzx & 0 & 0 & zzyy & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yzzz & 0 & 0 & yzxx & 0 & 0 & yzyy & 0 \\ 0 & zyyy & 0 & 0 & zyzz & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & xyyy & 0 & 0 & xyzz & 0 & 0 & xyxx & 0 & 0 \\ yxxx & 0 & 0 & yxyy & 0 & 0 & yxzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & zxyz \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & xzzz & 0 & 0 & xzxx & 0 & 0 & xzyy & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & yxyz \\ zxxx & 0 & 0 & zxyy & 0 & 0 & zxzz & 0 & 0 & 0 \end{pmatrix}$$

$$[[V^2]^2] \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & xzxx & 0 \\ & & & & & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 \\ & & 0 & zzyz & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & xzxy \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ & 0 & 0 & 0 & 0 & yyxy \\ & & 0 & 0 & 0 & zzxy \\ & & & 0 & yzxx & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzxz & 0 \\ & & & 0 & 0 & yzxy \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$[V^2]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy \\ 0 & 0 & 0 & 0 & yxz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & zzxy \\ 0 & 0 & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzxz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy \\ xzxx & xzyy & xzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 \end{pmatrix}$$

$$n=1 \quad \mathbf{D}_{1h}(\mathbf{C}_{1h}) = \mathbf{2}_x \mathbf{m}_y \mathbf{m}_z (\mathbf{m}_z) = \mathbf{2}_x' \mathbf{m}_y' \mathbf{m}_z$$

 $\mathbf{D}_{nh}(\mathbf{C}_{nh})$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & zxxz & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zyzy & 0 & 0 \\ & & & & & & & xzxz & 0 \\ & & & & & & & & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ & & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & zxxy & 0 & 0 & zxyx \\ & & & & & 0 & 0 & xyxz & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & xzyx \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & xxyx \\ & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ & & & 0 & yzzx & 0 & 0 & yzxx & 0 \\ & & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & zyxx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzzx & 0 & 0 & zzzz & 0 \\ & & & 0 & 0 & yzxy & 0 & 0 & yzyx \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & xyzy & 0 & 0 \\ & & & & & & 0 & 0 & zyyx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxxy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & 0 & xyxz & 0 & 0 & xzyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzzz & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyzy & 0 & 0 & xyzy & 0 & 0 \end{pmatrix}$$

$D_{nh}(C_{nh})$

$n=3 \quad D_{3h}(C_{3h}) = \bar{6}_z m_x 2_1(\bar{6}_z) = \bar{6}_z m_x' 2_1'$

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V^4	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & xxxx & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \\ 0 & 0 & 0 & zxxz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzzy & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyyx & 0 & 0 & xxyy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -xzxx & 0 & 0 & -xzxx \\ zxxx & -zxxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxxz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -zxxx & 0 & 0 & -zxxx \\ xzxx & -xzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxxz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & zxyx & 0 & 0 & zxyx & 0 & 0 \\ xyyx & xyyx & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -zxzy & 0 & 0 & -zxyx & 0 \\ 0 & 0 & 0 & -yzxz & 0 & 0 & -yzxz & 0 & 0 \\ -xyyy & -xyxx & -xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zyxx & 0 & 0 & zyxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyx & 0 \\ zyxx & -zyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ xzxx & -xzxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyx & 0 \end{pmatrix}$
	and: $xxxx=xxyy+xxyx+xxyx$		and: $xxxy-yyxy=xyyy-xyxx$	

$[V^4]$	$\begin{pmatrix} 3xxyy & 0 & 0 & yyzz & 0 \\ 0 & 3xxyy & 0 & yyzz & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -zxxx \\ zxxx & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -xxyz & 0 & 0 & 0 \end{pmatrix}$
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$V[V^3]$	$\begin{pmatrix} 3xxyy & 0 & 0 & xxyy & 0 & 0 & yyzz & 0 & 0 & yyxx \\ 0 & 3xxyy & 0 & 0 & yyzz & 0 & 0 & xxyy & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzxx & 0 & 0 & zzxx & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xzxx & 0 & 0 & -xzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -xzxx \\ zxxx & 0 & 0 & -zxxx & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 3xyxx & 0 & 0 & xyzz & 0 & 0 & xyxx & 0 & 0 \\ -3xyxx & 0 & 0 & -xyxx & 0 & 0 & -xyzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & zxyz \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & 0 & yzxx & 0 & 0 & -yzxx & 0 \\ 0 & -yzxx & 0 & 0 & 0 & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$
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$[[V^2]^2]$	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & yzyz & 0 \\ & & & & & xxyy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ & 0 & 0 & -xxyz & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & xxyz \\ & & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -yyxz & 0 \\ & 0 & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & yyxz \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$
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and $xxyy = \frac{1}{2}(xxxx - xxyy)$

