

Supplementary material

(to '*Ind_X*: program for indexing single crystal diffraction patterns' by A. Morawiec)

Tables below show high quality solutions (parts of output files) obtained by running *Ind_X* for data of Duisenberg (1992). In response to a recommendation by an anonymous reviewer, also results for unpublished "examples 6 and 7" are listed. In all cases, the limit on the allowed deviation of Miller indices from integers was the *Ind_X* default of 0.12; reflections within this limit are denoted by **-h** and reflections within half of the limit are denoted by **H-**. The unit of length is Ångström, and angles are in degrees.

Example 1:

Solution nr 2 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

-0.41624538	0.04542523	5.05672819
-2.24425594	-7.88469324	-0.11196100
-11.93485938	1.08178703	-0.99071084

		h	k	l		Real (hkl)				Error
1	H-	2	0	-8		2.01	0.00	-8.03		0.032
2	H-	2	-5	-7		2.00	-5.00	-6.96		0.037
3						1.97	-0.01	-9.20		0.205
4										
5	H-	1	-1	-7		0.99	-1.00	-6.94		0.058
6										
7	H-	1	0	-7		1.00	0.00	-7.03		0.032
8	H-	1	-2	-7		1.00	-2.00	-7.00		0.004
9	-h	2	-3	3		1.93	-3.00	2.93		0.093
10	H-	3	-4	0		3.01	-4.00	0.04		0.039
11	H-	1	-4	1		1.01	-4.00	0.95		0.055
12	H-	1	-4	0		0.99	-4.00	0.04		0.044
13	-h	1	-4	0		1.01	-4.00	-0.07		0.074
14	H-	2	-1	4		2.01	-1.00	4.02		0.026
15						1.93	0.00	4.22		0.232
16	H-	2	-3	-2		2.01	-3.00	-1.98		0.025
17	H-	3	1	6		3.02	1.00	5.99		0.021
18	-h	2	0	8		2.01	0.01	8.07		0.073
19										
20	H-	1	1	6		1.00	1.00	6.00		0.004
21	H-	1	4	-1		0.98	4.01	-0.99		0.026
22						1.97	1.99	2.22		0.226
23	H-	2	1	1		1.99	0.99	0.94		0.061
24	H-	3	2	1		3.00	2.00	0.99		0.011
25						3.04	1.99	-2.16		0.166

18h and 15H vectors out of 25. Quality : 0.684

Primitive cell :

5.074	8.199	12.025
79.26	89.99	89.99

Volume of the cell : 491.47

Example 2:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

22.39827411	23.67373447	19.86433441
-55.72013365	53.74568735	2.85453230
27.36701652	32.99645419	-66.67913680

	h	k	l	Real (hkl)			Error	
1	-h	0	4	7	-0.02	3.92	6.89	0.134
2	H-	3	15	11	3.01	15.01	10.99	0.017
3	H-	4	24	14	4.01	24.03	14.01	0.031
4	-h	1	6	7	0.99	5.94	6.91	0.105
5	H-	6	15	0	6.00	14.95	-0.04	0.059
6	-h	5	22	10	4.99	22.05	10.09	0.100
7	H-	4	27	15	4.01	27.00	14.97	0.037
8	-h	7	26	5	7.00	26.05	5.07	0.090
9	-h	9	27	-6	9.00	26.91	-6.10	0.131
10	-h	6	29	10	6.01	29.11	10.03	0.116
11	-h	5	24	11	4.99	24.03	11.08	0.090
12	-h	1	26	21	0.98	25.90	20.92	0.126
13	-h	6	13	-3	6.01	12.98	-3.08	0.079
14	H-	4	15	8	4.00	15.02	8.03	0.030
15	-h	3	5	-2	3.00	4.98	-2.06	0.066
16	H-	4	7	-3	4.00	6.99	-3.05	0.054
17	H-	4	8	-1	4.00	8.00	-1.02	0.020
18	H-	7	23	3	7.00	22.98	3.03	0.040
19	H-	6	10	-11	5.98	9.96	-11.02	0.051
20	H-	5	13	2	5.00	12.98	1.99	0.021
21	H-	6	20	5	6.00	20.00	5.05	0.049
22	-h	2	26	19	2.00	25.93	18.93	0.101
23	H-	8	28	1	8.01	28.01	1.02	0.023
24	-h	8	30	2	8.00	30.09	2.08	0.118
25	H-	3	13	9	3.00	13.00	9.01	0.008
26	H-	7	16	-5	7.00	15.95	-5.05	0.074

