1 Doing the $\mathcal{D}\mu$ integration

$$
\begin{aligned}
\frac{1}{\sqrt{\det A^{-1}}} &= \frac{1}{\sqrt{\frac{1}{\det A}}} \\
&= \sqrt{\det A} \\
&= \sqrt{\det\left[-2i\left(\mathcal{L}+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right)\right]} \\
&= \sqrt{\det\left(-2i\right)\det\left(\mathcal{L}+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right)} \\
&\propto \exp\frac{1}{2}\mathrm{Tr}\ln\left(\mathcal{L}+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right) \\
&\propto \exp\left[\frac{1}{2}\mathrm{Tr}\ln\left(1+\left(\mathcal{L}-1\right)+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right)\right] \\
&\propto \exp\left[\frac{1}{2}\mathrm{Tr}\left(\mathcal{L}-1+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}-\frac{1}{2}\left(\left(\mathcal{L}-1\right)+\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right)^{2}\right)+\ldots\right] \\
&\propto \exp\left[\frac{1}{2}\mathrm{Tr}\mathcal{L}+\frac{\kappa\sqrt{\delta\left(\mathbf{0}\right)}}{2}-\frac{1}{4}\mathrm{Tr}\left(\mathcal{L}-1\right)^{2}-\frac{1}{2}\mathrm{Tr}\left(\mathcal{L}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\right)+\frac{1}{2}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\delta\left(\mathbf{0}\right)-\frac{1}{4}\kappa^{2}\frac{\delta\left(\mathbf{0}\right)}{\delta\left(\mathbf{0}\right)}\right] \\
&\propto \exp\left[\frac{\lambda\left(\mathbf{0}\right)}{2}+\frac{\kappa\sqrt{\delta\left(\mathbf{0}\right)}}{2}-\frac{1}{4}\left(\mathrm{Tr}\mathcal{L}^{2}-2\mathrm{Tr}\mathcal{L}+\mathrm{Tr}1\right)-\frac{1}{2}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\lambda\left(\mathbf{0}\right)+\frac{\kappa\sqrt{\delta\left(\mathbf{0}\right)}}{2}-\frac{\kappa^{2}}{4}\right] \\
&\propto \exp\left[\frac{\lambda\left(\mathbf{0}\right)}{2}+\kappa\sqrt{\delta\left(\mathbf{0}\right)}-\frac{1}{4}\left(\frac{1}{4}\times 2\left\{\lambda^{t}\Sigma\lambda+\lambda^{t}\lambda\right\}-2\lambda\left(\mathbf{0}\right)\right)-\frac{1}{2}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\lambda\left(\mathbf{0}\right)-\frac{\kappa^{2}}{4}\right] \\
&\propto \exp\left[\lambda\left(\mathbf{0}\right)+\kappa\sqrt{\delta\left(\mathbf{0}\right)}-\frac{1}{8}\lambda^{t}\left(1+\Sigma\right)\lambda-\frac{1}{2}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\lambda\left(\mathbf{0}\right)-\frac{\kappa^{2}}{4}\right] \\
&\propto \exp\left[\lambda^{t}\delta+\kappa\sqrt{\delta\left(\mathbf{0}\right)}-\frac{1}{8}\lambda^{t}\left(1+\Sigma\right)\lambda-\frac{1}{2}\frac{\kappa}{\sqrt{\delta\left(\mathbf{0}\right)}}\lambda^{t}\delta-\frac{\kappa^{2}}{4}\right]
\end{aligned}
$$

