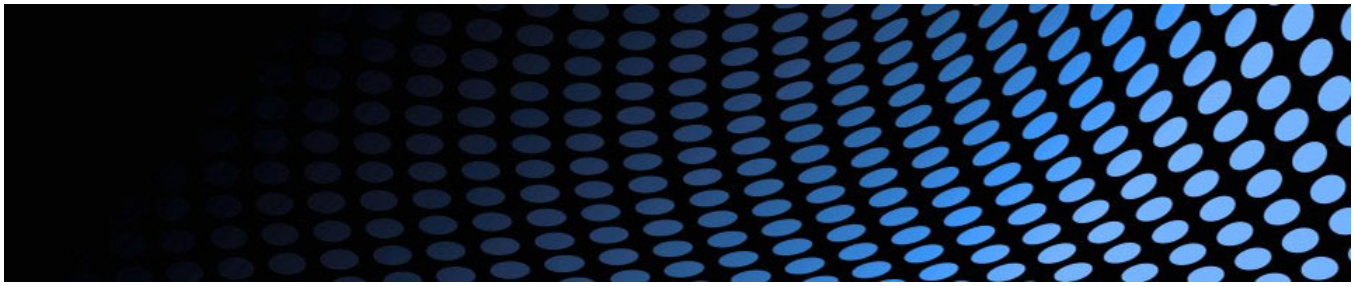

3D Visualization of neo-Eulerian and other rotation representations

By Marc De Graef

3D visualization of rotations and crystallographic fundamental zones

3D rotations are typically represented in terms of 3x3 rotation matrices that are based on an Euler angle triplet; the set of rotation matrices is then represented by SO(3). While Euler angles form the basis for the study of material textures, they suffer from well known issues, such as the lack of a unique representation of the identity rotation and gimbal lock. Visual representations of texture orientation distribution functions typically involve contour plots on multiple planar sections through Euler space, and it takes a bit of experience to be able to correctly interpret those; in addition, discovering crystal symmetry in Euler space plots is also not trivial. In this research project, we start from a number of alternative rotation/orientation representations, including the neo-Eulerian representation and the recently introduced cubochoric representation and combine them with modern 3D visualization tools to generate alternative ways to visualize orientation distribution functions.

In the tables below, links are provided to mp4 animations of the fundamental zones (FZs) for four different orientation representations: Rodrigues, homochoric, cubochoric, and stereographic. Two animations are available for each representation and each rotational point group symmetry; one is a solid (S) representation in which the FZ is filled with small solid spheres, the other is a wireframe (W) representation of the outline of the FZ. In the Rodrigues case, the space is infinite, so for the cyclic rotation groups the FZ stretches to infinity; for all other representations the FZ has a finite volume. It should be noted that all representations use the same length scale, so that the FZs in each representation have the correct relative size.

Solid (S) and wireframe (W) mp4 movies of the Fundamental zones of the 10 rotational point groups in four different orientation representations.

Representation	2	3	4	6	222	32	422	622	23	432
Rodrigues	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W

Homochoric	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W
Cubochoric	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W
Stereographic	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W

Solid (S) and wireframe (W) mp4 blue-red anaglyph movies of the Fundamental zones of the 10 rotational point groups in four different orientation representations.

Representation	**2**	**3**	**4**	**6**	**222**	**32**	**422**	**622**	**23**	**432**
Rodrigues	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W
Homochoric	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W
Cubochoric	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W
Stereographic	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W	S,W

The movies in the tables above all have "empty" fundamental zones. In the next table, we provide a few links to 3D stereographic projections of different sets of rotations around a given axis for both the cubic (octahedral **432**) and hexagonal (dihedral **622**) fundamental zones. Each orientation is represented by a colored sphere; spheres with identical colors are related to each other by the symmetry operations of the rotational point group. Note that the equivalent FZs have also been drawn. Each movie is available in standard and anaglyph versions.

Standard (S) and stereoscopic blue-red anaglyph (A) mp4 movies of equivalent rotations around the [100], [110], [111], and [123] axes for the octahedral and hexagonal symmetry groups (3D stereographic projections).

Fundamental Zone	**[100]**	**[110]**	**[111]**	**[123]**
Cubic 432	A,S	A,S	A,S	A,S
Hexagonal 622	A,S	A,S	A,S	A,S

As a second ilustration, consider all rotations with rotation axes in the *x*-*y* plane and a rotation angle of 45°; they are represented by quaternions of the type *[cos(pi/8), sin(pi/8)cos(theta), sin(pi/8)sin(theta), 0]*, where theta is the angle of the rotation axis with respect to the *x*-axis. In the stereographic projection, which is an equal-angle projection, those rotations are represented by a circle of 180 spheres (theta is incremented from 0° to 358° in steps of 2°). A stereographic projection of all the orientations that are equivalent to the initial set is available here in standard rendering, and here in red-blue anaglyph mode. Superposition of the cubic fundamental zone outlines results in the following standard and anaglyph representations.

Finally, we make a connection between the traditional Euler space representation of textures and the 3D visualizations introduced above. The links in the table below connect to movies that show how the Rodrigues fundamental zones are mapped

onto the primary Euler space. Note that all of the RFZs lie along a diagonal line through Euler space, and that the vertical (along Phi) surfaces are planar. The top surface is either planar (for the cyclic groups) or "tented" for the other groups.

Standard (S) and stereoscopic blue-red anaglyph (A) mp4 movies of the ten rotational point group Rodrigues FZs in the primary Euler space.

Fundamental Zone	**2**	**3**	**4**	**6**	**222**	**32**	**422**	**622**	**23**	**432**
Movie links	S,A	S,A	S,A	S,A	S,A	S,A	S,A	S,A	S,A	S,A