

Supporting information

for article

**Parameterization of the coupling
between strain and order parameter for LuF[SeO₃]**

O.V. Magdysyuk^a, M. Müller^a, R.E. Dinnebier^{a*},

Ch. Lipp^b, T. Schleid^b.

^{a)} *Max Planck Institute for Solid State Research, Stuttgart, Germany*

^{b)} *Institute for Inorganic Chemistry, University of Stuttgart, Germany*

* Correspondence author (e-mail: R.Dinnebier@fkf.mpg.de)

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Section S1. Calculation of the strain components.

For calculations of the strains the following formulas were used (Carpenter *et al.*, 1998):

$$e_1 = \frac{a}{a_0} \sin \gamma - 1$$

$$e_2 = \frac{b}{b_0} - 1$$

$$e_3 = \frac{c \sin \alpha \sin \beta^*}{c_0 \sin \beta_0^*} - 1$$

$$e_4 = \frac{c \cos \alpha}{c_0 \sin \beta_0^*} + \frac{a \cos \beta_0^* \cos \gamma}{a_0 \sin \beta_0^*}$$

$$e_5 = \frac{a \sin \gamma \cos \beta_0^*}{a_0 \sin \beta_0^*} - \frac{c \sin \alpha \cos \beta^*}{c_0 \sin \beta_0^*}$$

$$e_6 = \frac{a}{a_0} \cos \gamma$$

Asterics (*) refers to the reciprocal lattice angles.

Section S2. Simulated heating/cooling-Guinier pattern.

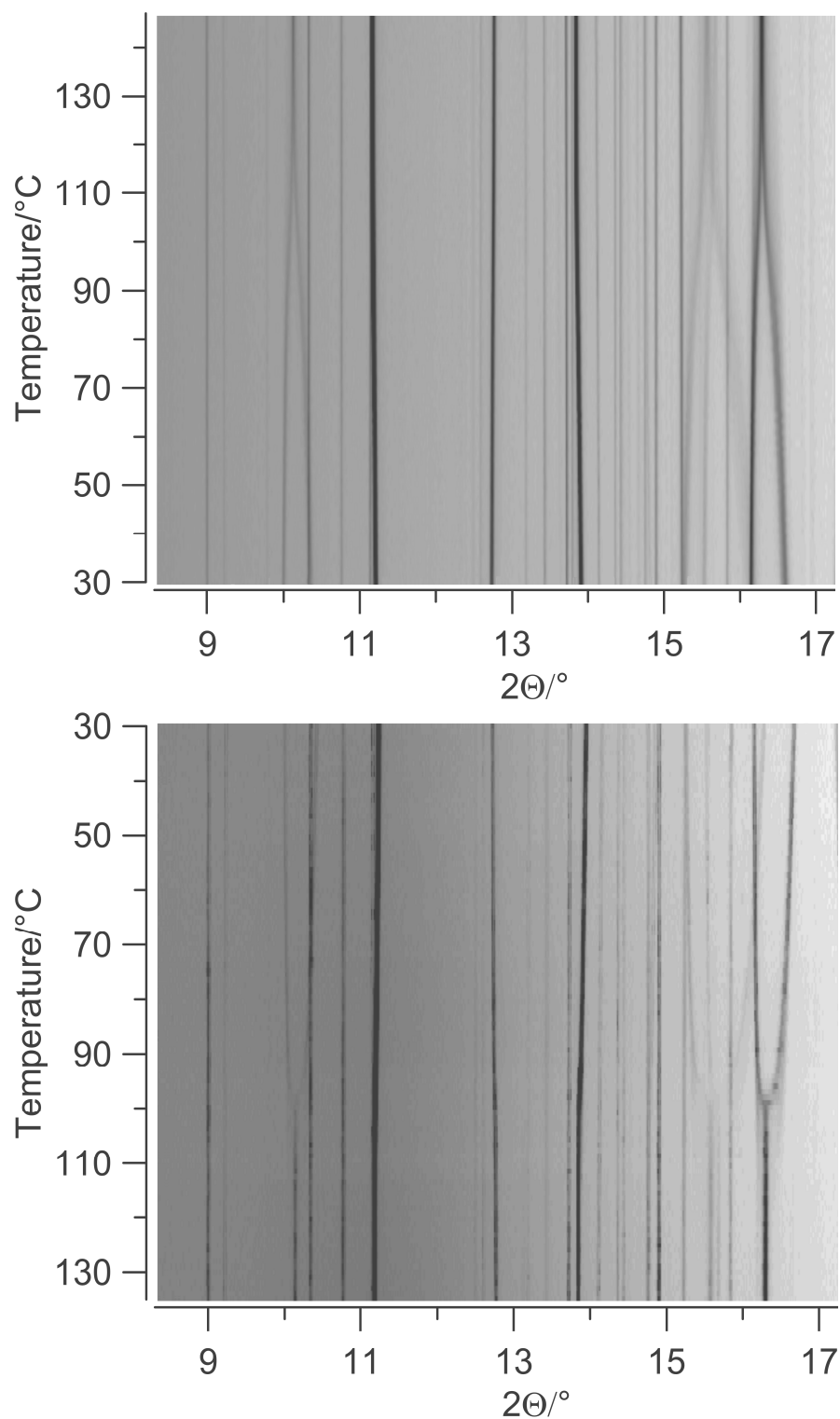


Fig. S2. Two-dimensional projection (simulated heating/cooling-Guinier pattern, prepared using Powder3D (Hinrichsen *et al.*, 2006)) of the observed scattered X-ray intensity for $\text{LuF}[\text{SeO}_3]$ as a function of diffraction angle (x-axis) and temperature (y-axis), top – on heating, bottom – on cooling.

Section S3. Temperature dependence of symmetry-adapted strains for LuF[SeO₃].