Then

$$
\begin{aligned}
I &= \int_{-\infty}^{\infty} d\kappa \exp\left[-\frac{1}{4}\kappa^2\right] \int \mathcal{D}\lambda \mathcal{D}\mu \, \exp\left[-\frac{1}{2}\mu^t A^{-1}\mu - i\lambda^t \left(Q - i\frac{\kappa}{2\sqrt{\delta(\mathbf{0})}}\delta\right)\right] \exp\left[-i\mu^t\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right) - \frac{1}{8}\lambda^t(1+\Sigma)\lambda\right] \times \\
&\quad \exp\left[i\mu^t \frac{1}{A}\frac{M\sqrt{N}}{\delta(\mathbf{0})}\frac{1}{A}\mu\right] \exp\left[\frac{1}{2}\frac{\delta}{\delta\rho}^t \mathcal{K}^{-1}\frac{\delta}{\delta\rho}\right] E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=\varphi} \\
\varphi &= -i\mathcal{K}^{-1}\mu \\
\mathcal{K} &= -2i\left(\mathcal{L} + \frac{\kappa}{\sqrt{\delta(\mathbf{0})}} + \frac{M\sqrt{N}}{\delta(\mathbf{0})}\right) \\
&= A - 2i\frac{M\sqrt{N}}{\delta(\mathbf{0})} \\
A &= -2i\left(\mathcal{L} + \frac{\kappa}{\sqrt{\delta(\mathbf{0})}}\right) \\
I &= \int d\kappa \, \exp\left[-\frac{1}{4}\kappa^2\right] \int \mathcal{D}\lambda \, \sqrt{\det A} \, \exp\left[-i\lambda^t\left(Q - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}\delta\right) - \frac{1}{8}\lambda^t(1+\Sigma)\lambda\right] \times \\
&\quad \exp\left[-\frac{1}{2}\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right) A\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)\right] \times \\
&\quad \exp\left[\frac{1}{2}\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right] \left\{\exp\left[i\mu^t\frac{1}{A}\frac{M\sqrt{N}}{\delta(\mathbf{0})}\frac{1}{A}\mu\right] \exp\left[\frac{1}{2}\frac{\delta}{\delta\rho}^t\mathcal{K}^{-1}\frac{\delta}{\delta\rho}\right] E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=-i\mathcal{K}^{-1}\mu}\right\}_{\mu=iA\sqrt{N}\left(1+\frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}+i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)} \\
&= \int d\kappa \, \exp\left[-\frac{1}{4}\kappa^2\right] \int \mathcal{D}\lambda \, \exp\left[-i\lambda^t\left(Q - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}\delta\right) - \frac{1}{8}\lambda^t(1+\Sigma)\lambda + \lambda^t\delta + \kappa\sqrt{\delta(\mathbf{0})} - \frac{1}{8}\lambda^t(1+\Sigma)\lambda - \frac{1}{2}\frac{\kappa}{\sqrt{\delta(\mathbf{0})}}\lambda^t\delta - \frac{\kappa^2}{4}\right] \times \\
&\quad \exp\left[\frac{1}{2}\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right) A\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)\right] \times \\
&\quad \exp\left[\frac{1}{2}\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right] \left\{\exp\left[i\mu^t\frac{1}{A}\frac{M\sqrt{N}}{\delta(\mathbf{0})}\frac{1}{A}\mu\right] \exp\left[\frac{1}{2}\frac{\delta}{\delta\rho}^t\mathcal{K}^{-1}\frac{\delta}{\delta\rho}\right] E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=-i\mathcal{K}^{-1}\mu}\right\}_{\mu=iA\sqrt{N}\left(1+\frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}+i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)} \\
&= \int d\kappa \, \exp\left[-\frac{1}{2}\kappa^2 + \kappa\sqrt{\delta(\mathbf{0})}\right] \int \mathcal{D}\lambda \, \exp\left[-i\lambda^t\left(Q + i\delta - \frac{i\kappa}{\sqrt{\delta(\mathbf{0})}}\delta\right) - \frac{1}{4}\lambda^t(1+\Sigma)\lambda + \lambda^t\delta\right] \times \\
&\quad \exp\left[\frac{1}{2}\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right) A\sqrt{N}\left(-1 - \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} - i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)\right] \times \\
&\quad \exp\left[\frac{1}{2}\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right] \left\{\exp\left[i\mu^t\frac{1}{A}\frac{M\sqrt{N}}{\delta(\mathbf{0})}\frac{1}{A}\mu\right] \exp\left[\frac{1}{2}\frac{\delta}{\delta\rho}^t\mathcal{K}^{-1}\frac{\delta}{\delta\rho}\right] E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=-i\mathcal{K}^{-1}\mu}\right\}_{\mu=iA\sqrt{N}\left(1+\frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}+i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)}
\end{aligned}
$$

$$= \int d\kappa \, \exp\left[-\frac{1}{2}\kappa^2 + \kappa\sqrt{\delta(\mathbf{0})}\right] \int \mathcal{D}\lambda \, \exp\left[-i\lambda^t\left(Q + i\delta - \frac{i\kappa}{\sqrt{\delta(\mathbf{0})}}\delta\right) - \frac{1}{4}\lambda^t(1+\Sigma)\lambda + \lambda^t\delta\right] \times$$

$$\exp\left[\left(iN\left(\int \lambda + \frac{\kappa}{\sqrt{\delta(\mathbf{0})}}\right)\right)\left(1 + \frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}} + i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)^2\right] \times$$

