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**Supporting information for article:**

**Pressure-dependent X-ray diffraction of the multiferroics  $RMn_2O_5$**

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# Supplemental Materials

## Pressure-dependent x-ray diffraction of multiferroic $\text{RMn}_2\text{O}_5$

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### REFINED PARAMETERS OF $\text{RMn}_2\text{O}_5$ (R = PR, SM, ND, TB AND DY)

We have refined the X-ray diffraction of  $\text{RMn}_2\text{O}_5$  (R = Pr, Sm, Nd, Tb and Dy) at different pressure (up to  $\sim 8$  GPa). The refined lattice parameters of  $\text{RMn}_2\text{O}_5$  (R = Pr, Sm, Nd, Tb and Dy) are listed in Tables I to V while their refined atomic parameters are given in Tables VI to VIII. As we mentioned in the article, for the compounds with R = Dy, Tb and Nd, the refinements were difficult, especially at high pressure. The peak intensities at medium-high angles of their high pressure diffractograms were not correctly fitted. So the analysis of the atomic position are limited below 4 GPa for which the refinements were of good quality and reliable.

TABLE I: Refined parameters of  $\text{PrMn}_2\text{O}_5$  at different pressures.  $\lambda = 0.71 \text{ \AA}$

Pressure (GPa)	0	1.11	1.88	3.01	4.03	4.96	6.30	7.05	8.28
<i>lattice parameters</i>									
<i>a</i> (Å)	7.566(6)	7.535(6)	7.527(0)	7.508(6)	7.491(4)	7.473(0)	7.457(3)5	7.446(5)	7.436(3)
<i>b</i> (Å)	8.654(4)	8.639(3)	8.634(4)7	8.626(6)	8.618(7)	8.610(3)	8.606(5)	8.603(2)	8.599(4)
<i>c</i> (Å)	5.715(4)	5.703(1)	5.699(9)	5.693(6)	5.688(2)	5.683(2)	5.680(4)	5.678(5)	5.678(1)
<i>Refinement</i>									
$\chi^2$	0.071	0.078	0.078	0.081	0.098	0.146	0.205	0.273	0.438
$R_p$ (%)	4.47	4.79	4.68	4.93	5.48	6.87	8.43	9.50	13.0
$R_{wp}$ (%)	4.84	5.03	5.08	5.32	5.96	7.54	9.19	10.80	14.1
$R_{exp}$ (%)	18.16	17.94	18.19	18.74	19.01	19.73	20.30	20.59	21.37
$R_{bragg}$ (%)	2.76	2.55	2.78	3.22	3.81	4.38	5.52	6.35	10.04

TABLE II: Refined parameters of  $\text{SmMn}_2\text{O}_5$  at different pressures.  $\lambda = 0.71 \text{ \AA}$

Pressure (GPa)	0	1.19	2.15	3.25	4.11	5.55	6.37
<i>lattice parameters</i>							
<i>a</i> (Å)	7.439(2)	7.410(4)	7.396(2)	7.380(0)	7.364(2)	7.347(5)	7.329(4)
<i>b</i> (Å)	8.584(3)	8.569(5)	8.562(1)	8.555(7)	8.549(0)	8.544(1)	8.537(1)
<i>c</i> (Å)	5.694(6)	5.683(9)	5.678(1)	5.672(1)	5.667(3)	5.664(0)	5.660(2)
<i>Refinement</i>							
$\chi^2$	0.029	0.031	0.017	0.017	0.016	0.026	0.023
$R_p$ (%)	7.04	7.60	5.48	5.18	5.10	4.87	4.59
$R_{wp}$ (%)	6.46	7.14	5.33	5.33	5.25	5.39	5.10
$R_{exp}$ (%)	37.88	40.79	40.40	40.08	41.62	33.80	33.88
$R_{bragg}$ (%)	5.41	4.39	1.64	1.76	1.55	1.67	1.39

TABLE III: Refined parameters of NdMn<sub>2</sub>O<sub>5</sub> at different pressures.  $\lambda = 0.71 \text{ \AA}$ 