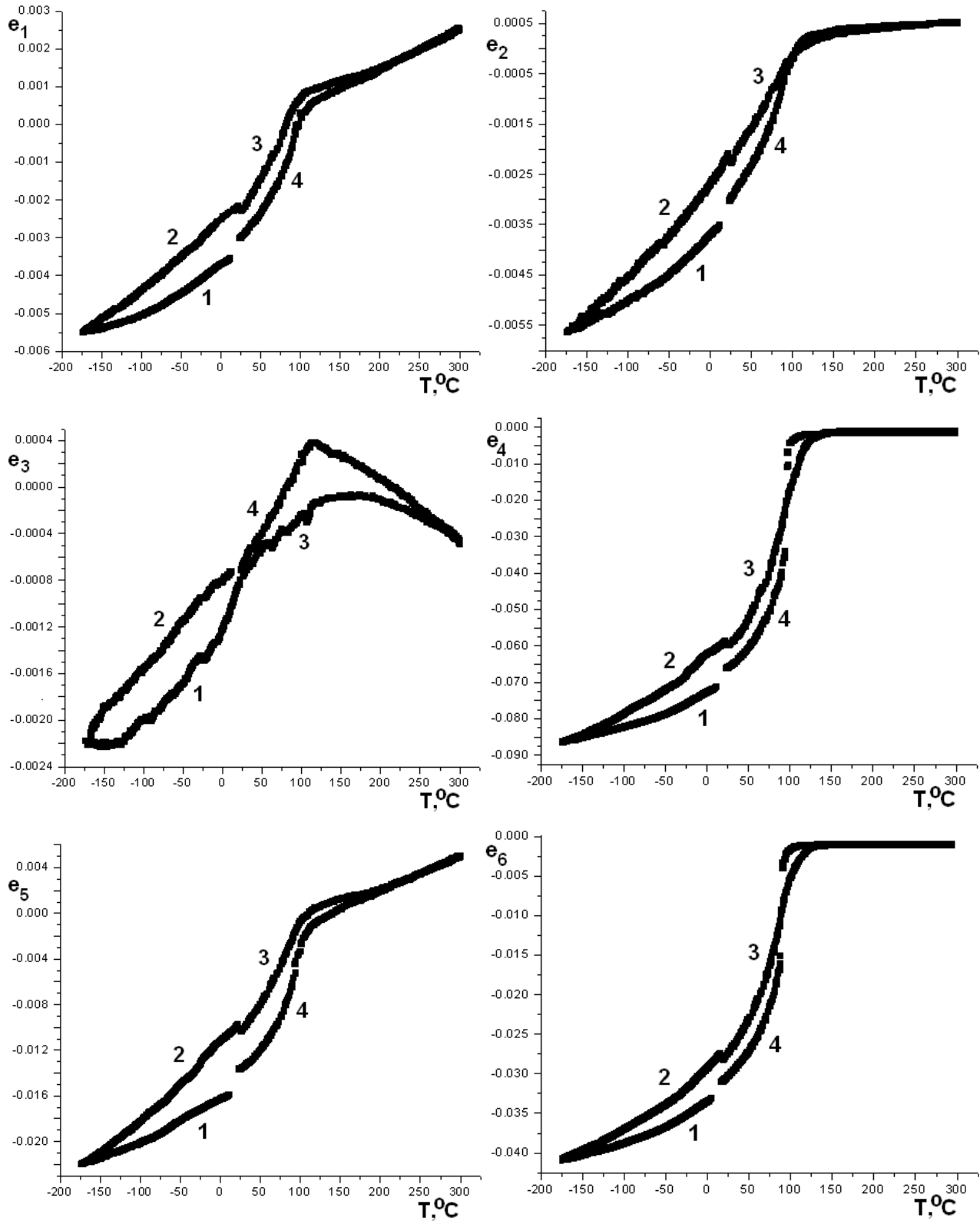


Fig. S3. Temperature dependence of symmetry-adapted strains for LuF[SeO₃]. Numbers show the sequence of cooling and heating: 1 – cooling from RT to -173°C, 2 – heating from -173°C to RT, 3 – heating from RT to 300°C, 4 – cooling from 300°C to RT.

Unit cell dimensions at RT before cooling (at the begin of step 1): 6.4504(1); 6.8460(1); 4.2812(1); 94.084(2); 96.509(2); 91.899(2); at RT after cooling (at the end of step 2): 6.4594(1); 6.8558(1); 4.2808(1); 93.364(3); 96.159(3); 91.574(2); at RT before heating (at the begin of step 3): 6.4586(1); 6.8545(1); 4.2810(1); 93.435(2); 96.174(2); 91.615(2); at RT after cooling (at the end of step 4): 6.4549(1); 6.8508(1); 4.2815(1); 93.690(3); 96.382(2); 91.711(1).

Section S4. Example of an input file for program Topas 4 which was used for the sequential Rietveld fit of the strain modes for LuF[SeO₃].

```

macro Out_file(file)
{
  out file append
    Out(Get(r_wp), "      \n %11.5f" ) ' _Rwp
    Out(Get(a), "%15.5f", "%9.5f") ' cell_length_a
    Out(Get(b), "%15.5f", "%9.5f") ' cell_length_b
    Out(Get(c), "%15.5f", "%9.5f") ' cell_length_c
    Out(Get(al), "%15.5f", "%9.5f") ' cell_angle_alpha
    Out(Get(be), "%15.5f", "%9.5f") ' cell_angle_beta
    Out(Get(ga), "%15.5f", "%9.5f") ' cell_angle_gamma
    Out(s1, "%12.5f", "%9.5f")
    Out(s2, "%12.5f", "%9.5f")
    Out(s3, "%12.5f", "%9.5f")
    Out(s4, "%12.5f", "%9.5f")
    Out(s5, "%12.5f", "%9.5f")
    Out(s6, "%12.5f", "%9.5f")
    Out(pe1, "%12.5f", "%9.5f")
    Out(pe2, "%12.5f", "%9.5f")
    Out(pe3, "%12.5f", "%9.5f")
    Out(pe4, "%12.5f", "%9.5f")
    Out(pe5, "%12.5f", "%9.5f")
    Out(pe6, "%12.5f", "%9.5f")
}

'{{{macros for calculations involving the lattice parameters

'Takes non-refinable undistorted cell parameters, and variable strains as
input.
'Returns strained cell parameters
macro straincell(pa0, pb0, pc0, pal0, pbe0, pga0)
{
  prm !radian 57.2957795
  prm palpha0 = pal0/radian;
  prm pbeta0 = pbe0/radian;
  prm pgamma0 = pga0/radian;
  'distorted parent-cell parameters
  prm pa = pa0*(pe1 + 1);: 6.44848
  prm pb = pb0*(pe2 + 1);: 6.84409
  prm pc = pc0*(pe3 + 1);: 4.27980
  prm palpha = palpha0 - pe4*Sin(palalpha0);
  prm pbeta = pbeta0 - pe5*Sin(pbeta0);
  prm pgamma = pgamma0 - pe6*Sin(pgamma0);
  prm pal = palpha*radian;: 90.07091
  prm pbe = pbeta*radian ;: 95.61740
  prm pga = pgamma*radian;: 90.03307
}

'Use basis-transformation matrix to transform from parent-cell to supercell
parameters
macro celltransform(t11, t12, t13, t21, t22, t23, t31, t32, t33)
{
  prm !radian 57.2957795

  'parent-lattice basis vectors in cartesian coords
  prm zzz = (Cos(palalpha) - Cos(pbeta)*Cos(pgamma))/Sin(pgamma);
  prm pv11 = pa;
  prm !pv21 0
  prm !pv31 0
}