26h and 14H vectors out of 26. Quality : 0.862

Primitive cell :

38.167	77.469	79.271
89.46	88.68	88.43

Volume of the cell : 234225.58

Example 3, solution a:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

```
-0.08757438  2.89242892  -3.33163710
18.02074562  -1.42129210  1.17612284
-3.89689517 -16.76216719 -11.52746836
```

		h	k	l		Real (hkl)		Error
1	H-	1	3	-7		1.00 3.02 -6.99		0.023
2	H-	1	-3	-4		1.00 -2.99 -4.00		0.008
3	H-	0	5	-5		0.00 5.00 -5.00		0.004
4	H-	1	-3	-8		1.00 -3.00 -8.00		0.007
5	H-	0	0	-8		0.00 0.00 -8.00		0.003
6	H-	0	-5	-7		0.00 -5.00 -7.00		0.003
7	H-	0	-5	-3		0.00 -5.00 -3.00		0.005
8	H-	0	0	-12		0.00 0.00 -11.99		0.006
9	H-	-1	4	-6		-1.00 4.02 -6.00		0.016
10	H-	-1	-6	-8		-1.00 -5.99 -8.00		0.012
11	H-	-1	-7	-6		-1.00 -7.01 -6.00		0.012
12	H-	0	0	-12		0.00 -0.01 -12.01		0.016
13	H-	0	5	-13		0.00 5.00 -13.00		0.007
14	H-	1	2	-13		1.00 2.00 -13.00		0.003
15	H-	1	0	-13		1.00 -0.04 -13.01		0.036
16	H-	1	-3	-12		1.00 -3.00 -11.99		0.005
17	H-	1	1	-11		1.00 0.98 -11.01		0.019
18	H-	-1	3	-8		-1.00 3.00 -8.00		0.007
19	H-	-1	4	-10		-1.00 4.01 -9.99		0.014
20	H-	0	10	-10		0.00 10.00 -10.00		0.003
21	H-	1	-7	-9		1.00 -6.98 -9.00		0.023
22	H-	1	3	-11		1.00 3.02 -11.00		0.025
23	H-	1	-3	-8		1.00 -3.00 -8.00		0.003
24	H-	1	2	-9		1.00 2.01 -9.00		0.006
25	H-	1	1	-7		1.00 0.98 -7.00		0.016

25h and 25H vectors out of 25. Quality : 1.000

Primitive cell :

```
4.413  18.115  20.713
99.20  96.11  96.90
```

Volume of the cell : 1609.27

Example 4:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

```
1.20056709  1.71918935 -10.94962184
-0.92071771 -22.25549176 -3.59537022
97.65751543 -5.77590232  9.77179118
```