Pressure (GPa)	0	0.76	1.43	2.37	3.05	3.84	4.62	5.64	6.54	7.28
<i>lattice parameters</i>										
$a$ (Å)	7.524(5)	7.510(9)	7.505(7)	7.497(0)	7.477(6)	7.461(2)	7.441(9)	7.425(6)	7.409(0)	7.397(6)
$b$ (Å)	8.624(8)	8.620(3)	8.616(2)	8.608(7)	8.601(9)	8.595(6)	8.593(0)	8.590(2)	8.588(3)	8.586(0)
$c$ (Å)	5.708(1)	5.703(6)	5.701(1)	5.696(5)	5.692(1)	5.688(0)	5.683(8)	5.683(4)	5.681(5)	5.683(7)
<i>Refinement</i>										
$\chi^2$	0.107	0.256	0.216	0.259	0.265	0.285	0.327	0.365	0.423	0.419
$R_p$ (%)	8.21	12.0	11.3	11.5	11.3	12.4	13.5	15.1	16.7	20.2
$R_{wp}$ (%)	8.79	13.3	12.6	12.4	13.5	14.6	15.9	17.1	18.8	21.6
$R_{exp}$ (%)	26.84	26.361	27.08	24.36	26.18	27.35	27.74	28.33	28.93	33.37
$R_{bragg}$ (%)	6.42	8.83	9.25	8.88	9.06	9.58	9.96	11.42	11.41	15.93

TABLE IV: Refined parameters of TbMn<sub>2</sub>O<sub>5</sub> at different pressures.  $\lambda = 0.71 \text{ \AA}$ 

Pressure (GPa)	0	0.81	1.96	2.66	3.61	4.69	5.96	6.88	7.80	8.60
<i>lattice parameters</i>										
$a$ (Å)	7.354(1)	7.338(1)	7.318(7)	7.301(7)	7.284(3)	7.267(3)	7.251(9)	7.240(2)	7.225(3)	7.203(5)
$b$ (Å)	8.545(1)	8.539(7)	8.533(5)	8.526(2)	8.519(5)	8.513(4)	8.508(7)	8.504(2)	8.496(8)	8.479(9)
$c$ (Å)	5.692(7)	5.688(9)	5.681(9)	5.676(3)	5.671(3)	5.666(9)	5.662(8)	5.660(1)	5.655(6)	5.643(9)
<i>Refinement</i>										
$\chi^2$	0.067	0.055	0.083	0.146	0.229	0.294	0.329	0.419	0.492	0.513
$R_p$ (%)	5.47	4.36	5.46	7.71	10.0	11.6	12.9	14.2	16.4	16.8
$R_{wp}$ (%)	5.42	4.71	5.88	8.12	10.5	12.3	13.3	15.40	17.0	17.7
$R_{exp}$ (%)	20.88	20.03	20.42	21.23	21.97	22.71	23.22	23.73	24.31	24.77
$R_{bragg}$ (%)	3.54	2.37	3.64	5.37	7.66	8.78	9.25	11.72	12.67	12.63

TABLE V: Refined parameters of DyMn<sub>2</sub>O<sub>5</sub> at different pressures.  $\lambda = 0.71 \text{ \AA}$ 

Pressure (GPa)	0	0.85	1.23	2.11	3.06	3.62	4.84	5.74	6.75	7.72
<i>lattice parameters</i>										
$a$ (Å)	7.299(3)	7.276(7)	7.261(3)	7.243(5)	7.221(1)	7.209(5)	7.190(0)	7.178(3)	7.164(7)	7.156(4)
$b$ (Å)	8.523(9)	8.509(8)	8.501(5)	8.492(3)	8.481(6)	8.476(5)	8.468(8)	8.464(7)	8.459(1)	8.455(9)
$c$ (Å)	5.679(9)	5.672(5)	5.668(2)	5.662(5)	5.655(9)	5.652(2)	5.646(5)	5.644(3)	5.639(8)	5.637(3)
<i>Refinement</i>										
$\chi^2$	0.192	0.188	0.239	0.218	0.253	0.263	0.275	0.308	0.340	0.418
$R_p$ (%)	10.7	10.6	12.6	11.8	13.1	13.1	13.5	13.6	14.2	15.3
$R_{wp}$ (%)	10.5	10.6	12.2	12.0	13.1	13.3	13.8	14.6	15.5	17.0
$R_{exp}$ (%)	23.97	24.49	24.98	25.66	25.98	26.04	26.27	26.36	26.52	26.32
$R_{bragg}$ (%)	8.06	8.13	10.54	9.16	10.73	10.66	10.99	10.27	10.63	10.83

TABLE VI: Refined atomic parameters and isotropic displacement (temperature) parameter (in  $\text{\AA}^2$ ) of  $\text{PrMn}_2\text{O}_5$  (left) and  $\text{SmMn}_2\text{O}_5$  (right) at different pressures.