```

```

    prm  pv12 = pb*cos(pgamma);
    prm  pv22 = pb*sin(pgamma);
    prm  !pv32  0
    prm  pv13 = pc*cos(pbeta);
    prm  pv23 = pc*zzz;
    prm  pv33 = pc*sqrt(1 - cos(pbeta)^2 - zzz^2);

    'supercell basis vectors in cartesian coords
    prm  sv11 = t11*pv11 + t12*pv12 + t13*pv13;
    prm  sv21 = t11*pv21 + t12*pv22 + t13*pv23;
    prm  sv31 = t11*pv31 + t12*pv32 + t13*pv33;
    prm  sv12 = t21*pv11 + t22*pv12 + t23*pv13;
    prm  sv22 = t21*pv21 + t22*pv22 + t23*pv23;
    prm  sv32 = t21*pv31 + t22*pv32 + t23*pv33;
    prm  sv13 = t31*pv11 + t32*pv12 + t33*pv13;
    prm  sv23 = t31*pv21 + t32*pv22 + t33*pv23;
    prm  sv33 = t31*pv31 + t32*pv32 + t33*pv33;

    'distorted supercell parameters
    prm  sa = sqrt(sv11^2+sv21^2+sv31^2);
    prm  sb = sqrt(sv12^2+sv22^2+sv32^2);
    prm  sc = sqrt(sv13^2+sv23^2+sv33^2);
    prm  salpha = arccos((sv12*sv13+sv22*sv23+sv32*sv33)/(sb*sc));
    prm  sbeta  = arccos((sv11*sv13+sv21*sv23+sv31*sv33)/(sa*sc));
    prm  sgamma = arccos((sv11*sv12+sv21*sv22+sv31*sv32)/(sa*sb));
    prm  sal = salpha*radian;
    prm  sbe = sbeta*radian;
    prm  sga = sgamma*radian;
}
}}}}

iters 300
do_errors
XYE(fileName)

bkg @ 11753.9469`_14.7944678 -7130.01235`_25.1300073 1935.93213`_23.0003751
561.944125`_21.3053992 -368.916997`_20.9094482 35.1972098`_19.8681934 -
24.8299646`_19.7882956 149.933676`_18.4599147 119.144117`_18.3542657
90.609349`_15.9106356 -52.0456283`_15.1624355
    start_X 4
    finish_X 30
    LP_Factor( 90)
    Zero_Error(, -0.0138)
    convolution_step 5
    Rp 217.5
    Rs 217.5
    Simple_Axial_Model( 0.001)
    lam
        ymin_on_ymax 0.0001
        la 1 lo 0.826401 lh 0.0001
    x_calculation_step 0.001
    str
        LVol_FWHM_CS_G_L( 1, 371.24214`_52.84462, 0.89, 358.49782`_45.08627,@,
411.79449`_49.86917,@, 9999.99999`_20342.19111)
        e0_from_strain( 0.04488`_12.28008,lg, 0.17951`_0.00427,lg,
0.17951`_0.00427)
        r_bragg 6.55700305
        phase_name "Structure"
        MVW( 641.847, 187.946784`_0.00573257063, 100.000`_0.000)
        scale @ 0.000567754063`_6.89e-006
        Phase_LAC_1_on_cm( 531.89653`_0.01622)
        Phase_Density_g_on_cm3( 5.67082`_0.00017)

```

space_group P-1

#define refine_strain_modes 'comment out this line for a traditional
supercell-parameter refinement.

```
' #ifdef refine_strain_modes
  prm s1 -0.00230`_0.00002 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_1(a)
  prm s2 -0.01046`_0.00005 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_2(a)
  prm s3 -0.00231`_0.00002 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_3(a)
  prm s4 -0.00080`_0.00001 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_4(a)
  prm s5 -0.02823`_0.00004 min -0.50 max 0.50
'P2_1/m[0,0,0]GM2+(a)strain_1(a)
  prm s6 -0.06016`_0.00006 min -0.50 max 0.50
'P2_1/m[0,0,0]GM2+(a)strain_2(a)
```

'unitless parent cell strains (in lattice rather than orthogonal
coords)

```
prm pe1 = + 1.00000*s1;: -0.00230`_0.00002
prm pe2 = + 1.00000*s3;: -0.00231`_0.00002
prm pe3 = + 1.00000*s4;: -0.00080`_0.00001
prm pe4 = + 1.00000*s6;: -0.06016`_0.00006
prm pe5 = + 1.00000*s2;: -0.01046`_0.00005
prm pe6 = + 1.00000*s5;: -0.02823`_0.00004
```

```
straincell(6.47340,6.87020,4.28430,90.00000,95.60200,90.00000)
celltransform(1,0,0,0,1,0,0,0,1)
```

```
a = sa;: 6.45854`_0.00013
b = sb;: 6.85435`_0.00012
c = sc;: 4.28088`_0.00007
a1 = sal;: 93.44678`_0.00326
be = sbe;: 96.19830`_0.00258
ga = sga;: 91.61764`_0.00237
```

```
'{{{mode definitions
  prm !a1 -0.01000 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[Lul:e]A'_1(a)
  prm !a2 -0.01291 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[Lul:e]A'_2(a)
  prm a3 -0.02820`_0.01233 min -1.41 max 1.41
'P2_1/m[0,0,0]GM2+(a)[Lul:e]A''(a)
  prm !a4 -0.02302 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[F1:e]A'_1(a)
  prm !a5 -0.06958 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[F1:e]A'_2(a)
  prm a6 0.07997`_0.07392 min -1.41 max 1.41
'P2_1/m[0,0,0]GM2+(a)[F1:e]A''(a)
  prm !a7 -0.00079 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[Sel:e]A'_1(a)
  prm !a8 0.03442 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[Sel:e]A'_2(a)
  prm a9 0.18568`_0.02063 min -1.41 max 1.41
'P2_1/m[0,0,0]GM2+(a)[Sel:e]A''(a)
  prm !a10 -0.03090 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[O1:e]A'_1(a)
  prm !a11 -0.04852 min -1.41 max 1.41
'P2_1/m[0,0,0]GM1+(a)[O1:e]A'_2(a)
```



```

    prm a12      0.23911`_0.05407 min  -1.41 max  1.41
'P2_1/m[0,0,0]GM2+(a)[O1:e]A''(a)
    prm !a13    -0.06769 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM1+(a)[O2:f]A_1(a)
    prm !a14    -0.00842 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM1+(a)[O2:f]A_2(a)
    prm !a15    -0.02061 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM1+(a)[O2:f]A_3(a)
    prm a16     -0.44042`_0.04677 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM2+(a)[O2:f]A_1(a)
    prm a17     -0.24993`_0.04396 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM2+(a)[O2:f]A_2(a)
    prm a18     0.18712`_0.08173 min  -2.00 max  2.00
'P2_1/m[0,0,0]GM2+(a)[O2:f]A_3(a)
'}}}