$$\exp\left[\frac{1}{2}\frac{\delta}{\delta\mu}^t A \frac{\delta}{\delta\mu}\right]\left\{\underbrace{\exp\left[i\mu^t\frac{1}{A}\frac{M\sqrt{N}}{\delta(\mathbf{0})}\frac{1}{A}\mu\right]}_{\mathcal{H}[\mu]}\underbrace{\exp\left[\frac{1}{2}\frac{\delta}{\delta\rho}^t\mathcal{K}^{-1}\frac{\delta}{\delta\rho}\right]E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=-i\mathcal{K}^{-1}\mu}}_{H[\mu]}\right\}_{\mu=iA\sqrt{N}\left(1+\frac{i\kappa}{2\sqrt{\delta(\mathbf{0})}}+i\lambda(\mathbf{0})\frac{\sqrt{N}}{2\delta(\mathbf{0})}\right)}$$

$$H[\mu] = E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]_{\rho=-i\mathcal{K}^{-1}\mu} - i\int d\mathbf{u}d\mathbf{v}\,\exp\left[2\pi i(\mathbf{h}\cdot\mathbf{u}+\mathbf{k}\cdot\mathbf{v})\right]\left\{\int d\mathbf{z}d\mathbf{y}\left[\mathcal{K}^{-1}_{\mathbf{z},\mathbf{z}+\mathbf{u}}\mathcal{K}^{-1}_{\mathbf{z}+\mathbf{v},\mathbf{y}}\mu(\mathbf{y}) + \mathcal{K}^{-1}_{\mathbf{z},\mathbf{z}+\mathbf{v}}\mathcal{K}^{-1}_{\mathbf{z}+\mathbf{u},\mathbf{y}}\mu(\mathbf{y}) + \mathcal{K}^{-1}_{\mathbf{z}+\mathbf{u},\mathbf{z}+\mathbf{v}}\mathcal{K}^{-1}_{\mathbf{z},\mathbf{y}}\mu(\mathbf{y})\right]\right\}$$

2 Calculation of $L_{\mathbf{h},\mathbf{k}}$.

$\langle\mathbf{x}|\mathbf{h}\rangle \equiv \exp[-2\pi i\mathbf{h}\cdot\mathbf{x}]$. Then $\langle\mathbf{h}|\mathbf{x}\rangle \equiv \overline{\langle\mathbf{x}|\mathbf{h}\rangle} = \exp[2\pi i\mathbf{h}\cdot\mathbf{x}] = \langle-\mathbf{x}|\mathbf{h}\rangle = \langle\mathbf{x}|-\mathbf{h}\rangle$

$$
\begin{aligned}
L_{\mathbf{h},\mathbf{k}} &= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|L|\mathbf{y}\rangle\,\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,L_{\mathbf{x},\mathbf{y}}\,\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\lambda\left(\mathbf{x}-\mathbf{y}\right)\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\int -d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\langle\mathbf{y}-\mathbf{x}+\mathbf{x}|\mathbf{k}\rangle \qquad \mathbf{u}=\mathbf{x}-\mathbf{y} \\
&= -\int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\int d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\langle-\mathbf{u}+\mathbf{x}|\mathbf{k}\rangle \\
&= -\int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|\mathbf{k}\rangle\int d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\underbrace{\langle-\mathbf{u}|\mathbf{k}\rangle}_{\langle\mathbf{k}|\mathbf{u}\rangle} \\
&= -\int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|\mathbf{k}\rangle\int d\mathbf{u}\ \langle\mathbf{k}|\mathbf{u}\rangle\,\langle\mathbf{u}|\lambda\rangle \\
&= -\underbrace{\int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|\mathbf{k}\rangle}_{\langle\mathbf{h}|\mathbf{k}\rangle}\langle\mathbf{k}|\lambda\rangle \\
&= -\left(2\pi\right)^3\delta\left(\mathbf{h}-\mathbf{k}\right)\hat{\lambda}\left(\mathbf{k}\right)
\end{aligned}
$$

$$
\begin{aligned}
L^t_{\mathbf{h},\mathbf{k}} &= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\left\langle\mathbf{x}|L^t|\mathbf{y}\right\rangle\,\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,L^t_{\mathbf{x},\mathbf{y}}\,\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}d\mathbf{y}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\lambda\left(\mathbf{y}-\mathbf{x}\right)\langle\mathbf{y}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\int d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\langle\mathbf{y}-\mathbf{x}+\mathbf{x}|\mathbf{k}\rangle \qquad \mathbf{u}=\mathbf{y}-\mathbf{x} \\
&= \int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\int d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\langle\mathbf{u}+\mathbf{x}|\mathbf{k}\rangle \\
&= \int d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|\mathbf{k}\rangle\int d\mathbf{u}\,\lambda\left(\mathbf{u}\right)\langle\mathbf{u}|\mathbf{k}\rangle \\
&= d\mathbf{x}\ \langle\mathbf{h}|\mathbf{x}\rangle\,\langle\mathbf{x}|\mathbf{k}\rangle\int d\mathbf{u}\ \langle-\mathbf{k}|\mathbf{u}\rangle\,\langle\mathbf{u}|\lambda\rangle \\
&= \langle\mathbf{h}|\mathbf{k}\rangle\,\langle-\mathbf{k}|\lambda\rangle \\
&= \left(2\pi\right)^3\delta\left(\mathbf{h}-\mathbf{k}\right)\hat{\lambda}\left(-\mathbf{k}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathbf{h},\mathbf{k}} &= \frac{1}{2}\left(L_{\mathbf{h},\mathbf{k}} + L^t_{\mathbf{h},\mathbf{k}}\right) \\
&= \frac{1}{2}(2\pi)^3\,\delta(\mathbf{h}-\mathbf{k})\left[\hat{\lambda}(-\mathbf{h}) - \hat{\lambda}(\mathbf{h})\right] \qquad \mathcal{L}_{\mathbf{h},\mathbf{k}} \text{ is diagonal}
\end{aligned}
$$

