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Supporting information for article:

**Unexpected Phase Transition Sequence in the Ferroelectric
Bi₄Ti₃O₁₂**

**Yuan-Yuan Guo, Alexandra S. Gibbs, J. Manuel Perez-Mato and Philip
Lightfoot**

Table S1 Crystallographic models derived from the $I4/mmm$ parent structure by action of each significant tilt mode or selected combinations.

Table S2 Constraints applied to the ‘simplified’ $B1a1$ model discussed in the text, such that all atoms except O1/O1’ confirm to the ideal $B2eb$ model.

Figure S1: Temperature dependence of β angle for the monoclinic phase below T_C .

Figure S2. Crystal structures of the $P4/mbm$ and $Cmce$ models for the intermediate phase at 685 °C, with atomic numbering for the $P4/mbm$ phase, and highlighting the differing X_2^+ modes and the M_1^+ modes for the $P4/mbm$ model.

Figure S3. Rietveld plots, $B1a1$ ‘constrained’ model at 20 °C

Figure S4. Rietveld plots, $B1a1$ ‘constrained’ model at 655 °C

Figure S5. Rietveld plots, $P4/mbm$ model at 685 °C

Figure S6. Rietveld plots, $I4/mmm$ model (+ $Bi_2Ti_2O_7$ impurity) at 1000 °C

Separate Files:

1. ISODISTORT output for refined $B1a1$ models (‘constrained’ and ‘independent’) at 20 °C (Isodistort_B1a1_RT_78v; Isodistort_B1a1_RT_101v) and refined $P4/mbm$ model (68 variables) at 685 °C (Isodistort_P4mbm_685C_68v).
2. CIF files for $B1a1$ (constrained) model at 20 °C (B1a1_RT_78v) and 655 °C (B1a1_655C_78v).
3. CIF files for comparative refined models at 685 °C (Table 1): $P4/mbm$, 68 variables (P4mbm_685C), $Cmce$ (Cmce_685C) $P4_2/ncm$ (P42ncm_685C) and $P4/nbm$, (P4nbm_685C).
4. CIF files for idealized models described in Table 4 (Cmce_X2+, P4mbm_X2+, P4mbm_X2+_M1+).
5. CIF file for $I4/mmm$ model at 1000 °C, with impurity phase included (I4mmm_1000C).

Table S1 Crystallographic models derived from the *I4/mmm* parent structure by action of each significant tilt mode or selected combinations.

Mode	OPD	Space group	basis	origin	s,i	k-active
\mathbf{X}_3^+	(a,a)	<i>P4₂/ncm</i>	(1,1,0) (-1,1,0) (0,0,1)	(0,0,0)	4,4	(1/2,1/2,0), (1/2,1/2,1)
\mathbf{X}_3^+	(o,a)	<i>Cmce</i>	(1,1,0) (0,0,1) (1,-1,0)	(0,0,0)	2,4	(1/2,1/2,1)
$^{\$}\mathbf{X}_3^+$	(a,b)	<i>Pccn</i>	(1,1,0) (-1,1,0) (0,0,1)	(0,0,0)	4,8	(1/2,1/2,0), (1/2,1/2,1)
\mathbf{X}_1^-	(a,a)	<i>P4/nbm</i>	(1,1,0) (-1,1,0) (0,0,1)	(0,1/2,0)	4,4	(1/2,1/2,0), (1/2,1/2,1)
\mathbf{X}_1^-	(0,a)	<i>Ccce</i>	(0,0,1) (1,1,0) (-1,1,0)	(0,1/2,0)	2,4	(1/2,1/2,1)
$^{\$}\mathbf{X}_1^-$	(a,b)	<i>Pban</i>	(1,1,0) (-1,1,0) (0,0,1)	(0,1/2,0)	4,8	(1/2,1/2,0), (1/2,1/2,1)
\mathbf{X}_2^+	(a,a)	<i>P4/mbm</i>	(1,1,0) (-1,1,0) (0,0,1)	(1/2,1/2,0)	4,4	(1/2,1/2,0), (1/2,1/2,1)
\mathbf{X}_2^+	(0,a)	<i>Cmce</i>	(0,0,1) (1,1,0) (-1,1,0)	(0,0,0)	2,4	(1/2,1/2,1)
$^{\$}\mathbf{X}_2^+$	(a,b)	<i>Pbam</i>	(1,1,0) (-1,1,0) (0,0,1)	(0,0,0)	4,8	(1/2,1/2,0), (1/2,1/2,1)
$^{\$\$}\mathbf{X}_2^+/\mathbf{X}_1^-$	(0;a b;0)	<i>Pcca</i>	(-1,1,0) (0,0,1) (1,1,0)	(1/4,1/4,1/4)	4,8	(1/2,1/2,0), (1/2,1/2,1)
$^{\$\$}\mathbf{X}_2^+/\mathbf{X}_3^+$	(0;a b;0)	<i>Pbca</i>	(-1,1,0) (0,0,1) (1,1,0)	(0,0,0)	4,8	(1/2,1/2,0), (1/2,1/2,1)