```

```
'{{{mode-amplitude to delta transformation
```

```

    prm Lu1_dx  = + 0.10923*a2;: -0.00141
    prm Lu1_dy  = + 0.10292*a3;: -0.00290`_0.00127
    prm Lu1_dz  = + 0.16505*a1;: -0.00165
    prm Fl_dx   = + 0.10923*a5;: -0.00760
    prm Fl_dy   = + 0.10292*a6;: 0.00823`_0.00761
    prm Fl_dz   = + 0.16505*a4;: -0.00380
    prm Sel_dx  = - 0.10923*a8;: -0.00376
    prm Sel_dy  = - 0.10292*a9;: -0.01911`_0.00212
    prm Sel_dz  = - 0.16505*a7;: 0.00013
    prm O1_dx   = + 0.10923*a11;: -0.00530
    prm O1_dy   = + 0.10292*a12;: 0.02461`_0.00557
    prm O1_dz   = + 0.16505*a10;: -0.00510
    prm O2_dx   = + 0.07724*a14 + 0.07724*a17;: -0.01995`_0.00340
    prm O2_dy   = + 0.07278*a15 + 0.07278*a18;: 0.01212`_0.00595
    prm O2_dz   = + 0.11671*a13 + 0.11671*a16;: -0.05930`_0.00546
    prm O3_dx   = + 0.07724*a14 - 0.07724*a17;: 0.01865`_0.00340
    prm O3_dy   = - 0.07278*a15 + 0.07278*a18;: 0.01512`_0.00595
    prm O3_dz   = + 0.11671*a13 - 0.11671*a16;: 0.04350`_0.00546

```

```
'}}}
```

```
'{{{distorted parameters
```

```

    prm Lu1_x   = 0.36836 + Lu1_dx;: 0.36695
    prm Lu1_y   = 1/4 + Lu1_dy;: 0.24710`_0.00127
    prm Lu1_z   = 0.46083 + Lu1_dz;: 0.45918
    prm Fl_x    = 0.35790 + Fl_dx;: 0.35030
    prm Fl_y    = 1/4 + Fl_dy;: 0.25823`_0.00761
    prm Fl_z    = -0.03820 + Fl_dz;: -0.04200
    prm Sel_x   = 0.14630 + Sel_dx;: 0.14254
    prm Sel_y   = 3/4 + Sel_dy;: 0.73089`_0.00212
    prm Sel_z   = 0.32170 + Sel_dz;: 0.32183
    prm O1_x    = 0.02340 + O1_dx;: 0.01810
    prm O1_y    = 1/4 + O1_dy;: 0.27461`_0.00557
    prm O1_z    = 0.41130 + O1_dz;: 0.40620
    prm O2_x    = 0.32160 + O2_dx;: 0.30165`_0.00340
    prm O2_y    = 0.92560 + O2_dy;: 0.93772`_0.00595
    prm O2_z    = 0.45060 + O2_dz;: 0.39130`_0.00546
    prm O3_x    = 0.32160 + O3_dx;: 0.34025`_0.00340
    prm O3_y    = 0.57440 + O3_dy;: 0.58952`_0.00595
    prm O3_z    = 0.45060 + O3_dz;: 0.49410`_0.00546

    prm !Lu1_occ = 1;: 1.00000
    prm !Fl_occ  = 1;: 1.00000
    prm !Sel_occ = 1;: 1.00000
    prm !O1_occ  = 1;: 1.00000
    prm !O2_occ  = 1;: 1.00000

```

```

        prm !O3_occ = 1;: 1.00000
'}}

```

Section S5. Example of an input file for program Topas 4 which was used for the sequential Le Bail fit of the strain modes for LuF[SeO₃].

```

macro Out_file(file)
{
  out file append
    Out(Get(r_wp), " \n %11.5f" ) ' _Rwp
    Out(Get(a), "%15.5f", "%9.5f") ' cell_length_a
    Out(Get(b), "%15.5f", "%9.5f") ' cell_length_b
    Out(Get(c), "%15.5f", "%9.5f") ' cell_length_c
    Out(Get(al), "%15.5f", "%9.5f") ' cell_angle_alpha
    Out(Get(be), "%15.5f", "%9.5f") ' cell_angle_beta
    Out(Get(ga), "%15.5f", "%9.5f") ' cell_angle_gamma
    Out(pe1, "%15.5f", "%9.5f")
    Out(pe2, "%15.5f", "%9.5f")
    Out(pe3, "%15.5f", "%9.5f")
    Out(pe4, "%15.5f", "%9.5f")
    Out(pe5, "%15.5f", "%9.5f")
    Out(pe6, "%15.5f", "%9.5f")
}

macro straincell(pa0, pb0, pc0, pal0, pbe0, pga0)
{
  prm !radian 57.2957795
  prm palpha0 = pal0/radian;
  prm pbeta0 = pbe0/radian;
  prm pgamma0 = pga0/radian;
  'distorted parent-cell parameters
  prm pa = pa0*(pe1 + 1);: 6.44848
  prm pb = pb0*(pe2 + 1);: 6.84409
  prm pc = pc0*(pe3 + 1);: 4.27980
  prm palpha = palpha0 - pe4*Sin(palpha0);
  prm pbeta = pbeta0 - pe5*Sin(pbeta0);
  prm pgamma = pgamma0 - pe6*Sin(pgamma0);
  prm pal = palpha*radian;: 90.07091
  prm pbe = pbeta*radian ;: 95.61740
  prm pga = pgamma*radian;: 90.03307
}

macro celltransform(t11, t12, t13, t21, t22, t23, t31, t32, t33)
{
  prm !radian 57.2957795
  'parent-lattice basis vectors in cartesian coords
  prm zzz = (Cos(palpha) - Cos(pbeta)*Cos(pgamma))/Sin(pgamma);

```

```

    prm  pv11 = pa;
    prm  !pv21  0
    prm  !pv31  0
    prm  pv12 = pb*cos(pgamma);
    prm  pv22 = pb*sin(pgamma);
    prm  !pv32  0
    prm  pv13 = pc*cos(pbeta);
    prm  pv23 = pc*zzz;
    prm  pv33 = pc*Sqrt(1 - Cos(pbeta)^2 - zzz^2);
    'supercell basis vectors in cartesian coords
    prm  sv11 = t11*pv11 + t12*pv12 + t13*pv13;
    prm  sv21 = t11*pv21 + t12*pv22 + t13*pv23;
    prm  sv31 = t11*pv31 + t12*pv32 + t13*pv33;
    prm  sv12 = t21*pv11 + t22*pv12 + t23*pv13;
    prm  sv22 = t21*pv21 + t22*pv22 + t23*pv23;
    prm  sv32 = t21*pv31 + t22*pv32 + t23*pv33;
    prm  sv13 = t31*pv11 + t32*pv12 + t33*pv13;
    prm  sv23 = t31*pv21 + t32*pv22 + t33*pv23;
    prm  sv33 = t31*pv31 + t32*pv32 + t33*pv33;
    'distorted supercell parameters
    prm  sa = Sqrt(sv11^2+sv21^2+sv31^2);
    prm  sb = Sqrt(sv12^2+sv22^2+sv32^2);
    prm  sc = Sqrt(sv13^2+sv23^2+sv33^2);
    prm  salpha = ArcCos((sv12*sv13+sv22*sv23+sv32*sv33)/(sb*sc));
    prm  sbeta  = ArcCos((sv11*sv13+sv21*sv23+sv31*sv33)/(sa*sc));
    prm  sgamma = ArcCos((sv11*sv12+sv21*sv22+sv31*sv32)/(sa*sb));
    prm  sal = salpha*radian;
    prm  sbe = sbeta*radian;
    prm  sga = sgamma*radian;
}
iters 100
do_errors
continue_after_convergence
XYE(fileName)
    bkg @ 11209.8127` -7332.04118` 2248.66707` 312.4385` -446.151343`
48.9687557` 92.0060604` 62.5305514` -97.8755136` -150.174412` 40.1732165`
start_X 4
LP_Factor( 90)
Zero_Error(, -0.0138)
convolution_step 5
Rp 217.5

