

TABLES:  
SCANNING OF TWO-DIMENSIONAL SPACE GROUPS

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Abstract

Tables of the scanning of two-dimensional space groups are presented to determine the frieze group symmetry of lines that transect two-dimensional crystals.

Two-Dimensional  
Space Group

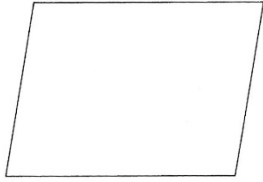
Direction **a**

**d**

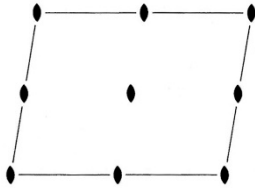
Linear  
Orbit  
**sd**

Frieze Group

1)  $p1$



2)  $p2$



$[n,m]$   $na + mb$

$pa - qb$

$0d, \frac{1}{2}d$

$\mu 211$

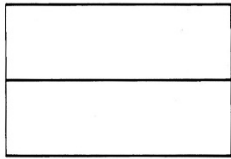
F2

$[sd, -sd]$

$\mu 1$

F1

3)  $pm$



$[0,1]$  **b**

**a**

$0d, \frac{1}{2}d$

$\mu 11m$

F4

$[sd, -sd]$

$\mu 1$

F1

$[1,0]$  **a**

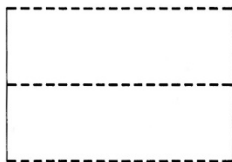
**b**

**sd**

$\mu 1m1$

F3

4)  $pg$



$[0,1]$  **b**

**a**

$0d, \frac{1}{2}d$

$\mu 11g$

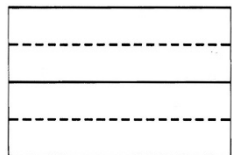
F5

$[sd, -sd]$

$\mu 1$

F1

5)  $cm$



$[0,1]$  **b**

**a**

$[0d, \frac{1}{2}d]$

$\mu 11m$

F4

$[\frac{1}{4}d, \frac{3}{4}d]$

$\mu 11g$

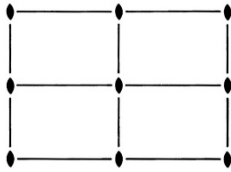
F5

$[\pm sd, (\pm s + \frac{1}{2})d]$   $\mu 1$

F1

[1,0] **a** **b** [sd, (s+½)d] /  $\mu$ 1m1 F3

6)  $p2mm$



[1,0] **a** **b** 0d, ½d /  $\mu$ 2mm F6

[sd, -sd] /  $\mu$ 1m1 F3

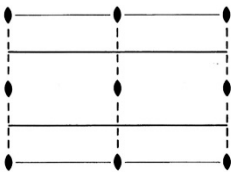
[0,1] **b** **a** 0d, ½d /  $\mu$ 2mm F6

[sd, -sd] /  $\mu$ 1m1 F3

[n,m] na + mb pa - qb 0d, ½d /  $\mu$ 211 F2

[sd, -sd] /  $\mu$ 1 F1

7)  $p2mg$



[1,0] **a** **b** 0d, ½d /  $\mu$ 2mg F7

[sd, -sd] /  $\mu$ 1m1(a<sub>F</sub>/4) F3

[0,1] **b** **a** [0d, ½d] /  $\mu$ 211 F2

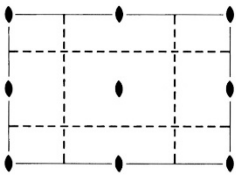
[¼d, ¾d] /  $\mu$ 11m F4

[±sd, (±s+½)d] /  $\mu$ 1 F1

[n,m] na + mb pa - qb 0d, ½d /  $\mu$ 211 F2

[sd, -sd] /  $\mu$ 1 F1

8)  $p2gg$

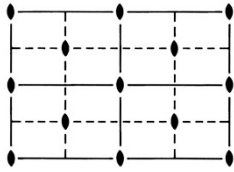


$[1,0]$	<b>a</b>	<b>b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\cancel{\mu}211$	F2
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\cancel{\mu}11g$	F5
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\cancel{\mu}1$	F1

$[0,1]$	<b>b</b>	<b>a</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\cancel{\mu}211$	F2
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\cancel{\mu}11g$	F5
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\cancel{\mu}1$	F1

$[n,m]$	$na + mb$	$pa - qb$	$0\mathbf{d}, \frac{1}{2}\mathbf{d}$	$\cancel{\mu}211$	F2
			$[s\mathbf{d}, -s\mathbf{d}]$	$\cancel{\mu}1$	F1

9)  $c2mm$



$[1,0]$	<b>a</b>	<b>b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\cancel{\mu}2mm$	F6
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\cancel{\mu}2mg(\mathbf{a}_F/4)$	F7
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\cancel{\mu}1m1$	F3