\$ These lower symmetry model were not considered.

\$\$ These models were used as a confirmatory check on the presence of only the X_2^+ mode at 685 °C (above T_C). For comparison these models produced χ^2 values of 6.17 and 6.30 (unstable); 76 and 75 variables, for *Pcca* and *Pbca* respectively. The best *P4/mbm* model gave $\chi^2 = 6.30$ for 72 variables.

Space groups are given in the standard setting, which may result in switching of the axes compared to the *B1a1* model.

OPD is the Order parameter direction. The parameters (s,i) represent, respectively, the *size* of the primitive unit cell of the subgroup relative to the parent space group and the *index* of the subgroup relative to the parent space group. For further details of these and other notations see the ISOTROPY website: stokes.byu.edu/iso/isodistorthelp.php

Table S2 Constraints applied to the ‘simplified’ *B1a1* model discussed in the text, such that all atoms except O1/O1’ confirm to the ideal *B2eb* model.

Atom 1	Parameter	Atom 2	Parameter
Bi1	x	Bi1a	x
Bi1	y	Bi1a	-y
Bi1	z	Bi1a	-z
Bi2	x	Bi2a	x
Bi2	y	Bi2a	-y
Bi2	z	Bi2a	-z
Ti2	x	Ti2a	x
Ti2	y	Ti2a	-y
Ti2	z	Ti2a	-z
O1	z	O1a	-z
O2	x	O2a	x
O2	y	O2a	-y
O2	z	O2a	-z
O3	x	O3a	x
O3	y	O3a	-y

O3	z	O3a	-z
O4	x	O4a	x
O4	y	O4a	-y
O4	z	O4a	-z
O5	x	O5a	x
O5	y	O5a	-y
O5	z	O5a	-z
O6	x	O6a	x
O6	y	O6a	-y
O6	z	O6a	-z

Note that the Ti2 y and z coordinates were also constrained to lie on the pseudo 2-fold axis.

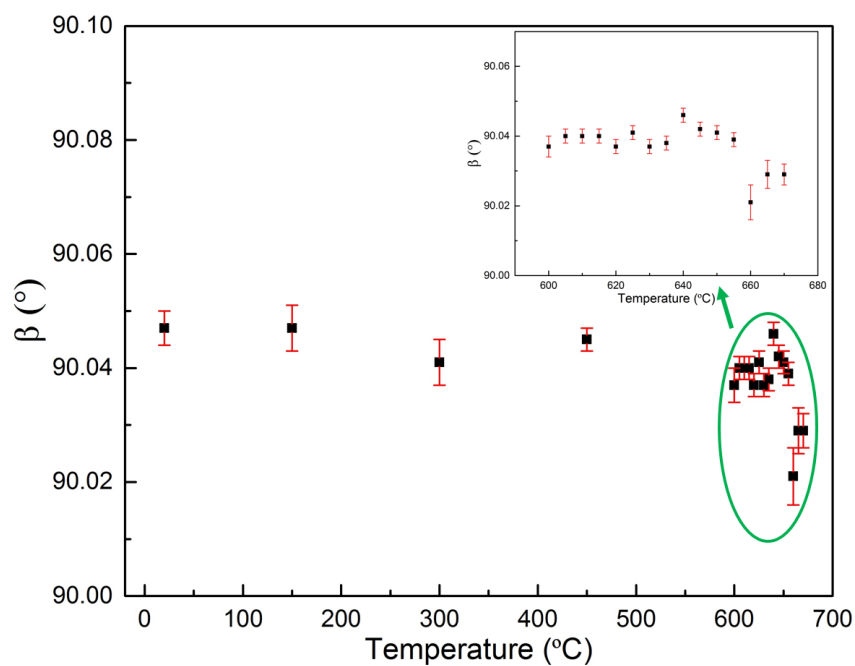


Figure S1: Temperature dependence of β angle for the monoclinic phase below T_C .