```

```

Rs 217.5
Simple_Axial_Model( 0.001)
lam
    ymin_on_ymax 0.0001
    la 1 lo 0.826401 lh 0.0001
x_calculation_step 0.001
hkl_Is
    lebail 1
    LVol_FWHM_CS_G_L( 1, 317.8518953, 0.89, 306.0315349,@, 350.4475805,@,
10000_LIMIT_MIN_0.3)
    e0_from_Strain( 0.06714911244,lg, 0.1637203233,lg, 0.1637203233)
    r_bragg 0.462166152
    phase_name "hkl_P-1"
    space_group "P-1"
hkl_m_d_th2 0 1 0 2 6.83792591 6.92873144 I 0.03167503434
hkl_m_d_th2 1 0 0 2 6.41686535 7.38399029 I 3.883647208
hkl_m_d_th2 1 -1 0 2 4.76291418 9.95375633 I 1.968085691
hkl_m_d_th2 1 1 0 2 4.59972572 10.3078327 I 1.666087449
hkl_m_d_th2 0 0 1 2 4.24731684 11.1657076 I 35.1017308
hkl_m_d_th2 1 0 -1 2 3.73559332 12.7011623 I 14.973385
hkl_m_d_th2 0 -1 1 2 3.71510625 12.7714939 I 0.5720678465
hkl_m_d_th2 0 1 1 2 3.50958157 13.5227985 I 0.3002439841
hkl_m_d_th2 0 2 0 2 3.41896296 13.8829575 I 41.48216495
hkl_m_d_th2 1 0 1 2 3.37526488 14.0635958 I 0.02639934675
hkl_m_d_th2 1 1 -1 2 3.32830381 14.263052 I 0.03559812789
hkl_m_d_th2 1 -1 -1 2 3.2304616 14.6973915 I 0.05614762095
hkl_m_d_th2 2 0 0 2 3.20843267 14.7988653 I 0.6354186041
hkl_m_d_th2 1 -1 1 2 3.11277413 15.2563143 I 5.681631878
hkl_m_d_th2 1 -2 0 2 3.0620892 15.5103836 I 3.931344037
hkl_m_d_th2 1 2 0 2 2.97459078 15.9695597 I 0.2948419239
hkl_m_d_th2 1 1 1 2 2.94725585 16.1186523 I 4.110764447
hkl_m_d_th2 2 -1 0 2 2.94439578 16.1344128 I 31.8610824
hkl_m_d_th2 2 1 0 2 2.86635399 16.5767517 I 21.50942402
hkl_m_d_th2 0 -2 1 2 2.74983549 17.28442 I 10.92717352
hkl_m_d_th2 2 0 -1 2 2.70705819 17.5596905 I 0.2522461304
hkl_m_d_th2 0 2 1 2 2.58443761 18.399889 I 5.025544893
hkl_m_d_th2 1 2 -1 2 2.56783724 18.5198841 I 4.494602893
hkl_m_d_th2 2 1 -1 2 2.5264852 18.8257408 I 10.55444544
hkl_m_d_th2 2 -1 -1 2 2.50760698 18.968771 I 8.191734326
hkl_m_d_th2 1 -2 1 2 2.48906946 19.1113625 I 0.625480755
hkl_m_d_th2 1 -2 -1 2 2.4787128 19.191967 I 6.415372667