Atom	$x$	$y$	$z$	B	Atom	$x$	$y$	$z$	B
<i>0 GPa</i>					<i>0 GPa</i>				
Pr	0.1457(5)	0.1735(2)	0	0.0062(1)	Sm	0.1411(7)	0.1748(5)	0	0.0079(9)
Mn1	0	0.5	0.2544(5)	0.0070(9)	Mn1	0	0.5	0.2556(2)	0.0063(7)
Mn2	0.4113(5)	0.3531(2)	0.5	0.0057(8)	Mn2	0.4131(7)	0.3542(1)	0.5	0.0081(2)
O1	0	0	0.2690(5)	0.005	O1	0	0	0.2527(1)	0.005
O2	0.1656(5)	0.4454(4)	0	0.005	O2	0.1760(5)	0.4694(6)	0	0.005
O3	0.1337(3)	0.4377(0)	0.5	0.005	O3	0.1429(7)	0.4339(5)	0.5	0.005
O4	0.4046(3)	0.2028(7)	0.2550(7)	0.005	O4	0.4016(3)	0.2054(6)	0.2522(0)	0.005
<i>1.11 GPa</i>					<i>1.19 GPa</i>				
Pr	0.1480(0)	0.1742(2)	0	0.0061(6)	Sm	0.1412(6)	0.1754(3)	0	0.0070(1)
Mn1	0	0.5	0.25497	0.0068(4)	Mn1	0	0.5	0.2510(7)	0.0052(7)
Mn2	0.41091	0.35138	0.5	0.0052(4)	Mn2	0.4149(1)	0.3558(8)	0.5	0.0083(1)
<i>1.88 GPa</i>					<i>2.15 GPa</i>				
Pr	0.1474(9)	0.1741(2)	0	0.0089(1)	Sm	0.1415(7)	0.1756(4)	0	0.0059(8)
Mn1	0	0.5	0.2550(2)	0.0096(2)	Mn1	0	0.5	0.2503(9)	0.0056(2)
Mn2	0.4118(4)	0.3507(2)	0.5	0.0089(0)	Mn2	0.4143(5)	0.3564(0)	0.5	0.0064(1)
<i>3.01 GPa</i>					<i>3.25 GPa</i>				
Pr	0.1475(4)	0.1746(2)	0	0.0117(2)	Sm	0.1416(0)	0.1754(6)	0	0.0057(8)
Mn1	0	0.5	0.25389	0.0120(8)	Mn1	0	0.5	0.2503(2)	0.0050(4)
Mn2	0.41217	0.35152	0.5	0.0135(3)	Mn2	0.4136(0)	0.3561(2)	0.5	0.0055(8)
<i>4.03 GPa</i>					<i>4.11 GPa</i>				
Pr	0.1472(8)	0.1745(5)	0	0.0138(3)	Sm	0.1415(8)	0.1754(2)	0	0.0098(6)
Mn1	0	0.5	0.2536(2)	0.0172(3)	Mn1	0	0.5	0.2502(3)	0.0087(5)
Mn2	0.4122(4)	0.3510(4)	0.5	0.0147(7)	Mn2	0.4138(7)	0.3559(0)	0.5	0.0094(7)
<i>4.96 GPa</i>					<i>5.55 GPa</i>				
Pr	0.1482(4)	0.1751(2)	0	0.0153(2)	Sm	0.1413(2)	0.1751(8)	0	0.0123(2)
Mn1	0	0.5	0.2520(6)	0.0182(3)	Mn1	0	0.5	0.2495(0)	0.015(4)
Mn2	0.4126(8)	0.3514(8)	0.5	0.0181(7)	Mn2	0.4136(0)	0.3565(1)	0.5	0.0011(3)
<i>6.30 GPa</i>					<i>6.37 GPa</i>				
Pr	0.1480(7)	0.1755(6)	0	0.0192(1)	Sm	0.1411(2)	0.1750(2)	0	0.0172(8)
Mn1	0	0.5	0.2508(1)	0.0212(7)	Mn1	0	0.5	0.2493(7)	0.0192(3)
Mn2	0.41322(2)	0.3523(6)	0.5	0.0225(8)	Mn2	0.4135(6)	0.3566(0)	0.5	0.0152(5)
<i>7.05 GPa</i>									
Pr	0.1485(1)	0.1752(7)	0	0.0242(1)					
Mn1	0	0.5	0.2502(6)	0.0252(4)					
Mn2	0.4125(7)	0.3529(3)	0.5	0.0252(1)					
<i>8.28 GPa</i>									
Pr	0.1484(4)	0.1755(9)	0	0.0282(3)					
Mn1	0	0.5	0.2508(4)	0.0312(6)					
Mn2	0.41173	0.331(2)	0.5	0.0352(5)					