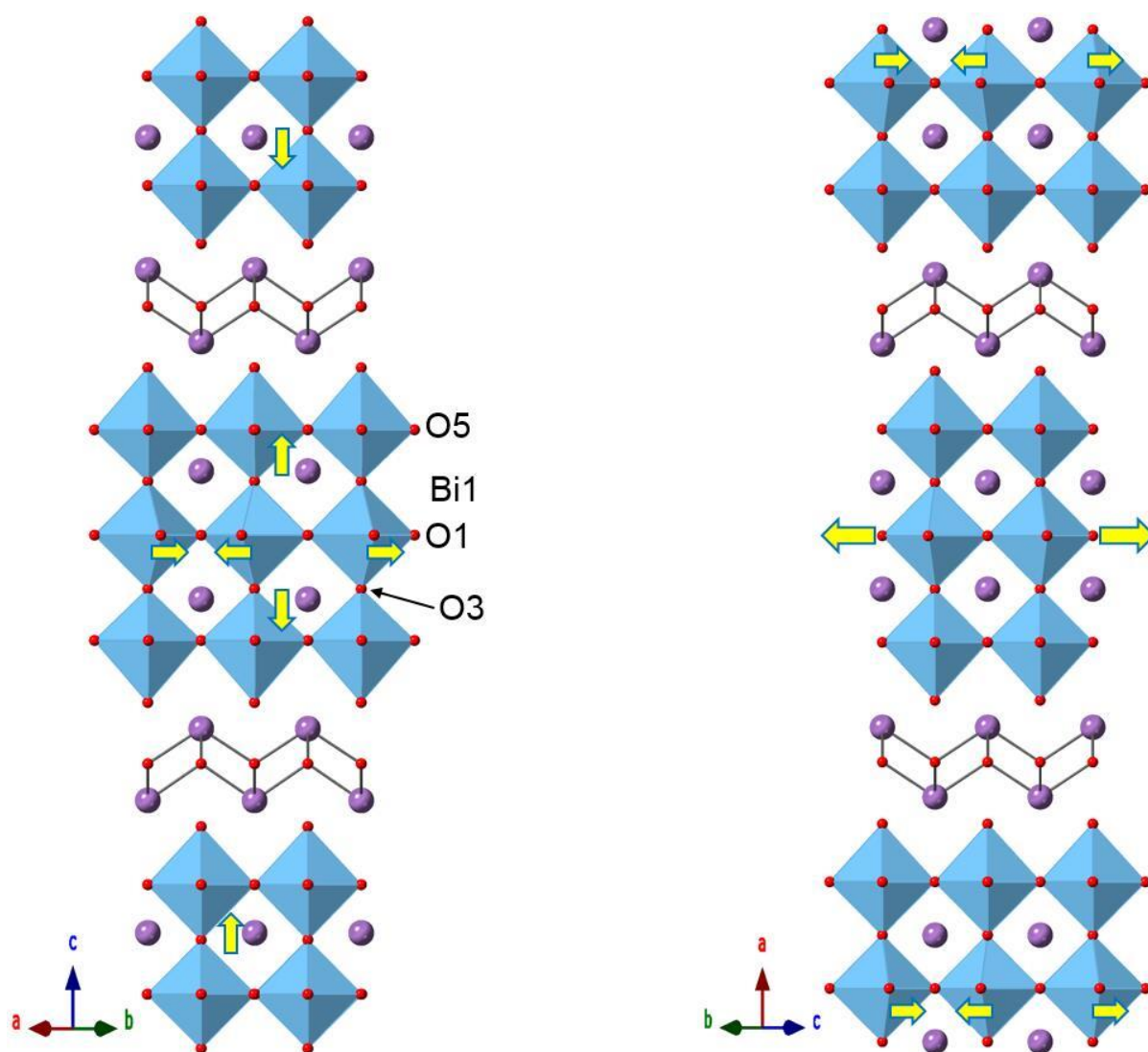


Figure S2: Crystal structures of the $P4/mbm$ (left) and $Cmce$ (right) models for the intermediate phase at 685 °C, with atomic numbering for the $P4/mbm$ phase, and highlighting the differing X_2^+ modes and the M_1^+ modes for the $P4/mbm$ model.