When λ is even: $\lambda(\mathbf{u}) = \lambda(-\mathbf{u})$ then

$$
\begin{aligned}
\hat{\lambda}(-\mathbf{h}) &= \langle -\mathbf{h}|\lambda\rangle \\
&= \int d\mathbf{u}\,\langle -\mathbf{h}|\mathbf{u}\rangle\langle \mathbf{u}|\lambda\rangle \\
&= \int d\mathbf{u}\,\langle \mathbf{h}|-\mathbf{u}\rangle\langle \mathbf{u}|\lambda\rangle \\
&= -\int d\mathbf{u}\,\langle \mathbf{h}|\mathbf{u}\rangle\langle -\mathbf{u}|\lambda\rangle \\
&= -\int d\mathbf{u}\,\langle \mathbf{h}|\mathbf{u}\rangle\langle \mathbf{u}|\lambda\rangle \\
&= -\langle \mathbf{h}|\lambda\rangle \\
&\equiv -\hat{\lambda}(\mathbf{h}) \\
\text{Then } \mathcal{L}_{\mathbf{h},\mathbf{k}} &= -(2\pi)^3\,\delta(\mathbf{h}-\mathbf{k})\,\hat{\lambda}(\mathbf{h})
\end{aligned}
$$

3 Final calculations. From now on we assume that λ is even!

1. $H_1^0[\mu]\big|_{\mu=A\alpha} = 0$
   Indeed

$$
\begin{aligned}
E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]\big|_{\rho=-iA^{-1}\mu}\Big|_{\mu=A\alpha} &= E_{\mathbf{h}}E_{\mathbf{k}}E_{-\mathbf{h}-\mathbf{k}}[\rho]\big|_{\rho=-iA^{-1}A\alpha} \\
&\propto \alpha^3\left(\underbrace{\int d\mathbf{u} < \mathbf{h}|\mathbf{u} >}_{=0}\right)\left(\underbrace{\int d\mathbf{v} < \mathbf{k}|\mathbf{v} >}_{=0}\right)\left(\underbrace{\int d\mathbf{z}\,\langle -\mathbf{h}-\mathbf{k}\mid \mathbf{z}\rangle}_{=0}\right) = 0
\end{aligned}
$$

2. $H_1^1[\mu]\big|_{\mu=A\alpha} = 0.$
   Indeed

$$
-i\int d\mathbf{u}d\mathbf{v}d\mathbf{z} < \mathbf{h}|\mathbf{u} >< \mathbf{k}|\mathbf{v} > \langle -\mathbf{h}-\mathbf{k}\mid \mathbf{z}\rangle \int d\mathbf{s}\,A^{-1}_{\mathbf{z},\mathbf{u}}\underbrace{\int d\mathbf{y}\,A^{-1}_{\mathbf{v},\mathbf{y}}A_{\mathbf{y},\mathbf{s}}}_{\delta(\mathbf{v}-\mathbf{s})}\alpha \propto \left(\underbrace{\int d\mathbf{v} < \mathbf{k}|\mathbf{v} >}_{=0}\right)\int d\mathbf{z}d\mathbf{u} < \mathbf{u}|\mathbf{h} > \langle -\mathbf{h}-\mathbf{k}\mid \mathbf{z}\rangle\,A^{-1}_{\mathbf{z},\mathbf{u}} = 0
$$

3. $H_2^0[\mu]_{\mu=A\alpha}=0$.
Indeed e.g.

$$i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3 \ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}> \mathcal{K}^{-1}_{\mathbf{z},\mathbf{x}_1}\mu(\mathbf{x}_1)\mathcal{K}^{-1}_{\mathbf{u},\mathbf{x}_2}\mu(\mathbf{x}_2)\mathcal{K}^{-1}_{\mathbf{v},\mathbf{x}_3}\mu(\mathbf{x}_3)=0$$