		h	k	l		Real (hkl)		Error
1	H-	0	-3	-21		0.00 -3.02 -21.06		0.059
2	H-	0	-3	-24		0.00 -3.01 -23.99		0.011
3						0.00 -3.04 -22.14		0.143
4	H-	0	-3	-23		0.00 -3.00 -22.98		0.023
5	H-	0	-5	-16		0.00 -5.00 -15.99		0.009
6	H-	0	-4	-26		0.00 -4.00 -25.95		0.050
7	H-	0	-1	-29		0.00 -1.01 -28.97		0.031
8	-h	0	-1	-31		-0.01 -0.89 -31.03		0.112
9	H-	0	0	-22		0.00 -0.01 -21.97		0.035
10	H-	-1	-4	18		-1.00 -3.99 17.98		0.022
11	H-	-1	-4	17		-1.00 -3.99 16.99		0.014
12	H-	-1	-5	11		-1.00 -4.99 11.00		0.007
13	H-	-1	-5	10		-1.00 -5.01 10.02		0.025
14	H-	-2	1	-19		-2.00 1.00 -18.97		0.025
15	H-	-2	0	-21		-2.00 0.00 -20.99		0.010
16	H-	-2	1	-23		-2.00 1.00 -22.96		0.037
17	H-	-2	0	-23		-2.00 0.00 -22.98		0.020
18	H-	-1	-3	27		-1.00 -2.99 27.00		0.015
19	H-	2	-2	19		2.00 -2.00 19.00		0.004
20	H-	2	-3	16		2.00 -3.00 16.01		0.011
21	H-	2	-4	11		2.00 -4.00 11.05		0.047
22	H-	2	-3	17		2.00 -3.00 17.01		0.013
23	H-	2	-4	9		2.00 -4.00 9.01		0.010
24	H-	2	-5	0		2.00 -5.01 0.01		0.014
25	H-	2	-4	15		2.00 -3.99 15.00		0.007

24h and 23H vectors out of 25. Quality : 0.948

Primitive cell :

```
11.149  22.563  98.315
89.91   89.98   90.00
```

Volume of the cell : 24730.50

The diffraction data of Example 3 were known to originate from an incommensurate structure. According to (Duisenberg 1992), a "supercell" fitting all reflections has the "volume $3942.00 = 49 \times 80.44$ ". The quite accurate *Ind_X* solution (*a*) shown above gives a smaller volume ($1609.27 \approx 20 \times 80.46$). If 100 is the upper limit on the cell volume, *Ind_X* gives the solution (*b*) with the volume of 80.43; see below. The relationship between the two solutions is $\mathbf{M}_a \approx \mathbf{T} \times \mathbf{M}_b$, where \mathbf{M}_a and \mathbf{M}_b are the matrices of direct lattice vectors (in rows) for each of the solutions, and

$$\mathbf{T} = \begin{bmatrix} 1 & -1 & 0 \\ -3 & -2 & 0 \\ 0 & 1 & 4 \end{bmatrix} ;$$

Miller indices of the solution (*a*) can be obtained by multiplying \mathbf{T} and the column of indices of the solution (*b*). The smallest-volume solution accepting all reflections with the default *Ind_X* parameters is listed below as (*c*).

Example 3, solution *b*:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

-3.63857749	1.44382309	-1.57287407
-3.55149346	-1.44846389	1.75859322
-0.08637420	-3.83006631	-3.31877056

		h	k	l		Real (hkl)		Error
1								
2	H-	1	0	-1		1.00 0.00 -1.00		0.001
3	H-	-1	-1	-1		-1.00 -1.00 -1.00		0.001
4	H-	1	0	-2		1.00 0.00 -2.00		0.001
5	H-	0	0	-2		0.00 0.00 -2.00		0.001
6	H-	1	1	-2		1.00 1.00 -2.00		0.001
7	H-	1	1	-1		1.00 1.00 -1.00		0.002
8	H-	0	0	-3		0.00 0.00 -3.00		0.002
9								
10								
11	H-	1	2	-2		1.00 2.00 -2.00		0.001
12	H-	0	0	-3		0.00 0.00 -3.00		0.003
13	H-	-1	-1	-3		-1.00 -1.00 -3.00		0.001
14	H-	0	-1	-3		0.00 -1.00 -3.00		0.000
15								
16	H-	1	0	-3		1.00 0.00 -3.00		0.001
17								
18	H-	-1	0	-2		-1.00 0.00 -2.00		0.001
19								
20	H-	-2	-2	-2		-2.00 -2.00 -2.00		0.001
21								
22								
23	H-	1	0	-2		1.00 0.00 -2.00		0.001
24	H-	0	-1	-2		0.00 -1.00 -2.00		0.000
25								