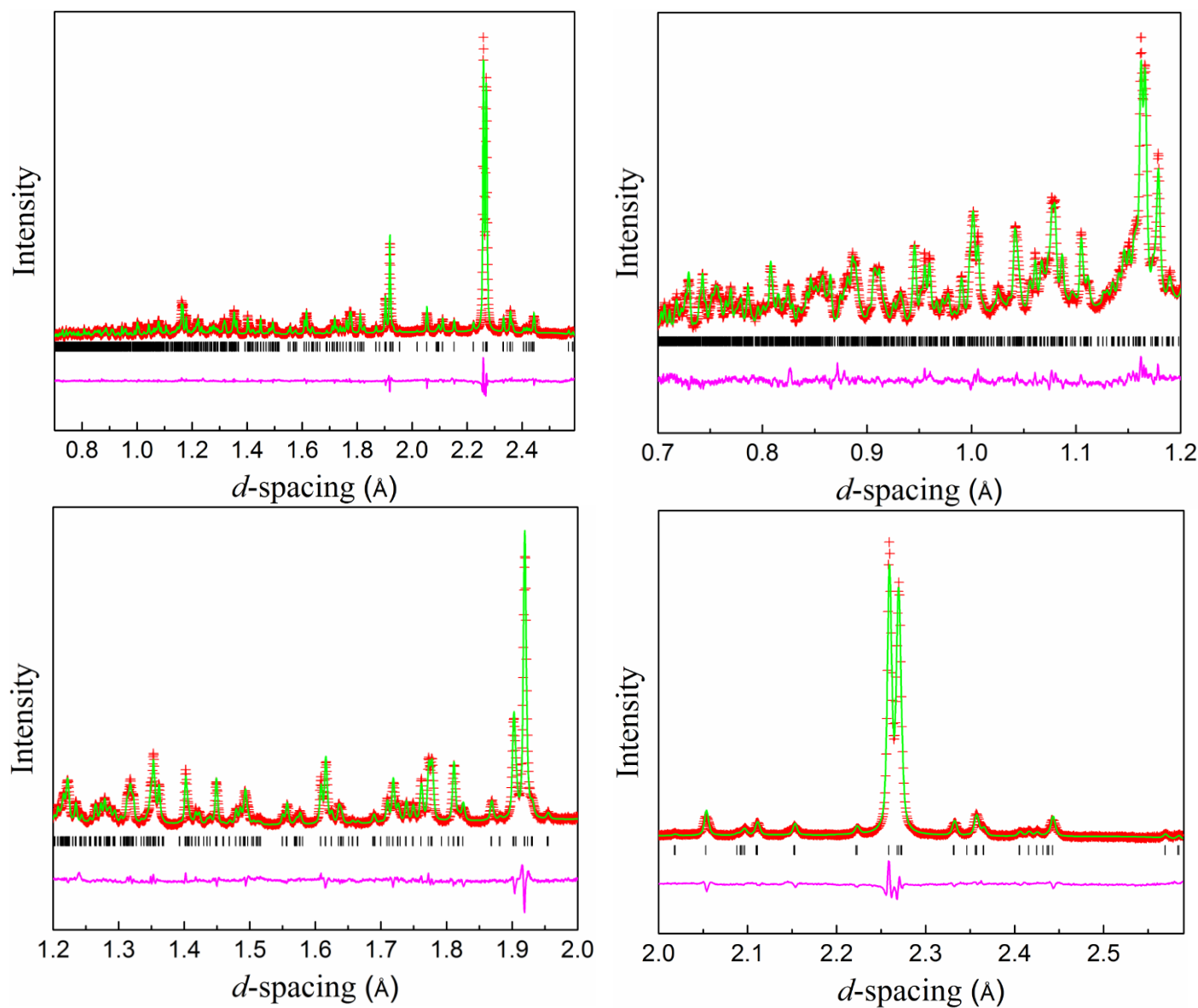


Figure S3. Full-range (top left) and expanded Rietveld plots (Bank 1), $B1a1$ 'constrained' model at 20 °C.