$[0,1]$	<b>b</b>	<b>a</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\cancel{\mu}2mm$	F6
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\cancel{\mu}2mg(\mathbf{a}_F/4)$	F7
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\cancel{\mu}1m1$	F3

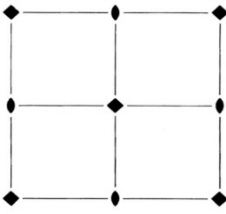
$[h,k]$	$n\hat{\mathbf{a}} + m\hat{\mathbf{b}}$	$p\hat{\mathbf{a}} - q\hat{\mathbf{b}}$	$0\mathbf{d}, \frac{1}{2}\mathbf{d}$	$\cancel{\mu}211$	F2
			$[s\mathbf{d}, -s\mathbf{d}]$	$\cancel{\mu}1$	F1

$$\hat{\mathbf{a}} = (\mathbf{a} + \mathbf{b})/2 \quad \hat{\mathbf{b}} = (\mathbf{a} - \mathbf{b})/2$$

If  $h, k$  even/odd or odd/even the  $n = h + k$  and  $m = h - k$ .

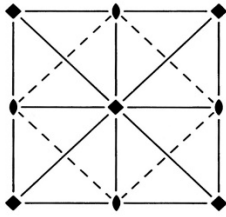
If  $h, k$  both odd, then  $n = (h + k)/2$  and  $m = (h - k)/2$ .

10)  $p4$



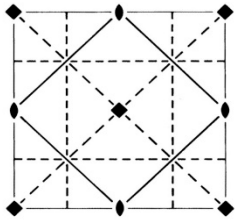
$[n,m]$	$na + mb$	$pa - qb$	$0d, \frac{1}{2}d$	$\mu 211$	F2
			$[sd, -sd]$	$\mu 1$	F1

11)  $p4mm$



$[1,0]$	<b>a</b>	<b>b</b>	$0d, \frac{1}{2}d$	$\mu 2mm$	F6
			$[sd, -sd]$	$\mu 1m1$	F3
$[0,1]$	<b>b</b>	<b>a</b>	$0d, \frac{1}{2}d$	$\mu 2mm$	F6
			$[sd, -sd]$	$\mu 1m1$	F3
$[1,1]$	<b>a+b</b>	<b>a-b</b>	$[0d, \frac{1}{2}d]$	$\mu 2mm$	F6
			$[\frac{1}{4}d, \frac{3}{4}d]$	$\mu 2mg(a_F/4)$	F7
			$[\pm sd, (\pm s + \frac{1}{2})d]$	$\mu 1m1$	F3
$[1,-1]$	<b>a-b</b>	<b>a+b</b>	$[0d, \frac{1}{2}d]$	$\mu 2mm$	F6
			$[\frac{1}{4}d, \frac{3}{4}d]$	$\mu 2mg(a_F/4)$	F7
			$[\pm sd, (\pm s + \frac{1}{2})d]$	$\mu 1m1$	F3
$[n,m]$	$na + mb$	$pa - qb$	$0d, \frac{1}{2}d$	$\mu 211$	F2
			$[sd, -sd]$	$\mu 1$	F1

12)  $p4gm$



$[1,0]$	<b>a</b>	<b>b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 211$	F2
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 11g$	F5
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\neq 1$	F1

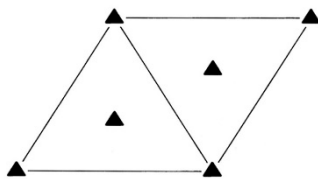
$[1,0]$	<b>b</b>	<b>a</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 211$	F2
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 11g$	F5
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\neq 1$	F1

$[1,1]$	<b>a+b</b>	<b>a-b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 2mg$	F7
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 2mm(\mathbf{a}_F/4)$	F6
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\neq 1m1(\mathbf{a}_F/4)$	F3

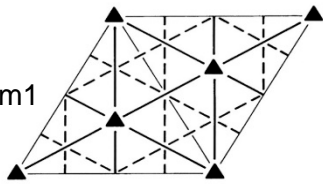
$[1,-1]$	<b>a-b</b>	<b>a+b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 2mg$	F7
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 2mm(\mathbf{a}_F/4)$	F6
			$[\pm s\mathbf{d}, (\pm s + \frac{1}{2})\mathbf{d}]$	$\neq 1m1(\mathbf{a}_F/4)$	F3

$[n,m]$	$na + mb$	$pa - qb$	$0\mathbf{d}, \frac{1}{2}\mathbf{d}$	$\neq 211$	F2
			$[s\mathbf{d}, -s\mathbf{d}]$	$\neq 1$	F1