TABLE VII: Refined atomic parameters and isotropic displacement (temperature) parameter (in  $\text{\AA}^2$ ) of  $\text{NdMn}_2\text{O}_5$  (left) and  $\text{TbMn}_2\text{O}_5$  (right) at different pressures.

Atom	$x$	$y$	$z$	B	Atom	$x$	$y$	$z$	B
<i>0 GPa</i>					<i>0 GPa</i>				
Nd	0.1422(9)	0.1745(3)	0	0.0089(1)	Tb	0.1404(4)	0.1719(9)	0	0.0089(8)
Mn1	0	0.5	0.2570(9)	0.0072(7)	Mn1	0	0.5	0.25024	0.0075(7)
Mn2	0.4165(5)	0.3661(5)	0.5	0.0095(3)	Mn2	0.4101(9)	0.3536(3)	0.5	0.0135(1)
O1	0	0	0.2349(7)	0.005	O1	0	0	0.2598(4)	0.005
O2	0.1571(6)	0.4469(4)	0	0.005	O2	0.1693(1)	0.4431(6)	0	0.005
O3	0.1524(6)	0.4374(4)	0.5	0.005	O3	0.1429(3)	0.4322(5)	0.5	0.005
O4	0.4009(3)	0.2067(3)	0.2515(6)	0.005	O4	0.3988(1)	0.2017(8)	0.2473(5)	0.005
<i>0.76 GPa</i>					<i>0.81 GPa</i>				
Nd	0.1399(2)	0.1739(9)	0	0.0076(5)	Tb	0.1407(8)	0.1725(1)	0	0.0067(2)
Mn1	0	0.5	0.2664(6)	0.0068(3)	Mn1	0	0.5	0.2498(3)	0.0060(5)
Mn2	0.4161(9)	0.3677(9)	0.5	0.0085(1)	Mn2	0.4109(1)	0.3550(9)	0.5	0.0083(7)
<i>1.43 GPa</i>					<i>1.96 GPa</i>				
Nd	0.1386(7)	0.1733(5)	0	0.0127(5)	Tb	0.1413(5)	0.1732(9)	0	0.0043(9)
Mn1	0	0.5	0.2648(1)	0.0146(4)	Mn1	0	0.5	0.2488(6)	0.0051(2)
Mn2	0.4190(2)	0.3711(3)	0.5	0.0175(3)	Mn2	0.4116(7)	0.3584(4)	0.5	0.0065(7)
<i>2.37 GPa</i>					<i>2.66 GPa</i>				
Nd	0.1385(0)	0.1732(5)	0	0.0247(4)	Tb	0.1425(2)	0.1742(4)	0	0.0038(1)
Mn1	0	0.5	0.2658(4)	0.0314(8)	Mn1	0	0.5	0.24644	0.0029(7)
Mn2	0.4187(4)	0.3721(3)	0.5	0.0294(2)	Mn2	0.41116	0.35908	0.5	0.0064(3)
<i>3.05 GPa</i>					<i>3.61 GPa</i>				
Nd	0.1432(9)	0.1769(1)	0	0.0328(6)	Tb	0.1428(3)	0.1747(9)	0	0.0018(9)
Mn1	0	0.5	0.2637(0)	0.0307(4)	Mn1	0	0.5	0.2449(7)	0.0027(3)
Mn2	0.4268(5)	0.3661(2)	0.5	0.0344(5)	Mn2	0.4116(5)	0.3584(3)	0.5	0.0058(9)

TABLE VIII: Refined atomic parameters and isotropic displacement (temperature) parameter (in  $\text{\AA}^2$ ) of  $\text{DyMn}_2\text{O}_5$  at different pressures.