$[V^2]^2$	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -yyxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzyy \\ -xzyy & xzyy & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yyxz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & yzxx & 0 & 0 \\ -xxyy & xxyy & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ 0 & 0 & 0 & -xxyz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxyz & 0 \end{pmatrix}$
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and $xxyy = \frac{1}{2}(xxxx - xxyy)$

and $xxyy = \frac{1}{2}(xxxx - xxyy)$

$D_{nh}(C_{nh})$

$n=3 \quad D_{3h}(C_{3h}) = \bar{6}_z m_x 2_1(\bar{6}_z) = \bar{6}_z m_x' 2_1'$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzzx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -xxyy \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & 0 & 0 & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & 0 & xxyz & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxzx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & -zxyz & 0 \\ 0 & 0 & 0 & 0 & zxyz & 0 & 0 & -yzzx & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $yyxy-xxyy=2xyxx$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyz & 0 \end{pmatrix}$$

$D_{nh}(D_n)$

$n=3 \quad D_{3h}(D_3) = \bar{6}_z m_x 2_1(3_2 2_1) = \bar{6}_z 'm_x' 2_1$

1

$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzzx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

a

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzzx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$

$$n=1 \quad \mathbf{D}_{1h}(\mathbf{C}_{1v}) = \mathbf{2}_x \mathbf{m}_y \mathbf{m}_z (\mathbf{m}_y) = \mathbf{2}_x' \mathbf{m}_y \mathbf{m}_z'$$

$$\mathbf{D}_{nh}(\mathbf{C}_{nv})$$

1

e

a

ae

$$V^4 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & xzxx & 0 & 0 & zxxx & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \\ 0 & 0 & 0 & zyyz & 0 & 0 & zyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & xzzx & 0 & 0 & xzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & yxxy & 0 & 0 & yxyx \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxxy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & xyzx & 0 & 0 & xyxz & 0 \\ zyxx & zyyy & zyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy & 0 & 0 & xzyx \\ 0 & 0 & 0 & 0 & yxzx & 0 & 0 & yxxz & 0 \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzzx & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 & xyzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zyxy & 0 & 0 & zyyx \\ xzxx & xzyy & xzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yxyz & 0 & 0 & yxzy & 0 & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzyy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & zxyy & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyyy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zyzy & 0 & 0 & zyxz & 0 \\ yxxx & yxyy & yxzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$[V^4] \begin{pmatrix} xxxx & 0 & 0 & yyyz & 0 \\ 0 & yyyy & 0 & zzyy & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yyyz & 0 & 0 \\ 0 & zzyy & 0 & 0 & 0 \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & xzzz & 0 & 0 \\ 0 & 0 & 0 & 0 & yyxz \\ zxxx & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & xxyy & 0 & 0 & 0 \\ yxxx & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zzyy \end{pmatrix}$$

$$V[V^3] \begin{pmatrix} xxxx & 0 & 0 & xxyy & 0 & 0 & xxzz & 0 & 0 & 0 \\ 0 & yyyy & 0 & 0 & yyzz & 0 & 0 & yyxx & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzyy & 0 & 0 & zzyy & 0 \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & yzzz & 0 & 0 & yzxx & 0 & 0 & yzyy & 0 \\ 0 & zyyy & 0 & 0 & zyzz & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & xzzz & 0 & 0 & xzxx & 0 & 0 & xzyy & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & yxyz \\ zxxx & 0 & 0 & zxyy & 0 & 0 & zzzz & 0 & 0 & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & xxyy & 0 & 0 & xyzz & 0 & 0 & xyxx & 0 & 0 \\ yxxx & 0 & 0 & yxyy & 0 & 0 & yxzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & zzyy \end{pmatrix}$$

$$[[V^2]^2] \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & xzxx & 0 \\ & & & & & xyxy \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 \\ & & 0 & zzyz & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & xzxy \\ & & & & & 0 \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzzx & 0 \\ & & & 0 & 0 & yzxy \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ & 0 & 0 & 0 & 0 & yyxy \\ & & 0 & 0 & 0 & zzyy \\ & & & 0 & yzxx & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$$