13)  $p3$

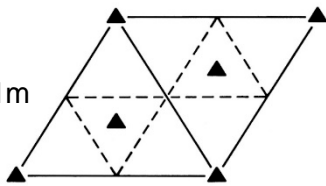


14)  $p3m1$



$[1,0]$	<b>a</b>	<b>a+2b</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3
$[0,1]$	<b>b</b>	<b>2a+b</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3
$[1,1]$	<b>a+b</b>	<b>a-b</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3
$[2,1]$	<b>2a+b</b>	<b>b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 11m$	F4
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 11g$	F5
			$[\pm\mathbf{sd}, (\pm\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1$	F1
$[1,2]$	<b>a+2b</b>	<b>a</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 11m$	F4
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 11g$	F5
			$[\pm\mathbf{sd}, (\pm\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1$	F1
$[1,-1]$	<b>a-b</b>	<b>a+b</b>	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$\neq 11m$	F4
			$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$\neq 11g$	F5
			$[\pm\mathbf{sd}, (\pm\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1$	F1

15)  $p31m$



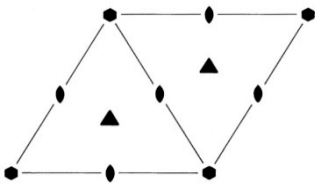
$[2,1]$	<b>2a+b</b>	<b>b</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3
$[1,2]$	<b>a+2b</b>	<b>a</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3
$[1,-1]$	<b>a-b</b>	<b>a+b</b>	$[\mathbf{sd}, (\mathbf{s}+\frac{1}{2})\mathbf{d}]$	$\neq 1m1$	F3

[1,0]	<b>a</b>	<b>a+2b</b>	[0d, ½d]	$\cancel{p}11m$	F4
			[¼d, ¾d]	$\cancel{p}11g$	F5
			[±sd, (±s+½)d]	$\cancel{p}1$	F1

[0,1]	<b>b</b>	<b>2a+b</b>	[0d, ½d]	$\cancel{p}11m$	F4
			[¼d, ¾d]	$\cancel{p}11g$	F5
			[±sd, (±s+½)d]	$\cancel{p}1$	F1

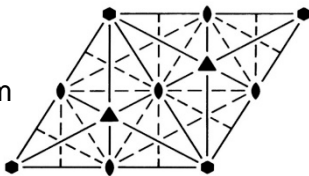
[1,1]	<b>a+b</b>	<b>a-b</b>	[0d, ½d]	$\cancel{p}11m$	F4
			[¼d, ¾d]	$\cancel{p}11g$	F5
			[±sd, (±s+½)d]	$\cancel{p}1$	F1

16)  $p6$



[n,m]	<b>na + mb</b>	<b>pa - qb</b>	0d, ½d	$\cancel{p}211$	F2
			[sd, -sd]	$\cancel{p}1$	F1

17)  $p6mm$



[1,0]	<b>a</b>	<b>a+2b</b>	[0d, ½d]	$\cancel{p}2mm$	F6
			[¼d, ¾d]	$\cancel{p}2mg(a_F/4)$	F7
			[±sd, (±s+½)d]	$\cancel{p}1m1$	F3

[2,1]	<b>2a+b</b>	<b>b</b>	[0d, ½d]	$\cancel{p}2mm$	F6
			[¼d, ¾d]	$\cancel{p}2mg(a_F/4)$	F7
			[±sd, (±s+½)d]	$\cancel{p}1m1$	F3



[1,1]	<b>a+b</b>	<b>a-b</b>	[0d, ½d]	$\neq 2\text{mm}$	F6
			[¼d, ¾d]	$\neq 2\text{mg}(a_F/4)$	F7
			[±sd, (±s+½)d]	$\neq 1\text{m1}$	F3
[1,2]	<b>a+2b</b>	<b>a</b>	[0d, ½d]	$\neq 2\text{mm}$	F6
			[¼d, ¾d]	$\neq 2\text{mg}(a_F/4)$	F7
			[±sd, (±s+½)d]	$\neq 1\text{m1}$	F3
[0,1]	<b>b</b>	<b>2a+b</b>	[0d, ½d]	$\neq 2\text{mm}$	F6
			[¼d, ¾d]	$\neq 2\text{mg}(a_F/4)$	F7
			[±sd, (±s+½)d]	$\neq 1\text{m1}$	F3
[1,-1]	<b>a-b</b>	<b>a+b</b>	[0d, ½d]	$\neq 2\text{mm}$	F6
			[¼d, ¾d]	$\neq 2\text{mg}(a_F/4)$	F7
			[±sd, (±s+½)d]	$\neq 1\text{m1}$	F3
[n,m]	<b>na + mb</b>	<b>pa - qb</b>	0d, ½d	$\neq 211$	F2
			[sd, -sd]	$\neq 1$	F1