Atom	$x$	$y$	$z$	B	Atom	$x$	$y$	$z$	B
<i>0 GPa</i>					<i>1.23 GPa</i>				
Dy	0.1417(6)	0.1790(6)	0	0.0126(3)	Dy	0.1424(7)	0.1798(5)	0	0.0097(8)
Mn1	0	0.5	0.2384(0)	0.0137(2)	Mn1	0	0.5	0.23856	0.0132(0)
Mn2	0.4135(8)	0.3476(8)	0.5	0.0145(8)	Mn2	0.41926	0.34441	0.5	0.0123(7)
O1	0	0	0.2893(5)	0.005	<i>2.11 GPa</i>				
O2	0.2406(9)	0.4459(2)	0	0.005	Dy	0.1414(5)	0.1798(3)	0	0.0073(8)
O3	0.1562(5)	0.4421(7)	0.5	0.005	Mn1	0	0.5	0.2345(8)	0.0104(3)
O4	0.3951(0)	0.2082(3)	0.2414(1)	0.005	Mn2	0.4149(4)	0.3412(2)	0.5	0.0114(2)
<i>0.85 GPa</i>					<i>3.06 GPa</i>				
Dy	0.1424(4)	0.1798(9)	0	0.0101(2)	Dy	0.1407(9)	0.1801(7)	0	0.0087(5)
Mn1	0	0.5	0.2387(7)	0.0140(1)	Mn1	0	0.5	0.2317(1)	0.0078(1)
Mn2	0.4169(5)	0.3452(4)	0.5	0.0155(2)	Mn2	0.4161(2)	0.3363(2)	0.5	0.00142(7)

## DFT OPTIMIZED GEOMETRIES

All the DFT optimized geometries can be found in the cif archives provided as supplementary independent files.

DyMn<sub>2</sub>O<sub>5</sub>: cif\_DyMn2O5\_Pm\_AFM.tar.gz

GdMn<sub>2</sub>O<sub>5</sub>: cif\_gdMn2O5\_Pm\_AFM.tar.gz

### MICROSCOPIC CONTENT OF EFFECTIVE MAGNETIC EXCHANGES : GENERAL DESCRIPTION

The effective exchange integrals between two magnetic sites bridged by a ligand can be expressed as the singlet-triplet energy difference when each magnetic site support one electron. Let us call  $d$  and  $d'$  the magnetic orbitals of the  $M$  and  $M'$  magnetic sites and  $p$  the ligand orbital bridging the interaction. The other electrons will be supposed to be in doubly occupied orbitals and ignored in the present representation.

The effective exchange integral can be calculated in a quasi-degenerate perturbative scheme in which the model space is restricted to the two main configurations of the two spin states, namely  $|p\bar{p}d\bar{d}'\rangle$  and  $|p\bar{p}d'd\rangle$ . In our notations the overlined orbitals support down spins and the unbarred ones up spins,  $d$  refers to the first magnetic atom while  $d'$  to the second. One thus have

$$J = J_d + J_{sd} + J_{sb} \quad (1)$$

where  $J_d = \langle d\bar{d}'|\hat{V}|d'\bar{d}\rangle$  is the first order term of the perturbation series, it corresponds to the direct Pauli exchange between the two magnetic orbitals

As well known from the 3<sup>rd</sup> Hund's rule this term is always ferromagnetic.

This term is small, but not always negligible

$$J_{sd} = -4 \frac{|\langle d|\hat{F}|d'\rangle|^2}{U}$$

is the second order term, it corresponds to the direct or Anderson's super-exchange and may also be found under the name of through-space super-exchange

When symmetry allowed, this term is always antiferromagnetic.

This term is negligible in bridged systems, due to the large M-M' bond length.

$$J_{sb} = -4 \frac{|\langle d|\hat{F}|p\rangle|^2 |\langle d'|\hat{F}|p\rangle|^2}{(U + \varepsilon_d - \varepsilon_p)^2 U} - 8 \frac{|\langle d|\hat{F}|p\rangle|^2 |\langle d'|\hat{F}|p\rangle|^2}{2(U + \varepsilon_d - \varepsilon_p)^3}$$

is the fourth order term, it corresponds to the through-bridge or through-ligands super-exchange

When symmetry allowed, this term is always antiferromagnetic and usually dominant.