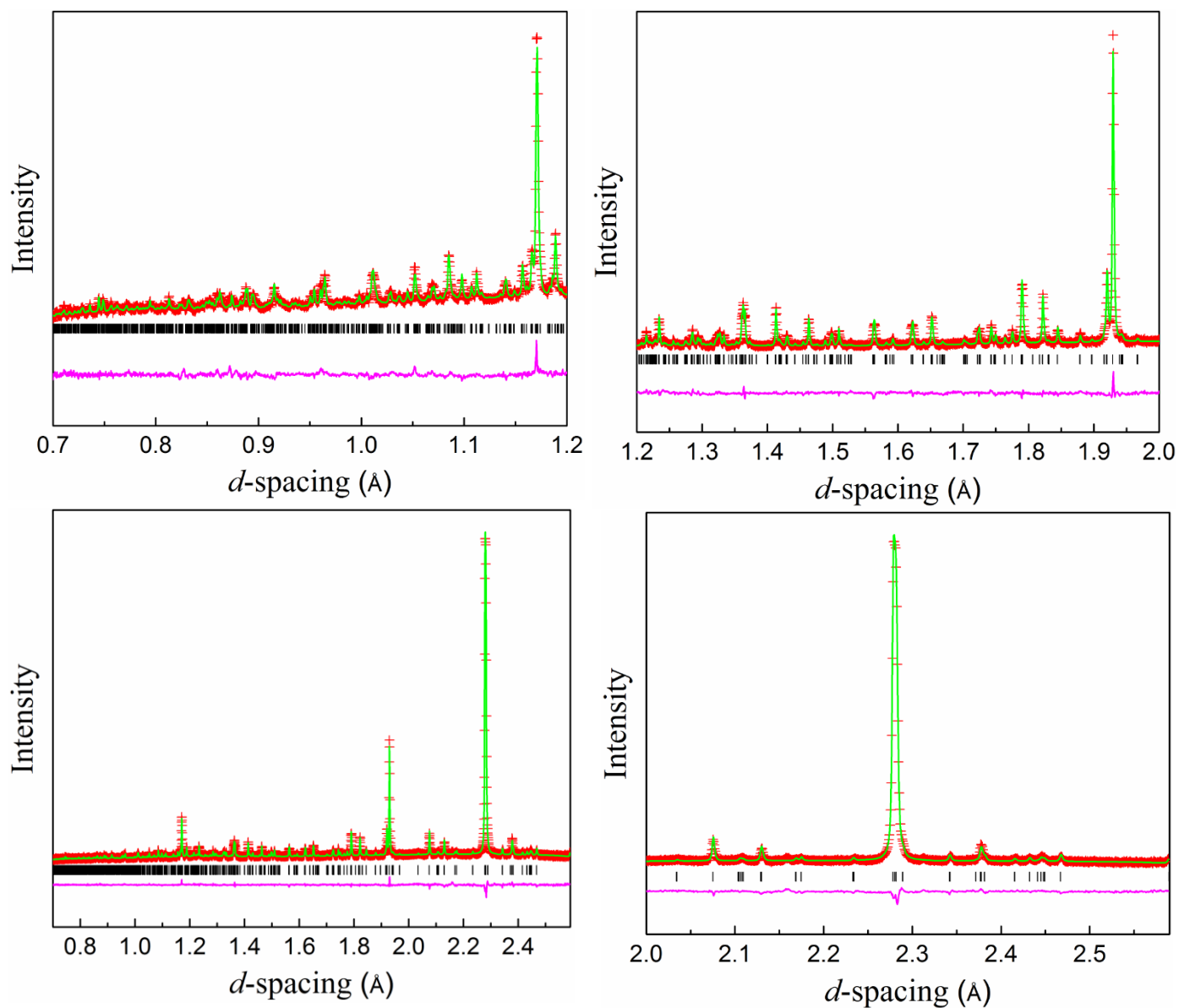


Figure S4. Full-range (top left) and expanded Rietveld plots (Bank 1), *B1a1* ‘constrained’ model at 655 °C.