Indeed at least one of the $\mathcal{K}$ equals A, say $\mathcal{K}^{-1}_{\mathbf{z},\mathbf{x}_1}=A^{-1}_{\mathbf{z},\mathbf{x}_1}$. Then at $\mu=A\alpha$

$$i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3 \ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}> A^{-1}_{\mathbf{z},\mathbf{x}_1}\mu(\mathbf{x}_1)\mathcal{K}^{-1}_{\mathbf{u},\mathbf{x}_2}\mu(\mathbf{x}_2)\mathcal{K}^{-1}_{\mathbf{v},\mathbf{x}_3}\mu(\mathbf{x}_3)\propto \underbrace{\int d\mathbf{s}\underbrace{\int d\mathbf{x}_1\, A^{-1}_{\mathbf{z},\mathbf{x}_1}A_{\mathbf{x}_1,\mathbf{s}}}_{\delta(\mathbf{z}-\mathbf{s})}}_{1}$$

So this becomes 0 since

$$\int d\mathbf{z} \ <-\mathbf{h}-\mathbf{k}|\mathbf{z}>=0$$

4. $\mu(\mathbf{y})$ at $\mu=A\alpha$ is a constant (independent of $\mathbf{y}$)
Indeed

$$\begin{aligned}
\mu(\mathbf{y}) &= \alpha\int d\mathbf{s}\, A_{\mathbf{y},\mathbf{s}} \\
&\propto \int d\mathbf{s}\int d\mathbf{p}d\mathbf{q} \ <\mathbf{y}|\mathbf{p}>\underbrace{<\mathbf{p}|A|\mathbf{q}>}_{A(\mathbf{p})\delta(\mathbf{p}-\mathbf{q})}<\mathbf{q}|\mathbf{s}> \\
&\propto \int d\mathbf{s}\int d\mathbf{p} \ <\mathbf{y}|\mathbf{p}>A(\mathbf{p})<\mathbf{p}|\mathbf{s}> \\
&\propto \int d\mathbf{p} \ <\mathbf{y}|\mathbf{p}>A(\mathbf{p})\underbrace{\int d\mathbf{s} \ <\mathbf{p}|\mathbf{s}>}_{\delta(\mathbf{p})} \\
&\propto A(\mathbf{0})
\end{aligned}$$

5. $\int d\mathbf{y}\, A^{-1}_{\mathbf{w},\mathbf{y}}$ is a constant (independent of $\mathbf{w}$).
Indeed

$$\begin{aligned}
\int d\mathbf{y}\, A^{-1}_{\mathbf{w},\mathbf{y}} &\propto \int d\mathbf{y}\int d\mathbf{p}d\mathbf{q} \ <\mathbf{w}|\mathbf{p}>\underbrace{<\mathbf{p}|A^{-1}|\mathbf{q}>}_{A^{-1}(\mathbf{p})\delta(\mathbf{p}-\mathbf{q})}<\mathbf{q}|\mathbf{y}> \\
&\propto \int d\mathbf{p} \ <\mathbf{w}|\mathbf{p}>A^{-1}(\mathbf{p})\underbrace{\int d\mathbf{y} \ <\mathbf{p}|\mathbf{y}>}_{\delta(\mathbf{p})} \\
&\propto A^{-1}(\mathbf{0})
\end{aligned}$$

6. $\int d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{w},\mathbf{y}}$ is independent of $\mathbf{w}$

7. $H^1_2[\mu]_{\mu=A\alpha} = 0$.
Indeed the $\delta(\mathbf{0})^{-1}$ (at $\mu = A\alpha$) part of

$$-i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}> \times \left[\int d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{z},\mathbf{u}}\mathcal{K}^{-1}_{\mathbf{v},\mathbf{y}}\mu(\mathbf{y}) + \int d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{z},\mathbf{v}}\mathcal{K}^{-1}_{\mathbf{u},\mathbf{y}}\mu(\mathbf{y}) + \int d\mathbf{z}d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{u},\mathbf{v}}\mathcal{K}^{-1}_{\mathbf{z},\mathbf{y}}\mu(\mathbf{y})\right]$$

gives 0. Indeed
$\mathcal{K}^{-1} = A^{-1} + \frac{1}{\delta(\mathbf{0})}2i\sqrt{N}A^{-1}MA^{-1} + O\left(\delta(\mathbf{0})^{-2}\right)$. Consider the case $\mu = \mathcal{K}\alpha$. Then e.g.

$$\int d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{z},\mathbf{u}}\mathcal{K}^{-1}_{\mathbf{v},\mathbf{y}}\mu(\mathbf{y}) = \int d\mathbf{y}\, \mathcal{K}^{-1}_{\mathbf{z},\mathbf{u}} \underbrace{\int d\mathbf{s}\, \mathcal{K}^{-1}_{\mathbf{v},\mathbf{y}}\mathcal{K}_{\mathbf{y},\mathbf{s}}}_{1}$$

and $\int d\mathbf{v}\ <\mathbf{k}|\mathbf{v}>= 0$.