```

hkl_m_d_th2 2 0 1 2 2.43472695 19.5420589 I 0.01653932826
 hkl_m_d_th2 2 -2 0 2 2.38145709 19.9836235 I 0.1609160045
 hkl_m_d_th2 2 -1 1 2 2.34062719 20.3358898 I 15.29571802
 hkl_m_d_th2 1 2 1 2 2.32342339 20.4880867 I 0.2304624266
 hkl_m_d_th2 2 2 0 2 2.29986286 20.7002735 I 0.6435761287
 hkl_m_d_th2 0 3 0 2 2.27930856 20.8890305 I 2.133650791
 hkl_m_d_th2 2 1 1 2 2.24942732 21.1696987 I 2.537921561
 hkl_m_d_th2 1 -3 0 2 2.17188907 21.9346733 I 5.45092177
 hkl_m_d_th2 3 0 0 2 2.13895512 22.2766914 I 13.74680978
 hkl_m_d_th2 2 2 -1 2 2.13375187 22.3317127 I 1.011896311
 hkl_m_d_th2 1 3 0 2 2.12456274 22.4295502 I 4.910475903
 hkl_m_d_th2 0 0 2 2 2.12365842 22.4392262 I 14.32599399
 hkl_m_d_th2 2 -2 -1 2 2.11111546 22.5742931 I 5.095830096
 hkl_m_d_th2 1 0 -2 2 2.08558846 22.8542938 I 8.260360785
 hkl_m_d_th2 0 -1 2 2 2.06554317 23.0791206 I 0.08161570324
 hkl_m_d_th2 0 -3 1 2 2.06365323 23.1005478 I 0.6368514118
 hkl_m_d_th2 3 -1 0 2 2.06203032 23.1189785 I 4.78373099
 hkl_m_d_th2 2 -2 1 2 2.04490328 23.3153172 I 3.571661755
 hkl_m_d_th2 1 1 -2 2 2.02383375 23.5615063 I 0.6194230323
 hkl_m_d_th2 3 1 0 2 2.02139831 23.5903015 I 6.838058186
 hkl_m_d_th2 3 0 -1 2 2.00067735 23.8381939 I 2.138912158
 hkl_m_d_th2 0 1 2 2 1.99262273 23.9359741 I 0.05554895191
 hkl_m_d_th2 1 3 -1 2 1.97712755 24.1263771 I 0.6722031757
 hkl_m_d_th2 1 -1 -2 2 1.96710289 24.251194 I 1.284730243
 hkl_m_d_th2 0 3 1 2 1.95733178 24.3741112 I 0.2216940918
 hkl_m_d_th2 1 0 2 2 1.95315433 24.4270458 I 2.875062992
 hkl_m_d_th2 1 -3 1 2 1.95222831 24.4388103 I 2.043713098
 hkl_m_d_th2 2 2 1 2 1.92684615 24.7658138 I 0.02774018865
 hkl_m_d_th2 3 -1 -1 2 1.92162049 24.8342361 I 16.54722922
 hkl_m_d_th2 3 1 -1 2 1.91873336 24.8722019 I 9.05852093
 hkl_m_d_th2 1 -3 -1 2 1.91575527 24.9114876 I 0.01173116786
 hkl_m_d_th2 1 -1 2 2 1.91320658 24.9452057 I 23.60225578
 hkl_m_d_th2 2 -3 0 2 1.88956165 25.2624798 I 29.56832132
 hkl_m_d_th2 2 0 -2 2 1.86779666 25.5618095 I 0.05974077687
 hkl_m_d_th2 0 -2 2 2 1.85755312 25.7051773 I 4.36158894
 hkl_m_d_th2 1 1 2 2 1.84475112 25.8866501 I 6.38744074
 hkl_m_d_th2 3 -2 0 2 1.84248817 25.9189968 I 8.95762995
 hkl_m_d_th2 1 3 1 2 1.83143497 26.0781727 I 9.407112421
 hkl_m_d_th2 3 0 1 2 1.8312993 26.0801392 I 10.03115965
 hkl_m_d_th2 2 3 0 2 1.828251 26.1243916 I 25.89582379
 hkl_m_d_th2 1 2 -2 2 1.82220745 26.2125778 I 13.62178355

```

hkl_m_d_th2 2 1 -2 2 1.81824589 26.27071 I 16.46117778
hkl_m_d_th2 3 -1 1 2 1.79481769 26.6198902 I 9.352267211e-005
hkl_m_d_th2 2 -1 -2 2 1.7857691 26.7572727 I 1.220059406
hkl_m_d_th2 3 2 0 2 1.78551149 26.7612057 I 2.084311552
hkl_m_d_th2 0 2 2 2 1.75479078 27.2386513 I 0.1200195344
hkl_m_d_th2 2 3 -1 2 1.75305951 27.2660713 I 20.78355162
hkl_m_d_th2 1 -2 2 2 1.74865687 27.3360519 I 2.077151171
hkl_m_d_th2 3 1 1 2 1.74418604 27.4074879 I 2.388976308
hkl_m_d_th2 1 -2 -2 2 1.74147427 27.4510002 I 8.668837292
hkl_m_d_th2 2 -3 -1 2 1.73423302 27.5678806 I 11.94817054
hkl_m_d_th2 3 -2 -1 2 1.72886372 27.6551952 I 0.8972588613
hkl_m_d_th2 3 2 -1 2 1.72466445 27.723875 I 0.2418645551
hkl_m_d_th2 2 -3 1 2 1.71871519 27.8217678 I 12.65468874
hkl_m_d_th2 0 4 0 2 1.70948148 27.9750957 I 38.6924784
hkl_m_d_th2 2 0 2 2 1.68763244 28.3447971 I 0.003445217182
hkl_m_d_th2 1 -4 0 2 1.66640663 28.7135372 I 5.179661842
hkl_m_d_th2 2 -1 2 2 1.66532505 28.7325859 I 6.513869393
hkl_m_d_th2 2 2 -2 2 1.66415191 28.7532768 I 1.521876791
hkl_m_d_th2 3 -2 1 2 1.65420151 28.9299927 I 10.34876388
hkl_m_d_th2 1 2 2 2 1.6476984 29.0466785 I 0.05979760466
hkl_m_d_th2 1 4 0 2 1.63770437 29.227869 I 2.089787806
hkl_m_d_th2 0 -4 1 2 1.62187767 29.5195274 I 18.25260649
hkl_m_d_th2 2 -2 -2 2 1.6152308 29.6437836 I 3.296554867
hkl_m_d_th2 2 3 1 2 1.6141026 29.664978 I 0.8524039247
hkl_m_d_th2 2 1 2 2 1.61287057 29.6881599 I 2.74473682
hkl_m_d_th2 0 -3 2 2 1.60537469 29.8299904 I 26.96598698
hkl_m_d_th2 4 0 0 2 1.60421634 29.8520298 I 3.543460163
hkl_m_d_th2 3 0 -2 2 1.59728301 29.9846439 I 0.001599724206
hkl_m_d_th2 3 -3 0 2 1.58763802 30.1711159 I 0.06687685381
hkl_m_d_th2 1 3 -2 2 1.57924962 30.3352146 I 0.2402028967
hkl_m_d_th2 3 2 1 2 1.5771724 30.3761311 I 0.3127680431
hkl_m_d_th2 1 4 -1 2 1.5757283 30.4046421 I 0.3293136094
hkl_m_d_th2 4 -1 0 2 1.57408154 30.4372196 I 0.3341454992
MVW( 0.000, 187.982346`, 0.000)
prm s1 -0.00385 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_1(a)
prm s2 -0.01547 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_2(a)
prm s3 -0.00380 min -0.50 max 0.50
'P2_1/m[0,0,0]GM1+(a)strain_3(a)

```

```

        prm  s4      -0.00105 min  -0.50 max  0.50
'P2_1/m[0,0,0]GM1+(a)strain_4(a)
        prm  s5      -0.03307 min  -0.50 max  0.50
'P2_1/m[0,0,0]GM2+(a)strain_1(a)
        prm  s6      -0.07091 min  -0.50 max  0.50
'P2_1/m[0,0,0]GM2+(a)strain_2(a)
        prm  pe1     = +  1.00000*s1;: -0.00385
        prm  pe2     = +  1.00000*s3;: -0.00380
        prm  pe3     = +  1.00000*s4;: -0.00105
        prm  pe4     = +  1.00000*s6;: -0.07091
        prm  pe5     = +  1.00000*s2;: -0.01547
        prm  pe6     = +  1.00000*s5;: -0.03307
        straincell(6.47340,6.87020,4.28430,90.00000,95.60200,90.00000)
        celltransform(1,0,0,0,1,0,0,0,1)
            a  = sa;:    6.44848
            b  = sb;:    6.84409
            c  = sc;:    4.27980
            a1 = sal;:   94.06284
            be = sbe;:   96.48413
            ga = sga;:   91.89477
Out_file(hkl.txt)

```


Section S6. Example of an input file for program Topas 4 which was used for the parametric Le Bail fit of the strain modes for LuF[SeO₃].

```

macro Out_file(file)
{
  out file append
    Out(Get(r_wp), " \n %11.5f" ) ' _Rwp
    Out(Get(a), "%15.5f", "%9.5f") ' cell_length_a
    Out(Get(b), "%15.5f", "%9.5f") ' cell_length_b
    Out(Get(c), "%15.5f", "%9.5f") ' cell_length_c
    Out(Get(al), "%15.5f", "%9.5f") ' cell_angle_alpha
    Out(Get(be), "%15.5f", "%9.5f") ' cell_angle_beta
    Out(Get(ga), "%15.5f", "%9.5f") ' cell_angle_gamma
    Out(pe1, "%15.5f", "%9.5f")
    Out(pe2, "%15.5f", "%9.5f")
    Out(pe3, "%15.5f", "%9.5f")
    Out(pe4, "%15.5f", "%9.5f")
    Out(pe5, "%15.5f", "%9.5f")
    Out(pe6, "%15.5f", "%9.5f")
}

'{{{macros for calculations involving the lattice parameters
'Takes non-refinable undistorted cell parameters, and variable strains as
input.
'Returns strained cell parameters
macro straincell(pa0, pb0, pc0, pal0, pbe0, pga0)
{
  prm !radian 57.2957795
  local palpha0 = pal0/radian;
  local pbeta0 = pbe0/radian;
  local pgamma0 = pga0/radian;
  'distorted parent-cell parameters
  local pa = pa0*(pe1 + 1);: 6.44848
  local pb = pb0*(pe2 + 1);: 6.84409
  local pc = pc0*(pe3 + 1);: 4.27980
  local palpha = palpha0 - pe4*Sin(palpha0);
  local pbeta = pbeta0 - pe5*Sin(pbeta0);
  local pgamma = pgamma0 - pe6*Sin(pgamma0);
  local pal = palpha*radian;: 90.07091
  local pbe = pbeta*radian ;: 95.61740
  local pga = pgamma*radian;: 90.03307
}