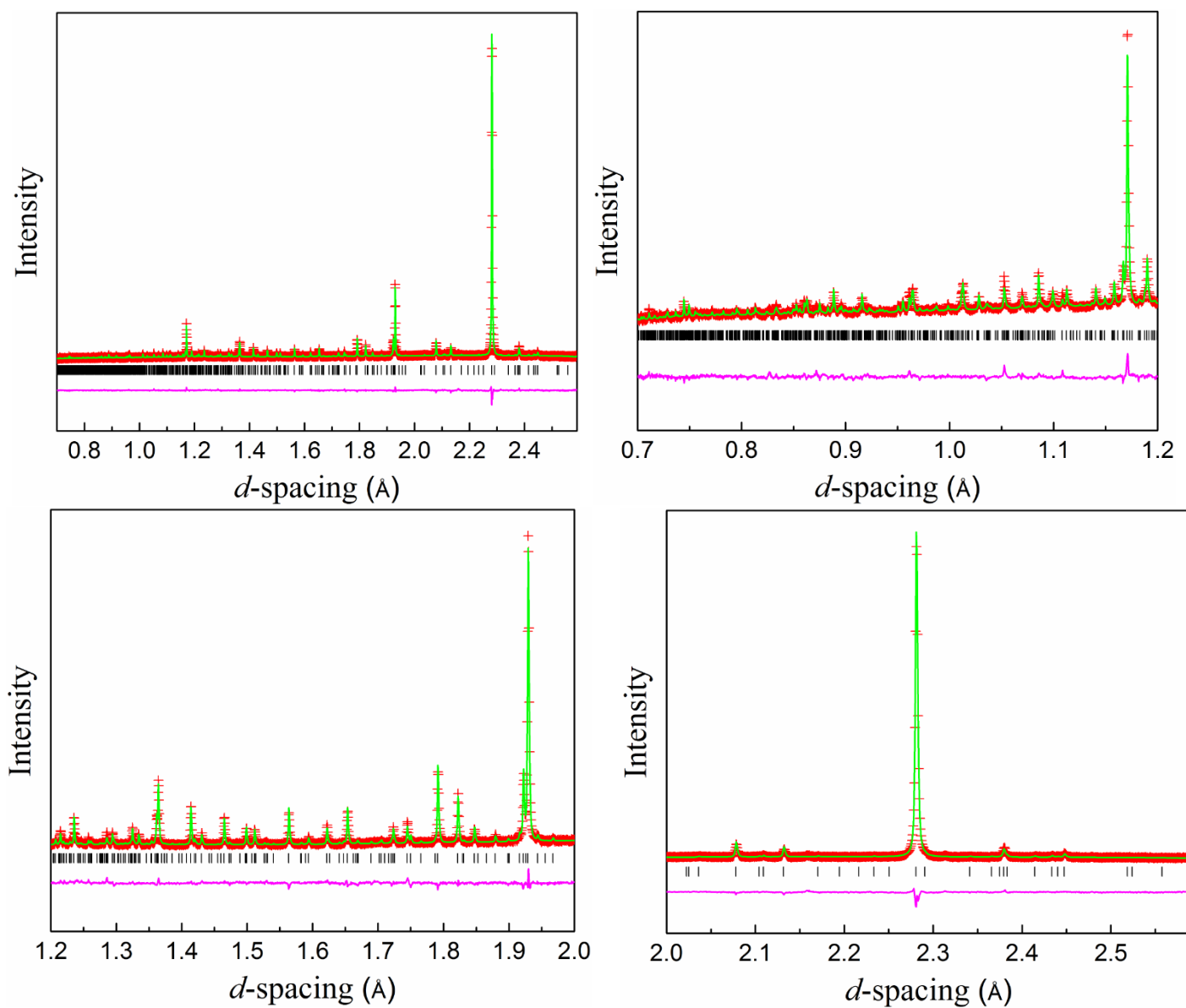


Figure S5. Full-range (top left) and expanded Rietveld plots (Bank 1), $P4/mbm$ model at 685 °C.