8. $\left.\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}H^0_1[\mu]\right|_{\mu=A\alpha} = 0$. Indeed
one of the μ's in $H^0_1[\mu]$ survives the differentiation and then we have the reasoning of nr. 1. Hence the contribution is at least of order $\delta(\mathbf{0})^{-1}$

9. $(H_1[\mu]\mathcal{H}_1[\mu] + H_2[\mu])_{\mu=A\alpha} = 0$ since $H_1[\mu]$ and $H_2[\mu]$ are 0 at $\mu = A\alpha$ (see above)

10. $\left.\frac{1}{2}\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\left(H^0_1[\mu]\mathcal{H}_1[\mu] + H^1_1[\mu]\mathcal{H}_1[\mu] + H^0_2[\mu]\right)\right|_{\mu=A\alpha} = 0$

11. $\frac{1}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^2\left(H^0_1[\mu]\mathcal{H}_1[\mu] + H^0_2[\mu]\right)_{\mu=A\alpha} = -\sqrt{N}\left(A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zv}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zu}}A^{-1}_{zv}\right)$.

   (a) $\left.\frac{1}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^2 \underbrace{H^0_1[\mu]\mathcal{H}_1[\mu]}_{\text{deg } 6}\right|_{\mu=A\alpha} = 0$

   (b) But

$$\begin{aligned}\frac{1}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^2 \underbrace{H^0_2[\mu]}_{\text{deg } 4} &= -\frac{\sqrt{N}}{2}\left(A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zv}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zu}}A^{-1}_{zv} + A^{-1}_{\mathbf{zv}}A^{-1}_{zu} + A^{-1}_{\mathbf{vu}}A^{-1}_{zv} + A^{-1}_{\mathbf{uv}}A^{-1}_{zu}\right)\\ &= -\sqrt{N}\left(A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zv}}A^{-1}_{\mathbf{u}v} + A^{-1}_{\mathbf{zu}}A^{-1}_{zv}\right)\end{aligned}$$

   Indeed

$$\begin{aligned} i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3 &< \mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}> \mathcal{K}^{-1}_{\mathbf{z},\mathbf{x}_1}\mu(\mathbf{x}_1)\mathcal{K}^{-1}_{\mathbf{u},\mathbf{x}_2}\mu(\mathbf{x}_2)\mathcal{K}^{-1}_{\mathbf{v},\mathbf{x}_3}\mu(\mathbf{x}_3)\\ \mathcal{K}^{-1} &= A^{-1} + \frac{1}{\delta(\mathbf{0})}2i\sqrt{N}A^{-1}MA^{-1}\end{aligned}$$

i. say

$$\frac{1}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^2 2i\sqrt{N}\left(A^{-1}MA^{-1}\right)_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(A^{-1}MA^{-1}\right)_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)A^{-1}_{\mathbf{zs}}\mu\left(\mathbf{s}\right)A^{-1}_{\mathbf{sx}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)A^{-1}_{\mathbf{zs}}A_{\mathbf{s},\mathbf{x}_1}A^{-1}_{\mathbf{sx}_1}A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}\underbrace{A_{\mathbf{s},\mathbf{x}_1}A^{-1}_{\mathbf{sx}_1}}_{\delta(\mathbf{0})}\underbrace{A^{-1}_{\mathbf{u},\mathbf{x}_2}A_{\mathbf{x}_2,\mathbf{x}_3}}_{\delta(\mathbf{u}-\mathbf{x}_3)}A^{-1}_{\mathbf{v},\mathbf{x}_3}$$

$$=-\frac{\sqrt{N}\delta\left(\mathbf{0}\right)}{4}A^{-1}_{\mathbf{zs}}A^{-1}_{\mathbf{v},\mathbf{u}}$$

$$\Longrightarrow< -\mathbf{h}-\mathbf{k}|\mathbf{z}> \underbrace{\int d\mathbf{s}\,A^{-1}_{\mathbf{zs}}}_{Cte}$$

$$=\int d\mathbf{z}\ < -\mathbf{h}-\mathbf{k}|\mathbf{z}>=0$$

$$\text{Indeed}\int \mathrm{d}\mathbf{s}\ \ A^{-1}_{\mathbf{zs}}\propto\int d\mathbf{s}\ <\mathbf{z}|\mathbf{p}>\underbrace{<\mathbf{p}|A^{-1}|\mathbf{q}>}_{A^{-1}(\mathbf{p})\delta(\mathbf{p}-\mathbf{q})}<\mathbf{q}|\mathbf{s}>$$