```

```

'Use basis-transformation matrix to transform from parent-cell to supercell
parameters
macro celltransform(t11, t12, t13, t21, t22, t23, t31, t32, t33)
{
    prm !radian 57.2957795
    'parent-lattice basis vectors in cartesian coords
    local zzz = (Cos(palpha) - Cos(pbeta)*Cos(pgamma))/Sin(pgamma);
    local pv11 = pa;
    local !pv21 0
    local !pv31 0
    local pv12 = pb*Cos(pgamma);
    local pv22 = pb*Sin(pgamma);
    local !pv32 0
    local pv13 = pc*Cos(pbeta);
    local pv23 = pc*zzz;
    local pv33 = pc*Sqrt(1 - Cos(pbeta)^2 - zzz^2);
    'supercell basis vectors in cartesian coords
    local sv11 = t11*pv11 + t12*pv12 + t13*pv13;
    local sv21 = t11*pv21 + t12*pv22 + t13*pv23;
    local sv31 = t11*pv31 + t12*pv32 + t13*pv33;
    local sv12 = t21*pv11 + t22*pv12 + t23*pv13;
    local sv22 = t21*pv21 + t22*pv22 + t23*pv23;
    local sv32 = t21*pv31 + t22*pv32 + t23*pv33;
    local sv13 = t31*pv11 + t32*pv12 + t33*pv13;
    local sv23 = t31*pv21 + t32*pv22 + t33*pv23;
    local sv33 = t31*pv31 + t32*pv32 + t33*pv33;
    'distorted supercell parameters
    local sa = Sqrt(sv11^2+sv21^2+sv31^2);
    local sb = Sqrt(sv12^2+sv22^2+sv32^2);
    local sc = Sqrt(sv13^2+sv23^2+sv33^2);
    local salpha = ArcCos((sv12*sv13+sv22*sv23+sv32*sv33)/(sb*sc));
    local sbeta = ArcCos((sv11*sv13+sv21*sv23+sv31*sv33)/(sa*sc));
    local sgamma = ArcCos((sv11*sv12+sv21*sv22+sv31*sv32)/(sa*sb));
    local sal = salpha*radian;
    local sbe = sbeta*radian;
    local sga = sgamma*radian;
}
}}}}
bootstrap_errors 100
approximate_A

```

```
prm !t_kr 120
```

```
prm pA_e1 0.049`  
prm pB_e1 757.51398`  
prm pA_e2 0.02951`  
prm pB_e2 750.70520`  
prm pA_e3 0.00188`  
prm pB_e3 1611.79064`  
prm pA_e4 0.68113`  
prm pB_e4 113.43002`  
prm pA_e5 0.90131`  
prm pB_e5 317.62995`  
prm pA_e6 0.94034`  
prm pB_e6 221.29718`  
prm up_beta 0.40009`
```

```
macro parametric {  
    local Tdiff = (t_kr - t_cur) / t_kr;  
    local ppA_e1 = pA_e1 * 0.00001;  
    local ppA_e2 = pA_e2 * 0.00001;  
    local ppA_e3 = pA_e3 * 0.00001;  
    local ppA_e4 = pA_e4 * 0.001;  
    local ppA_e5 = pA_e5 * 0.00001;  
    local ppA_e6 = pA_e6 * 0.0001;  
  
    local s1 = 0.000052109 + t_cur * 0.00000733868 - ((ppA_e1*Tdiff)^(up_beta))  
* (1- (2/ (Exp(2*Tdiff*pB_e1* ((ppA_e1*Tdiff)^(up_beta)) )+1 ) ) ) ;  
    local s3 = -0.000470013 + t_cur * 0.00000495331 - ((ppA_e2*Tdiff)^(up_beta))  
* (1- (2/ (Exp(2*Tdiff*pB_e2* ((ppA_e2*Tdiff)^(up_beta)) )+1 ) ) ) ;  
    local s4 = -0.000280688 + t_cur * 0.00000131401 - ((ppA_e3*Tdiff)^(up_beta))  
* (1- (2/ (Exp(2*Tdiff*pB_e3* ((ppA_e3*Tdiff)^(up_beta)) )+1 ) ) ) ;  
    local s6 = 0 - ((ppA_e4*Tdiff)^(up_beta)) * (1- (2/ (Exp(2*Tdiff*pB_e4*  
((ppA_e4*Tdiff)^(up_beta)) )+1 ) ) ) ;  
    local s2 = -0.00239 + t_cur * 0.0000240263 - ((ppA_e5*Tdiff)^(up_beta))  
* (1- (2/ (Exp(2*Tdiff*pB_e5* ((ppA_e5*Tdiff)^(up_beta)) )+1 ) ) ) ;  
    local s5 = 0 - ((ppA_e6*Tdiff)^(up_beta)) * (1- (2/ (Exp(2*Tdiff*pB_e6*  
((ppA_e6*Tdiff)^(up_beta)) )+1 ) ) ) ;  
  
    local pe1 = + 1.00000*s1;: -0.00385  
    local pe2 = + 1.00000*s3;: -0.00380  
    local pe3 = + 1.00000*s4;: -0.00105
```

```

local pe4      = + 1.00000*s6;: -0.07091
local pe5      = + 1.00000*s2;: -0.01547
local pe6      = + 1.00000*s5;: -0.03307
straincell(6.47340,6.87020,4.28430,90.00000,95.60200,90.00000)
celltransform(1,0,0,0,1,0,0,0,1)
a = sa;
b = sb;
c = sc;
al = sal;
be = sbe;
ga = sga;
}
do_errors
XYE(fileName)
      bkg @ 11709.2632` -7270.52638` 1807.12667` 559.698689` -327.151394`
59.2198455` -27.9445084` 134.076664` 129.903703` 104.599382` -82.3954619`
      start_X 4
      LP_Factor( 90)
      Zero_Error(, -0.0138)
      convolution_step 5
      Rp 217.5
      Rs 217.5
      Simple_Axial_Model( 0.001)
      lam
          ymin_on_ymax 0.0001
          la 1 lo 0.826401 lh 0.0001
      x_calculation_step 0.001

      local !t_cur 26.4

      hkl_Is
          lebail 1
          LVol_FWHM_CS_G_L( 1, 371.73081`, 0.89, 358.97949`,@,
412.35940`_LIMIT_MIN_0.3,@, 10000.00000`_LIMIT_MIN_0.3)
          e0_from_Strain( 0.04225`,lg32, 0.16898`,lg32, 0.16898`)
          r_bragg 0.391664467
          phase_name "hkl_P-1"
          space_group "P-1"
          load hkl_m_d_th2 I
          {
              0 1 0 2 6.83805 6.92861 0.024746976