In the above formula  $\hat{V}$  is the electron-electron repulsion operator and  $U$  its integral when two electrons are in the same magnetic orbital,  $\hat{F}$  is the Fock operator which diagonal part can be associated with the orbital energies ( $\varepsilon$ ) and the extradiagonal part to the hopping integrals. A graphical representation of the  $J_d$ ,  $J_{sd}$  and  $J_{sb}$  terms can be found in Figure 1a, Figure 1b and Figure 1c.

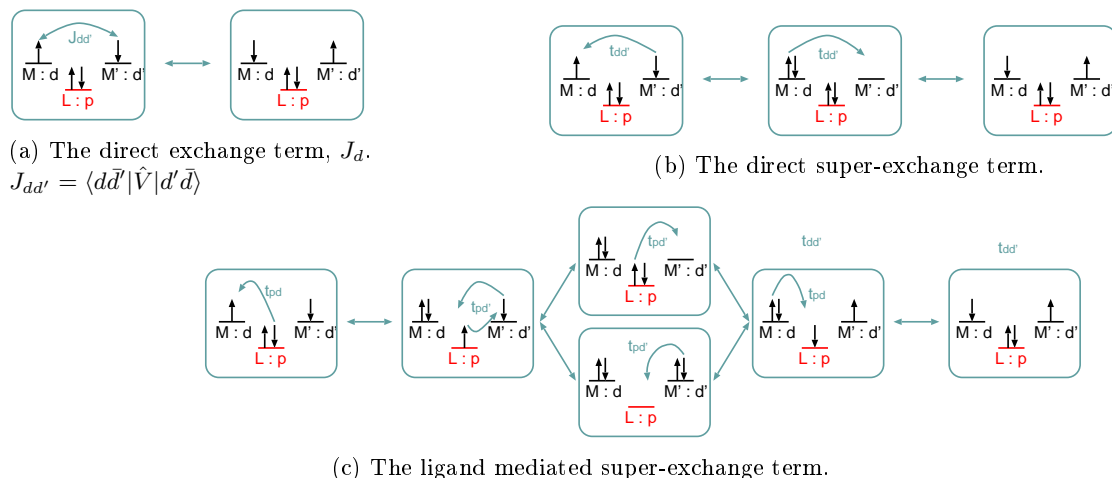


FIG. 1: Schematic representation of the different terms in an effective exchange integral between 2 sites, each supporting one magnetic electron.

When there are more than one electron and one magnetic orbital per magnetic site, the exchange integral is composed of all possible different terms of the above kinds. The latter should be summed up with appropriate coefficients (that depend on the number of magnetic electrons and orbitals on each magnetic atoms).

### THE $J_1$ AND $J_2$ EFFECTIVE MAGNETIC INTEGRALS IN THE $\text{RMn}_2\text{O}_5$ COMPOUNDS

The  $J_1$  and  $J_2$  magnetic integrals couple the  $\text{Mn}^{4+}$  ions along the  $\mathbf{c}$  direction. The through-bridge super-exchange terms are mediated by two O3 atoms for  $J_2$  and two O2 oxygens for  $J_1$ . The super-exchange paths are pictured in Figure 2.

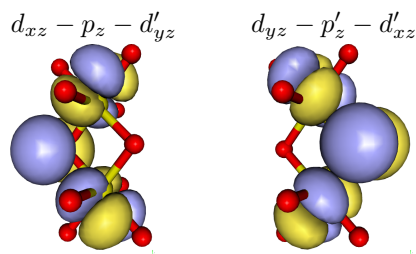


FIG. 2: Schematic representation of super-exchange path for the  $J_1$  and  $J_2$  magnetic integrals.

As the dependence of the  $\langle d | \hat{F} | p \rangle$  factors (in the  $J_{sb}$  super-exchange term) to the geometrical parameters is similar to the dependence of the overlaps between the magnetic orbital,  $d$ , and the ligands bridging-orbital,  $p$ , it is easy to see in Figure 2 that  $J_1$  and  $J_2$  do not depend on the  $\text{Mn}^{4+}\text{-O-Mn}^{4+}$  angle, but only on the  $\text{Mn}^{4+}\text{-O}$  distances ( $J_{sb}$ ) and to a lesser extent on the  $\text{Mn}^{4+}\text{-Mn}^{4+}$  distances ( $J_d$ ).