$$[V^2]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 \\ zzxx & zzyy & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & xzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy \end{pmatrix}$$

$$e \begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzxy \\ 0 & 0 & 0 & 0 & xyxz & 0 \end{pmatrix}$$

$$a \begin{pmatrix} 0 & 0 & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy \\ xzxx & xzyy & xzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 \end{pmatrix}$$

$$ae \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & yyxy \\ 0 & 0 & 0 & 0 & 0 & zzyy \\ 0 & 0 & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & xzyy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n=1 \quad \mathbf{D}_{1h}(\mathbf{C}_{1v}) = \mathbf{2}_x \mathbf{m}_y \mathbf{m}_z (\mathbf{m}_y) = \mathbf{2}_x' \mathbf{m}_y \mathbf{m}_z'$$

$$\mathbf{D}_{nh}(\mathbf{C}_{nv})$$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & zxxz & 0 \\ & & & & & xyxy & 0 & 0 & xyyx \\ & & & & & & zyzy & 0 & 0 \\ & & & & & & & xzxz & 0 \\ & & & & & & & & yxyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ & & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & zxyy & 0 & 0 & zxyx \\ & & & & & 0 & 0 & xyxz & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & xzyx \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & zzzx & 0 & 0 & zzzx & 0 \\ & & & 0 & 0 & yzxy & 0 & 0 & yzyx \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & xyzy & 0 & 0 \\ & & & & & & 0 & 0 & zyyx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & xxyx \\ & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ & & & 0 & yzzx & 0 & 0 & yzxz & 0 \\ & & & & 0 & 0 & zxzy & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & zyxz & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & yyyy & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzzx & zzyy & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzzy & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyyx \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & yyyz & 0 & 0 & yyzy & 0 & 0 \\ 0 & 0 & 0 & zzyz & 0 & 0 & zzyz & 0 & 0 \\ yzxx & yzyy & yzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zxyy & 0 & 0 & zxyx \\ 0 & 0 & 0 & 0 & xyxz & 0 & 0 & xyxz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & yyzx & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & zzzx & 0 & 0 & zzzx & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxy & 0 & 0 & yzyx \\ zxxx & zxyy & zxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & xyyz & 0 & 0 & xyzy & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & xxyx \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & yyyx \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & zzyx \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxz & 0 \\ 0 & 0 & 0 & xzyz & 0 & 0 & zxzy & 0 & 0 \\ xyxx & xyxy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$D_{nh}(C_{nv})$

$n=3 \quad D_{3h}(C_{3v}) = \bar{6}_z m_x 2_1(3_z m_x) = \bar{6}_z 'm_x 2_1'$

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V^4	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ yyxx & xxxx & yyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & zxzx & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \\ 0 & 0 & 0 & zxzx & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & yzyz & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy & 0 & 0 & xyxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -xzxx & 0 & 0 & -xzxx \\ zxzx & -zxzx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxxz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -zxzx & 0 & 0 & -zxzx \\ xzxx & -xzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxxz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zyxx & 0 & 0 & zyxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyz & 0 \\ zyxx & -zyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ xzxx & -xzxx & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & yzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ xyxx & xyxy & xyzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -zxzy & 0 & 0 & -zxzy & 0 \\ 0 & 0 & 0 & 0 & -yzxz & 0 & 0 & -yzxz & 0 \\ -xyyy & -xyxx & -xyzz & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
	and: $xxxx=xxyy+xyxy+xyxy$			and: $xxxy-yyxy=xyxy-xyxx$