16h and 16H vectors out of 25. Quality : 0.640

Primitive cell :

4.219	4.219	5.069
89.95	89.99	63.06

Volume of the cell : 80.43

Example 3, solution c:

Solution nr 9 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

0.08764924	-2.89243836	3.33158017
-7.09574696	3.84139431	3.45898860
10.99137244	12.92062820	8.06968392

	h	k	l	Real (hkl)			Error
1 H-	-1	0	7	-1.00	0.05	6.94	0.076
2 H-	-1	2	2	-1.00	2.00	2.00	0.007
3 H-	0	-1	6	0.00	-1.00	6.00	0.003
4 H-	-1	3	5	-1.00	3.00	4.99	0.005
5 H-	0	2	6	0.00	1.99	6.01	0.010
6 H-	0	4	3	0.00	3.99	3.01	0.016
7 H-	0	3	0	0.00	2.99	0.01	0.010
8 H-	0	3	9	0.00	2.99	9.01	0.015
9 H-	1	0	6	1.00	0.02	5.97	0.037
10 H-	1	5	3	1.00	5.02	2.98	0.025
11 H-	1	5	1	1.00	4.98	1.02	0.032
12 H-	0	3	9	0.00	2.99	9.02	0.019
13 H-	0	1	12	0.00	0.99	12.01	0.013
14 H-	-1	2	11	-1.00	2.00	11.00	0.003
15 -h	-1	3	10	-1.00	2.92	10.09	0.121
16 H-	-1	4	8	-1.00	4.00	8.00	0.003
17 H-	-1	2	9	-1.00	1.96	9.04	0.057
18 H-	1	1	7	1.00	0.98	7.02	0.028
19 H-	1	1	9	1.00	1.02	8.97	0.036
20 H-	0	-2	12	0.00	-2.00	12.00	0.003
21 H-	-1	5	4	-1.00	5.04	3.96	0.058
22 H-	-1	1	10	-1.00	1.04	9.96	0.062
23 H-	-1	3	5	-1.00	3.00	5.00	0.004
24 H-	-1	1	8	-1.00	1.00	7.99	0.008
25 H-	-1	1	6	-1.00	0.97	6.04	0.049

25h and 24H vectors out of 25. Quality : 0.988

Primitive cell :

4.413	8.779	18.785
90.15	96.60	90.31

Volume of the cell : 722.90

Examples 6 and 7 are described at <http://www.crystal.chem.uu.nl/distr/dirax/examples.html> as "inaccurate data" and "not very accurate data", respectively. *DirAx* solutions to these examples are available at the same web page. The first solution given by *Ind_X* to example 6 is similar to that given by *DirAx*; in both cases, one reflection is not indexed. Example 7 is fully indexable (solution no 1). However, solutions 2 and 3 of *Ind_X* indicate that the data originate from a twinned crystal. Only solutions similar to 1 and 2 are listed at <http://www.crystal.chem.uu.nl/distr/dirax/examples.html> and, in consequence, the possibility of twinning is overlooked.

Example 6:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

-0.22772791	0.41128973	3.87525272
-0.28186242	-5.94601285	1.00932446
16.69620088	-2.75388915	1.71750269