### THE $J_3$ EFFECTIVE MAGNETIC INTEGRALS IN THE $\text{RMn}_2\text{O}_5$ COMPOUNDS

The  $J_3$  magnetic integrals couple the  $\text{Mn}^{4+}$  and  $\text{Mn}^{3+}$  ions along a direction roughly  $\mathbf{b}$ . These integrals are associated to the inter-chain coupling. In a non-polar Pbam group their contribution to the magnetic energy (in and Heisenberg Hamiltonian scheme) is zero, while in the polar phase, the  $J_3$  integrals differentiate to lift the magnetic frustration, into a  $J_{3FM}$  and  $J_{3AFM}$  according to the FM/AFM ordering of the  $\text{Mn}^{4+}$  and  $\text{Mn}^{3+}$  ions. The through-bridge super-exchange terms are mediated by a O4 oxygen. The super-exchange paths are pictured in Figure 3.

One sees from Figure 3 that while the  $d'_{x'y'} - p_y - d_{yz}$  super-exchange path does not depend on the  $\text{Mn}_1\text{-O4-Mn}_2$  angle,  $\alpha_3$ , this is not the case for the two other ones. Indeed the  $d'_{x'z'} - p_x - d_{xz}$  super-exchange path varies as  $\cos^2 \alpha_3$  and the  $d'_{y'z'} - p_x - d_{xz}$  super-exchange path varies as  $\sin^2 \alpha_3$ . Of course all three terms depend on the Mn-O distances, however usually the angle dependence brings larger variations of the magnetic integrals than distance ones.

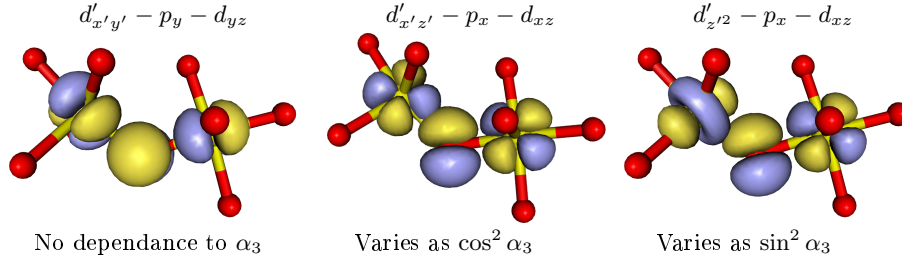


FIG. 3: Schematic representation of super-exchange paths for the  $J_3$  magnetic integrals. The  $z'$  axis for the square-pyramid is the apical one, the  $x'$  axis being the Mn-O axis toward the bridging oxygen O4. Similarly one can name  $z$  the axis between the  $\text{Mn}^{4+}$  ion in the octahedron and the O4 oxygen,  $y$  the axis perpendicular to the figures and  $x$  the second axis within the figure.

### THE $J_4$ EFFECTIVE MAGNETIC INTEGRALS IN THE $\text{RMn}_2\text{O}_5$ COMPOUNDS

The  $J_4$  magnetic integrals couple the  $\text{Mn}^{4+}$  and  $\text{Mn}^{3+}$  ions along the chain that run along roughly **a**. This is one of the two integrals associated with the intra-chain coupling, the second largest AFM exchange in the system. The through-bridge super-exchange terms are mediated by a O3 oxygen. The super-exchange paths are pictured in Figure 4.

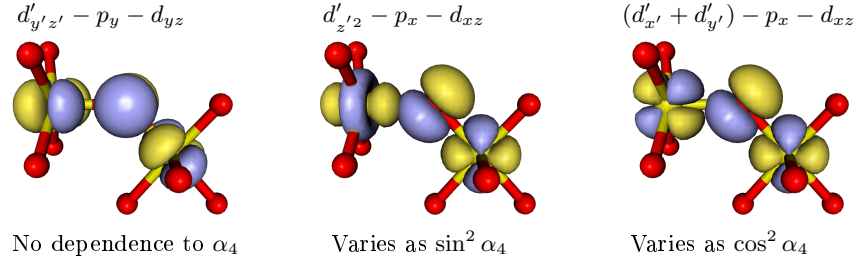


FIG. 4: Schematic representation of super-exchange paths for the  $J_4$  magnetic integrals. The  $z'$  axis for the square-pyramid is the apical one, the  $x'$  and  $y'$  axes being along the other Mn-O bonds. Similarly one can name  $z$  the axis between the  $\text{Mn}^{4+}$  ion in the octahedron and the O3 oxygen,  $y$  the axis perpendicular to the figures and  $x$  the second axis within the figure.