$[V^4]$	$\begin{pmatrix} 3xxyy & 0 & 0 & yyzz & 0 \\ 0 & 3xxyy & 0 & yyzz & 0 \\ 0 & 0 & zzzz & xxyy & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -zxzx \\ zxzx & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & xxyz \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -xyz & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
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$V[V^3]$	$\begin{pmatrix} 3xxyy & 0 & 0 & xxyy & 0 & 0 & yyzz & 0 & 0 & yyxx \\ 0 & 3xxyy & 0 & 0 & yyzz & 0 & 0 & xxyy & 0 & 0 \\ 0 & 0 & zzzz & 0 & 0 & zzxx & 0 & 0 & zzxx & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xzxx & 0 & 0 & -xzxx & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -xzxx \\ zxzx & 0 & 0 & -zxzx & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & 0 & yzxx & 0 & 0 & -yzxx & 0 \\ 0 & -yzxx & 0 & 0 & 0 & 0 & 0 & zyxx & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 3xyxx & 0 & 0 & xyzz & 0 & 0 & xyxx & 0 & 0 \\ -3xyxx & 0 & 0 & -xyxx & 0 & 0 & -xyzz & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & zxyz \end{pmatrix}$
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$[[V^2]^2]$	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 \\ & & & & yzyz & 0 \\ & & & & & xyxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ & 0 & 0 & -xxyz & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & xxyz \\ & & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -yyxz & 0 \\ & 0 & 0 & 0 & yyxz & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & yyxz \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}$
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and $xyxy = \frac{1}{2}(xxxx - xxyy)$

$[V^2]^2$	$\begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 \\ 0 & 0 & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xyxy \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -yyxz & 0 \\ 0 & 0 & 0 & 0 & yyxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xzyy \\ -xzyy & xzyy & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yyxz & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 \\ 0 & 0 & 0 & -xxyz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxyz & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & xxyy \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -xxyy \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -xxyy & xxyy & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
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and $xyxy = \frac{1}{2}(xxxx - xxyy)$

and $xyxy = \frac{1}{2}(xxxx - xxyy)$

$D_{nh}(C_{nv})$

$n=3 \quad D_{3h}(C_{3v}) = \bar{6}_z m_x 2_1(3_z m_x) = \bar{6}_z 'm_x 2_1'$

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$$[(V^2)]^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & yzyz & 0 & 0 & yzzy & 0 & 0 \\ & & & & zxzx & 0 & 0 & yzzy & 0 \\ & & & & & xxyy & 0 & 0 & xyyx \\ & & & & & & zxzx & 0 & 0 \\ & & & & & & & yzyz & 0 \\ & & & & & & & & xyxy \end{pmatrix}$$

e

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & -xxxz & 0 & 0 & -xxxz \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & -xxzx & 0 & 0 \\ & & & & & & 0 & 0 & -xxzx \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

a

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ & 0 & 0 & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & xxzy & 0 & 0 & xxzy \\ & & & & & 0 & 0 & xxyz & 0 \\ & & & & & & 0 & 0 & 0 \\ & & & & & & & 0 & xxyz \\ & & & & & & & & 0 \end{pmatrix}$$

ae

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & -xxxz \\ & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & -xxxz \\ & & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ & & & 0 & yzzx & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -yzzx & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{pmatrix}$$

and $xxxx=xxyy+yzyz+zyzy$

$$[V^2]V^2 \begin{pmatrix} xxxx & xxyy & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ xxyy & xxxx & xxzz & 0 & 0 & 0 & 0 & 0 & 0 \\ zzxx & zzxx & zzzz & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & yzyz & 0 & 0 & zxzx & 0 & 0 \\ 0 & 0 & 0 & 0 & zxzx & 0 & 0 & yzyz & 0 \\ 0 & 0 & 0 & 0 & 0 & xxyy & 0 & 0 & xxyy \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & xxzx & 0 & 0 & xxxz & 0 \\ 0 & 0 & 0 & 0 & -xxzx & 0 & 0 & -xxxz & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -zxxx \\ zxxx & -zxxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -xxxz & 0 & 0 & -xxzx & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & xxyz & 0 & 0 & xxzy & 0 & 0 \\ 0 & 0 & 0 & -xxyz & 0 & 0 & -xxzy & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ yzxx & -yzxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & yzxx & 0 & 0 & yzxx \\ 0 & 0 & 0 & 0 & xxzy & 0 & 0 & xxyz & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & xxxy & 0 & 0 & -yyxy \\ 0 & 0 & 0 & 0 & 0 & yyxy & 0 & 0 & -xxxz \\ 0 & 0 & 0 & 0 & 0 & zzxy & 0 & 0 & -zzxy \\ 0 & 0 & 0 & 0 & yzzx & 0 & 0 & -zxyz & 0 \\ 0 & 0 & 0 & 0 & zxyz & 0 & 0 & -yzzx & 0 \\ xyxx & -xyxx & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $xxxx=xxyy+2xyxy$

and $yyxy-xxxz=2xyxx$