$$=<\mathbf{z}|\mathbf{p}>A^{-1}\left(\mathbf{p}\right)\underbrace{\int d\mathbf{s}\ <\mathbf{p}|\mathbf{s}>}_{\delta(\mathbf{p})}$$

$$=A^{-1}\left(\mathbf{0}\right)$$

ii. Next

$$-\frac{\sqrt{N}}{4}\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)A^{-1}_{\mathbf{zs}}\mu\left(\mathbf{s}\right)A^{-1}_{\mathbf{sx}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}A_{s,x_2}A^{-1}_{\mathbf{sx}_1}A^{-1}_{\mathbf{u},\mathbf{x}_2}\underbrace{A_{x_1,x_3}A^{-1}_{\mathbf{v},\mathbf{x}_3}}_{\delta(v-x_1)}$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}A_{s,x_2}A^{-1}_{\mathbf{s}v}A^{-1}_{\mathbf{u},\mathbf{x}_2}$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}A^{-1}_{\mathbf{s}v}\underbrace{A_{s,x_2}A^{-1}_{\mathbf{u},\mathbf{x}_2}}_{\delta(s-u)}$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v}$$

$$\Longrightarrow \mathbf{2}\times-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v}=-\frac{\sqrt{N}}{2}A^{-1}_{\mathbf{zu}}A^{-1}_{\mathbf{u}v}$$

iii. also

$$-\frac{\sqrt{N}}{4}\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)A^{-1}_{\mathbf{zs}}\mu\left(\mathbf{s}\right)A^{-1}_{\mathbf{sx}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}A_{sx_3}A^{-1}_{\mathbf{sx}_1}\underbrace{A_{x_1x_2}A^{-1}_{\mathbf{u},\mathbf{x}_2}}_{\delta(u-x_1)}A^{-1}_{\mathbf{v},\mathbf{x}_3}$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zs}}A_{sx_3}A^{-1}_{\mathbf{s}u}A^{-1}_{\mathbf{v},\mathbf{x}_3}$$

$$=-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zv}}A^{-1}_{vu}$$

$$\Longrightarrow \mathbf{2}\times-\frac{\sqrt{N}}{4}A^{-1}_{\mathbf{zv}}A^{-1}_{\mathbf{u}v}=-\frac{\sqrt{N}}{2}A^{-1}_{\mathbf{zv}}A^{-1}_{\mathbf{u}v}$$

12. $\frac{1}{8}\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)^{2}\underbrace{H^{1}_{1}\left[\mu\right]\mathcal{H}_{1}\left[\mu\right]}_{\text{deg }4}\Bigg|_{\mu=A\alpha}=0$

Indeed

(a) We have

$$\frac{1}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^2 - i\int d\mathbf{u}d\mathbf{v}d\mathbf{z} \ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|z>\times$$

$$\left[\int d\mathbf{y}\,A^{-1}_{\mathbf{z},\mathbf{u}}A^{-1}_{\mathbf{v},\mathbf{y}}\mu\left(\mathbf{y}\right)+\int d\mathbf{y}\,A^{-1}_{\mathbf{z},\mathbf{v}}A^{-1}_{\mathbf{u},\mathbf{y}}\mu\left(\mathbf{y}\right)+\int d\mathbf{y}\,A^{-1}_{\mathbf{u},\mathbf{v}}A^{-1}_{\mathbf{z},\mathbf{y}}\mu\left(\mathbf{y}\right)\right]i\sqrt{N}\mu^t\frac{1}{A}M\frac{1}{A}\mu$$

$$=\frac{\sqrt{N}}{8}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\int d\mathbf{u}d\mathbf{v}d\mathbf{z}\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|z>\times$$

$$\left[\int d\mathbf{y}\,A^{-1}_{\mathbf{z},\mathbf{u}}A^{-1}_{\mathbf{v},\mathbf{y}}\mu\left(\mathbf{y}\right)+\int d\mathbf{y}\,A^{-1}_{\mathbf{z},\mathbf{v}}A^{-1}_{\mathbf{u},\mathbf{y}}\mu\left(\mathbf{y}\right)+\int d\mathbf{y}\,A^{-1}_{\mathbf{u},\mathbf{v}}A^{-1}_{\mathbf{z},\mathbf{y}}\mu\left(\mathbf{y}\right)\right]\int dsdtdp\,\mu\left(s\right)A^{-1}_{st}\mu\left(t\right)A^{-1}_{tp}\mu\left(p\right)$$

$$\text{one term }\Longrightarrow= A^{-1}_{\mathbf{z},\mathbf{u}}\underbrace{A^{-1}_{\mathbf{v},\mathbf{y}}\underbrace{A_{ys}A^{-1}_{st}}_{\delta(t-y)}}_{\int dy\,A^{-1}_{\mathbf{v},\mathbf{y}}}\underbrace{A_{tp}A^{-1}_{tp}}_{\delta(\mathbf{0})}$$

$$\underbrace{\int d\mathbf{v}\ <\mathbf{k}|\mathbf{v}>\int dy\,A^{-1}_{\mathbf{v},\mathbf{y}}}_{0}$$