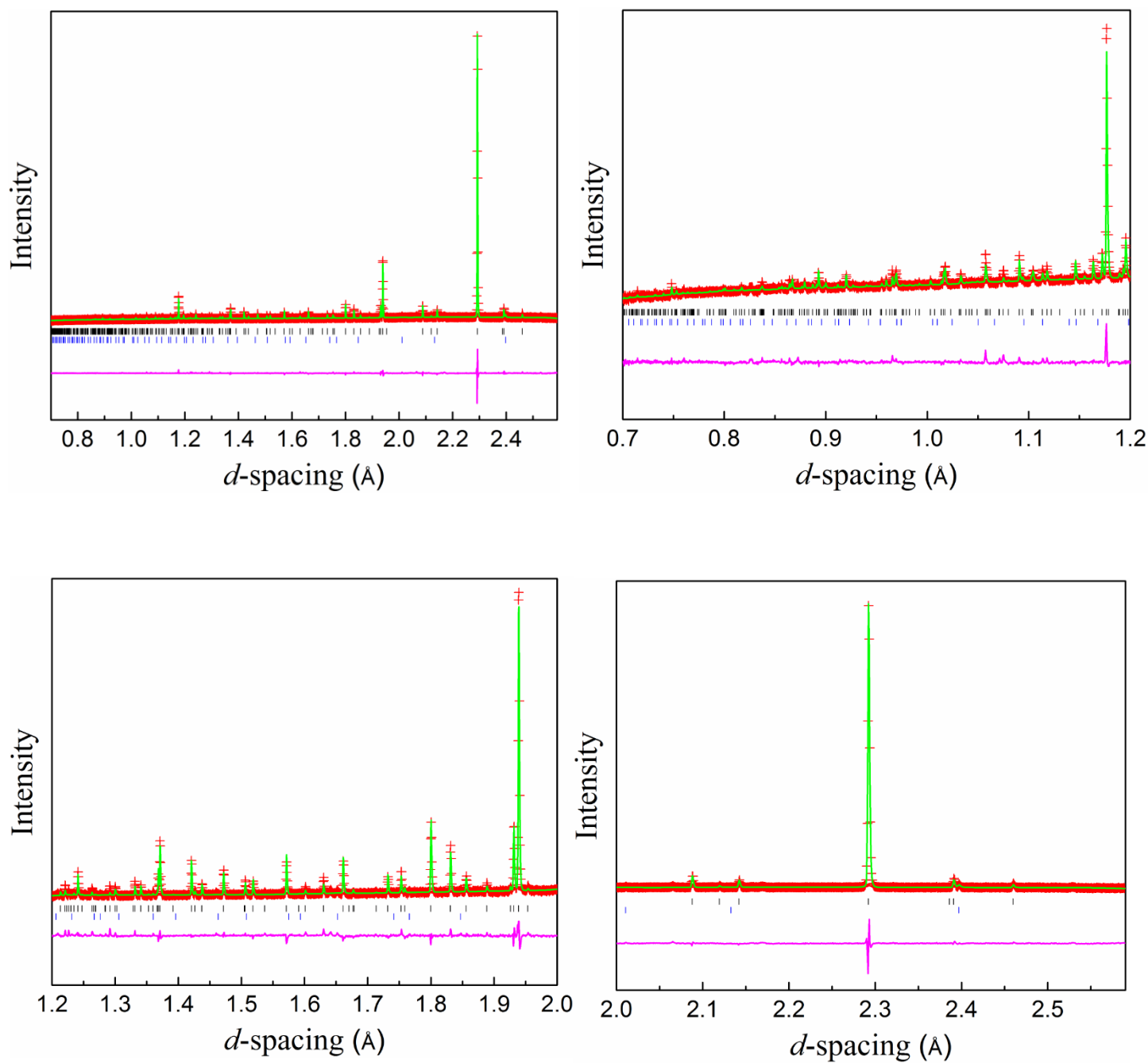


Figure S6. Full-range (top left) and expanded Rietveld plots (Bank 1), $I4/mmm$ model (+ $\text{Bi}_2\text{Ti}_2\text{O}_7$ impurity) at 1000 °C.