	h	k	l	Real (hkl)			Error
1 H-	2	0	10	2.00	0.00	10.00	0.007
2 H-	2	2	9	2.00	2.00	9.01	0.011
3 H-	1	-1	13	1.00	-1.01	13.00	0.007
4 H-	0	2	6	-0.03	2.00	5.98	0.033
5 H-	0	3	4	-0.03	3.00	3.96	0.049
6 H-	0	1	9	-0.02	1.00	9.01	0.022
7 H-	1	1	5	0.99	1.00	4.98	0.021
8 H-	1	0	5	0.99	0.01	5.00	0.016
9 H-	2	-2	-2	2.02	-2.00	-2.02	0.028
10							
11 H-	1	-2	2	0.99	-2.00	2.01	0.012
12 H-	0	-2	-6	-0.02	-2.00	-6.00	0.022
13 H-	0	-1	-3	-0.02	-1.01	-2.99	0.026
14 H-	1	-1	-1	1.01	-1.00	-1.01	0.013
15 H-	0	-3	5	-0.01	-3.00	5.02	0.022
16 H-	0	-3	-4	-0.02	-3.00	-3.99	0.020
17 H-	2	1	-7	2.00	1.00	-7.00	0.005
18 H-	1	4	-1	0.98	4.00	-0.99	0.019
19 H-	2	3	-2	2.00	3.00	-1.99	0.013
20 H-	1	5	4	0.98	5.00	4.05	0.052
21 -h	1	4	-4	0.99	4.01	-4.12	0.120
22 H-	1	0	-8	0.99	0.00	-8.00	0.012
23 -h	1	2	-5	0.98	2.01	-5.12	0.117
24 H-	1	4	-7	0.98	4.02	-6.97	0.041
25 H-	0	-1	-6	-0.03	-1.01	-6.02	0.035

24h and 22H vectors out of 25. Quality : 0.936

Primitive cell :

3.904	6.038	17.009
82.50	88.51	86.28

Volume of the cell : 396.55

Example 7:

Solution nr 1 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

1.84344954	3.25703922	8.82866263
22.22374211	1.60108683	-5.22574341
16.66747588	-35.84753215	9.80100707

		h	k	l		Real (hkl)		Error
1	H-	4	10	24		4.01 9.99 23.98		0.024
2	H-	6	12	16		6.00 12.00 16.02		0.018
3	H-	4	12	22		4.01 11.99 21.97		0.034
4	H-	4	8	20		4.00 8.00 19.99		0.015
5	H-	6	7	1		6.00 7.02 1.04		0.049
6	H-	7	10	6		6.99 10.02 6.00		0.021
7	H-	5	15	15		5.00 14.99 14.98		0.017
8	H-	5	15	16		4.99 14.99 15.96		0.041
9	H-	4	-14	-16		4.00 -14.00 -16.03		0.033
10	H-	5	-11	-17		5.00 -11.00 -17.01		0.014
11	H-	5	-13	-21		5.00 -13.00 -21.02		0.023
12	H-	7	4	-14		7.00 4.00 -14.00		0.005
13	H-	6	2	-12		6.01 2.00 -11.97		0.029
14	H-	4	-13	-9		4.00 -13.00 -9.06		0.055
15	H-	5	-9	-14		5.00 -9.00 -13.96		0.041
16	H-	7	-4	-18		7.00 -4.01 -17.96		0.044
17	H-	2	-4	30		2.00 -4.00 30.02		0.020
18	H-	2	-5	29		2.00 -5.00 29.03		0.027
19	H-	2	-16	8		1.99 -16.01 8.00		0.010
20	H-	2	-16	10		1.99 -16.00 9.96		0.038
21	H-	1	-10	20		1.00 -9.99 19.99		0.013
22	H-	1	-12	16		1.00 -11.99 16.03		0.034
23	H-	2	-17	-5		2.01 -17.01 -5.04		0.039
24	H-	1	-13	8		1.00 -13.00 8.06		0.058
25	H-	1	-12	10		1.00 -11.99 10.01		0.012

25h and 25H vectors out of 25. Quality : 1.000

Primitive cell :

9.589	22.886	40.730
73.69	89.93	89.99

Volume of the cell : 8578.63

Example 7:

Solution nr 2 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
transforms RL vectors to Miller indices :

```
-1.84244579  -3.25710729  -8.82838295
-2.78455888  -18.72373301  7.51579641
22.22271405   1.60042838  -5.22626771
```

	h	k	l	Real (hkl)			Error
1	H-	-4	7 10	-4.01	6.99	9.99	0.015
2	H-	-6	2 12	-5.99	2.01	12.00	0.010
3	H-	-4	5 12	-4.01	4.98	11.99	0.019
4	H-	-4	6 8	-4.00	5.99	8.00	0.012
5	H-	-6	-3 7	-6.00	-2.99	7.02	0.022
6	H-	-7	-2 10	-6.99	-2.01	10.02	0.024
7	H-	-5	0 15	-5.00	-0.01	14.99	0.011
8							
9	H-	-4	-1 -14	-4.00	-1.01	-13.99	0.015
10	H-	-5	-3 -11	-5.00	-3.00	-11.00	0.003
11	H-	-5	-4 -13	-5.00	-4.00	-13.00	0.006
12	H-	-7	-9 4	-7.00	-9.00	4.00	0.003
13	H-	-6	-7 2	-6.01	-6.99	2.00	0.014
14	H-	-4	2 -13	-4.00	1.98	-13.00	0.023
15							
16	H-	-7	-7 -4	-7.00	-6.97	-4.01	0.030
17	H-	-2	17 -4	-2.00	17.01	-4.00	0.013
18	H-	-2	17 -5	-2.00	17.02	-5.00	0.018
19	H-	-2	12 -16	-2.00	12.01	-16.01	0.012
20	H-	-2	13 -16	-1.99	12.99	-16.00	0.015
21	H-	-1	15 -10	-1.00	14.99	-9.99	0.012
22	H-	-1	14 -12	-1.00	14.01	-11.99	0.019
23	H-	-2	6 -17	-2.01	5.99	-17.01	0.015
24							
25	H-	-1	11 -12	-1.00	11.00	-11.99	0.009

22h and 22H vectors out of 25. Quality : 0.880

Primitive cell :

```
9.589  20.367  22.885
106.34  90.00  90.07
```

Volume of the cell : 4288.81

Example 7:

Solution nr 3 out of 16.

Direct basis vectors (in rows) = inverse of UB matrix;
 transforms RL vectors to Miller indices :

```

    1.84235217    3.25747591    8.82920015
    8.34452110   -17.93160379    4.89513606
    22.22639387    1.60217130   -5.22661136
  
```

	h	k	l	Real (hkl)			Error	
1	H-	4	12	10	4.01	12.00	9.99	0.012
2	H-	6	8	12	6.00	8.01	12.00	0.014
3	H-	4	11	12	4.01	10.99	11.99	0.014
4	H-	4	10	8	4.00	10.00	8.00	0.006
5								
6	H-	7	3	10	6.99	3.00	10.02	0.022
7								
8	H-	5	8	15	4.99	7.99	15.00	0.016
9	H-	4	-8	-14	4.00	-8.03	-14.00	0.028
10								
11								
12	H-	7	-7	4	7.00	-7.00	4.00	0.003
13	H-	6	-6	2	6.01	-5.99	2.00	0.012
14								
15	H-	5	-7	-9	5.00	-6.99	-9.00	0.012
16	H-	7	-9	-4	7.00	-8.99	-4.01	0.016
17	H-	2	15	-4	2.00	15.01	-4.01	0.014
18								
19	H-	2	4	-16	2.00	3.99	-16.01	0.013
20	H-	2	5	-16	1.99	4.98	-16.00	0.026
21	H-	1	10	-10	1.00	9.99	-9.99	0.010
22	H-	1	8	-12	1.00	8.01	-11.99	0.018
23								
24	H-	1	4	-13	1.00	4.02	-13.00	0.025
25	H-	1	5	-12	1.00	5.00	-12.00	0.005

18h and 18H vectors out of 25. Quality : 0.720

Primitive cell :

```

    9.590    20.375    22.889
    73.67    89.99    89.95
  
```

Volume of the cell : 4291.67

Output of LePage.exe for solution no. 3 of example 7 (cf. solution no 2):

	LATO	LATT	A	B	C	ALFA	BET	GAM	VOL
INPUT CELL		P	9.590	20.375	22.889	73.67	89.99	89.95	4292.00
REDUC CELL		P	9.590	20.375	22.889	73.67	89.99	89.95	4292.00
CONV. CELL	M	P	20.375	9.590	22.889	90.01	106.33	89.95	4292.00