13. $\frac{1}{48}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^3\left(H_1^1\left[\mu\right]\mathcal{H}_1\left[\mu\right]\right)\Big|_{\mu=A\alpha}=-\frac{\sqrt{N}}{3}\int d\mathbf{u}d\mathbf{v}d\mathbf{z}\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>\left(A^{-1}_{zu}A^{-1}_{uv}+A^{-1}_{zv}A^{-1}_{uv}+A^{-1}_{uz}A^{-1}_{zv}\right)$

(a) Indeed

$$\frac{1}{48}\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)^3 i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>A^{-1}_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)\times$$

$$i\sqrt{N}\mu^t\frac{1}{A}M\frac{1}{A}\mu$$

$$=-\frac{\sqrt{N}}{48}\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^t A\frac{\delta}{\delta\mu}\right)\times$$

$$A^{-1}_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)\int dsdtdp\,\mu\left(s\right)A^{-1}_{st}\mu\left(t\right)A^{-1}_{tp}\mu\left(p\right)$$

$$\Longrightarrow= A^{-1}_{\mathbf{z},\mathbf{x}_1}A_{x_1s}A^{-1}_{\mathbf{u},\mathbf{x}_2}A_{x_2t}A^{-1}_{\mathbf{v},\mathbf{x}_3}A_{x_3p}A^{-1}_{st}A^{-1}_{tp}$$

$$=\delta\left(z-s\right)\delta\left(u-t\right)\delta\left(v-p\right)A^{-1}_{st}A^{-1}_{tp}$$

$$=-\frac{\sqrt{N}}{48}\int d\mathbf{u}d\mathbf{v}d\mathbf{z}\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>A^{-1}_{zu}A^{-1}_{uv}$$

Thus

$$\underbrace{6\times 2\times\left(-\frac{\sqrt{N}}{48}\right)}_{-\frac{\sqrt{N}}{3}}\int d\mathbf{u}d\mathbf{v}d\mathbf{z}\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>\left(A^{-1}_{zu}A^{-1}_{uv}+A^{-1}_{zv}A^{-1}_{uv}+A^{-1}_{uz}A^{-1}_{zv}\right)$$

(b) Indeed e.g.

$$\frac{1}{48}\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)^{3}i\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>A^{-1}_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)\times$$

$$i\sqrt{N}\mu^{t}\frac{1}{A}M\frac{1}{A}\mu$$

$$=-\frac{\sqrt{N}}{48}\int d\mathbf{u}d\mathbf{v}d\mathbf{z}d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\ <\mathbf{h}|\mathbf{u}><\mathbf{k}|\mathbf{v}><-\mathbf{h}-\mathbf{k}|\mathbf{z}>\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)\left(\frac{\delta}{\delta\mu}^{t}A\frac{\delta}{\delta\mu}\right)\times$$

$$A^{-1}_{\mathbf{z},\mathbf{x}_1}\mu\left(\mathbf{x}_1\right)A^{-1}_{\mathbf{u},\mathbf{x}_2}\mu\left(\mathbf{x}_2\right)A^{-1}_{\mathbf{v},\mathbf{x}_3}\mu\left(\mathbf{x}_3\right)\int dsdtdp\,\mu\left(s\right)A^{-1}_{st}\mu\left(t\right)A^{-1}_{tp}\mu\left(p\right)$$

$$\Longrightarrow=\underbrace{A^{-1}_{\mathbf{z},\mathbf{x}_1}A_{x_1x_2}}_{\delta(z-x_2)}A^{-1}_{\mathbf{u},\mathbf{x}_2}\underbrace{A^{-1}_{\mathbf{v},\mathbf{x}_3}A_{x_3s}}_{\delta(v-s)}A^{-1}_{st}\underbrace{A_{tp}A^{-1}_{tp}}_{\delta(\mathbf{0})}$$

$$\Longrightarrow=\delta\left(\mathbf{0}\right)A^{-1}_{\mathbf{u},\mathbf{z}}\underbrace{\int dv\ <\mathbf{k}|\mathbf{v}>\underbrace{\int dt\,A^{-1}_{vt}}_{\neq f(v)}}_{0}$$