```

1	0	0	2	6.41715	7.38367	3.91833743
1	-1	0	2	4.76305	9.95348	1.95279846
1	1	0	2	4.59989	10.30747	2.19829103
0	0	1	2	4.24747	11.16532	35.4301596
1	0	-1	2	3.73571	12.70076	15.0139436
0	-1	1	2	3.71500	12.77186	0.514215051
0	1	1	2	3.50987	13.52167	0.301598166
0	2	0	2	3.41902	13.88271	41.7200628
1	0	1	2	3.37541	14.06300	0.00965341159
1	1	-1	2	3.32825	14.26329	0.015741823
1	-1	-1	2	3.23069	14.69633	0.0622192814
2	0	0	2	3.20857	14.79821	0.783841376
1	-1	1	2	3.11276	15.25639	6.2565887
1	-2	0	2	3.06215	15.51007	3.94958044
1	2	0	2	2.97467	15.96912	0.262972008
1	1	1	2	2.94748	16.11742	0.682246278
2	-1	0	2	2.94450	16.13383	35.3165712
2	1	0	2	2.86647	16.57605	21.2494837
0	-2	1	2	2.74972	17.28512	11.0410951
2	0	-1	2	2.70715	17.55908	0.287663831
0	2	1	2	2.58465	18.39836	5.03708848
1	2	-1	2	2.56776	18.52045	4.52744489
2	1	-1	2	2.52650	18.82560	10.5987918
2	-1	-1	2	2.50775	18.96768	8.29112031
1	-2	1	2	2.48901	19.11184	0.714758468
1	-2	-1	2	2.47890	19.19052	8.70722257
2	0	1	2	2.43484	19.54117	0.0045915165
2	-2	0	2	2.38152	19.98307	0.115084831
2	-1	1	2	2.34067	20.33554	15.3077497
1	2	1	2	2.32361	20.48646	0.203657649
2	2	0	2	2.29994	20.69953	0.654754332
0	3	0	2	2.27935	20.88865	2.14257628
2	1	1	2	2.24957	21.16830	3.8556419
1	-3	0	2	2.17193	21.93427	5.47330838
3	0	0	2	2.13905	22.27571	13.4310804
2	2	-1	2	2.13374	22.33188	1.59925821
1	3	0	2	2.12461	22.42902	2.29348514
0	0	2	2	2.12373	22.43843	17.0513902
2	-2	-1	2	2.11125	22.57285	5.01758677
1	0	-2	2	2.08566	22.85355	10.8027288
0	-1	2	2	2.06554	23.07919	0.0270627272

0	-3	1	2	2.06358	23.10133	0.546757305
3	-1	0	2	2.06211	23.11806	4.94936706
2	-2	1	2	2.04490	23.31537	3.63708017
1	1	-2	2	2.02383	23.56159	0.246170742
3	1	0	2	2.02149	23.58926	7.08766494
3	0	-1	2	2.00075	23.83729	2.21456699
0	1	2	2	1.99276	23.93434	0.0381457235
1	3	-1	2	1.97707	24.12703	0.884439619
1	-1	-2	2	1.96723	24.24963	1.59091136
0	3	1	2	1.95747	24.37234	0.290096333
1	0	2	2	1.95323	24.42607	1.49976884
1	-3	1	2	1.95218	24.43945	3.7225812
2	2	1	2	1.92698	24.76405	0.0357697151
3	-1	-1	2	1.92172	24.83298	15.3702146
3	1	-1	2	1.91878	24.87165	10.5021546
1	-3	-1	2	1.91588	24.90979	0.000554760994
1	-1	2	2	1.91322	24.94506	23.8592822
2	-3	0	2	1.88960	25.26190	29.9266132
2	0	-2	2	1.86786	25.56098	0.0611182032
0	-2	2	2	1.85750	25.70593	4.5347546
1	1	2	2	1.84487	25.88488	6.23538139
3	-2	0	2	1.84255	25.91813	9.53611491
1	3	1	2	1.83156	26.07629	10.3017915
3	0	1	2	1.83138	26.07892	6.34041753
2	3	0	2	1.82831	26.12359	29.0195537
1	2	-2	2	1.82216	26.21331	14.5588336
2	1	-2	2	1.81825	26.27061	15.8913536
3	-1	1	2	1.79487	26.61911	6.28905689e-006
2	-1	-2	2	1.78587	26.75572	1.35247204
3	2	0	2	1.78558	26.76012	1.94795434
0	2	2	2	1.75494	27.23635	0.00078025157
2	3	-1	2	1.75304	27.26639	21.2171818
1	-2	2	2	1.74862	27.33659	1.66479242
3	1	1	2	1.74429	27.40586	1.67610647
1	-2	-2	2	1.74161	27.44878	9.36480741
2	-3	-1	2	1.73434	27.56618	11.9615474
3	-2	-1	2	1.72896	27.65365	1.35138474
3	2	-1	2	1.72468	27.72359	0.239750293
2	-3	1	2	1.71870	27.82206	12.735513
0	4	0	2	1.70951	27.97458	39.181141
2	0	2	2	1.68770	28.34357	0.00092284731

1	-4	0	2	1.66644	28.71302	4.26644262
2	-1	2	2	1.66535	28.73210	8.31600931
2	2	-2	2	1.66412	28.75377	0.716769313
3	-2	1	2	1.65422	28.92959	10.6894532
1	2	2	2	1.64783	29.04427	0.0229028229
1	4	0	2	1.63774	29.22722	2.86629758
0	-4	1	2	1.62184	29.52029	18.6687268
2	-2	-2	2	1.61535	29.64162	3.37294527
2	3	1	2	1.61421	29.66295	0.617988899
2	1	2	2	1.61297	29.68622	2.88230491
0	-3	2	2	1.60531	29.83119	29.1986885
4	0	0	2	1.60429	29.85070	1.49953387
3	0	-2	2	1.59734	29.98363	0.000422094507
3	-3	0	2	1.58768	30.17026	0.0319418927
1	3	-2	2	1.57919	30.33638	0.163447899
3	2	1	2	1.57727	30.37417	0.210912489
1	4	-1	2	1.57570	30.40529	0.235003204
4	-1	0	2	1.57415	30.43595	0.26096073

}

MVW(0.000, 188.005031`, 0.000)

parametric

Out_file(hkl_param.txt)