The analysis of Figure 4 shows that the  $d'_{y'z'} - p_y - d_{yz}$  super-exchange path does not depend on the  $\text{Mn}_1\text{-O3-Mn}_2$  angle,  $\alpha_4$ , while this is not the case for the two other ones. The  $d'_{z'2} - p_x - d_{xz}$  super-exchange path is expected to be the largest term as it involves  $\sigma$  type of overlap ( $d'_{z'2} - p_x$ ), while all the other super-exchange path are associated with  $\pi$  type of transfer integrals. It varies as  $\sin^2 \alpha_4$ . Finally the  $(d'_{x'} + d'_{y'}) - p_x - d_{xz}$  path varies as  $\cos^2 \alpha_4$ .

### THE $J_5$ EFFECTIVE MAGNETIC INTEGRALS IN THE $\text{RMn}_2\text{O}_5$ COMPOUNDS

The  $J_5$  magnetic integrals couple two  $\text{Mn}^{3+}$  ions roughly along the **b** direction. This is the other intra-chain exchange integral and the largest AFM one. The through-bridge super-exchange terms are mediated by two O1 oxygens. The super-exchange paths are pictured in Figure 5.



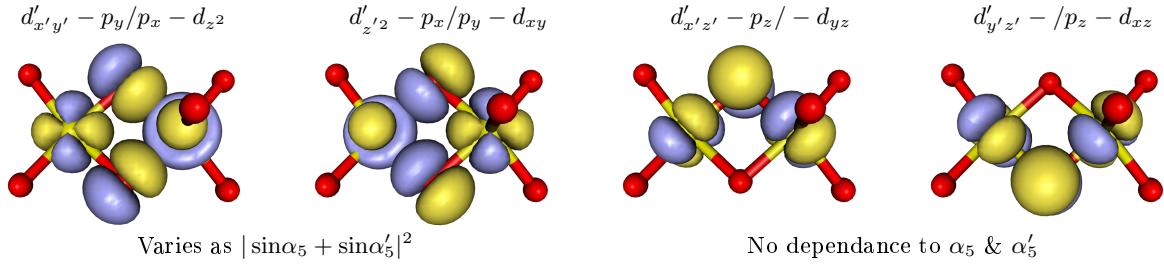


FIG. 5: Schematic representation of super-exchange paths for the  $J_5$  magnetic integrals. The  $z'$  and  $z$  axes of the two square-pyramid are chosen as the apical ones, that are roughly perpendicular to the figure. The  $x'$  and  $x$  axes are along the other Mn-O bonds roughly directed north-east in the figure, while the  $y'$  and  $y$  axes are along the other Mn-O bonds roughly directed south-east in the figure.  $\alpha_5$  will denote the  $\text{Mn}^{3+}\text{-O1-Mn}^{3+}$  angle with the top oxygen on the figure, while  $\alpha'_5$  is the  $\text{Mn}^{3+}\text{-O1-Mn}^{3+}$  angle with the bottom oxygen.

The  $J_5$  effective magnetic integrals has four through-bridge super-exchange terms. As a consequence  $J_5$  is the largest AFM magnetic coupling in the  $\text{RMn}_2\text{O}_5$  compounds. Out of these two are independent of the  $\text{Mn}^{3+}\text{-O1-Mn}^{3+}$  angles,  $\alpha_5$  and  $\alpha'_5$ , namely the  $d'_{x'z'} - p_z/ - d_{yz}$  and  $d'_{y'z'} - /p_z - d_{xz}$  paths (see Figure 5). The two other paths,  $d'_{x'y'} - p_y/p_x - d_{z^2}$  and  $d'_{z'^2} - p_x/p_y - d_{xy}$ , are maximum for  $90^\circ$  angles and vary as  $|\sin\alpha_5 + \sin\alpha'_5|^2$ .